Bayesian analysis improvements in the light of the new LHC measurements

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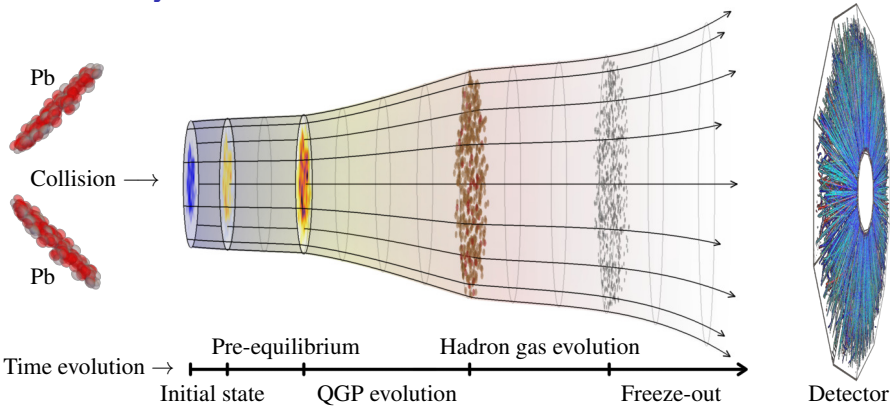
XIVth Quark Confinement and the Hadron Spectrum
University of Stavange, Stavange, Norway
1-6 August, 2022

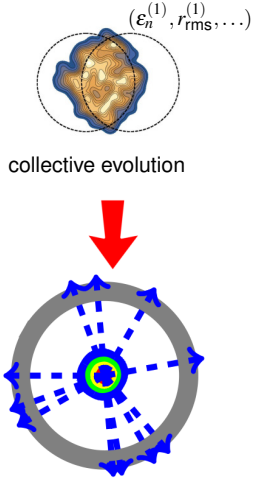
State-of-the-art heavy-ion collision models



Hybrid Model	=	IP-Glasma	$\mathbb{1}$		VISH2+1		UrQMD
		T_RENTo	Free-streaming		MUSIC		SMASH
		MC-Glauber	KøMPøST	$\otimes$	Trajectum	$\otimes$	B3D
		MC-KLN	Gauge/Gravity		VH2+1		⋮
		⋮	⋮		⋮		

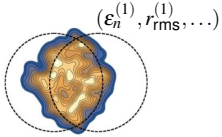
[References in the backup slides]

Flow harmonics in a nutshell!

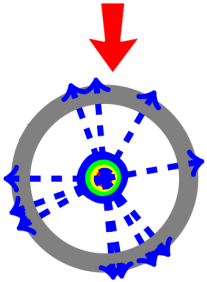


Flow harmonics in a nutshell!

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1 + \sum_{n=1}^{\infty} 2\, v_n \cos\left[n(\varphi - \psi_n)\right]\right]$$



collective evolution



Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients $(\eta/s,\ \zeta/s, \ldots)$, ...

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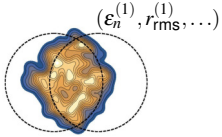

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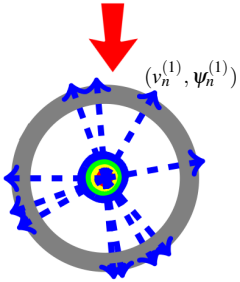

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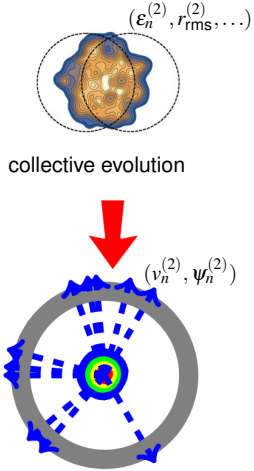

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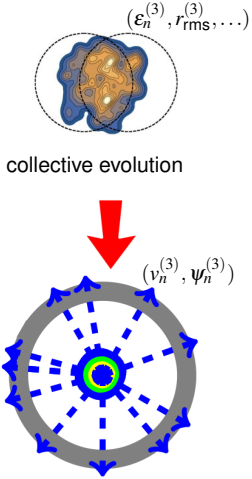

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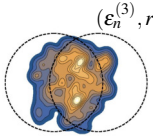

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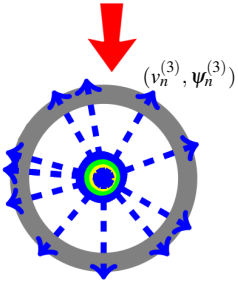

Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients $(\eta/s,\ \zeta/s, \ldots)$, ...

$$p([p_T], v_n, \psi_n, \ldots)$$

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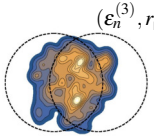

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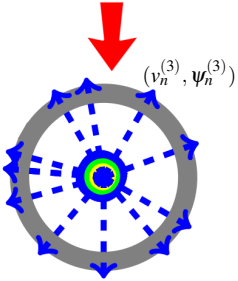
Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients $(\eta/s,\ \zeta/s, \ldots)$, ...

$$p([p_T], v_n, \psi_n, \ldots)$$

- Concentrate on a single harmonic flow amplitude $p(v_n)$,



$$v_n\{2\} \equiv \left(\langle v_n^2 \rangle\right)^{1/2}, \qquad v_n\{4\} \equiv \left(-\langle v_n^4 \rangle + 2\langle v_n^2 \rangle^2\right)^{1/4}, \qquad \cdots$$

[Borghini, Dinh, Ollitrault, PRC, 64, 054901 (2001)]

Theoretical models Vs experimental data

Initial state parameters

$N(\sqrt{s_{NN}})$	Overall normalization
p	Entropy deposition parameter
w	Gaussian nucleon width
	⋮

Pre-equilibrium parameters

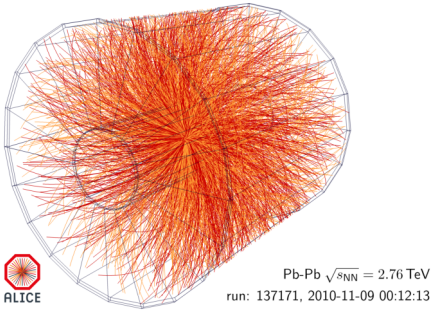
τ_{fs}	Free-streaming time
	⋮

QGP evolution parameters

$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{slope}$	Slope of $\eta/s(T)$ above T_c
$(\eta/s)_{curve}$	Curvature of $\eta/s(T)$ above T_c
	⋮

Hadronic gas evolution parameters

	⋮



Experimental observables

dN/dy	Particle yields, $\pi^{\pm}$, $k^{\pm}$, ...
$\langle p_T \rangle$	Mean transverse momentum, $\pi^{\pm}$, $k^{\pm}$, ...
$v_n\{2\}$	Anisotropic flow two-particle correlation
$v_n\{4\}$	Anisotropic flow four-particle correlation
	⋮

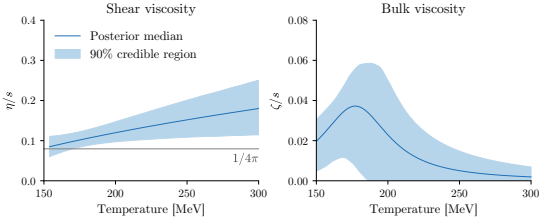
- What is the optimal value for the parameters to reproduce the experimental data, and how can we improve it?
- How much the models are applicable in small systems (Pb–Pb, Xe–Xe, ..., O–O, ..., Pb–p, p–p)?

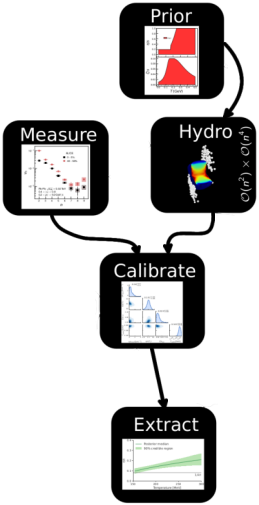
Model in the light of experimental data

- The initial the degree of belief to model parameters, *prior* distribution: $P(\text{theory})$
- Likelihood: $P(\text{Data}|\text{Theory})$
- Updated belief in the light of data. *posterior* distribution: $P(\text{Theory}|\text{Data})$.

Bayes's theorem: $P(\textit{Theory}|\textit{Data}) \propto P(\textit{Data}|\textit{Theory})P(\textit{Theory})$

Ref. [1]:





- Theoretical developments: collectivity **[2]**, jet-quenching **[3]**, nucleon substructure **[4]**

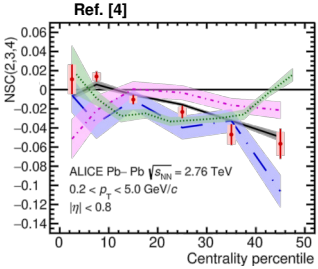
[1] Bernhard, PhD Thesis, arXiv: 1804.06469; Bernhard, Moreland, Bass, Nature Phys. 15 (2019) 11, 1113-1117
[2] Auvinen, et al., PRC 102 (2020) 044911, Nijs et al., PRL 126 (2021) 202301, JETSCAPE, PRC 103 (2021) 054904
[3] JETSCAPE, PRC 104 (2021), 024905
[4] Mäntysaari, Schenke, Shen, Zhao, arXiv: 2202.01998

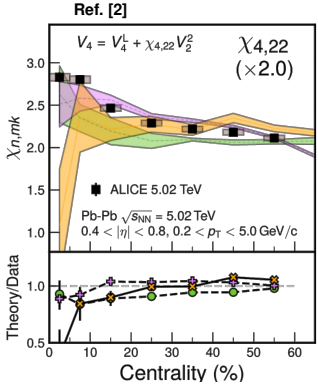
Many new measurements

Observable	
Single-harmonic observables [1, 2]	$v_2\{2\}, \ldots, v_7\{2\}$
Symmetric cumulants [3]	$\mathrm{NSC}(2,3)$, $\mathrm{NSC}(2,4)$, $\mathrm{NSC}(3,4)$
Higher-order symmetric cumulants [4]	$\mathrm{NSC}(2,3,4)$, $\mathrm{NSC}(2,3,5)$
Symmetry plane correlations [2,5]	$\rho_{4,22}, \rho_{5,23}, \rho_{6,222}, \rho_{6,33}$
Non-linear mode couplings [2,6]	$\chi_{4,22}, \chi_{5,23}, \chi_{6,222}, \chi_{6,33}$
Symmetry plane correlation (GE) [7]	$\langle\cos(4\psi_2 - 4\psi_4)\rangle_{\mathrm{GE}}, \ldots$
Asymmetric cumulants [8]	$\mathrm{AC}_{2,1}(m,n), \ldots$

$$\mathrm{NSC}(m,n) = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}, \quad \chi_{4,22} = \frac{\langle v_2^2 v_4 \cos(4\psi_2 - 4\psi_4)\rangle}{\langle v_2^4\rangle}, \quad \ldots$$

[1] Borghini, et al., PRC 64 (2001) 054901, ALICE Collaboration, PRL, 107 (2011) 032301, ALICE Collaboration, PRL, 116 (2016) 13, 132302.
[2] ALICE Collaboration, JHEP 05 (2020) 085, ALICE Collaboration, 773 (2017) 68-80.
[3] Bilandzic, et al., PRC 89 (6) (2014) 064904, ALICE Collaboration, PRC 97 (2018) 2, 024906, PRL, 117 (2016) 182301.
[4] Mordasini, et al., PRC 102 (2) (2020) 024907, ALICE Collaboration, PRL, 127 (2021) 9, 092302.
[5] Bhalerao, et al., PLB, 742 (2015) 94-98, Yan, et al. PLB, 744 (2015) 82-87.
[6] Qiu, et al., PLB, 717 (2012) 261-265, ATLAS Collaboration, PRC, 90 (2014) 2, 024905.
[7] A. Bilandzic, M. Lesch, SFT, PRC 102, 024910 (2020)
[8] A. Bilandzic, M. Lesch, C. Mordasini, SFT, PRC 105, 024912 (2022)





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C. Mordasini's talk

$$\mathrm{NSC}(m,n) = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}, \quad \chi_{4,22} = \frac{\langle v_2^2 v_4 \cos(4\psi_2 - 4\psi_4)\rangle}{\langle v_2^4\rangle}, \quad \ldots$$

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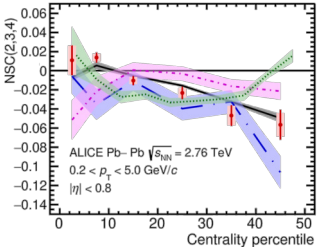

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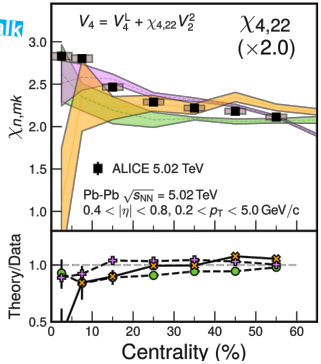

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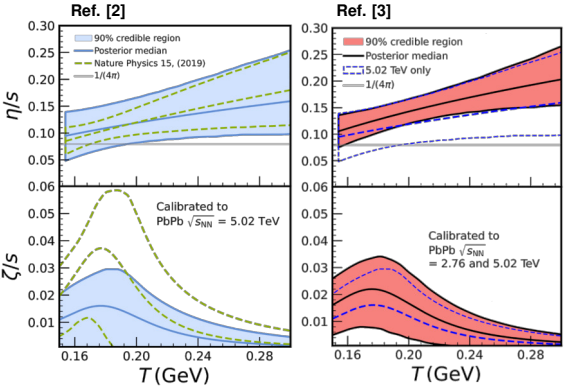
[8] A. Bilandzic, M. Lesch, C. Mordasini, SFT, PRC 105, 024912 (2022)

Transport properties of QGP

T$_R$ENTo$\otimes$ Free-streaming $\otimes$ VISH2+1 $\otimes$ UrQMD



Significant improvement in uncertainties, especially in bulk viscosity.

	Ref. [1]	Ref. [2] New!	Ref. [3] New!
2.76 TeV	PID multi. N_{ch} PID $\langle p_T \rangle$ $\delta p_T / \langle p_T \rangle$ E_T $v_2, \ldots, v_4$		N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ NSC$(3,2)$, NSC$(4,3)$ NSC$(2,3,4)$, NSC$(2,3,5)$ $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$
5.02 TeV	N_{ch} $v_2, \ldots, v_4$	PID multi. N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ $v_5, \ldots, v_7$ NSC$(3,2)$ to NSC$(4,3)$ $\chi_{4,22}$ to $\chi_{6,mk}$	PID multi. N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ $v_5, \ldots, v_7$ NSC$(3,2)$ to NSC$(4,3)$ $\chi_{4,22}$ to $\chi_{6,mk}$ $\rho_{4,22}$ to $\rho_{6,mk}$

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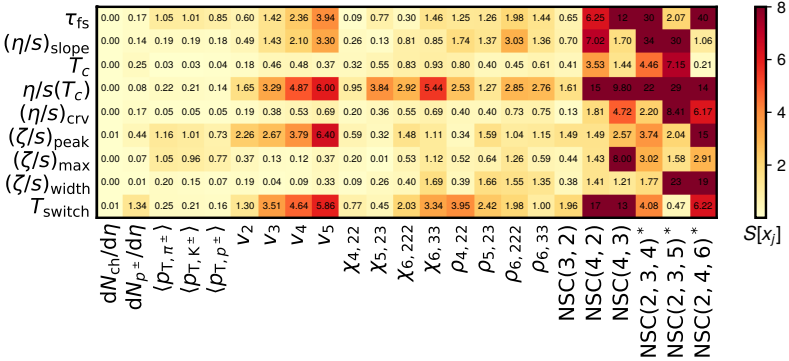
[3] J.E. Parkkila, A. Onnerstad, SFT, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv: 2111.08145.

Sensitivity index $S_{\hat{O}}[x_j] = \frac{1}{\delta}\frac{|\hat{O}(\vec{x}')-\hat{O}(\vec{x})|}{\hat{O}(\vec{x})}$ of the input parameters



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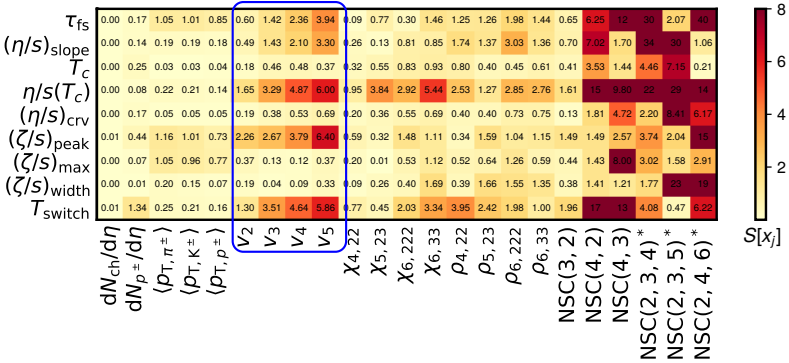

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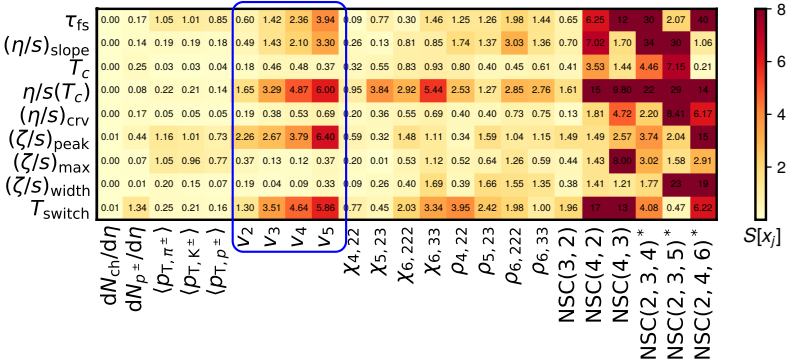

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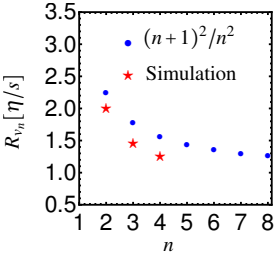

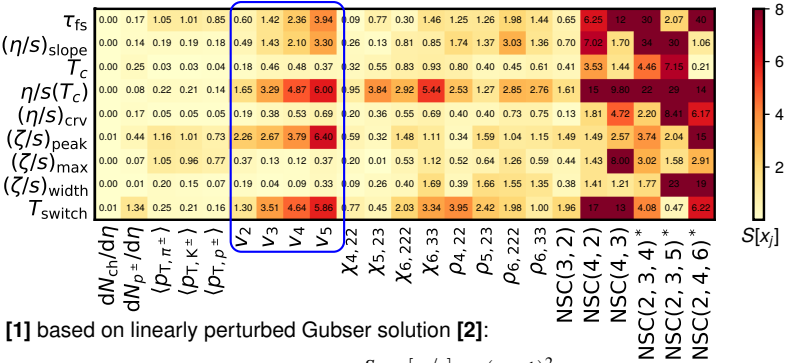

- Sensitivity of v_n: Teaney and Yan **[1]** based on linearly perturbed Gubser solution **[2]**:

$$v_n(\eta/s) \sim v_n(0)e^{-\lambda n^2 \eta/s}, \quad \rightarrow \quad S_{v_n}[\eta/s] \sim -\lambda\, n^2\, \eta/s, \quad \rightarrow \quad R_{v_n}[\eta/s] = \frac{S_{v_{n+1}}[\eta/s]}{S_{v_n}[\eta/s]} \sim \frac{(n+1)^2}{n^2},$$

what about $R_n[\zeta/s]$?

[1] Teaney, Yan, PRC 86 (2012) 044908
[2] Gubser, Yarom, Nucl.Phys.B 846 (2011) 469-511

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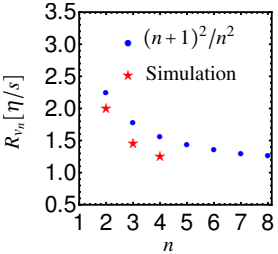

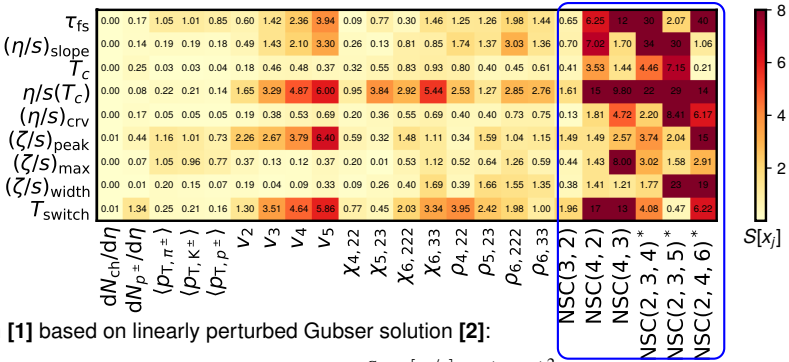

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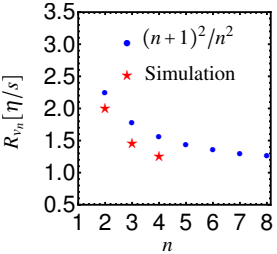
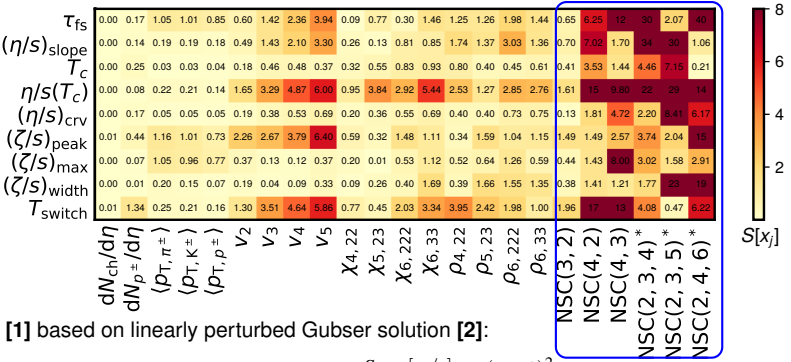




- Sensitivity of v_n: Teaney and Yan **[1]** based on linearly perturbed Gubser solution **[2]**:

$$v_n(\eta/s) \sim v_n(0)e^{-\lambda n^2 \eta/s}, \quad \rightarrow \quad S_{v_n}[\eta/s] \sim -\lambda\, n^2\, \eta/s, \quad \rightarrow \quad R_{v_n}[\eta/s] = \frac{S_{v_{n+1}}[\eta/s]}{S_{v_n}[\eta/s]} \sim \frac{(n+1)^2}{n^2},$$

what about $R_n[\zeta/s]$?

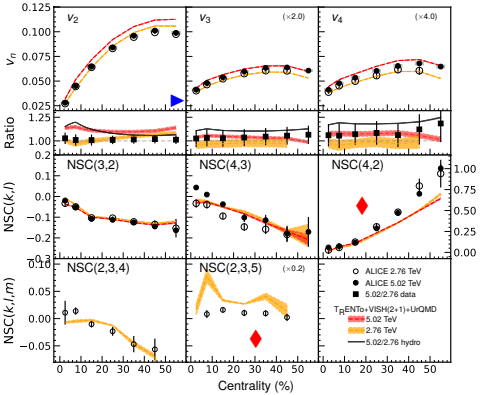
- Sensitivity of normalized symmetric cumulants:

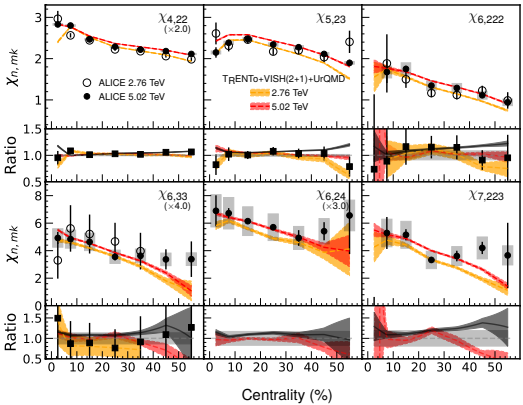
$$\mathrm{NSC}(m,n) = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}, \quad \rightarrow \quad S_{\mathrm{NSC}(m,n)}[\eta/s] \sim \mathbf{0} + \textbf{non-linear!?}$$

[1] Teaney, Yan, PRC 86 (2012) 044908
[2] Gubser, Yarom, Nucl.Phys.B 846 (2011) 469-511

Maximum A Posteriori parametrization

Overall agreement, with only few discrepancies

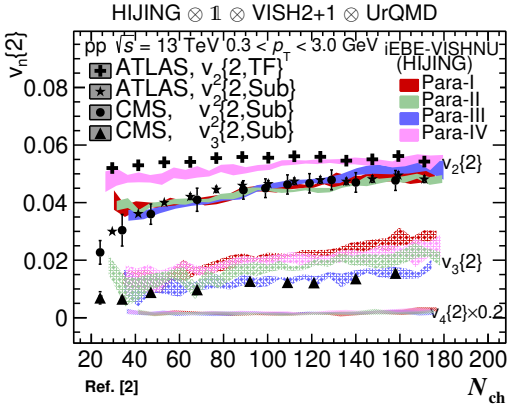


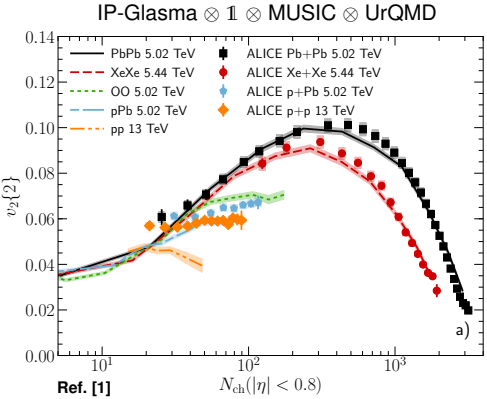


- ► The energy dependence of v_2.
- ♦ Deviation from simulation and data in NSC$(2,4)$ and NSC$(2,3,5)$.
  More plots in **Refs. [1,2]**.

[1] J.E. Parkkila, A. Onnerstad, D.J. Kim, Phys.Rev.C 104 (2021) 5, 054904.
[2] J.E. Parkkila, A. Onnerstad, SFT, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv: 2111.08145.

Pushing the model's limit to its extreme!





- Pre-equilibrium dynamics is missing in these models.
- The model predictions are worsen at lower multiplicities.
- Removing the non-flow effects are challenging, especially at low multiplicities.

[1] Schenke, Shen, Tribedy, PRC 102 (2020) 044905
[2] Zhao, Zhou, Xu, Deng, Song, PLB 780 (2018) 495-500

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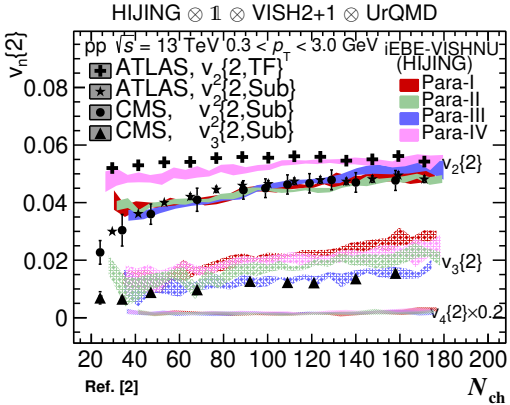

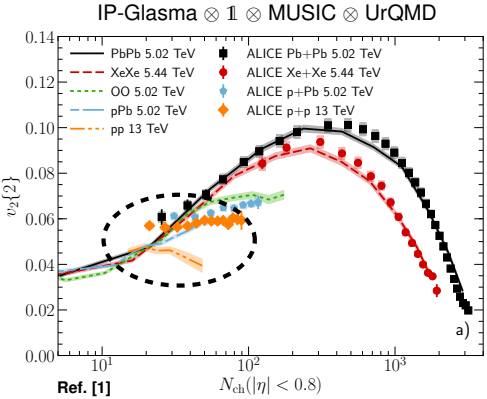

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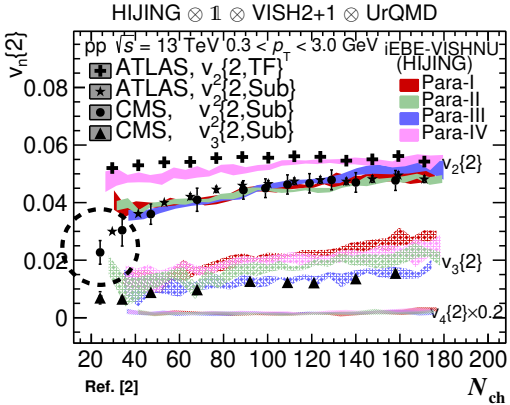

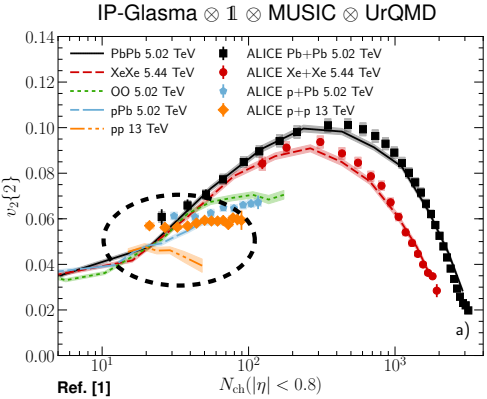

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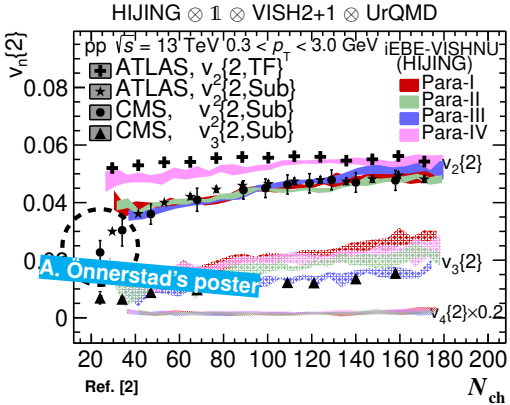

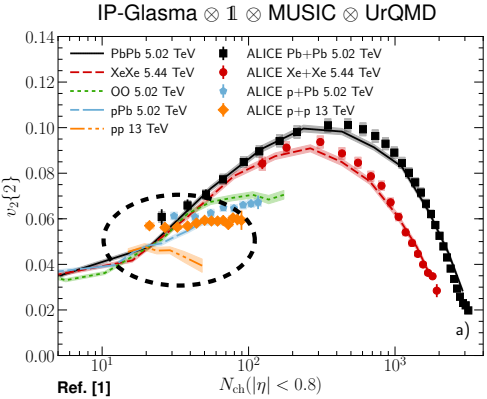

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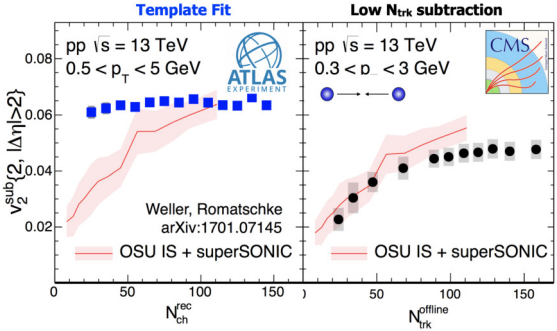
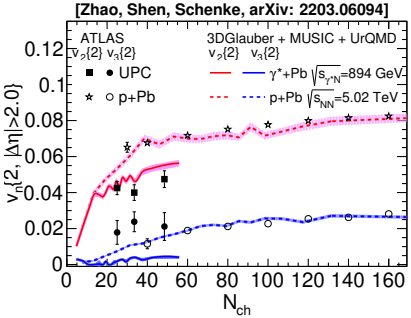
[2] Zhao, Zhou, Xu, Deng, Song, PLB 780 (2018) 495-500

What is happening in small systems / low multiplicities?





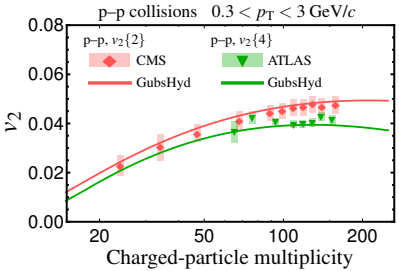
- In superSOINUC: as a pre-equilibrium stage $T^{\mu\nu}_{\text{AdS/CFT}}$ is matched to $T^{\mu\nu}_{\text{hydro}}$ at τ_{hydro}.
- In 3DGlauber⊗MUSIC⊗UrQMD: A 3D dynamical initial state is considered.

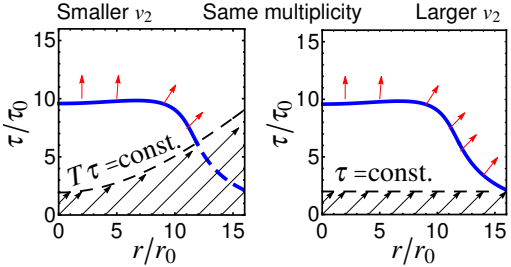
- IP-Glasma ⊗ 𝟙 ⊗ MUSIC ⊗ UrQMD
- HIJING ⊗ 𝟙 ⊗ VISH2+1 ⊗ UrQMD
- T$_R$ENTo⊗ Free-streaming ⊗ VISH2+1 ⊗ UrQMD

- MC-Glauber (OSU) ⊗ Gage/Gravity ⊗ VH2+1 ⊗ B3D
- 3DGlauber⊗Dynamical IS⊗MUSIC⊗UrQMD

Small system and GubsHyd

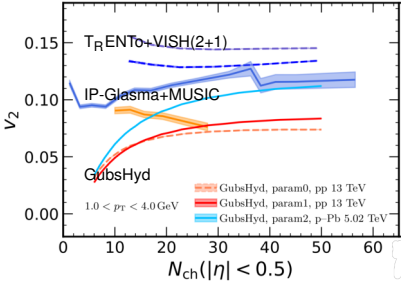
[SFT, PRC 102 (2020) 024910]





In GubsHyd:

- The hydrodynamic evolution starts at $T\tau$ =const. Evidences:
  - Attractors in Gubser flow using Kin. Theory, $w(T\tau) = w_0$ **[1]**
  - Non-hydrodynamic modes decay time: $e^{-z_0 T\tau}$ **[2]**
  - "Inhomogeneous longitudinal cooling"!? **[3]**
- Cold corona region contributes to the multiplicity.
- Compared to τ =const., less evolution time → smaller v_n.



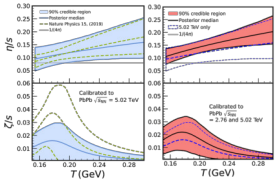
[1] Behtash, Cruz-Camacho, Martinez, PRD 97 (2018) 044041
[2] Heller, Janik, Witaszczyk, PRL 108 (2013) 211602
[3] Ambrus, Schlichting, Werthmann, PRD 105 (2022) 014031

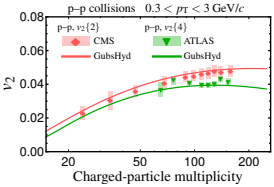
Summary

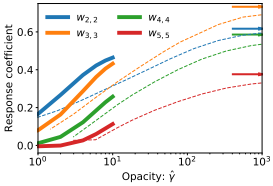
- Importance of observables to understand the models. We need to choose cleverly!
- Improvement of the transport coefficient uncertainties.
- Including small system information into a Bayesian analysis needs a careful consideration.
- The pre-equilibrium dynamics could have a substantial influence in small system collisions.

Outlook

- Observables sensitive to initial state: isobar ratio
- Collective models with dynamical pre-equilibrium
- Framework beyond hydrodynamics in an event-by-event basis







[Kurkela, SFT, Wiedemann, Wu, PLB 811 (2020) 135901]

Thank You!

Backup Slides

References of slide in page 2

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[SMASH] J. Weil et al., Phys. Rev. C 94, 054905 (2016)

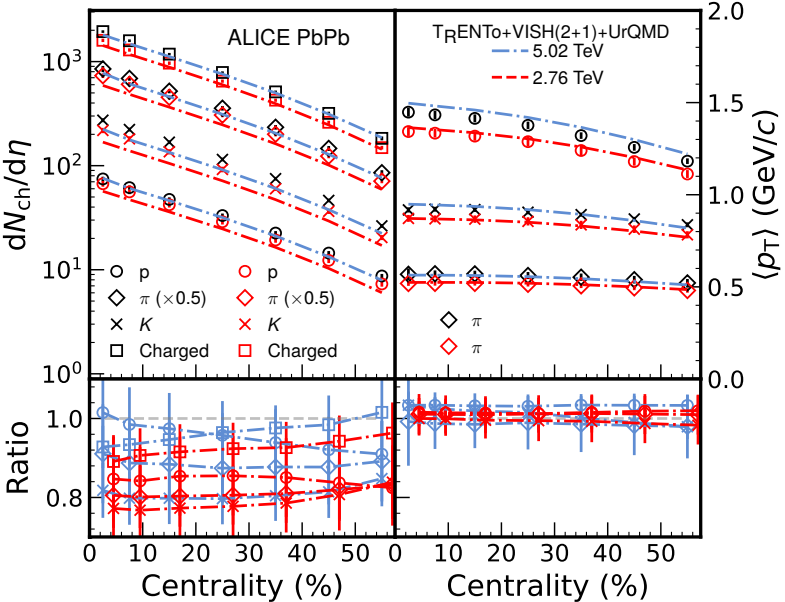
[B3D] John Novak, Kevin Novak, Scott Pratt, Joshua Vredevoogd, Chris Coleman-Smith, and Robert Wolpert, Phys. Rev. C 89, 034917 (2014)

MAP parameters

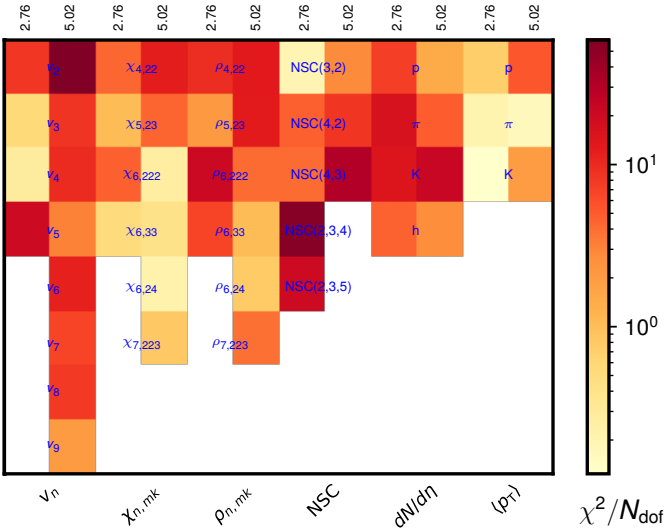
Parameter	Description	Range	MAP
N(2.76 TeV)	Overall normalization (2.76 TeV)	[11.152, 18.960]	14.373
N(5.02 TeV)	Overall normalization (5.02 TeV)	[16.542, 25]	21.044
p	Entropy deposition parameter	[0.0042 , 0.0098]	0.0056
σ_k	Std. dev. of nucleon multiplicity fluctuations	[0.5518, 1.2852]	1.0468
$d^3_{\rm min}$	Minimum volume per nucleon	$[0.889^3, 1.524^3]$	1.2367^3
$\tau_{\rm fs}$	Free-streaming time	[0.03, 1.5]	0.71
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$	[0.135, 0.165]	0.141
$\eta/s(T_c)$	Minimum $\eta/s(T)$	[0, 0.2]	0.093
$(\eta/s)_{\rm slope}$	Slope of $\eta/s(T)$ above T_c	[0, 4]	0.8024
$(\eta/s)_{\rm curve}$	Curvature of $\eta/s(T)$ above T_c	$[-1.3, 1]$	0.1568
$(\zeta/s)_{\rm peak}$	Temperature of $\zeta/s(T)$ maximum	[0.15, 0.2]	0.1889
$(\zeta/s)_{\rm max}$	Maximum $\zeta/s(T)$	[0, 0.1]	0.01844
$(\zeta/s)_{\rm width}$	Width of $\zeta/s(T)$ peak	[0, 0.1]	0.04252
$T_{\rm switch}$	Switching / particlization temperature	[0.135, 0.165]	0.1595

$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\rm slope}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\rm curve}}, \qquad (\zeta/s)(T) = \frac{(\zeta/s)_{\rm max}}{1 + \left(\frac{T-(\zeta/s)_{T_{\rm peak}}}{(\zeta/s)_{\rm width}}\right)^2}.$$

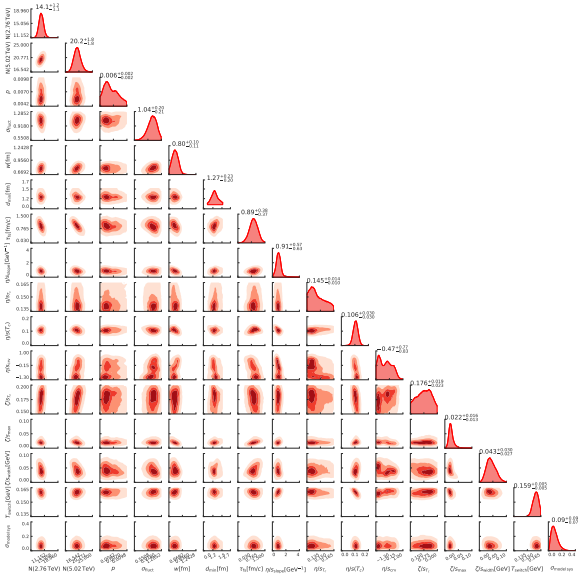
MAP parametrization



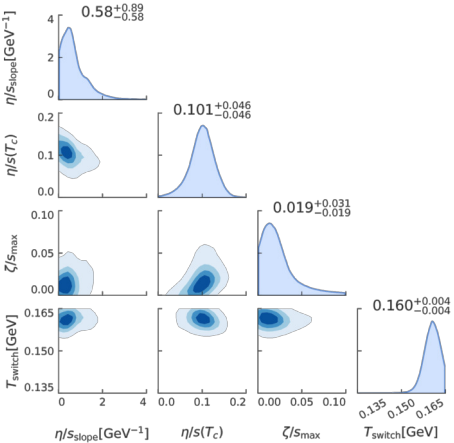
Chi-square

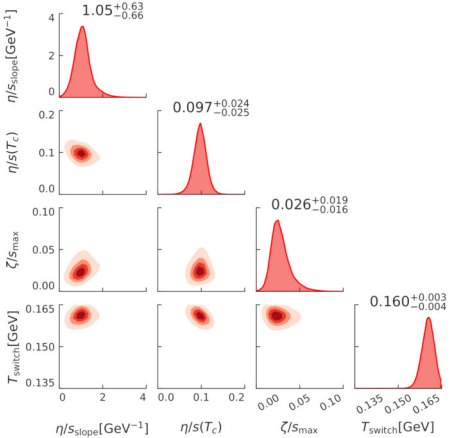

2.76
5.02
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
$\chi_{4,22}$
$\chi_{5,23}$
$\chi_{6,222}$
$\chi_{6,33}$
$\chi_{6,24}$
$\chi_{7,223}$
$\rho_{4,22}$
$\rho_{5,23}$
$\rho_{6,222}$
$\rho_{6,33}$
$\rho_{6,24}$
$\rho_{7,223}$
NSC(3,2)
NSC(4,2)
NSC(4,3)
NSC(2,3,4)
NSC(2,3,5)
p
π
K
h
10^1
10^0
v_n
$\chi_{n,mk}$
$\rho_{n,mk}$
NSC
dN/dη
$\langle p_T \rangle$
χ^2/N_{dof}

Posterior distribution



Posterior distribution

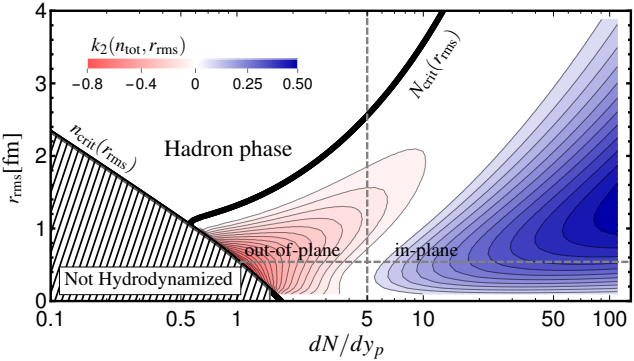


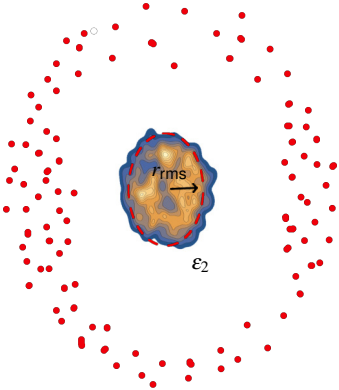


GubsHyd:

[SFT, Phys.Rev.C 104 (2021) 5, 054906]

A semi-analytical toy model based on the analytical Gubser hydrodynamic solution





$n_{\rm tot} \equiv dN/dy_p$

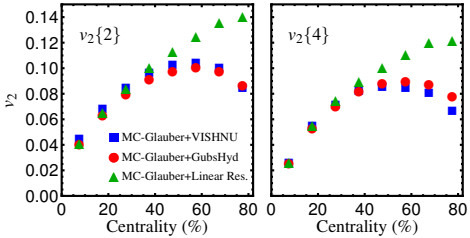
Linear response approximation $v_2 \simeq k_2 \varepsilon_2$

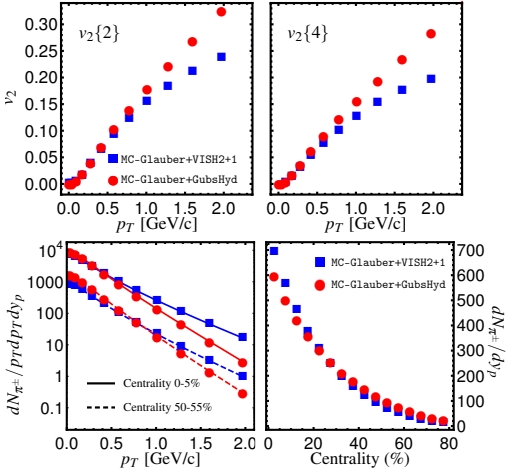
$$v_2 \simeq k_2(n_{\rm tot}, r_{\rm rms})\varepsilon_2$$

VALIDATION: MC-Glauber+VISH(2+1) Vs MC-Glauber+GubsHyd

Pb–Pb collision, $\sqrt{s_{NN}} = 2.76$ *TeV*

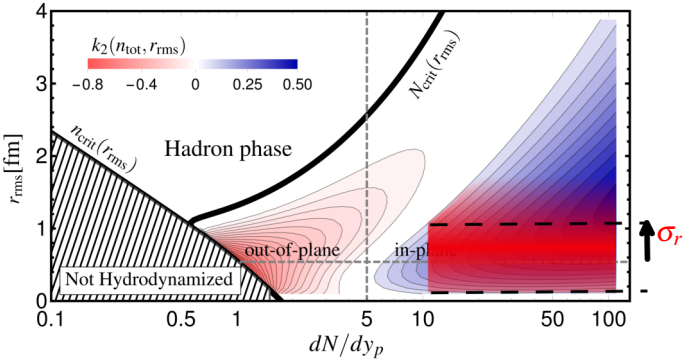
- **Same initial state** set for both and **GubsHyd**





GubsHyd for small system collisions

We model the initial state fluctuation



We assume ε_2 and $r_{\rm rms}$ fluctuate as follows:

$$p_{\rm init}(\varepsilon_2, r_{\rm rms}) = \left[\frac{r_{\rm rms}}{{\sigma_r}^2} e^{-r_{\rm rms}^2/2{\sigma_r}^2}\right] \times \left[\frac{\varepsilon_2}{{\sigma_\varepsilon}^2} e^{-\varepsilon_2^2/2{\sigma_\varepsilon}^2}\left[1 + \frac{\Gamma_2^\varepsilon}{2} L_2(\varepsilon_2^2/2{\sigma_\varepsilon}^2) + \cdots\right]\right]$$

Then the two particle correlation is given by

$$p_{\rm final}(v_2):$$

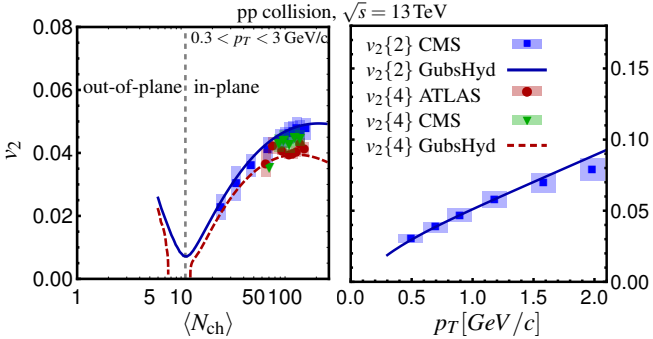
$$v_2\{2\} = \chi\, \sigma_\varepsilon \sqrt{2\langle k_2^2(n_{\rm ntot}, r_{\rm rms})\rangle_r}$$

$$v_2\{4\} = \chi\, \sigma_\varepsilon \left[8\langle k_2^2\rangle_r^2 - 4(2+\Gamma_2^\varepsilon)\langle k_2^4\rangle_r\right]^{1/4}$$

Two- and four-particle correlation of proton–proton collision

[SFT, Phys.Rev.C 104 (2021) 5, 054906]

- By fitting the model to data, we obtain the free parameters:



$$p_{final}(v_2) \longrightarrow p_{init}(\varepsilon_2, r_{rms})$$

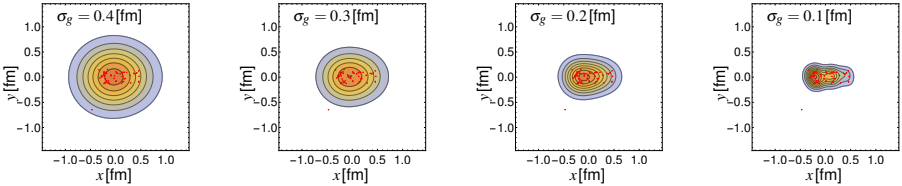
The initial state fluctuation should has the following properties:

$$\sigma_r \approx 0.4 \text{ [fm]}$$

$$\chi\,\sigma_\varepsilon \approx 0.097$$

$$\Gamma_2^\varepsilon \equiv -\left(\frac{\varepsilon_2\{4\}}{\varepsilon_2\{2\}}\right)^4 \approx -0.75$$

Do AMPT (only for initial state) and T$_R$ENTo fulfill these conditions?



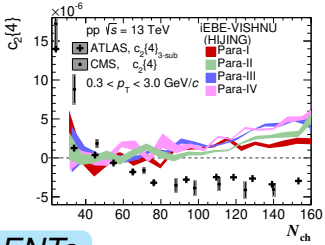
- To cancel out the effect of $\chi\,\sigma_\varepsilon$, we define $\Gamma_2^v = -\left(\frac{v_2\{4\}}{v_2\{2\}}\right)^4 \equiv \frac{c_2\{4\}}{c_2^2\{2\}}$

σ_g [fm]	σ_r^{AMPT} [fm]	$\Gamma_2^{\varepsilon(\text{AMPT})}$	$\Gamma_2^{v(\text{AMPT+Gubs})}$
0.5	0.48	0.53	0.80
0.4	0.41	0.18	0.53
0.3	0.35	-0.17	0.26
0.2	0.30	-0.48	0.01
0.1	0.26	-0.73	-0.20

$\sigma_r \approx 0.4$ [fm]

$\Gamma_2^{\varepsilon} \approx -0.75$

TENSION!



A similar behavior is observed for T$_R$ENTo.

Scanning the parameters