Recent developments in the study of flow fluctuations in heavy-ion collisions

Cindy Mordasini
University of Jyväskylä
XVth Quark Confinement and Hadron Spectrum — 01st of August 2022

Evolution of a heavy-ion collision

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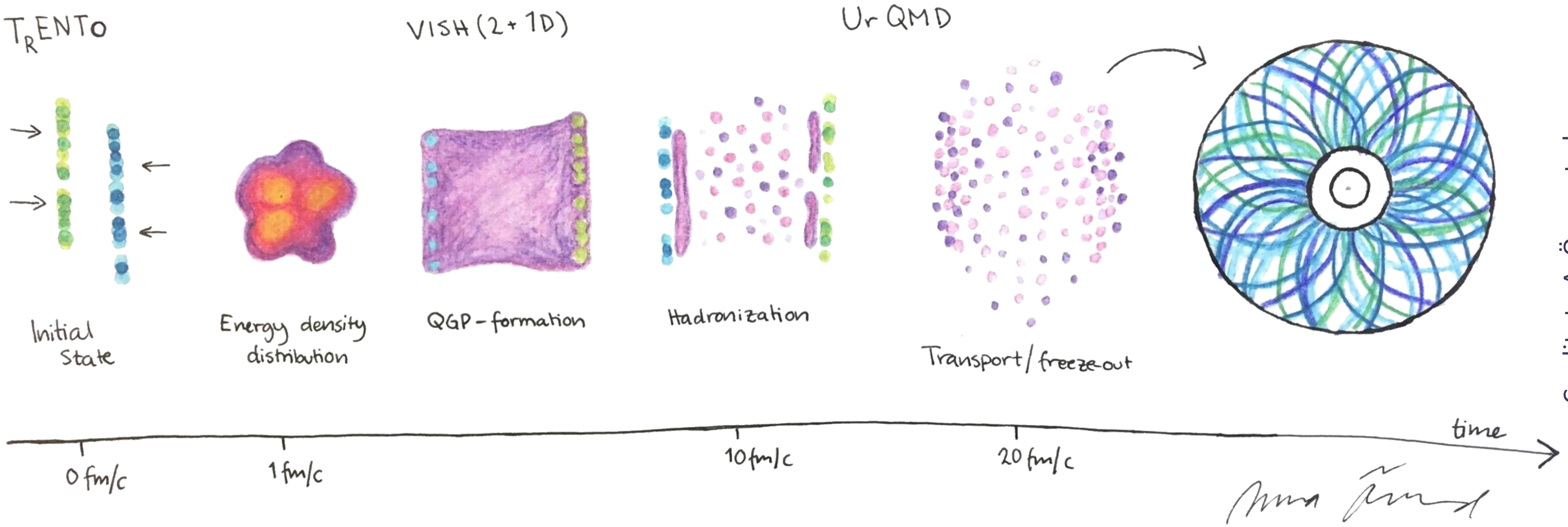

Credits to A. Önnerstad

- Distribution of detected final state particles → QGP transport properties?

Anisotropic flow

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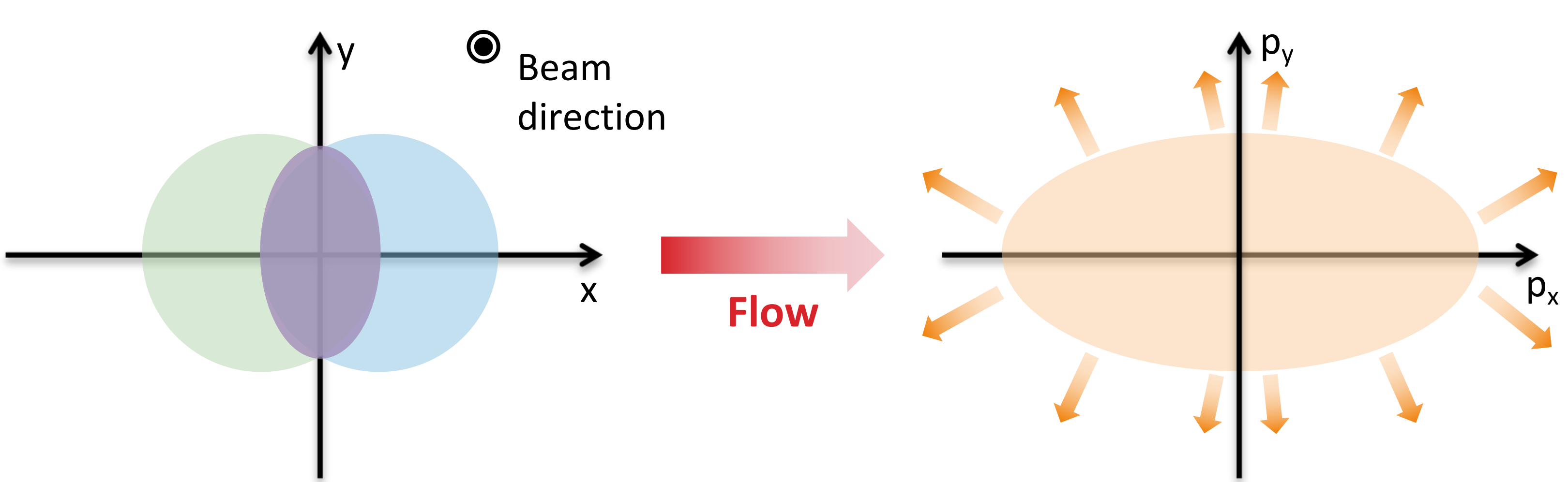
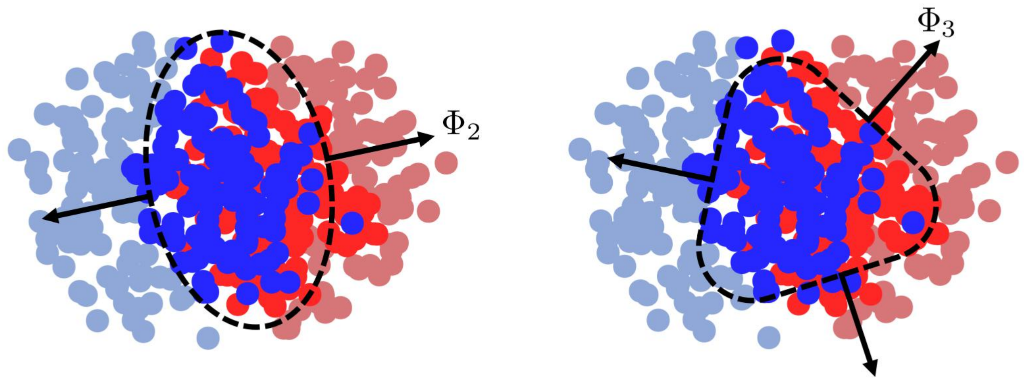


→ Anisotropic flow = medium response to the initial geometry

- Final azimuthal distribution given by Fourier series

$$f(\varphi) = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos\left(n(\varphi - \Psi_n)\right)\right)$$



A. Poskanzer, S. Voloshin, Z. Phys. C **70**, 665-672 (1996)

Credits to M. Lesch

Starting point of this talk

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- Broad range of flow observables developed over the years
  - flow amplitudes v_n with k-particle cumulants — $v_n\{k\}$
  - symmetric cumulants — $SC(m,n)$
  - non-linear flow modes — $\chi_{n,mk}$
  - symmetry plane correlations
  - Pearson Correlation Coefficient — $\rho(v_2^2, [p_T])$
  - ...

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Symmetric cumulants

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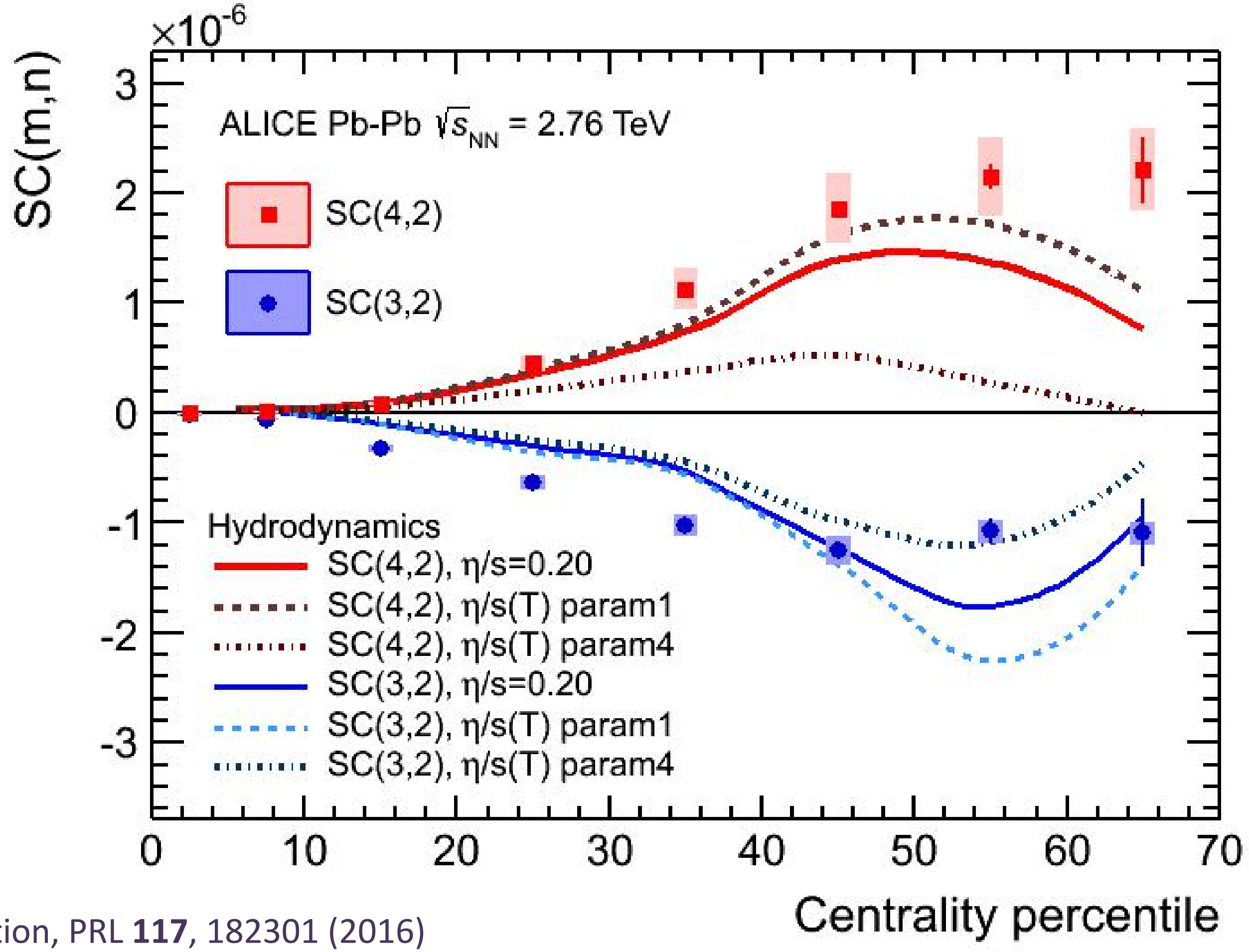
- Measurement of genuine correlations between two different harmonics m and n

$$\mathrm{SC}(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

- More sensitive to $\eta/s(T)$ than individual flow harmonics



ALICE Collaboration, PRL **117**, 182301 (2016)

Symmetry plane correlations

- Initial problem: Symmetry planes cannot be measured individually

$\longrightarrow$ Measurement of symmetry plane correlations (SPC)

$$\langle \cos\left(a_1 n_1 \Psi_{n_1} + \ldots + a_k n_k \Psi_{n_k}\right)\rangle$$

- SPC sensitive to the linear and non-linear response of the system
  - example:

$$v_2^2 v_4 e^{i4(\Psi_4-\Psi_2)} = \boxed{\omega_2^2 \omega_4 c_2^2 c_4 e^{i4(\phi_4-\phi_2)}} + \boxed{\omega_{422}\omega_2^2 c_2^2}$$

$$\boxed{\omega_{422} = 0} \qquad \langle \cos[4(\Psi_4 - \Psi_2)]\rangle = \langle \cos[4(\phi_4 - \phi_2)]\rangle$$

$$\boxed{\omega_{422} \neq 0} \qquad \langle \cos[4(\Psi_4 - \Psi_2)]\rangle \neq \langle \cos[4(\phi_4 - \phi_2)]\rangle$$

Scalar product (SP) method

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- Ideally, SPC would be measured event-by-event
$\longrightarrow$ Unstable method due to statistical uncertainties
- Limitation overcome by averaging over many events
$\longrightarrow$ Scalar product (SP) method
- Example:

$$\langle \cos[4(\Psi_4 - \Psi_2)] \rangle_{\mathrm{SP}} = \frac{\langle v_2^2 v_4 \cos[4(\Psi_4 - \Psi_2)] \rangle}{\sqrt{\langle v_2^4 \rangle \langle v_4^2 \rangle}}$$

STAR Collaboration, PRC **66**, 034904 (2002)
R.S. Bhalerao, J.-Y. Ollitrault and S. Pal, PRC **88**, 024909 (2013)

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- Problem: built-in bias due to neglected correlations between flow amplitudes

i.e.

$$\langle v_m^2 v_n^2\rangle \neq \langle v_m^2\rangle\langle v_n^2\rangle$$

STAR Collaboration, PRC **66**, 034904 (2002)
R.S. Bhalerao, J.-Y. Ollitrault and S. Pal, PRC **88**, 024909 (2013)

Summary of the situation

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- On the flow amplitudes side: Better constrain of the QGP parameters from $SC(m,n)$
$\longrightarrow$ Can we generalise them to more harmonics? To higher moments?
- On the symmetry planes side: Bias in the SP method + less estimators and studies than for the flow amplitudes
$\longrightarrow$ Can we develop less biased methods to improve the measurements?

Generalisation of the symmetric cumulants

Higher order symmetric cumulants

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- Traditional approach to build $SC(m,n)$

Obtain φ → Insert in cumulant → Rewrite for v_n

Higher order symmetric cumulants

- Traditional approach to build $SC(m,n)$

Obtain φ
→
Insert in cumulant
→
Rewrite for v_n

- **But** $SC(m,n)$ accidentally cumulant of $v_n{}^2$

Higher order symmetric cumulants

- Traditional approach to build $SC(m,n)$

Obtain φ → Insert in cumulant → Rewrite for v_n

- **But** $SC(m,n)$ accidentally cumulant of $v_n{}^2$

$\longrightarrow$ Starting point for the new approach

Take $v_n{}^2$ → Insert in cumulant → Rewrite for φ

- New theory applicable to any number of harmonics

3-harmonic symmetric cumulants

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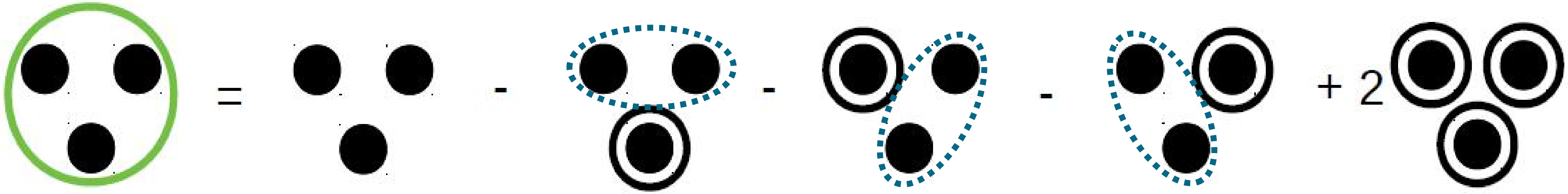
- First natural step to generalise *SC(m,n)*
- Potential to learn more about the origins of the correlations, the nonlinear response,...



$$\mathrm{SC}(k,l,m) = \langle v_k^2 v_l^2 v_m^2\rangle - \langle v_k^2 v_l^2\rangle\langle v_m^2\rangle - \langle v_k^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_l^2 v_m^2\rangle\langle v_k^2\rangle + 2\,\langle v_k^2\rangle\langle v_l^2\rangle\langle v_m^2\rangle$$

Normalised Symmetric Cumulants

$$\mathrm{NSC}(k,l,m) = \frac{\mathrm{SC}(k,l,m)}{\langle v_k^2\rangle\langle v_l^2\rangle\langle v_m^2\rangle}$$

CM, A. Bilandzic, D. Karakoç, S.F. Taghavi, PRC **102**, 024907 (2020)

First measurements of *SC(k,l,m)*

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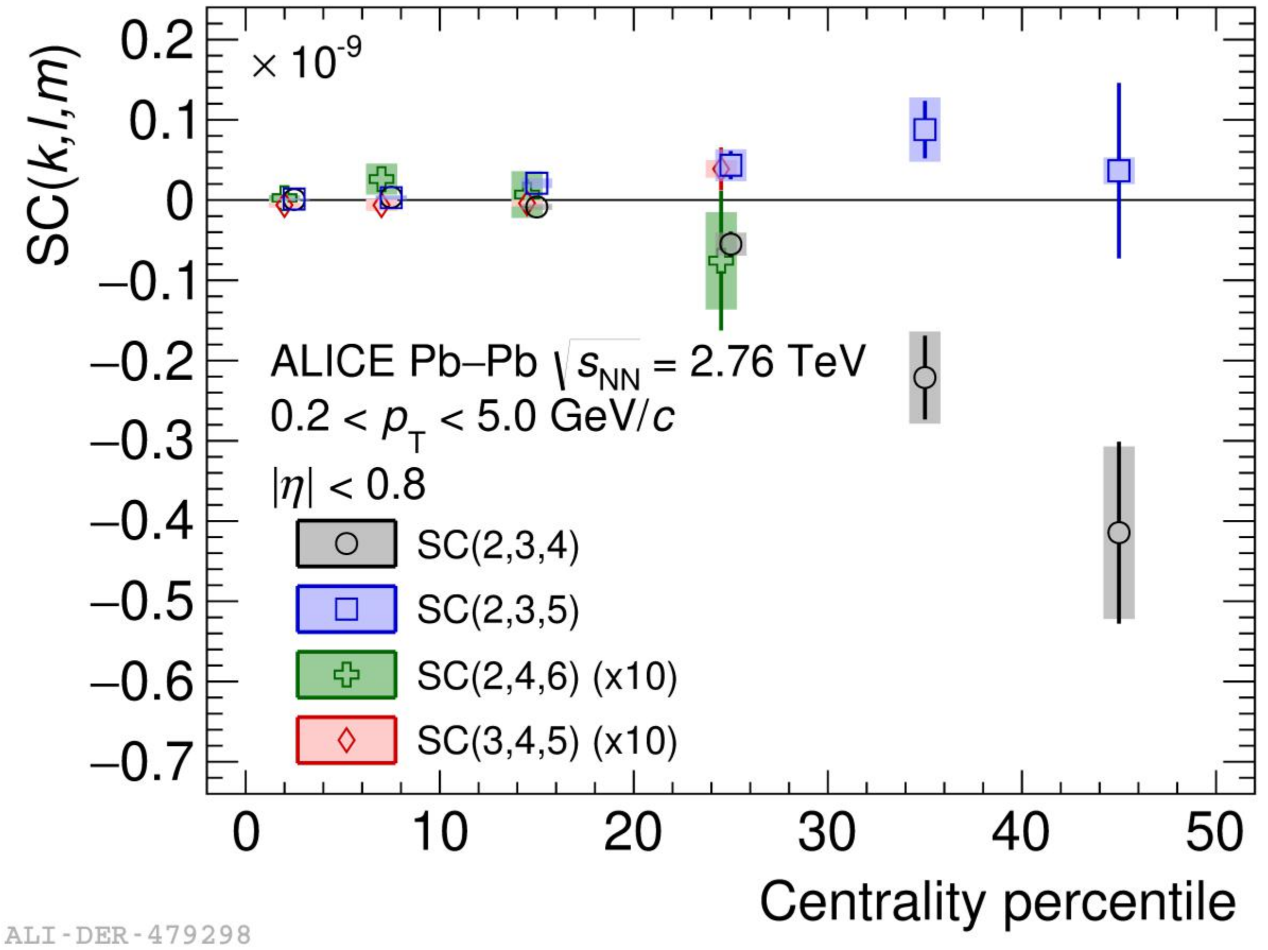
- Various combinations measured in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 TeV by ALICE
- Ordering of the magnitude with the chosen harmonics
- Signatures cannot be predicted from measurements of $SC(m,n)$



ALICE Collaboration, PRL **127**, 092302 (2021)

Comparison with theory

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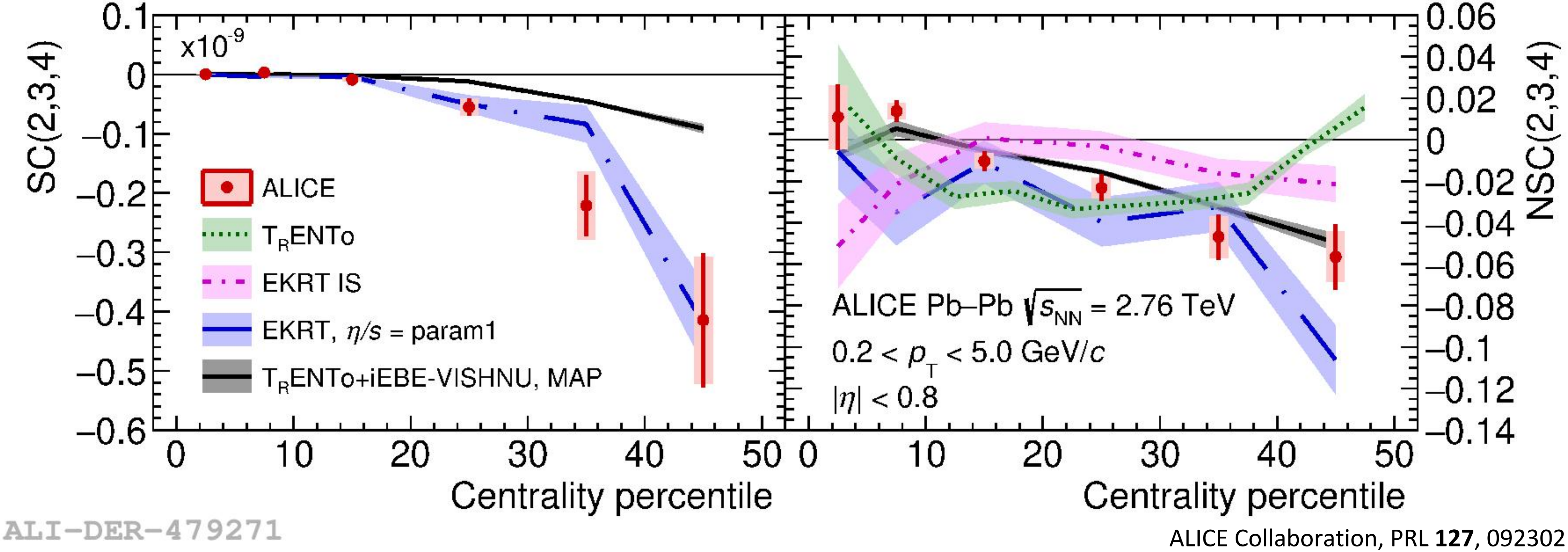
- Sensitivity of the data to the predictions from different models
- Signs of non-linear response when comparing initial and final states in models



ALICE Collaboration, PRL **127**, 092302 (2021)

NSC(k,l,m) in Bayesian analysis

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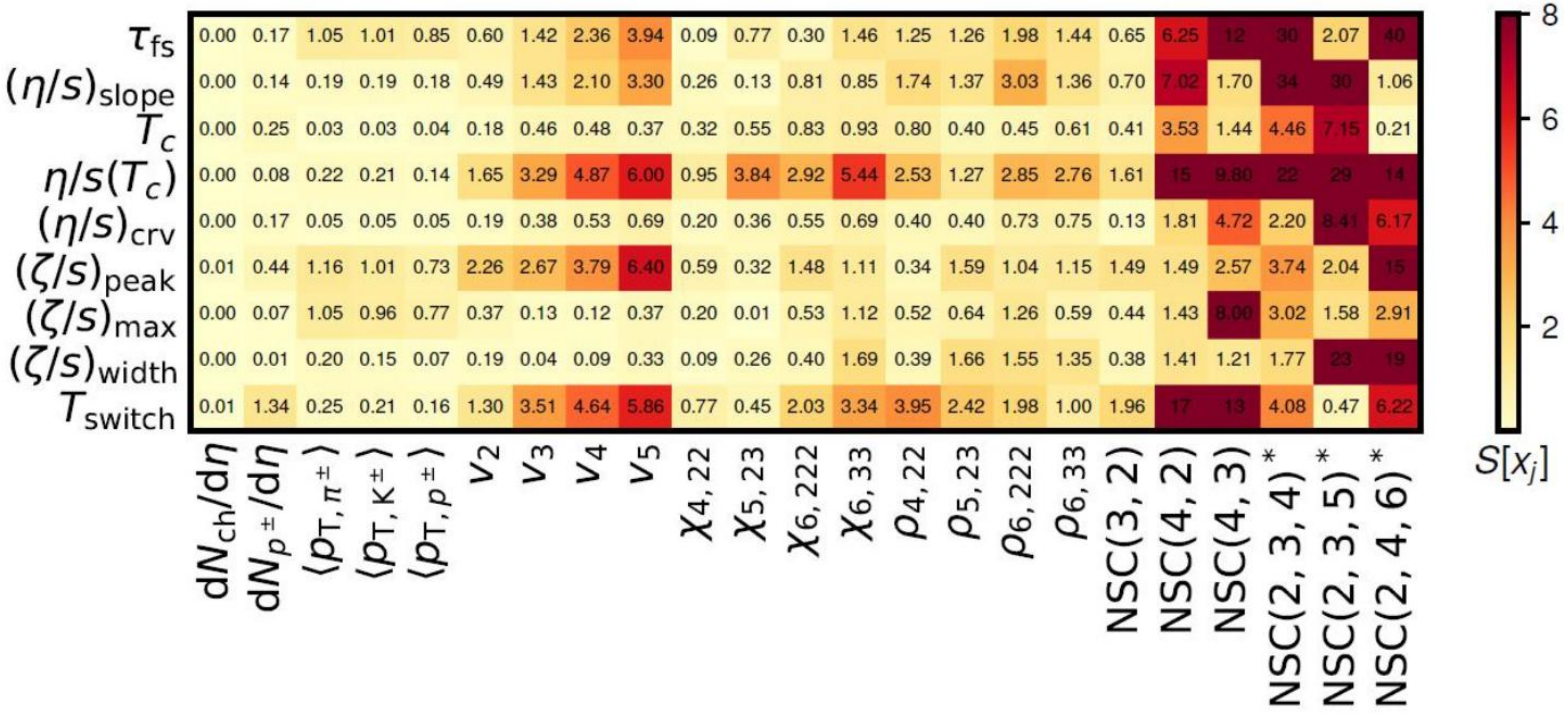
- NSC(k,l,m) measurements included in latest Bayesian analysis

→ Higher order flow observables more sensitive to QGP transport properties!

→ What about the genuine correlations between different moments of flow harmonics?



Talks by D.J. Kim (01.08) and S.F. Taghavi (02.08)

J.E. Parkkila, A. Önnerstad, S.F. Taghavi, CM, A. Bilandzic, D.J. Kim, arXiv:2111.08145 (Submitted to PLB)

Asymmetric cumulants

- Symmetric cumulants: $\langle v_m{}^2 v_n{}^2 \rangle_c$, $\langle v_k{}^2 v_l{}^2 v_m{}^2 \rangle_c$,…
- What if we take a higher moment of $v_m{}^2$?

Asymmetric cumulants

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- Symmetric cumulants: $\langle {v_m}^2 {v_n}^2 \rangle_c$, $\langle {v_k}^2 {v_l}^2 {v_m}^2 \rangle_c$,…
- What if we take a higher moment of ${v_m}^2$?

$\longrightarrow$ Asymmetric cumulants: $\langle {v_m}^4 {v_n}^2 \rangle_c$, $\langle {v_m}^6 {v_n}^2 \rangle_c$, $\langle {v_k}^4 {v_l}^2 {v_m}^2 \rangle_c$,…

- Example: "lowest order" asymmetric cumulant

$$
\begin{aligned}
\mathrm{AC}_{2,1}(m,n) &\equiv \langle (v_m^2)^2 v_n^2 \rangle_c = \langle v_m^4 v_n^2 \rangle_c \\
&= \langle v_m^4 v_n^2 \rangle - \langle v_m^4 \rangle \langle v_n^2 \rangle \\
&\quad - 2\langle v_m^2 v_n^2 \rangle \langle v_m^2 \rangle + 2\langle v_m^2 \rangle^2 \langle v_n^2 \rangle
\end{aligned}
$$

$$
\mathrm{NAC}_{2,1}(m,n) = \frac{\mathrm{AC}_{2,1}(m,n)}{\langle v_m^2 \rangle^2 \langle v_n^2 \rangle}
$$

A. Bilandzic, M. Lesch, CM, S.F. Taghavi, PRC **105**, 024912 (2022)

Predictions from theory

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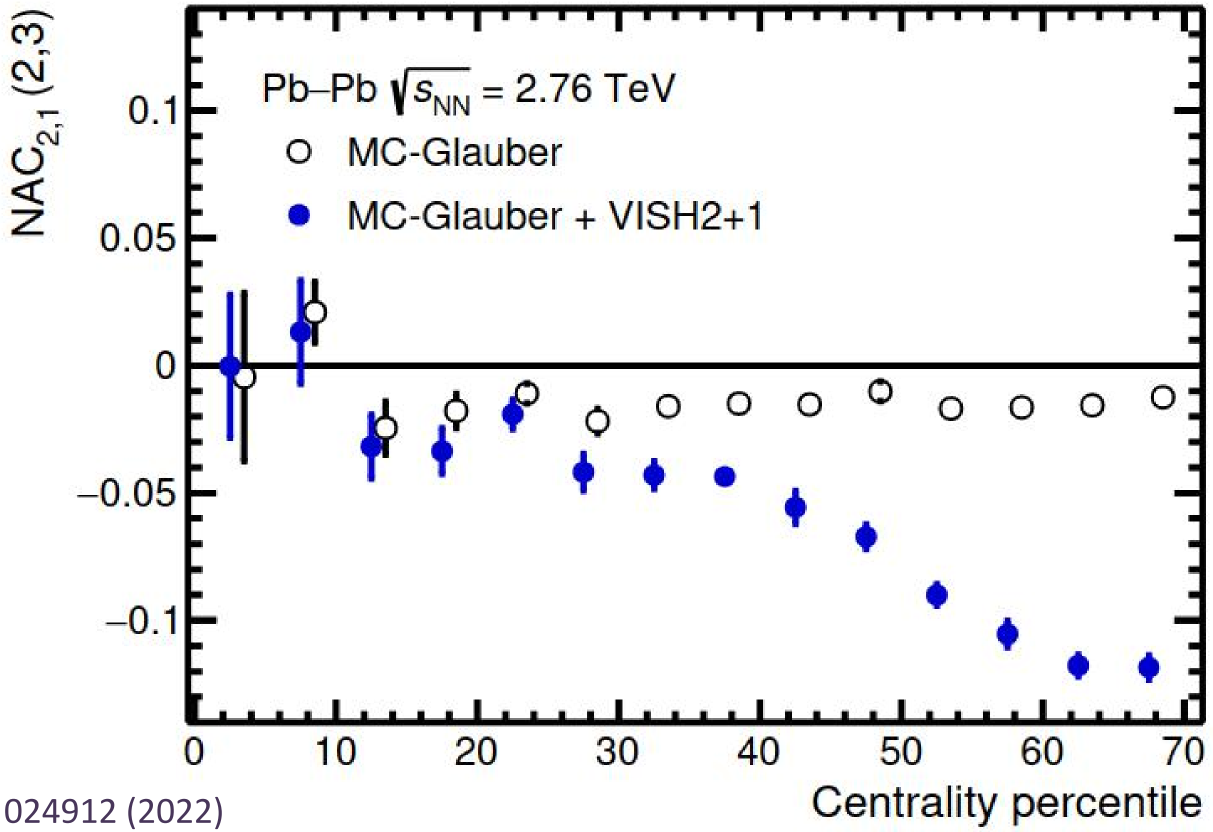
- Predictions from MC-Glauber-VISH2+1 in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV for different combinations and orders

- $v_2 \propto \varepsilon_2$ and $v_3 \propto \varepsilon_3$
$\longrightarrow$ Initial and final state should agree

- Agreement only up to 25%
$\longrightarrow$ Linear reponse dominates in central collisions

- Non-linear effects above 25%
$\longrightarrow$ Impact from ε_2^3?



A. Bilandzic *et al.*, PRC **105**, 024912 (2022)

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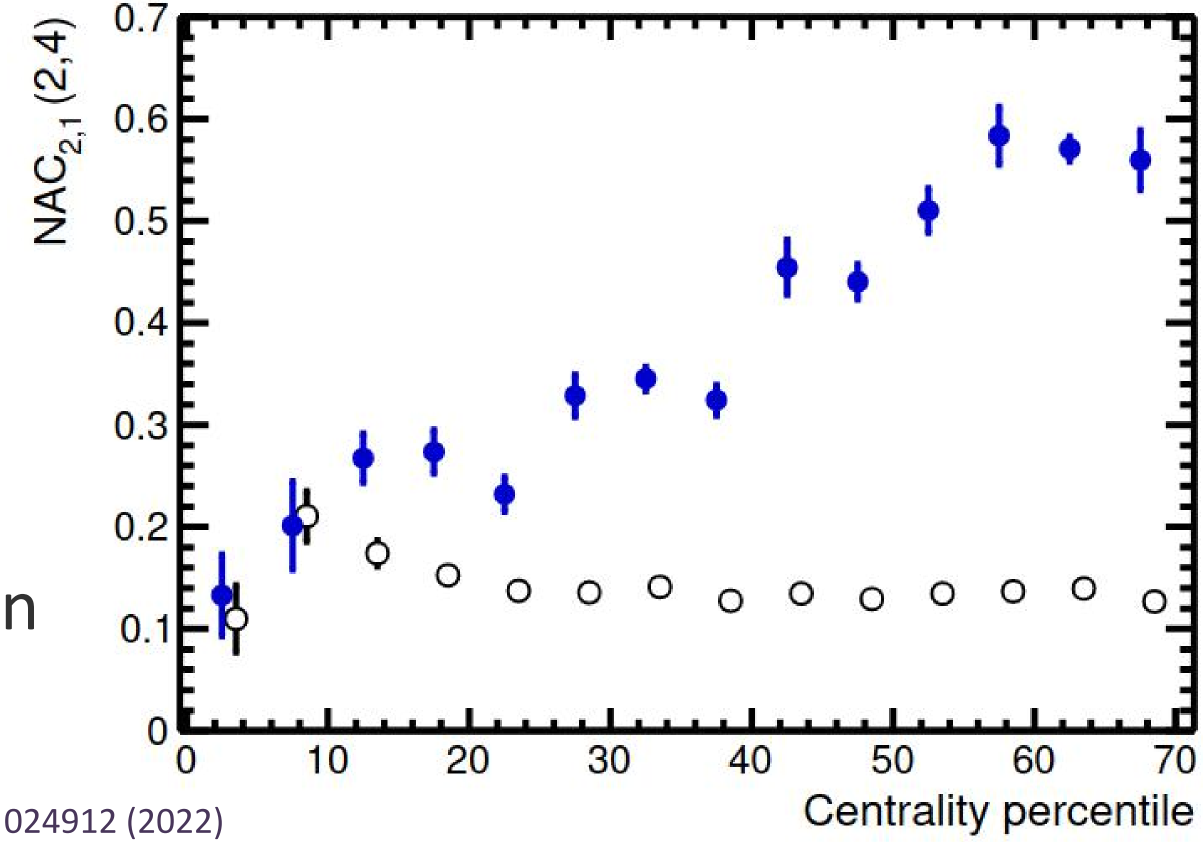
- Predictions from MC-Glauber-VISH2+1 in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 TeV for different combinations and orders
- Non-linear response dominates from 10% centrality already
- Contribution from ε_2^2 to v_4 visible at lower centralities

→ (N)AC can help to probe higher order terms of the non-linear response between harmonics



A. Bilandzic *et al.*, PRC **105**, 024912 (2022)

New method for the SPC

Gaussian Estimator (GE) for SPC

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- Main idea: Approximate the flow fluctuations using a 2D Gaussian distribution
- Readings: A. Bilandzic, M. Lesch and S.F.Taghavi, PRC **102**, 024910 (2020)

and M. Lesch, Master thesis (2021)

- Final expression for the SPC

$$\langle \cos[4(\Psi_4 - \Psi_2)] \rangle_{\mathrm{GE}} = \sqrt{\frac{\pi}{4}} \frac{\langle v_2^2 v_4 \cos[4(\Psi_4 - \Psi_2)] \rangle}{\sqrt{\langle v_2^4 v_4^2 \rangle}}$$

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- Final expression for the SPC

$$\langle\cos[4(\Psi_4-\Psi_2)]\rangle_{\mathrm{GE}} = \sqrt{\frac{\pi}{4}}\,\frac{\langle v_2^2 v_4 \cos[4(\Psi_4-\Psi_2)]\rangle}{\sqrt{\langle v_2^4 v_4^2\rangle}}$$

→ Joined mean of flow amplitudes in the denominator

Reminder: SP method

$$\langle\cos[4(\Psi_4-\Psi_2)]\rangle_{\mathrm{SP}} = \frac{\langle v_2^2 v_4 \cos[4(\Psi_4-\Psi_2)]\rangle}{\sqrt{\langle v_2^4\rangle\langle v_4^2\rangle}}$$

Comparison between SP and GE

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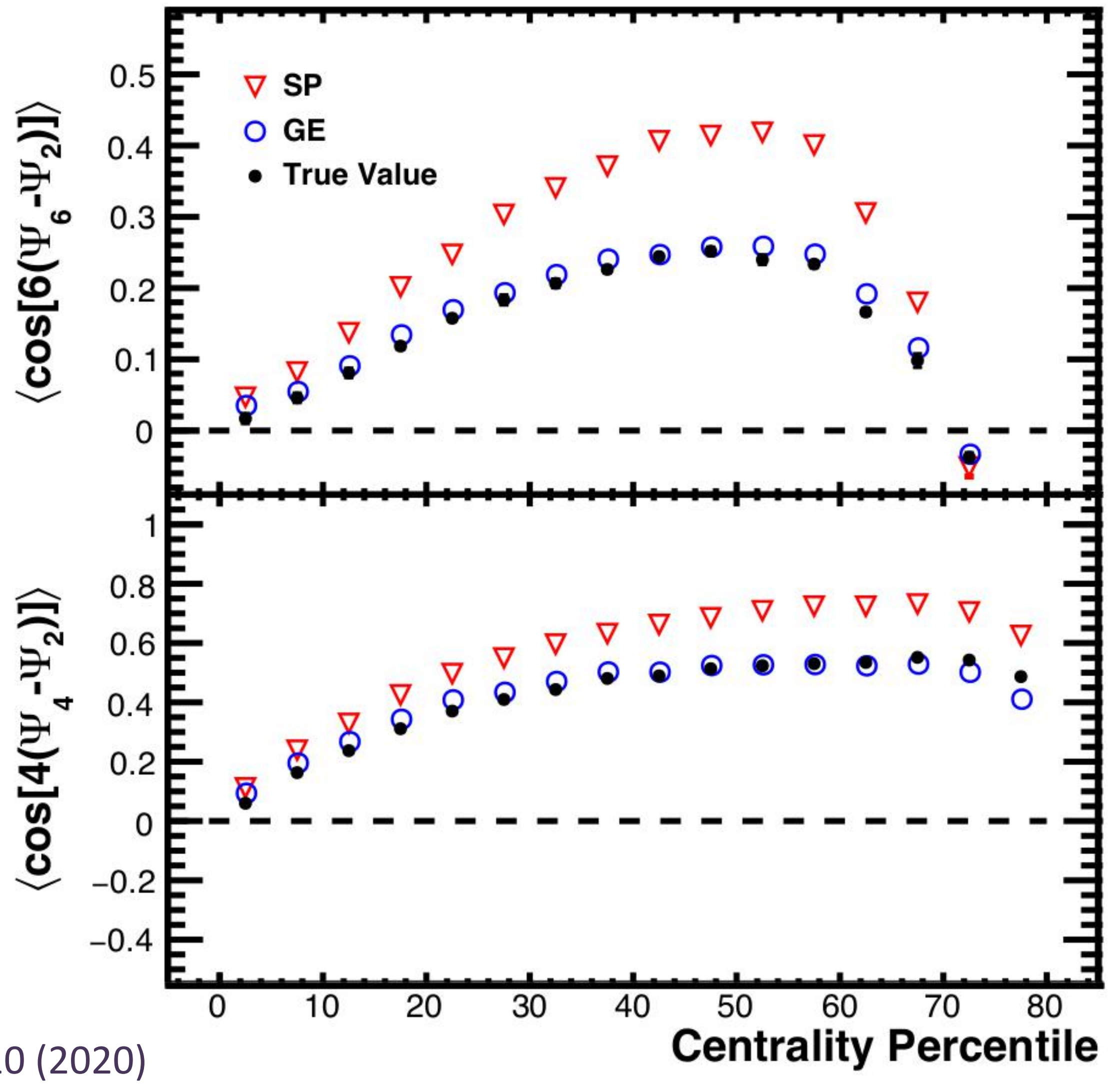
- Theoretical predictions from iEBE-VISHNU of Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 TeV
  - true value: calculated from v_n and Ψ_n in VISHNU
  - SP and GE: calculated with multiparticle correlations techniques and previous formulas
- Good agreement of the GE with the true value
  → GE overcomes the bias present in SP method



A. Bilandzic, M. Lesch and S.F. Taghavi, PRC **102**, 024910 (2020)

To summarise...

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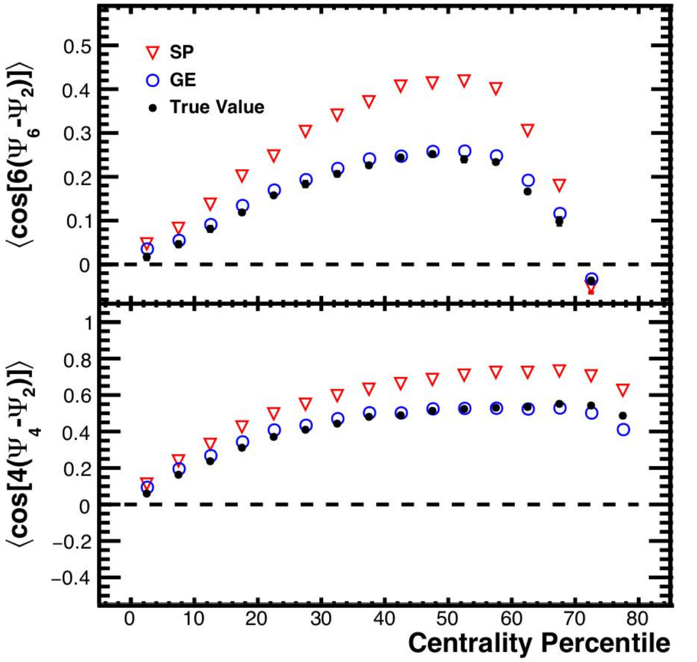
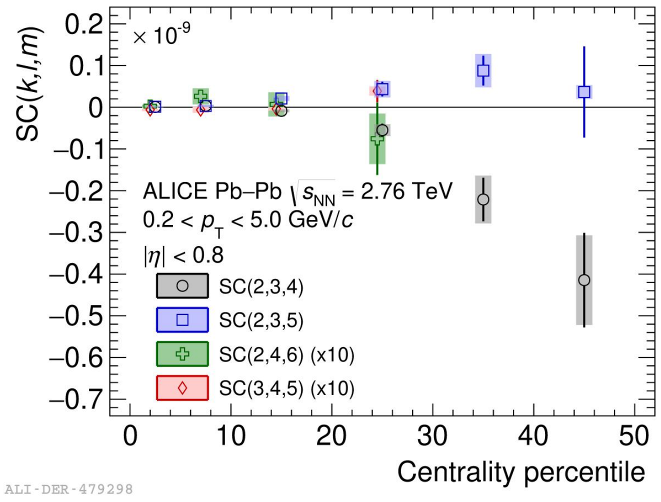
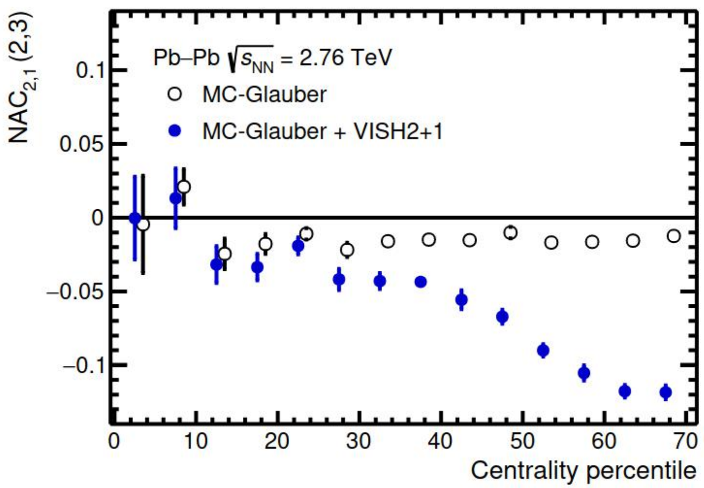
- Many new observables designed to probe the flow fluctuations
  - flow amplitudes: higher order SC, AC
  - symmetry planes: GE method
- $SC(k,l,m)$ already included in latest Bayesian analyses

→ High sensitivity to QGP transport parameters

  - AC and improved SPC will be used as input in future analyses







Thank you for your attention!

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Backup slides

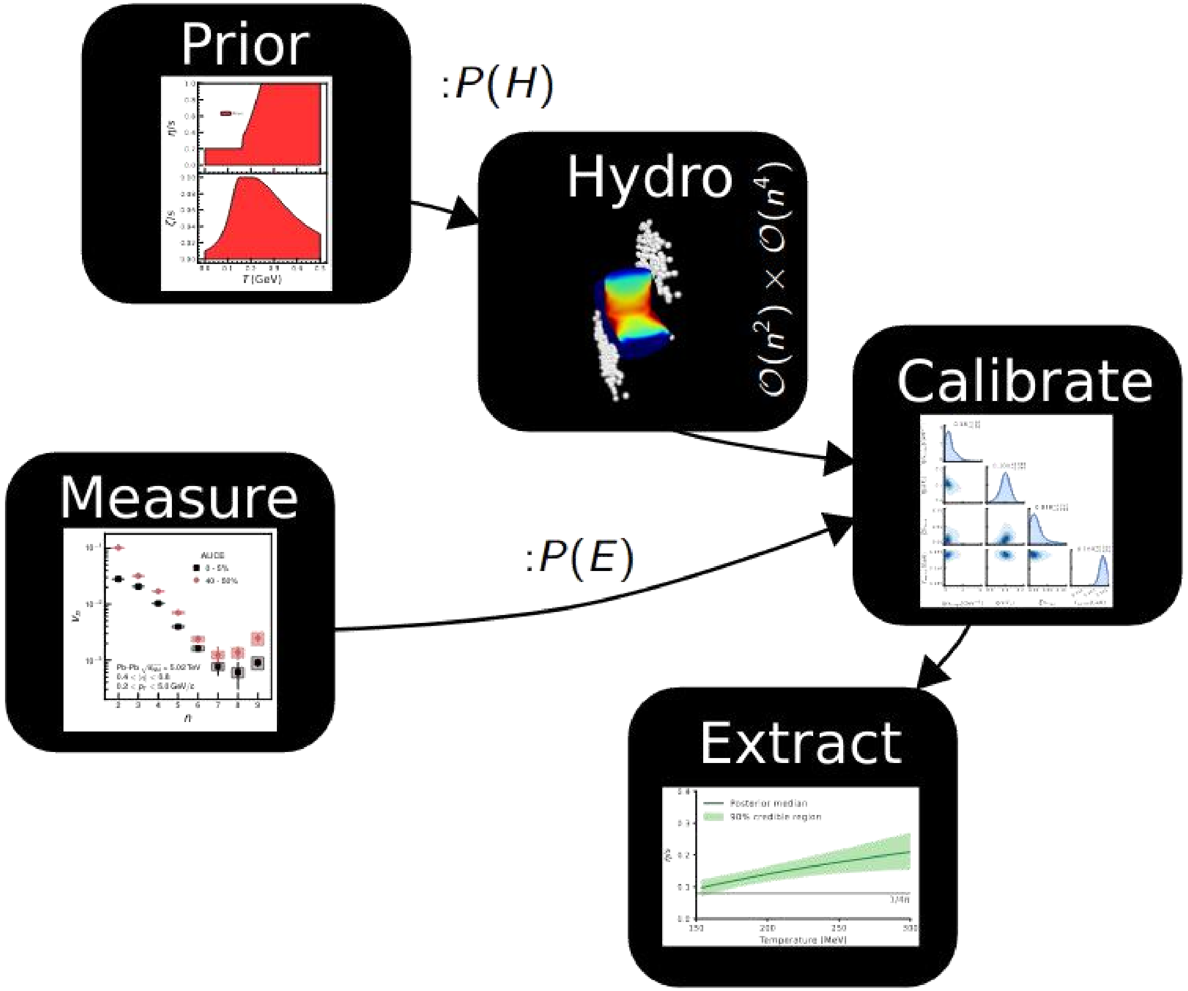
Bayesian parameter estimation

- Bayes' theorem

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

$\longrightarrow$ Find set of model parameters that fit best the experimental data

$\longrightarrow$ Use as input experimental data sensitive to QGP parameters



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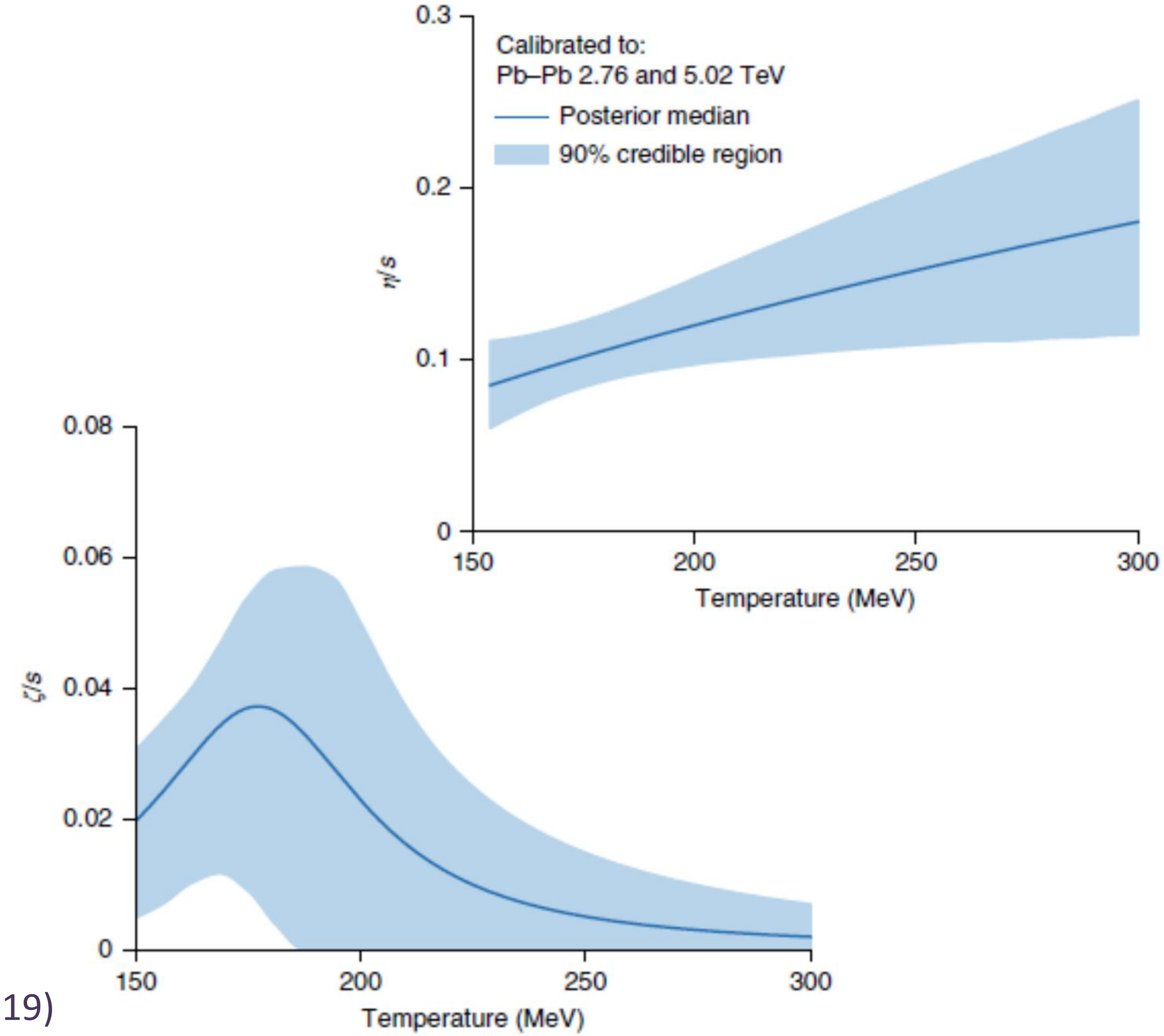
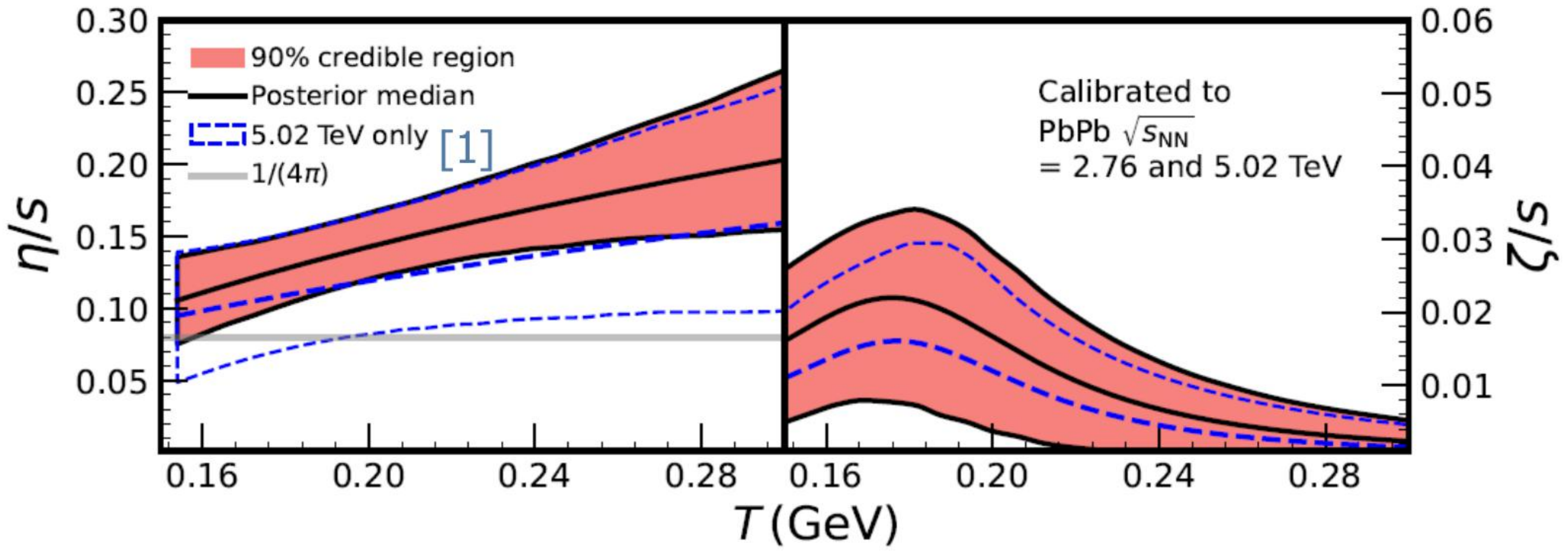


J.E. Bernhard, J.S. Moreland, S.A. Bass, Nature Phys. **15**, 1113-1117 (2019)

Bayesian parameter estimation

- Current knowledge on QGP properties from Bayesian analysis

→ But uncertainties quite large



J.E. Parkkila, A. Önnerstad, S.F. Taghavi, CM, A. Bilandzic, D.J. Kim, arXiv:2111.08145 (Submitted to PLB)