Studying the 3+1D structure of the Glasma using the weak field approximation

Andreas Ipp, Markus Leuthner, **David I. Müller**, Sören Schlichting, Pragya Singh

Institute for Theoretical Physics, TU Wien, Austria

dmueller@hep.itp.tuwien.ac.at

Quark Confinement and the Hadron Spectrum 2022

August 2, 2022

Based on

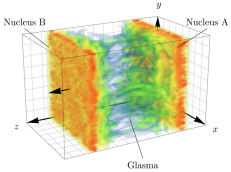
A. Ipp, D. I. Müller, S. Schlichting, P. Singh,
"Space-time structure of 3+1D color fields in high energy nuclear collisions"
Phys.Rev.D 104 (2021) 11, 114040 2109.05028

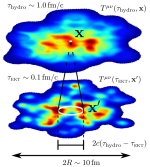
TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

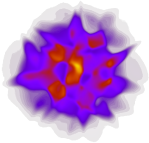
Der Wissenschaftsfonds.

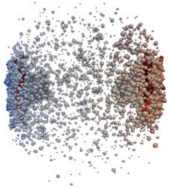
CRC-TR 211 Strong-interaction matter under extreme conditions

Stages of relativistic heavy-ion collisions









CGC & Glasma → Non-eq. QGP → Eq. QGP → Hadron gas

CGC & Glasma	Non-eq. QGP	Eq. QGP	Hadron gas
$\tau \lesssim 0.2\,\frac{\mathrm{fm}}{c}$	$0.2\,\frac{\mathrm{fm}}{c} \lesssim \tau \lesssim 1\,\frac{\mathrm{fm}}{c}$	$1\,\frac{\mathrm{fm}}{c} \lesssim \tau \lesssim 10\,\frac{\mathrm{fm}}{c}$	$10\,\frac{\mathrm{fm}}{c} \lesssim \tau$
Classical Yang-Mills	Kinetic Theory	Rel. Viscous Hydrodynamics	Kinetic Theory

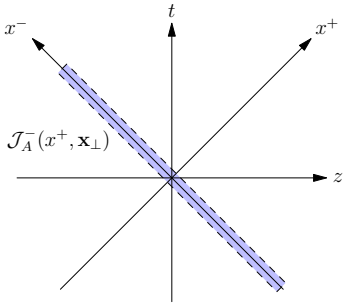
[image credits on last slide]

Color glass condensate

Color glass condensate (CGC): effective theory for high energy QCD

Separation of scales

- Hard partons: classical color currents $\mathcal{J}^\mu$ drawn from probability functional $W[\mathcal{J}]$
- Soft partons: classical color fields $\mathcal{A}_\mu \propto 1/g$ (large occupation number)



(analogous for "B" with $x^+ \to x^-$)

- Yang-Mills eqs. $\mathcal{D}_\mu \mathcal{F}_A^{\mu\nu} = \mathcal{J}_A^\nu$ with $\partial_\mu \mathcal{A}_A^\mu = 0$
- Poisson equation

$$-\Delta_\perp \mathcal{A}_A^-(x^+, \mathbf{x}_\perp) = \mathcal{J}_A^-(x^+, \mathbf{x}_\perp), \qquad \mathcal{A}_A^+ = \mathcal{A}_A^i = 0$$

- Solution with infrared regulator m

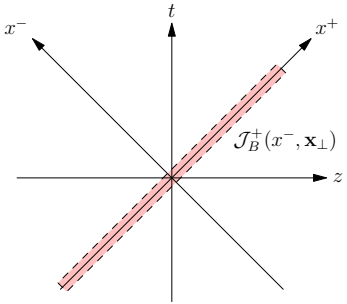
$$\mathcal{A}_A^-(x^+, \mathbf{x}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{\tilde{\mathcal{J}}_A^-(x^+, \mathbf{k}_\perp)}{\mathbf{k}_\perp^2 + m^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

Color glass condensate

Color glass condensate (CGC): effective theory for high energy QCD

Separation of scales

- Hard partons: classical color currents $\mathcal{J}^\mu$ drawn from probability functional $W[\mathcal{J}]$
- Soft partons: classical color fields $\mathcal{A}_\mu \propto 1/g$ (large occupation number)



(analogous for "B" with $x^+ \to x^-$)

- Yang-Mills eqs. $\mathcal{D}_\mu \mathcal{F}_B^{\mu\nu} = \mathcal{J}_B^\nu$ with $\partial_\mu \mathcal{A}_B^\mu = 0$
- Poisson equation

$$-\Delta_\perp \mathcal{A}_B^+(x^-, \mathbf{x}_\perp) = \mathcal{J}_B^+(x^-, \mathbf{x}_\perp), \qquad \mathcal{A}_B^- = \mathcal{A}_B^i = 0$$

- Solution with infrared regulator m

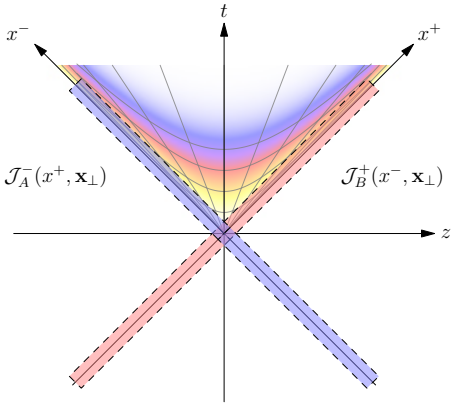
$$\mathcal{A}_B^+(x^-, \mathbf{x}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{\tilde{\mathcal{J}}_B^+(x^-, \mathbf{k}_\perp)}{\mathbf{k}_\perp^2 + m^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

The Glasma

Glasma: color fields produced in the collision of two CGCs

$$D_\mu F^{\mu\nu}(x^+, x^-, \mathbf{x}_\perp) = \mathcal{J}_A^\mu(x^+, \mathbf{x}_\perp) + \mathcal{J}_B^\mu(x^-, \mathbf{x}_\perp)$$

Problem: no general analytic solution to Yang-Mills equations for the collision problem



2+1D (boost-invariant) Glasma

$$\mathcal{J}_A^-(x^+, \mathbf{x}_\perp) = \delta(x^+)\rho_A(\mathbf{x}_\perp)$$

- Initial conditions on the future light cone [1]
- 2+1D classical lattice simulations, e.g. [2, 3]
- Small τ expansion, e.g. [4]
- Weak field approximation, e.g. [5]

3+1D Glasma

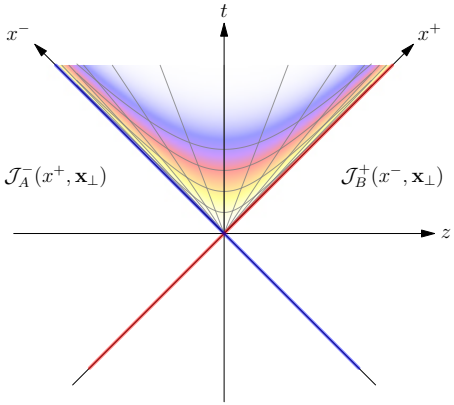
- 3+1D classical lattice simulations [6, 7]
- Weak field approximation [8] (this talk)

The Glasma

Glasma: color fields produced in the collision of two CGCs

$$D_\mu F^{\mu\nu}(x^+, x^-, \mathbf{x}_\perp) = \mathcal{J}_A^\mu(x^+, \mathbf{x}_\perp) + \mathcal{J}_B^\mu(x^-, \mathbf{x}_\perp)$$

Problem: no general analytic solution to Yang-Mills equations for the collision problem



2+1D (boost-invariant) Glasma

$$\mathcal{J}_A^-(x^+, \mathbf{x}_\perp) = \delta(x^+)\rho_A(\mathbf{x}_\perp)$$

- Initial conditions on the future light cone [1]
- 2+1D classical lattice simulations, e.g. [2, 3]
- Small τ expansion, e.g. [4]
- Weak field approximation, e.g. [5]

3+1D Glasma

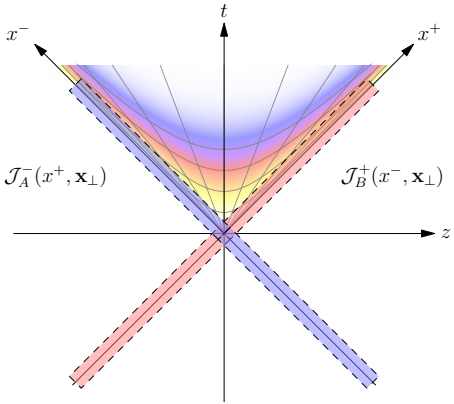
- 3+1D classical lattice simulations [6, 7]
- Weak field approximation [8] (this talk)

The Glasma

Glasma: color fields produced in the collision of two CGCs

$$D_\mu F^{\mu\nu}(x^+, x^-, \mathbf{x}_\perp) = \mathcal{J}_A^\mu(x^+, \mathbf{x}_\perp) + \mathcal{J}_B^\mu(x^-, \mathbf{x}_\perp)$$

Problem: no general analytic solution to Yang-Mills equations for the collision problem



2+1D (boost-invariant) Glasma

$$\mathcal{J}_A^-(x^+, \mathbf{x}_\perp) = \delta(x^+)\rho_A(\mathbf{x}_\perp)$$

- Initial conditions on the future light cone [1]
- 2+1D classical lattice simulations, e.g. [2, 3]
- Small τ expansion, e.g. [4]
- Weak field approximation, e.g. [5]

3+1D Glasma

- 3+1D classical lattice simulations [6, 7]
- Weak field approximation [8] (this talk)

Weak field approximation

Simple idea: linearize Yang-Mills equations and solve order by order

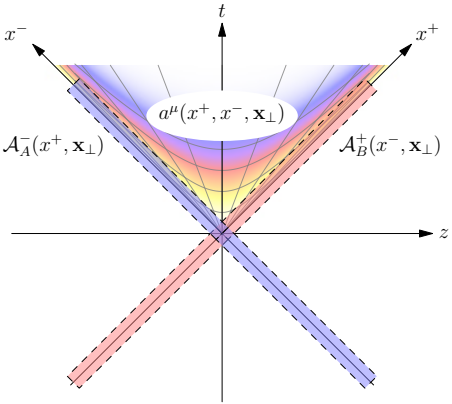
$$D_\mu F^{\mu\nu} = J^\nu \quad D_\mu J^\mu = 0 \quad \partial_\mu A^\mu = 0$$

- Background + perturbation split

$$\begin{aligned} A^\mu(x) &= \mathcal{A}_A^\mu(x) + \mathcal{A}_B^\mu(x) \quad + \quad a^\mu(x) \\ J^\mu(x) &= \underbrace{\mathcal{J}_A^\mu(x) + \mathcal{J}_B^\mu(x)}_{\text{background}} \quad + \quad \underbrace{j^\mu(x)}_{\text{perturbation}} \end{aligned}$$

- Expand in powers of $\mathcal{J}_A$ and $\mathcal{J}_B$
- Solve for a^μ and j^μ at each order $\mathcal{O}(\mathcal{J}_A^n \mathcal{J}_B^m)$
- Leading order = dilute limit $\mathcal{O}(\mathcal{J}_A \mathcal{J}_B)$

$$\partial_\nu \partial^\nu a^\mu(x) = \mathcal{S}^\mu[\mathcal{J}_A, \mathcal{J}_B]$$



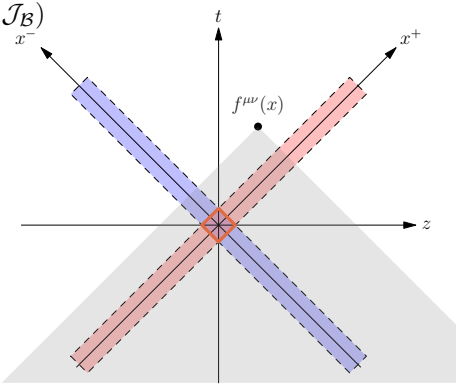
Glasma field strength tensor

Perturbative field strength tensor at leading order $\mathcal{O}(\mathcal{J}_\mathcal{A}\mathcal{J}_\mathcal{B})$

$$f^{+-} = -\frac{g}{2\pi}\int_{\mathbf{u}_\perp}\int_{-\infty}^{+\infty} d\eta_z\, V$$

$$f^{\pm i} = +\frac{g}{2\pi}\int_{\mathbf{u}_\perp}\int_{-\infty}^{+\infty} d\eta_z\, \left(V^{ij} \mp \delta^{ij} V\right)\frac{w^j}{\sqrt{2}} e^{\pm\eta_z}$$

$$f^{ij} = -\frac{g}{2\pi}\int_{\mathbf{u}_\perp}\int_{-\infty}^{+\infty} d\eta_z\, V^{ij}$$



with

$$V = f_{abc}t^c\, \partial^i \mathcal{A}_A^{a,-}(x^+ - \frac{|\mathbf{x}_\perp - \mathbf{u}_\perp|}{\sqrt{2}}e^{+\eta_z}, \mathbf{u}_\perp)\, \partial^i \mathcal{A}_B^{b,+}(x^- - \frac{|\mathbf{x}_\perp - \mathbf{u}_\perp|}{\sqrt{2}}e^{-\eta_z}, \mathbf{u}_\perp)$$

$$V^{ij} = f_{abc}t^c\left(\partial^i \mathcal{A}_A^{a,-}(\dots)\,\partial^j \mathcal{A}_B^{b,+}(\dots) - \partial^j \mathcal{A}_A^{a,-}(\dots)\,\partial^i \mathcal{A}_B^{b,+}(\dots)\right)$$

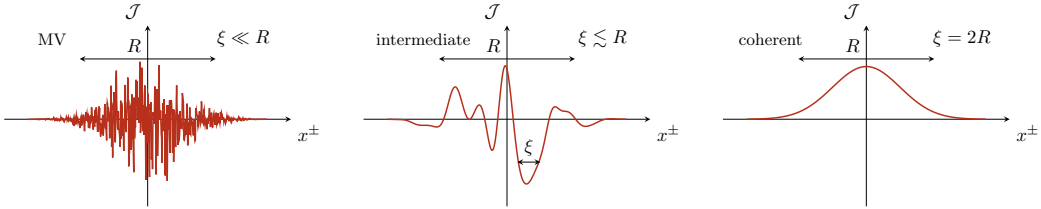
Perturbative energy-momentum tensor $t^{\mu\nu} = 2\mathrm{Tr}\left[f^{\mu\rho} f_\rho{}^\nu + \frac{1}{4} g^{\mu\nu} f^{\rho\sigma} f_{\rho\sigma}\right]$

Nuclear model with longitudinal structure

Simple generalization of McLerran-Venugopalan (MV) model with non-trivial longitudinal structure

$$\left\langle \mathcal{J}_A^{a,-}(x^+,\mathbf{x}_\perp)\right\rangle = 0$$

$$\left\langle \mathcal{J}_A^{a,-}(x^+,\mathbf{x}_\perp)\mathcal{J}_A^{b,-}(x'^+,\mathbf{x}'_\perp)\right\rangle = \underbrace{g^2\mu^2}_{\text{strength of color charges } Q_s \propto g^2\mu} \quad \delta^{ab} \quad \underbrace{T_R\left(\frac{x^+ + x'^+}{2}\right)}_{\text{longitudinal profile Gaussian of width } R} \quad \underbrace{U_\xi(x^+ - x'^+)}_{\text{long. correlations Gaussian of width } \xi} \quad \underbrace{\delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}'_\perp)}_{\text{transverse correlations}}$$



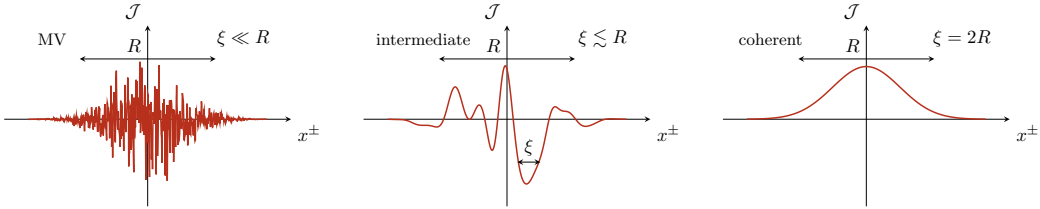
Transverse momentum modes are suppressed below infrared regulator m

Nuclear model with longitudinal structure

Simple generalization of McLerran-Venugopalan (MV) model with non-trivial longitudinal structure

$$\left\langle \mathcal{J}_A^{a,-}(x^+,\mathbf{x}_\perp)\right\rangle = 0$$

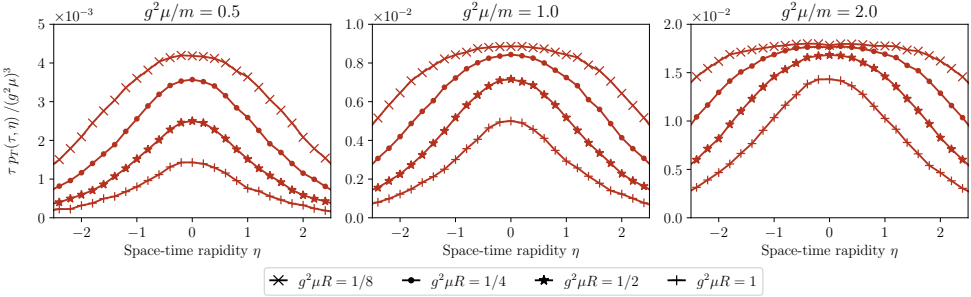
$$\left\langle \mathcal{J}_A^{a,-}(x^+,\mathbf{x}_\perp)\mathcal{J}_A^{b,-}(x'^+,\mathbf{x}'_\perp)\right\rangle = \underbrace{g^2\mu^2}_{\text{strength of color charges } Q_s \propto g^2\mu} \quad \delta^{ab} \quad \underbrace{T_R\left(\frac{x^+ + x'^+}{2}\right)}_{\text{longitudinal profile Gaussian of width } R} \quad \underbrace{U_\xi(x^+ - x'^+)}_{\text{long. correlations Gaussian of width } \xi} \quad \underbrace{\delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}'_\perp)}_{\text{transverse correlations}}$$



tractable with lattice simulations

Transverse momentum modes are suppressed below infrared regulator m

Comparison to 3+1D lattice simulations for coherent nuclei ($\xi = 2R$)



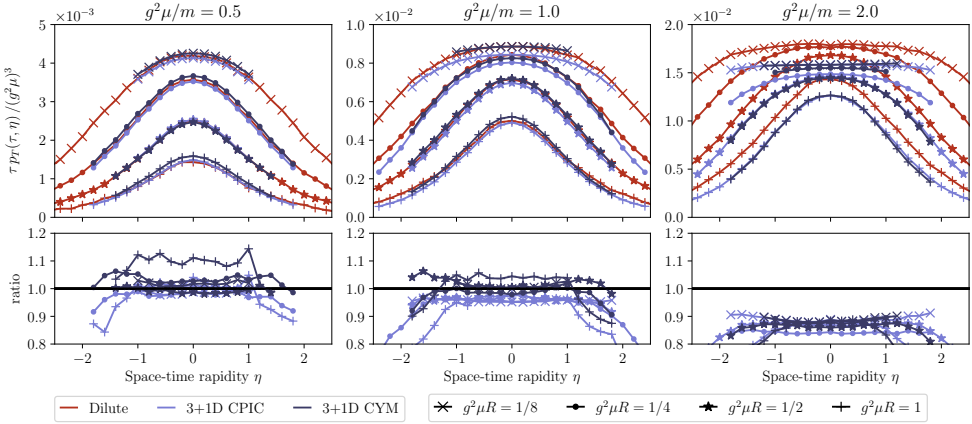
weak fields

strong(er) fields

(realistic value $g^2\mu/m \sim 10$)

- Transverse pressure $p_T(\tau,\eta) = \langle t^{xx}\rangle = \langle t^{yy}\rangle$
- Non-linearity parameter $g^2\mu/m$
- Dilute approximation overestimates at higher $g^2\mu/m$, rapidity profile less affected
- Fast numerical evaluation: hours (desktop) vs. days or weeks (cluster)

Comparison to 3+1D lattice simulations for coherent nuclei ($\xi = 2R$)

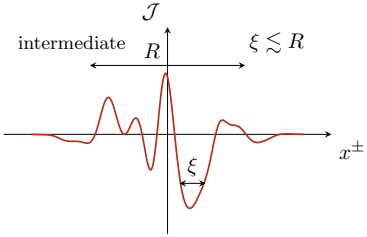


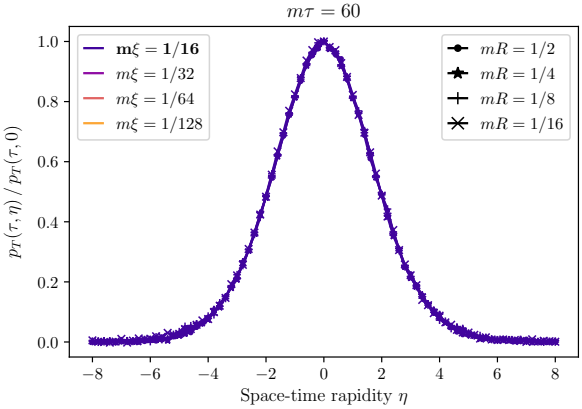
- Transverse pressure $p_T(\tau,\eta) = \langle t^{xx}\rangle = \langle t^{yy}\rangle$
- Non-linearity parameter $g^2\mu/m$
- Dilute approximation overestimates at higher $g^2\mu/m$, rapidity profile less affected
- Fast numerical evaluation: hours (desktop) vs. days or weeks (cluster)

What determines the width of the rapidity profile?

What is the role of longitudinal scales R, ξ and the infrared scale m?

- Consider late times $\tau \gg R$: free-streaming
- Width is almost purely determined by $m\xi$ for $\xi \lesssim R$
- Longitudinal correlations within nucleus determine the width in rapidity

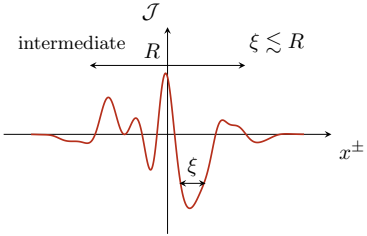


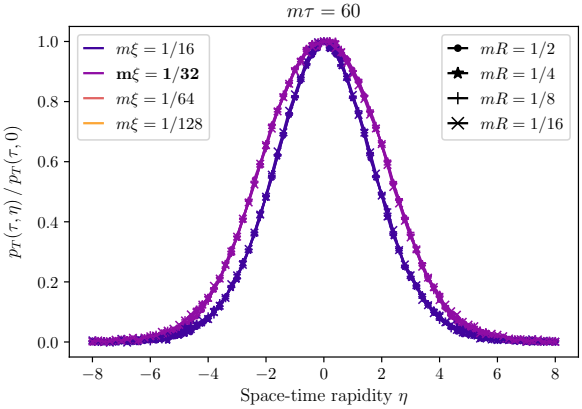


What determines the width of the rapidity profile?

What is the role of longitudinal scales R, ξ and the infrared scale m?

- Consider late times $\tau \gg R$: free-streaming
- Width is almost purely determined by $m\xi$ for $\xi \lesssim R$
- Longitudinal correlations within nucleus determine the width in rapidity

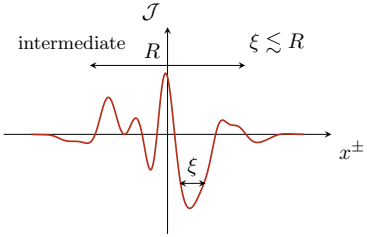


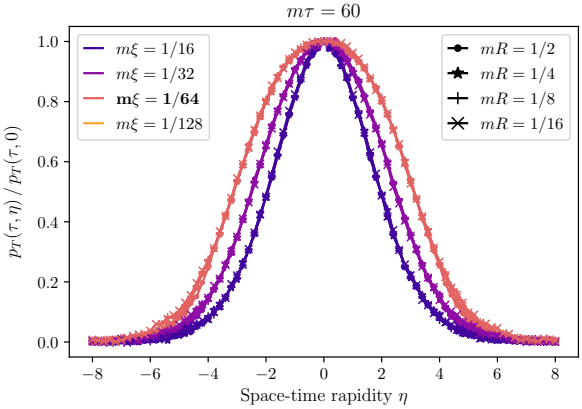


What determines the width of the rapidity profile?

What is the role of longitudinal scales R, ξ and the infrared scale m?

- Consider late times $\tau \gg R$: free-streaming
- Width is almost purely determined by $m\xi$ for $\xi \lesssim R$
- Longitudinal correlations within nucleus determine the width in rapidity

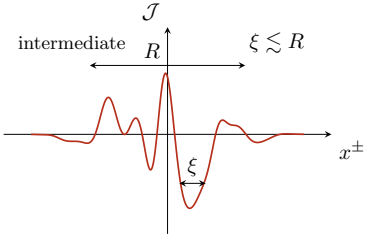


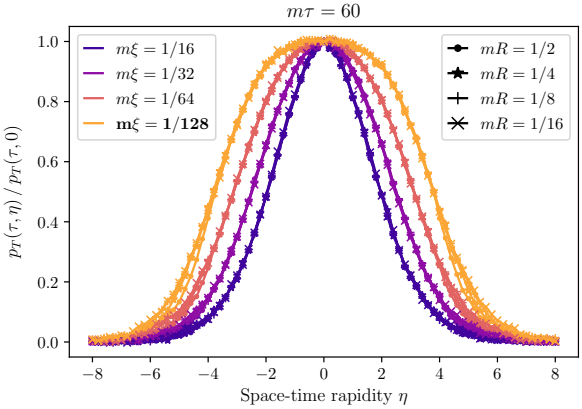


What determines the width of the rapidity profile?

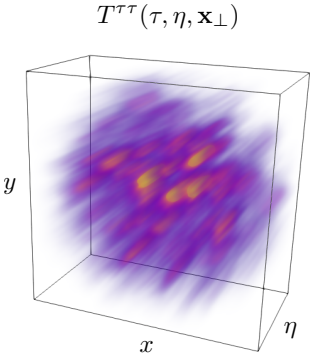
What is the role of longitudinal scales R, ξ and the infrared scale m?

- Consider late times $\tau \gg R$: free-streaming
- Width is almost purely determined by $m\xi$ for $\xi \lesssim R$
- Longitudinal correlations within nucleus determine the width in rapidity





Conclusions and outlook



What we have so far

- Field strength tensor of the Glasma 3+1D in the dilute limit
- Rapidity profile determined by longitudinal color correlations
- Substantial speedup vs. lattice simulations

What we are working on

- More sophisticated nuclear models (realistic geometry, hot spots, PDFs)
- Monte Carlo integration on GPUs
- Angular momentum of the Glasma
- Coupling to effective kinetic theory and hydrodynamics

References

[1] A. Kovner, L. D. McLerran and H. Weigert, *Gluon production from non-Abelian Weizsäcker-Williams fields in nucleus-nucleus collisions*, Phys. Rev. D 52 (1995), 6231-6237, hep-ph/9502289.

[2] A. Krasnitz and R. Venugopalan, *Non-perturbative computation of gluon mini-jet production in nuclear collisions at very high-energies*, Nucl. Phys. B 557 (1999), 237, hep-ph/9809433.

[3] T. Lappi, *Production of gluons in the classical field model for heavy ion collisions*, Phys. Rev. C 67 (2003), 054903, hep-ph/0303076.

[4] G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, *Early-time dynamics of gluon fields in high energy nuclear collisions*, Phys. Rev. C 92 (2015) no.6, 064912, 1507.03524.

[5] P. Guerrero-Rodríguez and T Lappi, *Evolution of initial stage fluctuations in the glasma*, Phys. Rev. D 104 (2021) no.1, 014011, 2102.09993.

[6] A. Ipp and D. I. Müller, *Progress on 3+1D Glasma simulations*, Eur. Phys. J. A 56 (2020) no.9, 243, 2009.02044.

[7] S. Schlichting and P. Singh, *3-D structure of the Glasma initial state: Breaking boost-invariance by collisions of extended shock waves in classical Yang-Mills theory*, Phys. Rev. D 103 (2021) no.1, 014003, 2010.11172.

[8] A. Ipp, D. I. Müller, S. Schlichting and P. Singh, *Space-time structure of (3 + 1)D color fields in high energy nuclear collisions*, Phys. Rev. D 104 (2021) no.11, 114040, 2109.05028.

Image credits

- 3+1D Glasma image:

  A. Ipp and D. Müller, *Broken boost invariance in the Glasma via finite nuclei thickness*, Phys. Lett. B 771 (2017), 74-79, 1703.00017

- Kinetic theory image:

  A. Kurkela, A. Mazeliauskas, J. F. Paquet, S. Schlichting and D. Teaney, *Matching the Nonequilibrium Initial Stage of Heavy Ion Collisions to Hydrodynamics with QCD Kinetic Theory*, Phys. Rev. Lett. 122 (2019) no.12, 122302, 1805.01604

- Hydrodynamics image (adapted):

  B. Schenke, S. Jeon and C. Gale, *Elliptic and triangular flow in event-by-event (3+1)D viscous hydrodynamics*, Phys. Rev. Lett. 106 (2011), 042301, 1805.01604

- Hadron gas image (adapated):

  F. Becattini, J. Liao and M. Lisa (eds), *Strongly Interacting Matter under Rotation*, Lecture Notes in Physics, vol 987, Springer, Cham