Aspects of chiral transition in a Hadron Resonance Gas Model

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Based on 2206.04579 [hep-ph]

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XVth Quark Confinement and the Hadron Spectrum
Stavanger, August-2022

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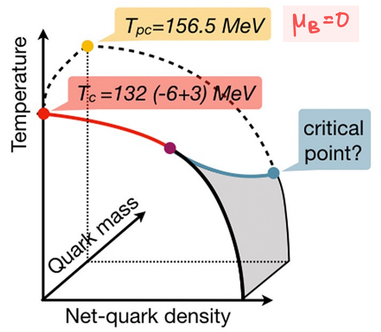
Introduction

- In QCD with 2 massless quarks, $SU(2)_v \times SU(2)_A \times U(1)_v \rightarrow$ Exact symmetry
- With physical mass, $SU(2)_v \times SU(2)_A \times U(1)_v \rightarrow$ Approximate (good)
- This symmetry is spontaneously broken to $SU(2)_v \times U(1)_v$

- For 2 flavors at non-zero mass, the chiral symmetry is restored via analytic crossover at $T_c = 156.5(1.5)$ MeV. [HotQCD 2018]
- How far can we estimate T_c and pseudo-critical line in Hadron Resonance Gas(HRG) model?



Chiral Condensate in the Hadron Resonance Gas model

Earlier results from χ_{PT} and HRG:

- The earliest estimation of pseudo-critical temperature, done within the NNLO chiral perturbation theory(χ_{PT}) gave $T_c =$ 250 MeV.

  [P. Gerber, H. Leutwyler 1989]

- Lowered to about 190 MeV with inclusion of heavier hadrons.

- Recent studies within the HRG have found a higher $T_c \sim 170$ MeV.

  [J. Jankowski et al. 2013, A. N Tawfik, N. Magdy 2015]

Renormalized chiral condensate

- We can define the renormalized chiral condensate from the pressure as,

$$\boxed{-m_s\left[\langle\bar{\psi}\psi\rangle_{l,T}-\langle\bar{\psi}\psi\rangle_{l,0}\right]=-m_s\frac{\partial P}{\partial m_l}}$$

- The normalization is not unique [BMW 2010] ,

$$\langle\bar{\psi}\psi\rangle_R=-\frac{m_l}{m_\pi^4}\left[\langle\bar{\psi}\psi\rangle_{l,T}-\langle\bar{\psi}\psi\rangle_{l,0}\right] .$$

- A natural choice for dimensionless condensate [HotQCD 2012],

$$\Delta_R^l=d+m_s r_1^4\left[\langle\bar{\psi}\psi\rangle_{l,T}-\langle\bar{\psi}\psi\rangle_{l,0}\right]$$

- Using low energy constant of $SU(2)\ \chi_{PT}$, $\Sigma^{1/3}=272(5)$ MeV, $m_s=92.2(1.0)$ MeV, and $r_1=0.3106$ fm, one gets $d=0.022791$. [FLAG 2022],

Chiral condensate in HRG model

- The renormalized chiral condensate,

$$m_s \frac{\partial P}{\partial m_l} = -\frac{m_s}{m_l} \sum_{\alpha} \frac{g_\alpha}{2\pi^2} \int_0^\infty dp \; p^2 \; n_\alpha \; (E_\alpha) \frac{1}{2E_\alpha} m_l \frac{\partial M_\alpha^2}{\partial m_l}.$$

- The non-trivial ingredient is $\partial M_\alpha^2 / \partial m_l$.

Pseudoscalar ground states

- From $SU(2)$ χ_{PT},

$$M_\pi^2 = M^2 \left[1 - \frac{1}{2}\zeta\ \bar{l}_3 + \mathcal{O}(\zeta^2)\right] \quad , \quad \zeta = \frac{M^2}{16\pi^2 F_\pi^2}$$

- Kaon properties are predicted well from 2+1 χ_{PT}
  [RBC 2014, Durr 2015]
  $M_K^2 = B_K(m_s)m_s \left[1 + \frac{\lambda_1(m_s)+\lambda_2(m_s)}{F^2} M^2\right]$
  $M^2 = 2Bm_l,\ B = \Sigma/F^2$

- From LQCD the pion mass is consistent with LO result $M_\pi^2 \approx 2Bm_l$. [RQCD Bali et al. 2016] .

Sigma terms for Heavier hadrons

$$\sigma_\alpha = m_l \frac{\partial M_\alpha}{\partial m_l}\Big|_{m_l = m_l^{phys}} = m_l \langle \alpha | \bar{u}u + \bar{d}d | \alpha \rangle = M_\pi^2 \frac{\partial M_\alpha}{\partial M_\pi^2}\Big|_{M_\pi = M_\pi^{phys}}.$$

N	Λ	Σ	Ξ
44(3)(3)	31(1)(2)	25(1)(1)	15(1)(1)
Δ	Σ^*	Ξ^*	Ω^-
29(9)(3)	18(6)(2)	10(3)(2)	5(1)(1)

The sigma terms of ground state baryons have been only recently calculated with precision. [Copeland et al. 2021] .

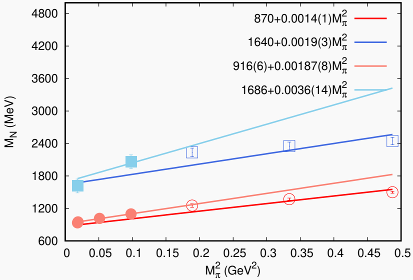
New development from our work:

- We have done extensive compilation of the LQCD results to find $M_\pi^2 \frac{\partial M_\alpha}{\partial M_\pi^2}$ at a constant m_s, set at the physical value.
- For the first time, σ terms for η, $\rho(770)$, $K^*(892)$, and η' have been calculated from LQCD data.

  [RQCD Bali et al. 2016, D. Guo et al. 2016, RQCD Bali et al. 2021] .
- We have assigned sigma terms for all meson resonances,
  - Iso-vector mesons $\rightarrow \sigma_{\rho(770)}$
  - Open strange mesons $\rightarrow \sigma_{K^*(892)}$.
  - Iso-scalar mesons $\rightarrow$ corresponding ground states mesons σ terms.
- $f_0(500)$ is not considered as cancellation by the repulsive interactions [Broniowski et al. 2015].

Sigma terms for Baryon resonances: Nucleons

- It is difficult to measure baryon resonances in LQCD as they are close to the scattering state and resonances.
- For the excited N state, the fit to 2+1 flavor LQCD data gives $\sigma = 68(27)$ MeV.
- Within large errors is consistent with the sigma term of its ground state.

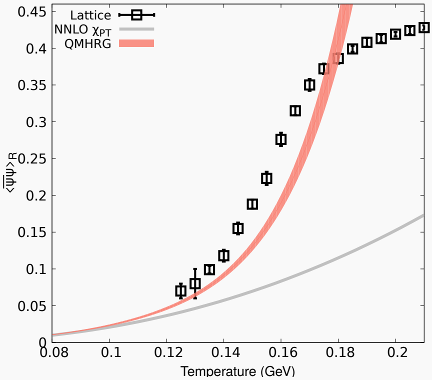


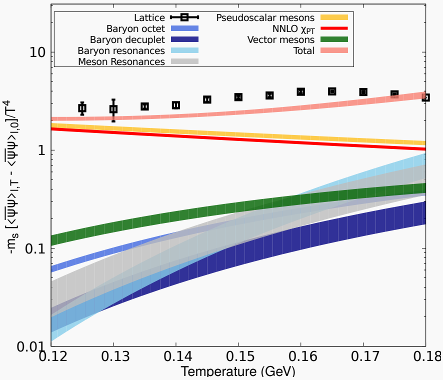
Sigma terms for Baryon resonances

- We have considered the σ terms for all resonances (even for strange baryons) to be same as the ground state.
- To reliably account for large uncertainty in σ of high mass resonances, we have taken the relative errors in the σ-terms of excited states to be 50%.
- However such large uncertainty contributes to only 10% of the total error in the renormalized chiral condensate as the dominant contribution comes from ground state pseudo-scalar mesons.

Results

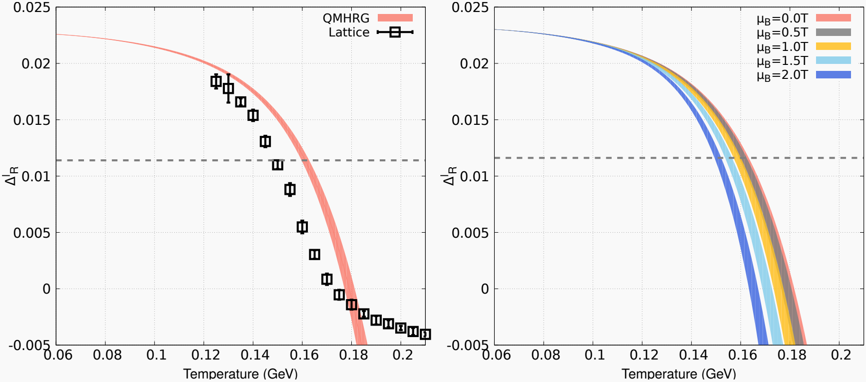
Chiral condensate: LQCD vs. HRG model





HRG model calculations are consistent with LQCD continuum estimates till T∼ 140MeV.

$\Delta_R^l = d + m_s r_1^4 \left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]$



- On the lattice Δ_R^l goes to half of its low-temperature value at T_c.
- We use this fact to estimate T_c from our HRG model calculations.

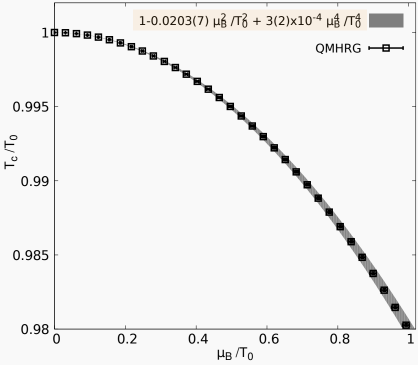
Summary of results

- Our improved HRG calculation gives $T_c = 161.2 \pm 1.7$ MeV at $\mu_B = 0$.
- Lattice QCD results on T_c in the continuum limit, $T_c = 156.5 \pm 1.5$ MeV [HotQCD 2018, BMW 2020]

Curvature of the pseudo-critical line

We extract κ_2 and κ_4 by fitting $T_c(\mu_B)$ for $0 < \mu_B / T_c(\mu_B = 0) < 1$.

- Our estimation $\kappa_2 = 0.0203(7)$.
- $\kappa_4 = -3(2) \times 10^{-4}$ is quite noisy.
- Highlight that our results are in very good agreement with LQCD estimates of $\kappa_2 = 0.012(4)$ [HotQCD 2018] and $0.0153(18)$ [BMW 2020], $\kappa_4 = 0$.

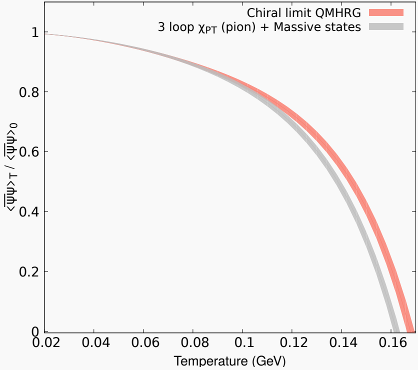


Transition in the chiral limit

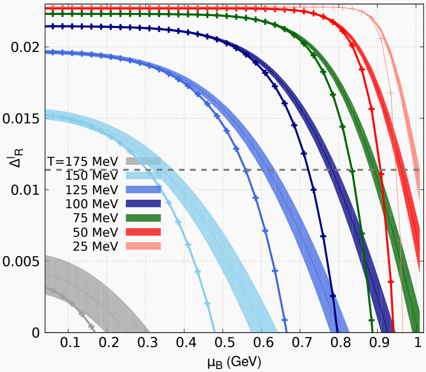
- 3-loop χ_{PT} for pions + hadrons, gave a $T_c^0 \sim 170$ MeV .

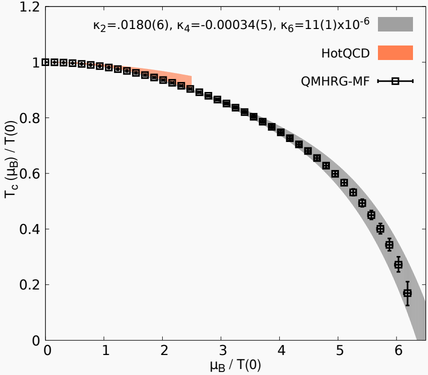
  [P. Gerber, H. Leutwyler 1989]

- 3-loop χ_{PT} + our improved HRG estimates lower it to 162 MeV.
- LQCD predicts $T_c = 132^{+3}_{-6}$ MeV [HotQCD 2019].
- Need to go beyond 3-loop χ_{PT} and include temperature dependent width of the critical f_0 mode to improve the agreement



For high density, we need to include repulsive interaction among baryons via mean field. [Huovinen, Petreczky 2018]





Summary and outlook

Summary and outlook

- We have studied chiral observables for physical hadrons within the HRG model.
- For the first time, precise values of σ terms for ρ, η, K^*, isoscalar mesons and ground state baryons have been included.
- This has successfully improved the T_c from HRG model, bringing it closer to the LQCD estimates.
- Curvature coefficients κ_2, κ_4 are very close to lattice results than previous estimates.
- With these excellent agreements at $\mu_B \approx 0$, we have extended our formalism to larger values of μ_B. [P.Petreczky, to appear]