Medium induced jet broadening in a quantum computer

4th August 2022, Confinement XV

João Barata, BNL

Based on: 2104.04661, with C. Salgado

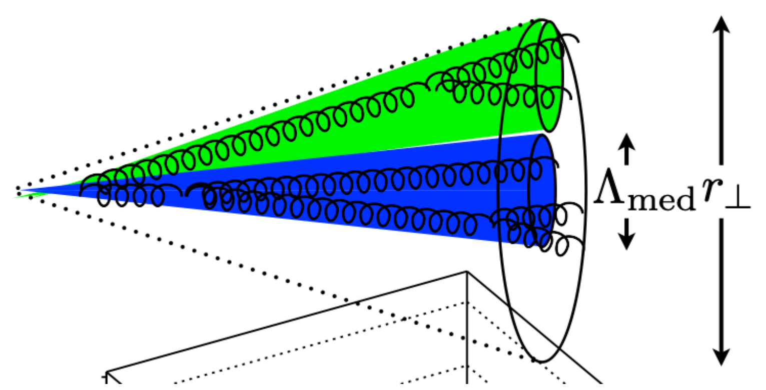
2208.XXXX, with C. Salgado, M. Li, W. Qian, X. Du

Why quantum computing for jets?

See also talks by L. Apolinário, P. Caucal, M. A. Escobedo, A. Sadofyev, …

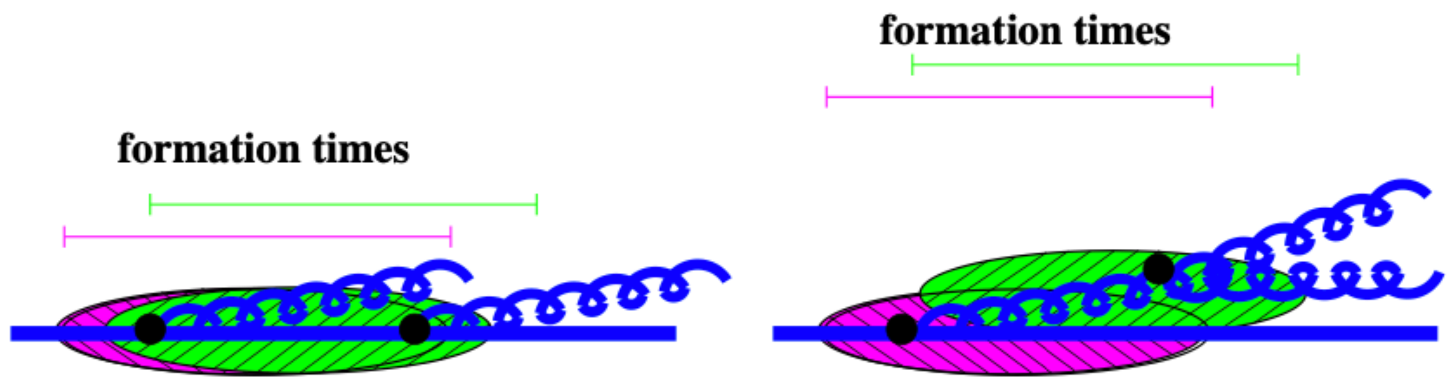
Many of the pheno relevant effects in jet quenching have a quantum origin

e.g.

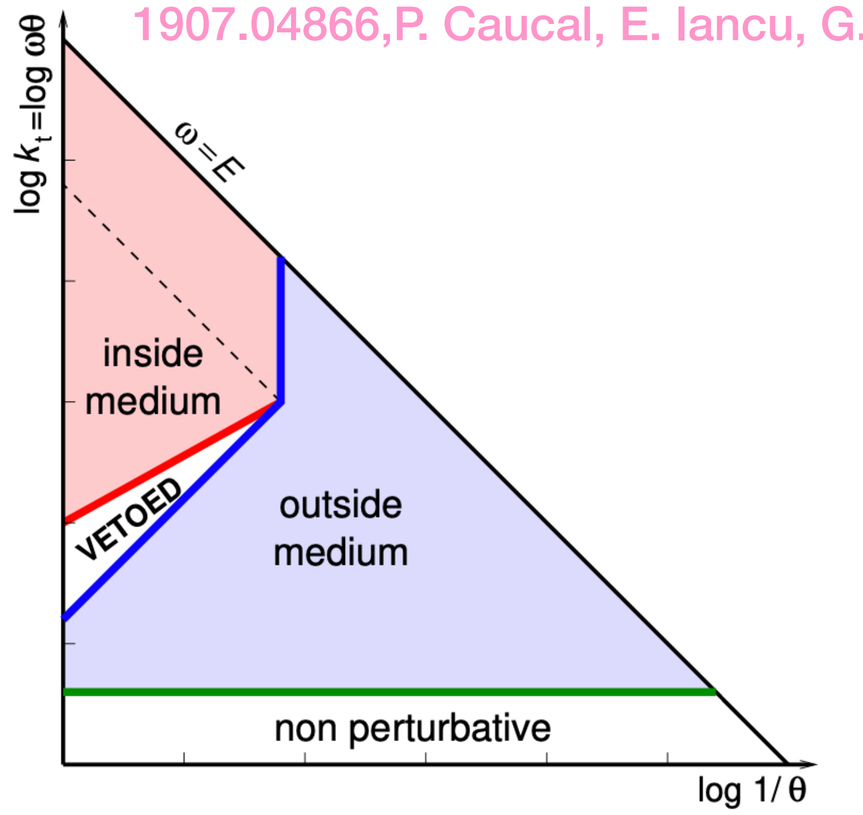


1210.7765, J. Casalderrey-Solana , Y. Mehtar-Tani, C.Salgado, K. Tywoniuk

1501.04964, P. Arnold, S. Iqbal



1907.04866,P. Caucal, E. Iancu, G. Soyez



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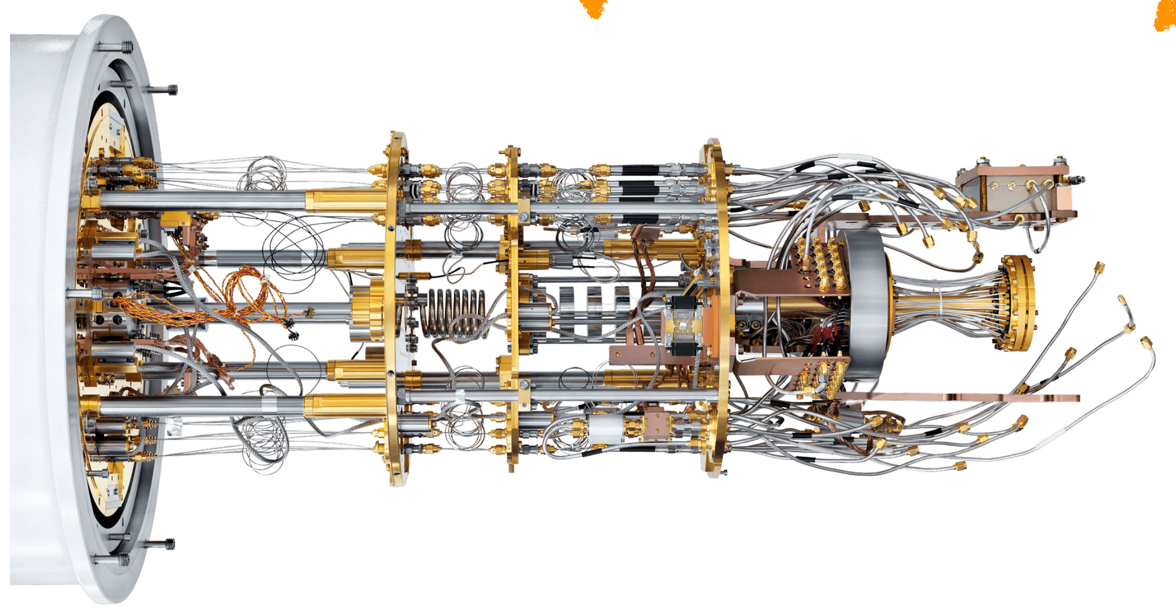

Qcomputers should be able to handle quantum systems naturally!

What can we learn about jets from these machines ?

Jet quenching in a QC: a possible approach

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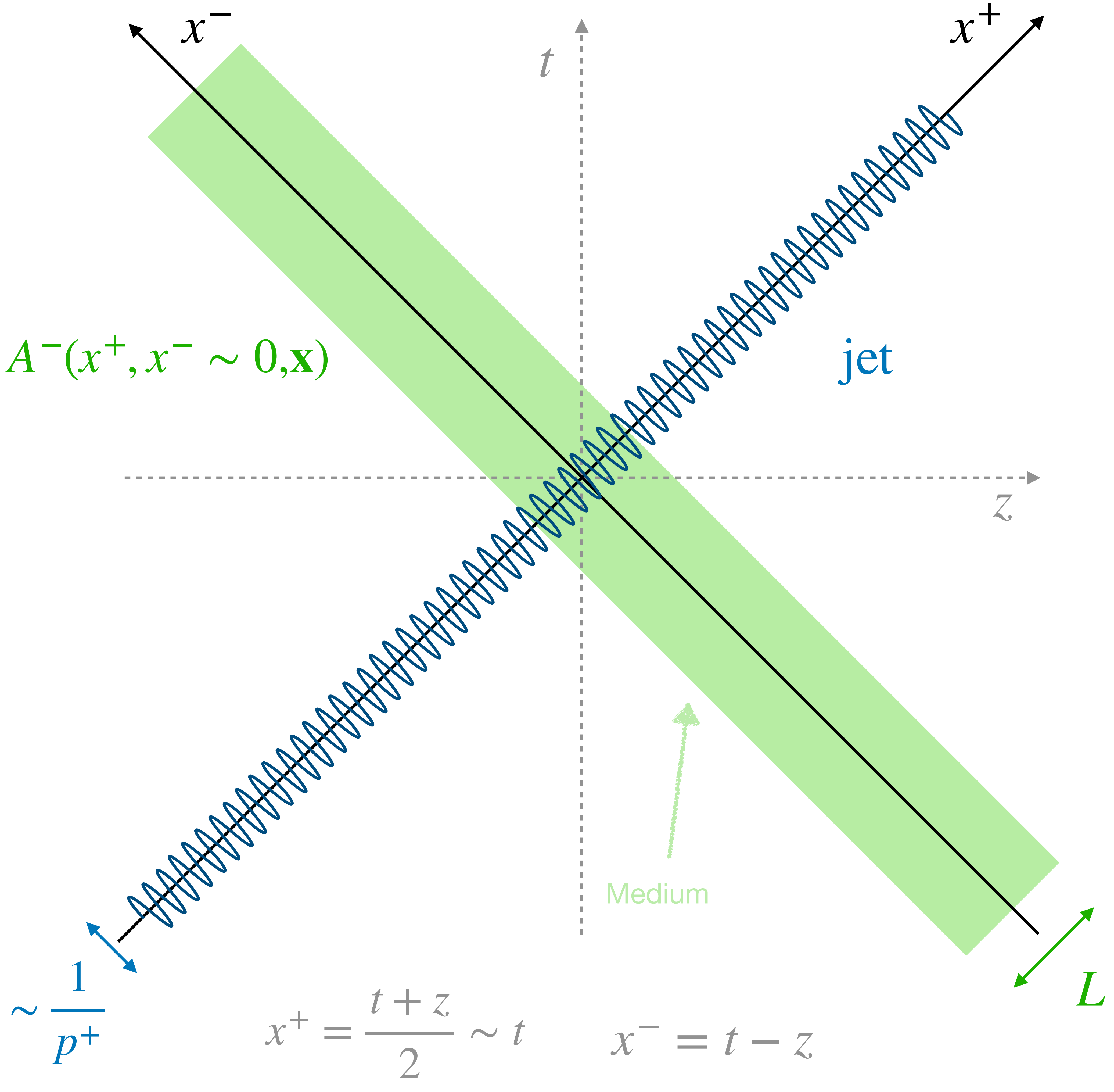
See also talk by D. Muller



Integrating out x^- the **quark propagator** satisfies

$$\left(i\partial_t + \frac{\partial_{\boldsymbol{x}}^2}{2\omega} + g\mathcal{A}^-(t,\boldsymbol{x}) \cdot T\right) G(t,\boldsymbol{x};0,\boldsymbol{y}) = i\delta(t)\delta(\boldsymbol{x}-\boldsymbol{y})$$

Parton evolution is equivalent to **2+1d non-rel. QM**

$$\mathcal{H}(t) = \underbrace{\frac{\boldsymbol{p}^2}{2\omega}}_{\text{p-space}} + \underbrace{g\mathcal{A}^-(t,\boldsymbol{x}) \cdot T}_{\text{x-space}} = \mathcal{H}_K + \mathcal{H}_\mathcal{A}(t)$$

Consider the **simplest** case:

1. $|q\rangle$ Fock space only
2. $T = 1$
3. Stochastic background (hybrid approach)

The quantum simulation algorithm

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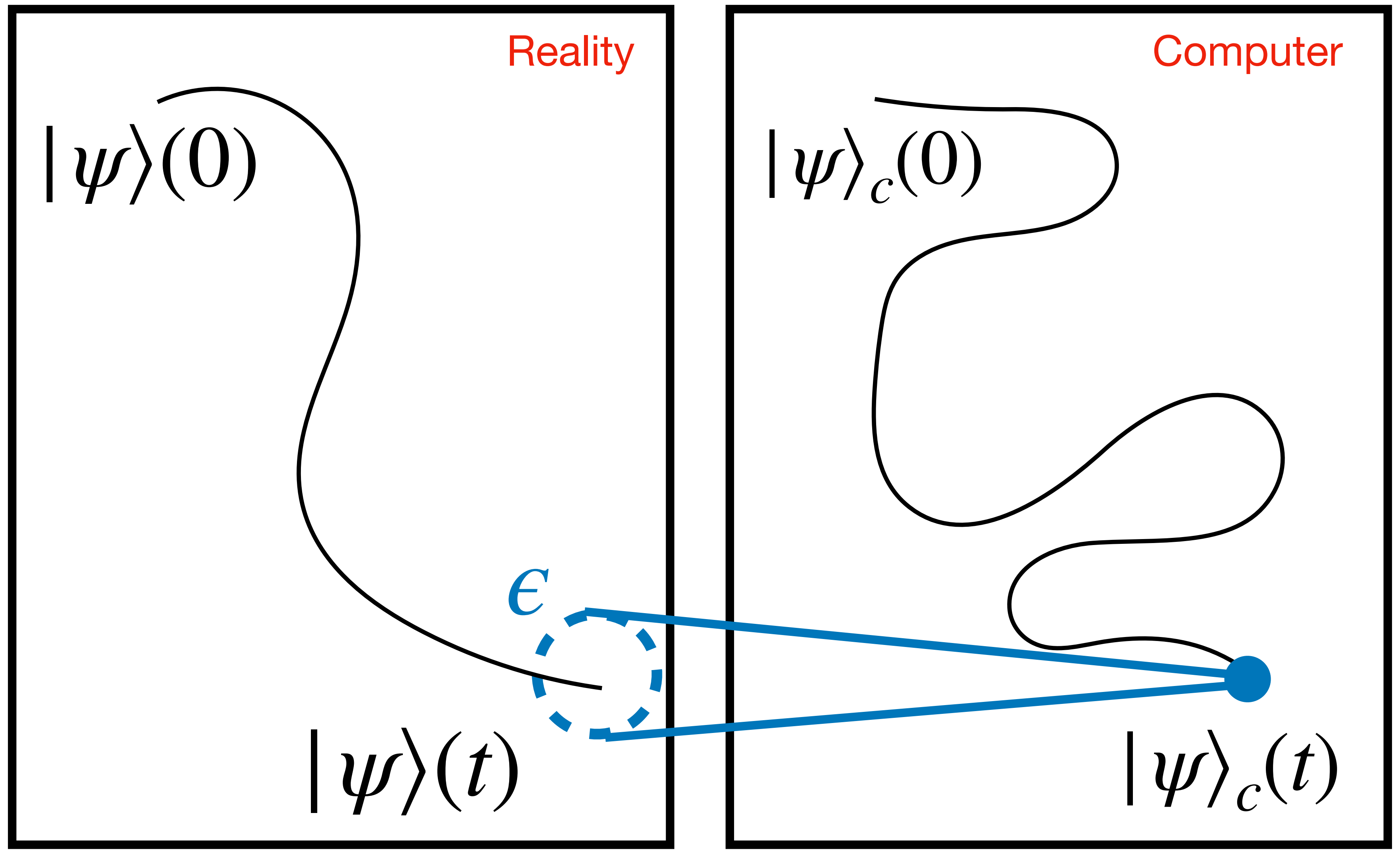
QComputers can **efficiently** simulate real time evolution ruled by:

$$|\psi\rangle(t) = \exp(-iHt)\,|\psi\rangle(0)$$

See also talk by Z. Davoudi



The **5** main steps of the **Quantum Simulation Algorithm**:

1. Provide $H = \sum_k H_k$ and $\psi(0)$
2. Encode the physical d.o.f's in terms of qubits and decompose H_k in terms of gates
3. Prepare the initial wave function from a fiducial state ($|0\rangle^{\otimes n_{\text{qubits}}}$)
4. Time evolve according to $\exp(-iHt)$
5. Implement a measurement protocol

Set up the algorithm

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1. Provide $H = H_K + H_A(t)$ and $\psi(0) = \psi(\mathbf{p} = 0)$ + ensemble of $\{A, p_A\}$

2. Encode the physical d.o.f's in terms of qubits and write H in terms of gates

Introduce 2d spatial lattice with $N_s = 2^{n_Q}$ sites per dimension

$$|\mathbf{x}\rangle = |x_1, x_2\rangle = a_\perp |n_1, n_2\rangle$$

such that

$$H = \frac{\boldsymbol{P}^2}{2E} + gA(t, \boldsymbol{X}) \cdot T = H_K + H_A(t)$$

$a_\perp$
Lattice spacing

$$\hat{P}|p\rangle = p|p\rangle \qquad \hat{X}|x\rangle = x|x\rangle \qquad x, p \in \mathbb{Z}$$

3. Prepare the initial wave function from a fiducial state $(|0\rangle^{\otimes n_{\text{qubits}}})$

Set up the algorithm

4. Time evolve according to U

Assuming that field is static we use

$$U(L_\eta;0) = \prod_{k=1}^{N_t} U(x_k^+;x_{k-1}^+) \longrightarrow U(x_k^+ + \delta x^+;x_k^+) \approx U_K(\delta x^+)\,U_A(\delta x^+,x_k^+)$$

$$\equiv \exp\left\{-i\delta x^+ \frac{\hat{\boldsymbol{p}}^2}{2p^+}\right\} \exp\left\{-ig\delta x^+ \hat{A}_a^-(x_k^+)T^a\right\}$$

Implement operators with a Fourier Transform in between

$$\exp\left\{-i\delta x^+ \frac{\hat{\boldsymbol{p}}^2}{2p^+}\right\} \quad |\psi_{\mathbf{p}}\rangle \qquad |\mathbf{p}\rangle \xrightarrow[\text{qFT}]{} |\mathbf{x}\rangle \qquad \exp\left\{-ig\delta x^+ \hat{A}_a^-(x_k^+)T^a\right\} \quad |\psi_{\mathbf{x}}\rangle$$

Set up the algorithm

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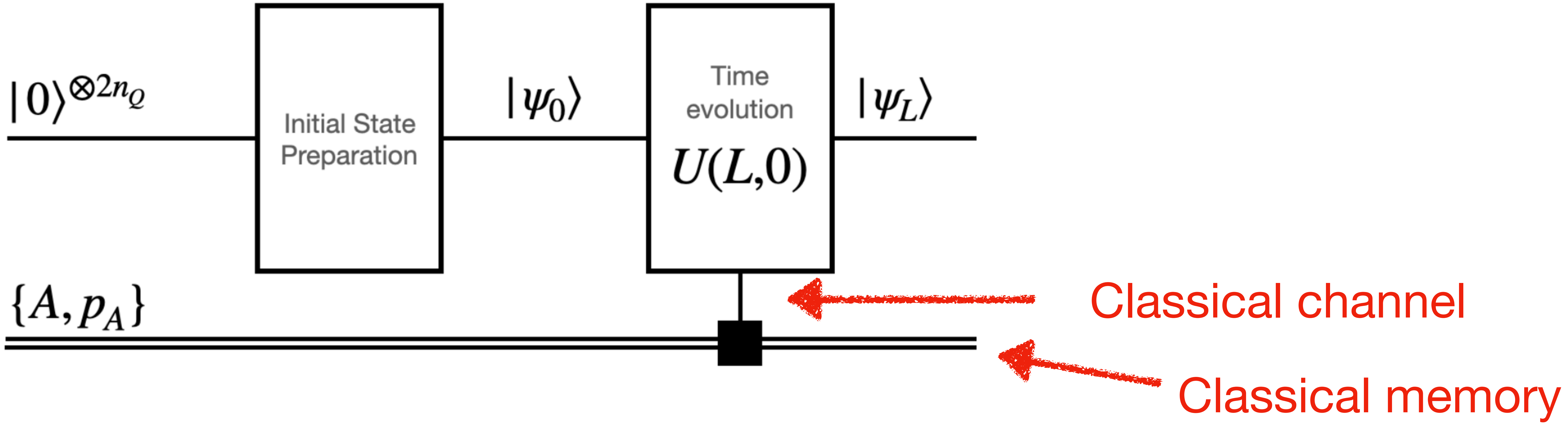
4. Time evolve according to U

Field insertions require probing the field value. This is done classically



Requires $\mathcal{O}(N_{\text{states}})$ field evaluations; **Ok for resolving parton evolution**

Major limitation of the approach due to classical treatment of medium

Can be made more efficient with further discretization of the field values

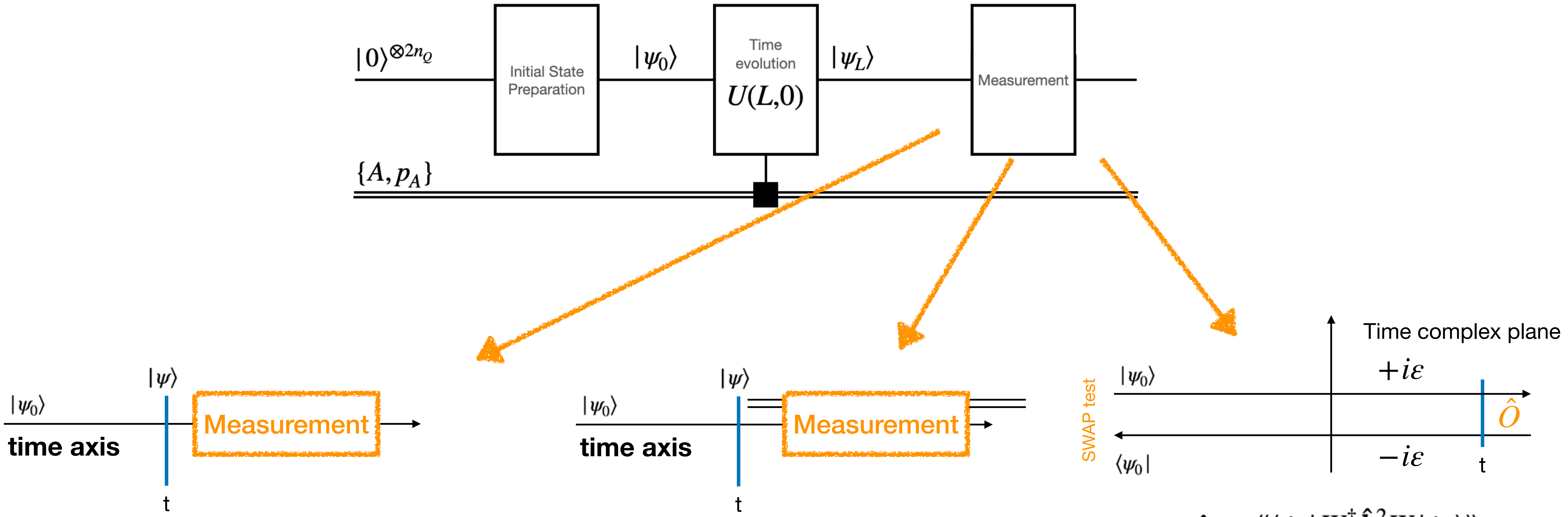
Set up the algorithm

5. Implement a measurement protocol

$$|\psi_L\rangle = \sum_{\boldsymbol{q}} \psi_L^{\boldsymbol{q}} \, |\boldsymbol{q}\rangle$$

Generic final state

Due to quantumness, measurement needs optimization: interference experiment



Brute force: repeat the experiment many times

Hadamard test: interfere real and imaginary part

$$\hat{q}t = \langle\!\langle \langle\psi_0| \, W^\dagger \hat{\boldsymbol{k}}^2 W \, |\psi_0\rangle \rangle\!\rangle$$

SWAP test: find overlap of 2 wave-functions

Rough estimate: for Gaussian errors and an uniform distribution, statistical error of 10% requires $\sim 10^2 \times$ #states shots

Rough estimate: $\sim 10^2 \times \#\mathcal{O}(1)$ shots

Numerical results

ongoing with C. Salgado, M. Li, X. Du, W. Qian

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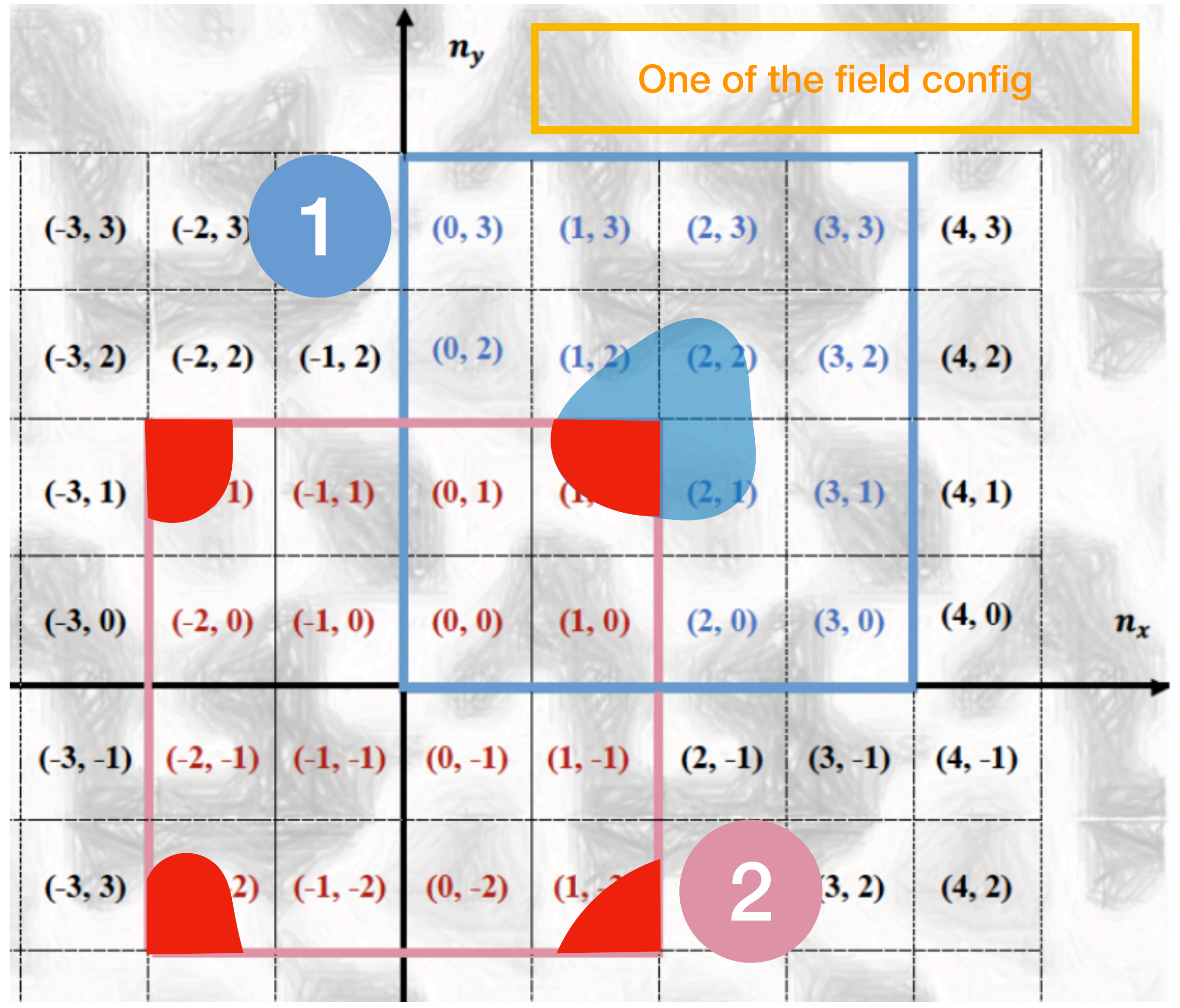
Set up:



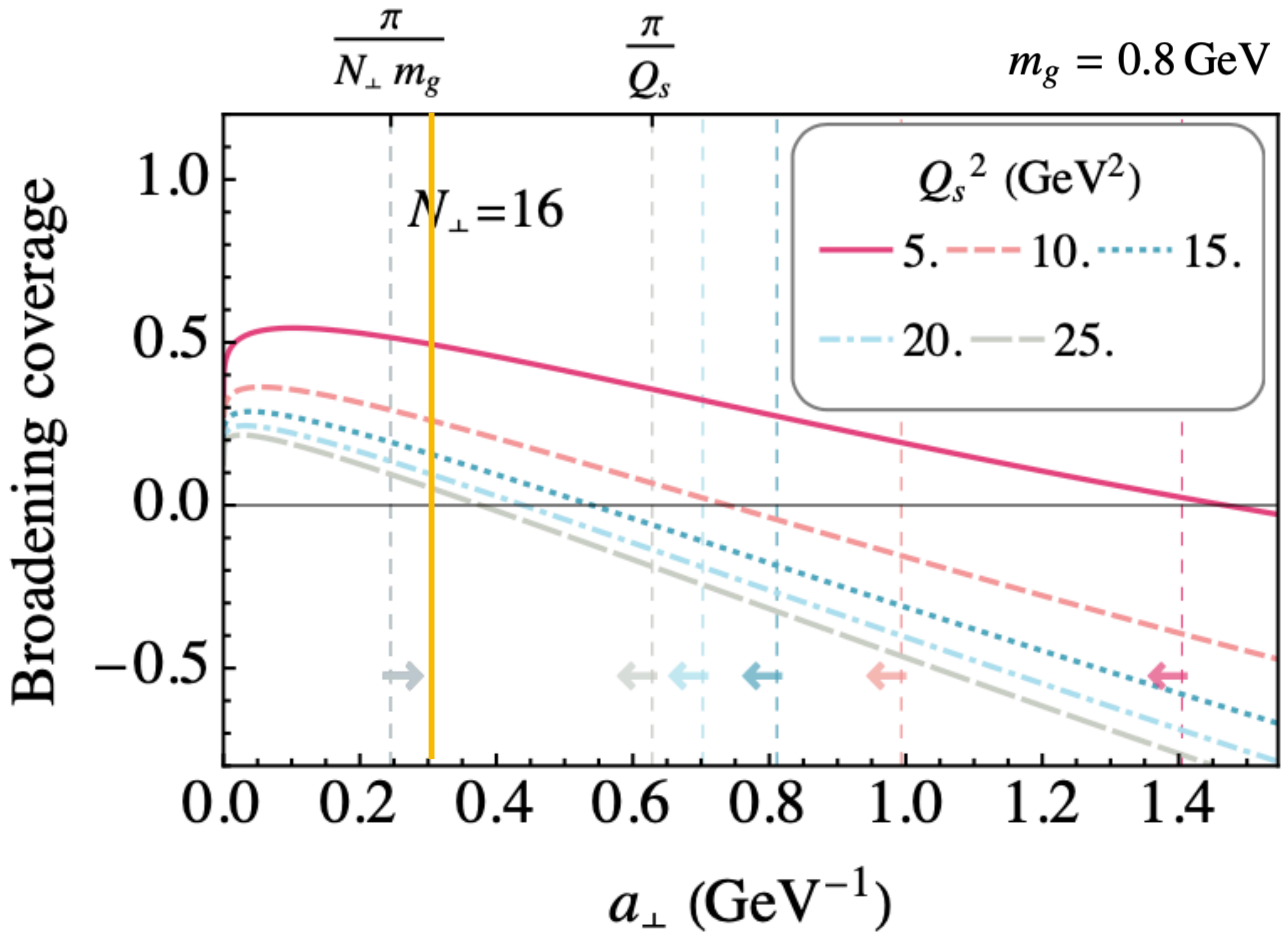
1. $T = 1$ (no colors) mostly
2. Static brick of length 10 fm
3. 5/6 qubits per spatial dimension (1024/4096 states in total).
4. We use 5 field configurations. These are determined by lattice spacing and the field strength

Determined by saturation scale: $$g^2\tilde{\mu} = \sqrt{\frac{2\pi Q_s^2}{C_F L_\eta}}$$

Determined by lattice saturation conditions:

$$\frac{\pi}{N_\perp m_g} \ll a_\perp \ll \frac{\pi}{Q_s}$$ (relevant physical region is covered)

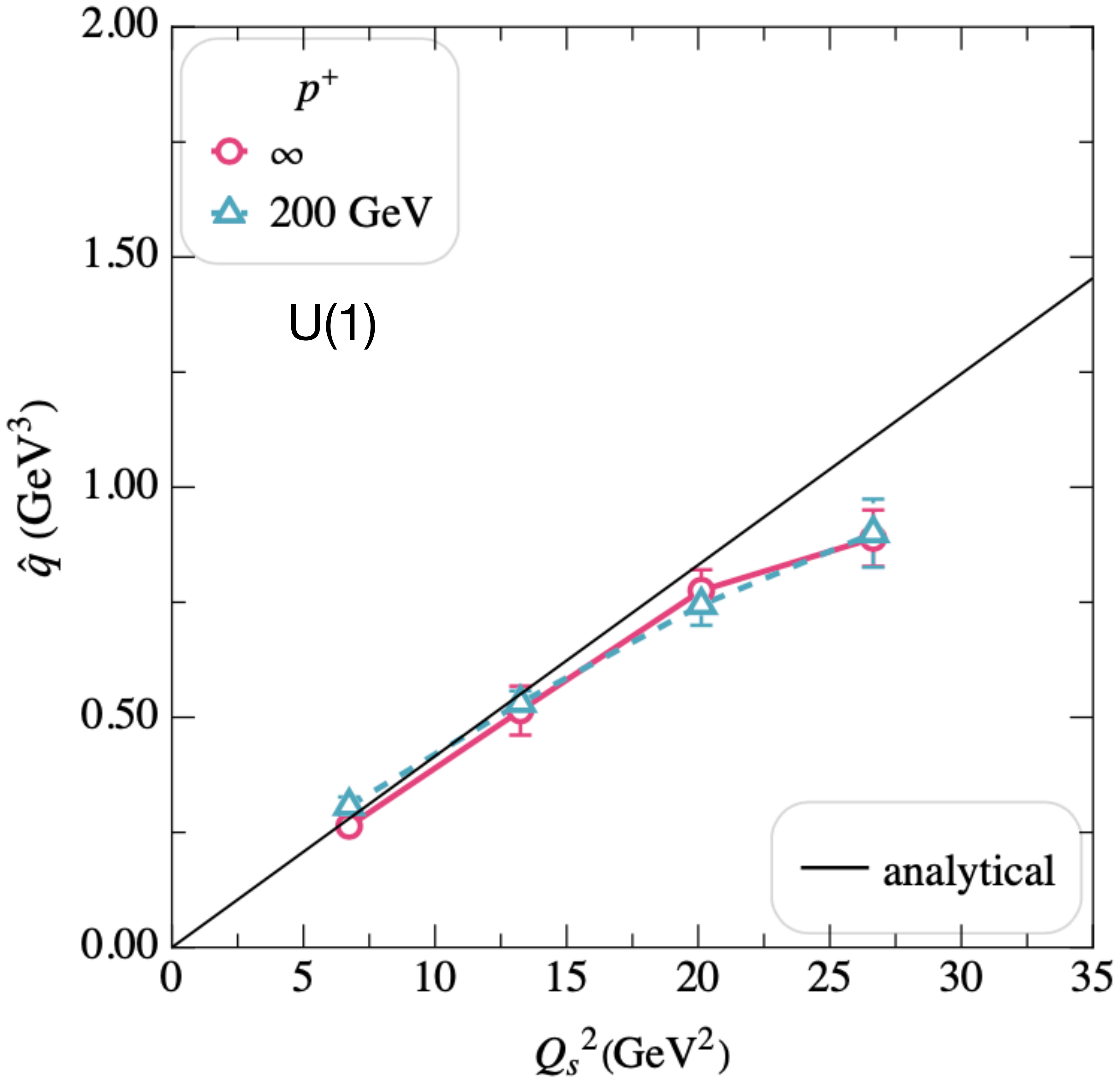
$$a_\perp^2 Q_s^2 < \frac{4\pi^2}{3}\left[\log\left(\frac{1}{a_\perp^2 m_g^2/\pi^2}+1\right) - \frac{1}{1+a_\perp^2 m_g^2/\pi^2}\right]^{-1}$$ (edge effects are absent)



Numerical results

The jet quenching parameter on the lattice is easily obtained analytically:

$$\hat{q} = \frac{1}{t}\int_{p,x,y} p^2 e^{-ip\cdot(y-x)} \langle\langle \mathcal{W}^\dagger(y)\mathcal{W}(x)\rangle\rangle = g^2 \langle\langle \nabla_x \mathcal{A}(0)\cdot\nabla_x\mathcal{A}(0)\rangle\rangle = \frac{g^4}{4\pi} C_F \tilde{\mu}^2 \left\{ \log\left(1+\frac{\frac{\pi^2}{a_\perp^2}}{m_g^2}\right) - \frac{1}{1+\frac{a_\perp^2 m_g^2}{\pi^2}} \right\}$$



In accordance with expected result w/wo kinetic terms

Deviation at large saturation values due to lattice

Numerical results

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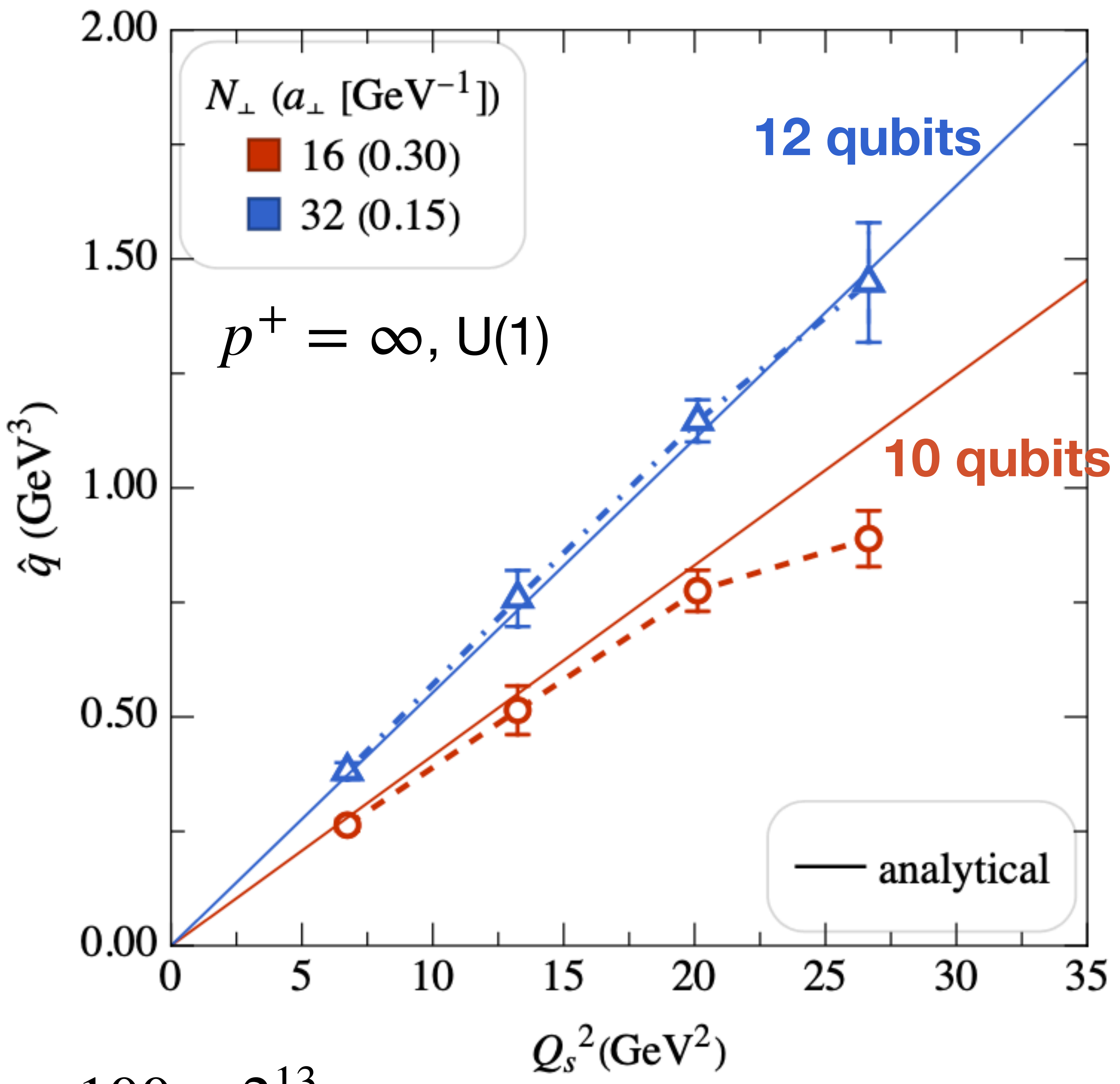
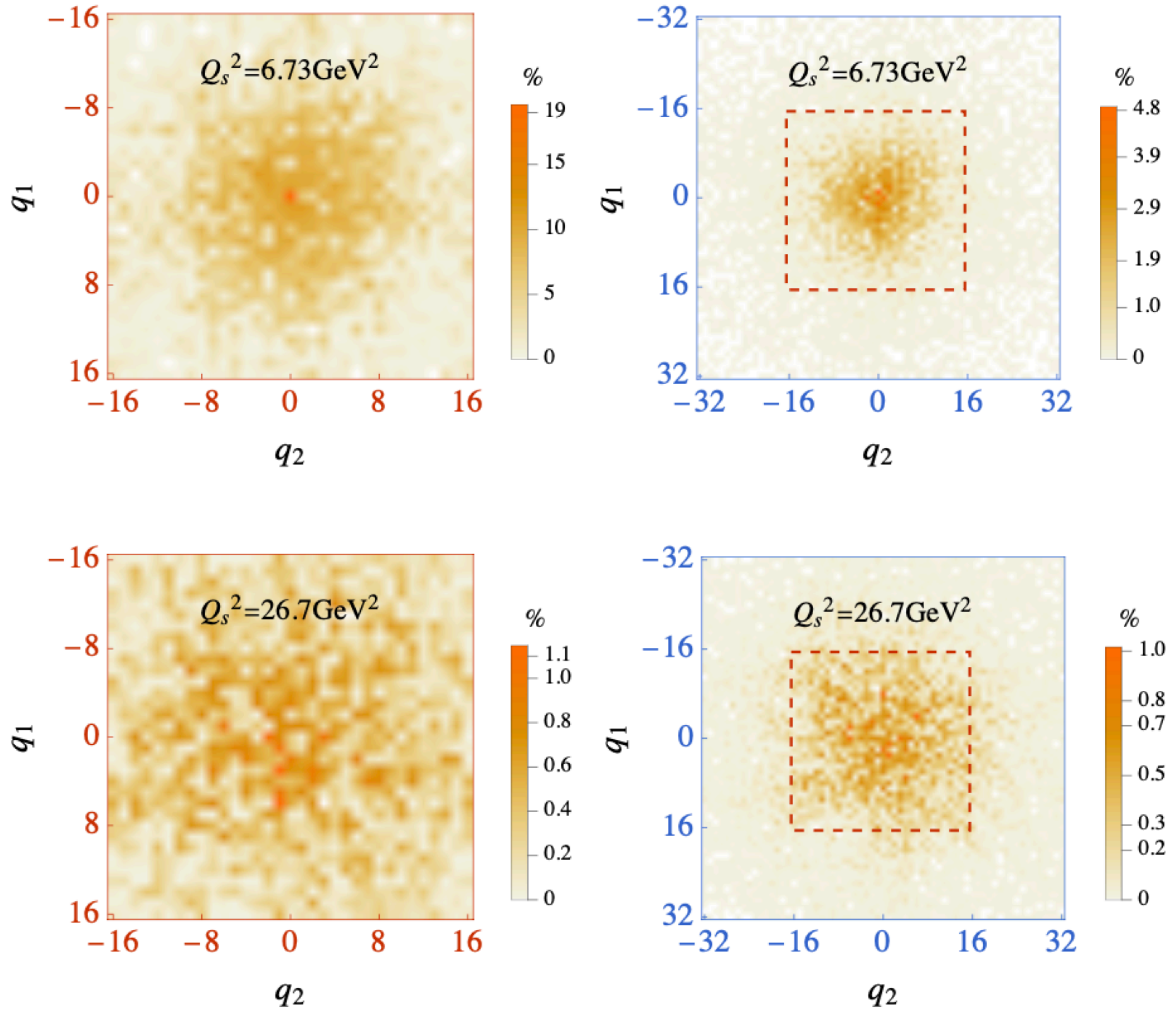
Same result but for two different lattices at infinite jet energy



shots=100×2^{13}



Numerical results

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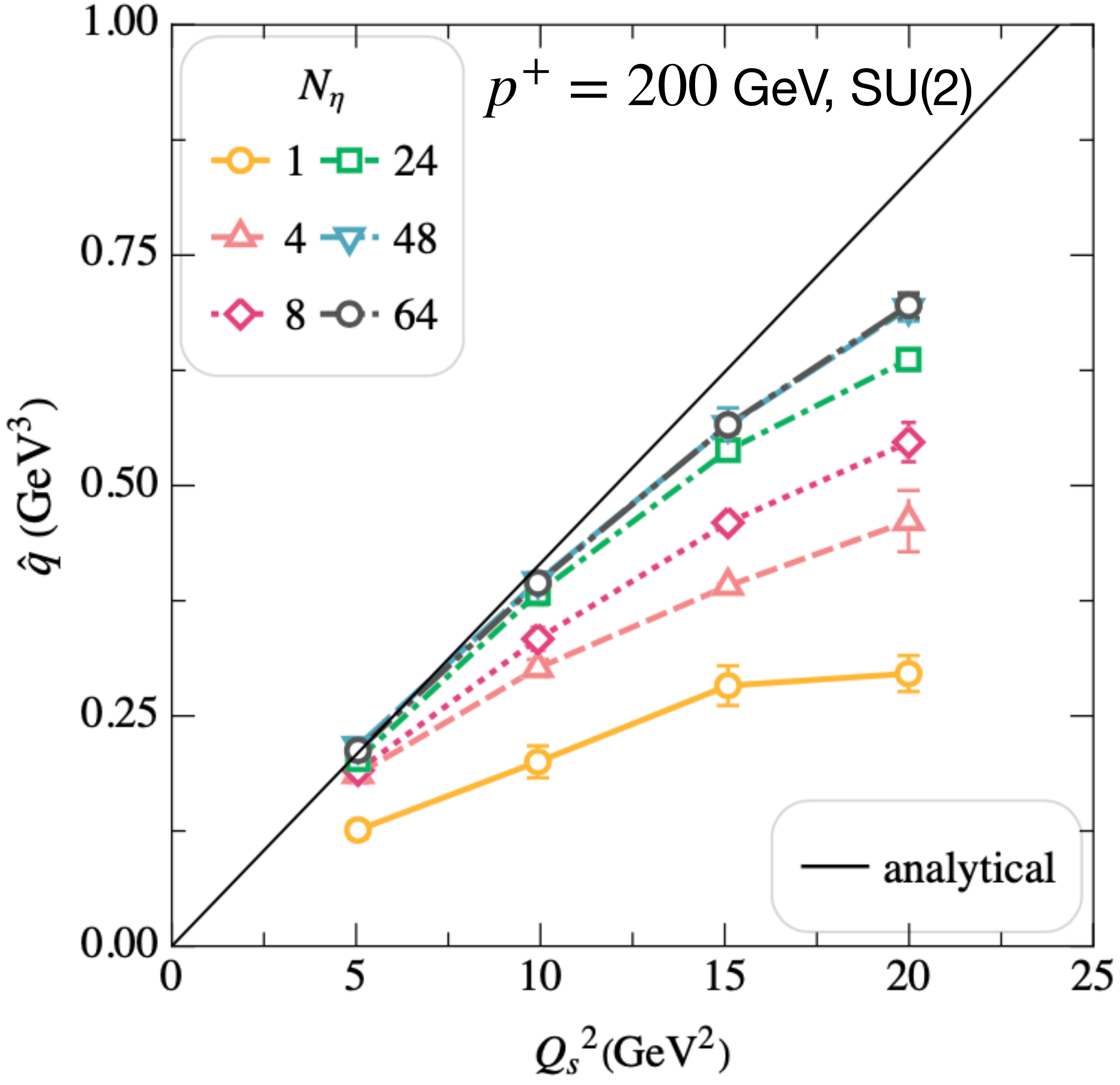
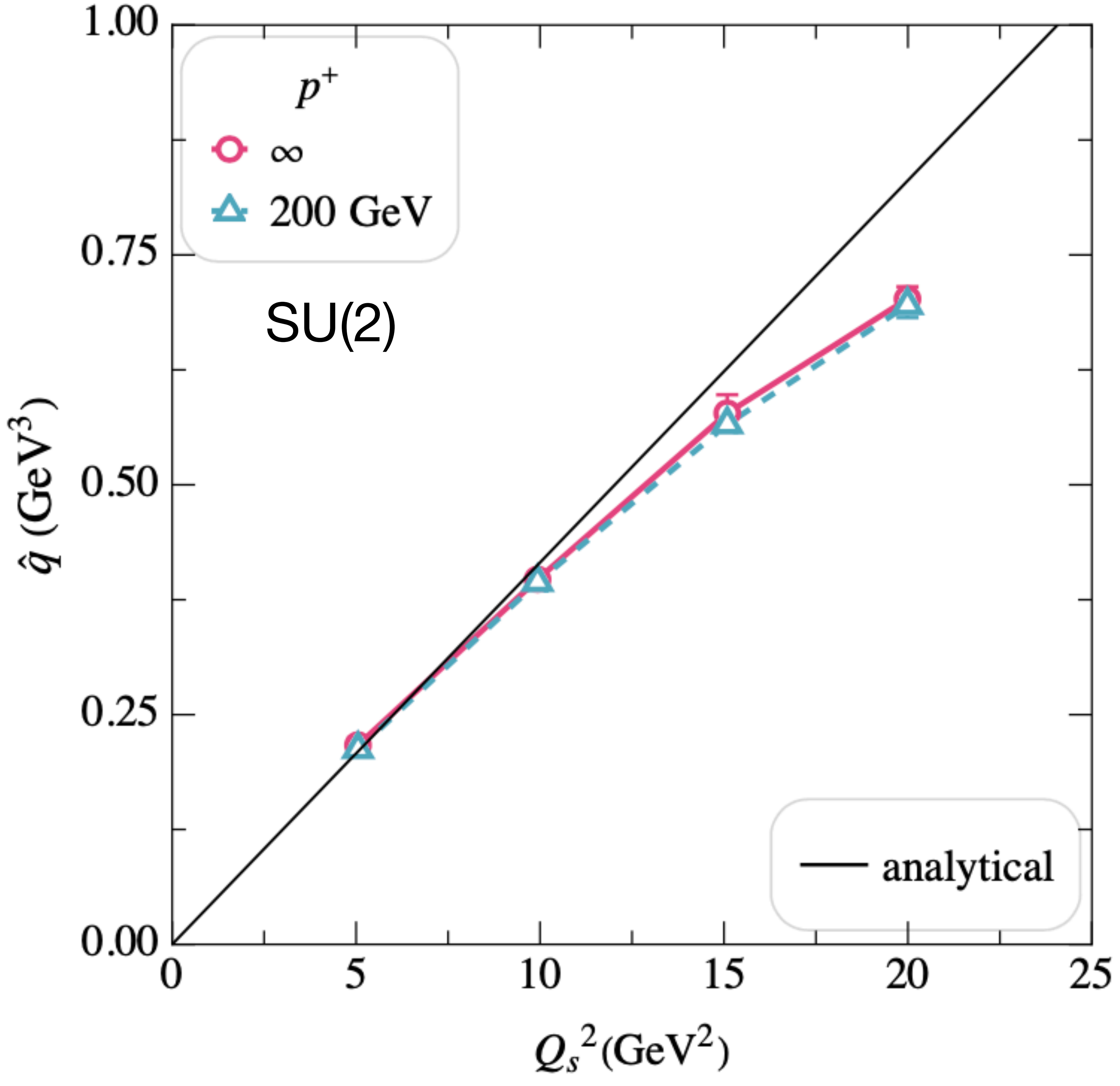
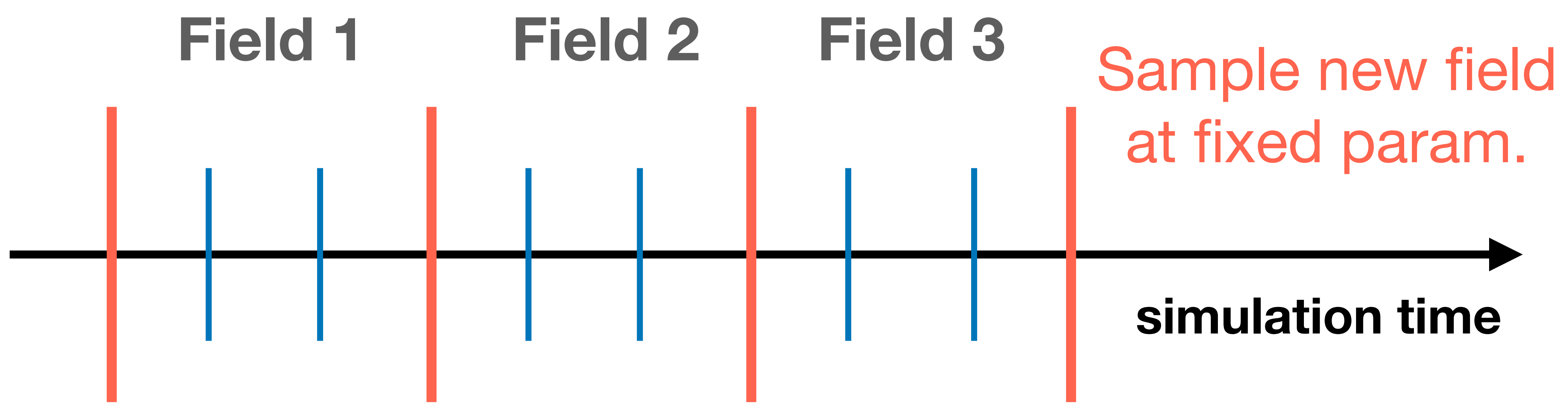
Same results but for a SU(2) background





Random medium leads to emergent time dependence

$$\langle\langle \rho_a(x^+, \boldsymbol{x}) \rho_b(y^+, \boldsymbol{y}) \rangle\rangle = g^2 \mu^2(\boldsymbol{x}) \delta_{ab} \delta^2(\boldsymbol{x} - \boldsymbol{y}) \underline{\delta(x^+ - y^+)}$$



Product formula decomposition

Numerical results

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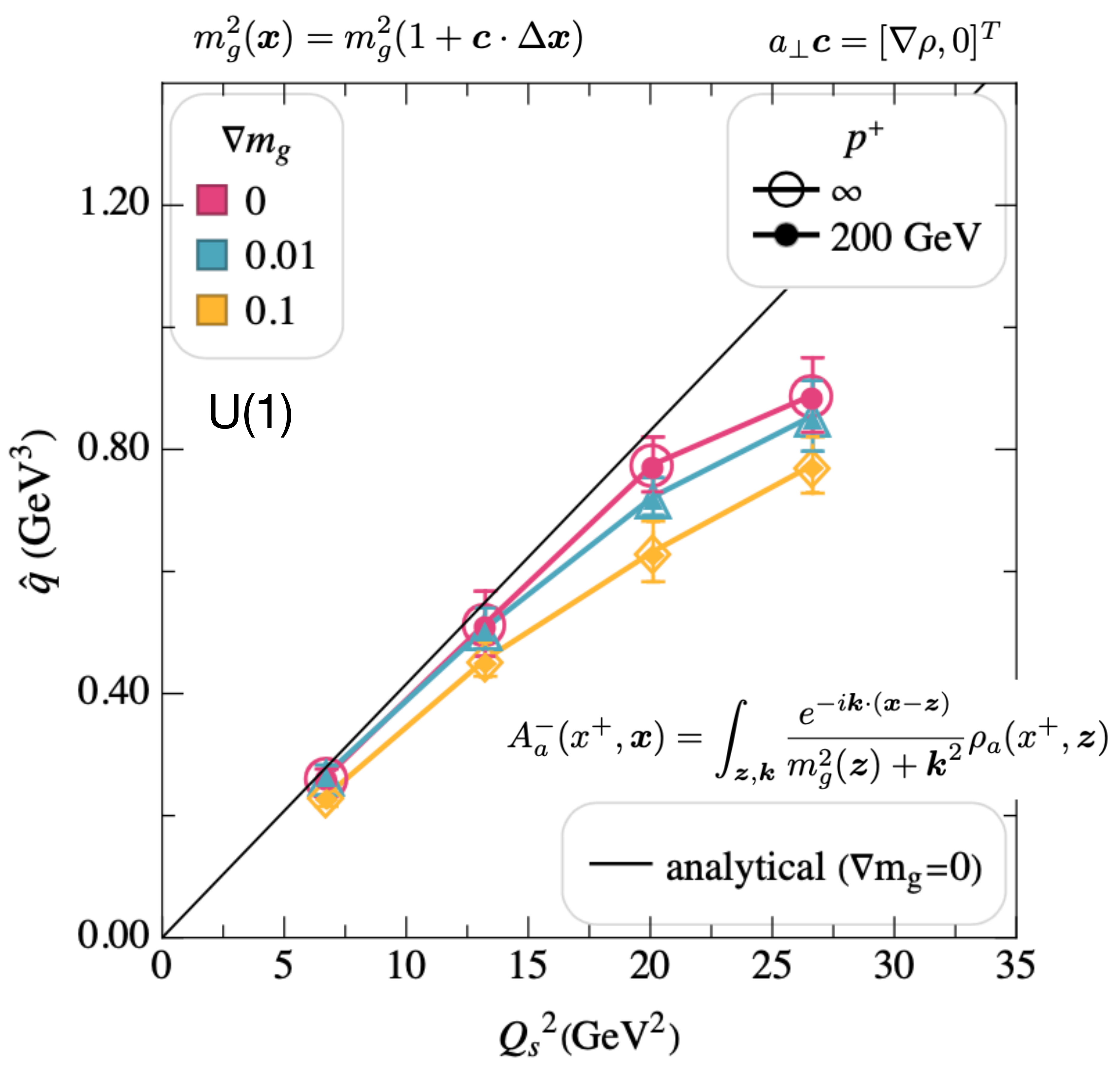
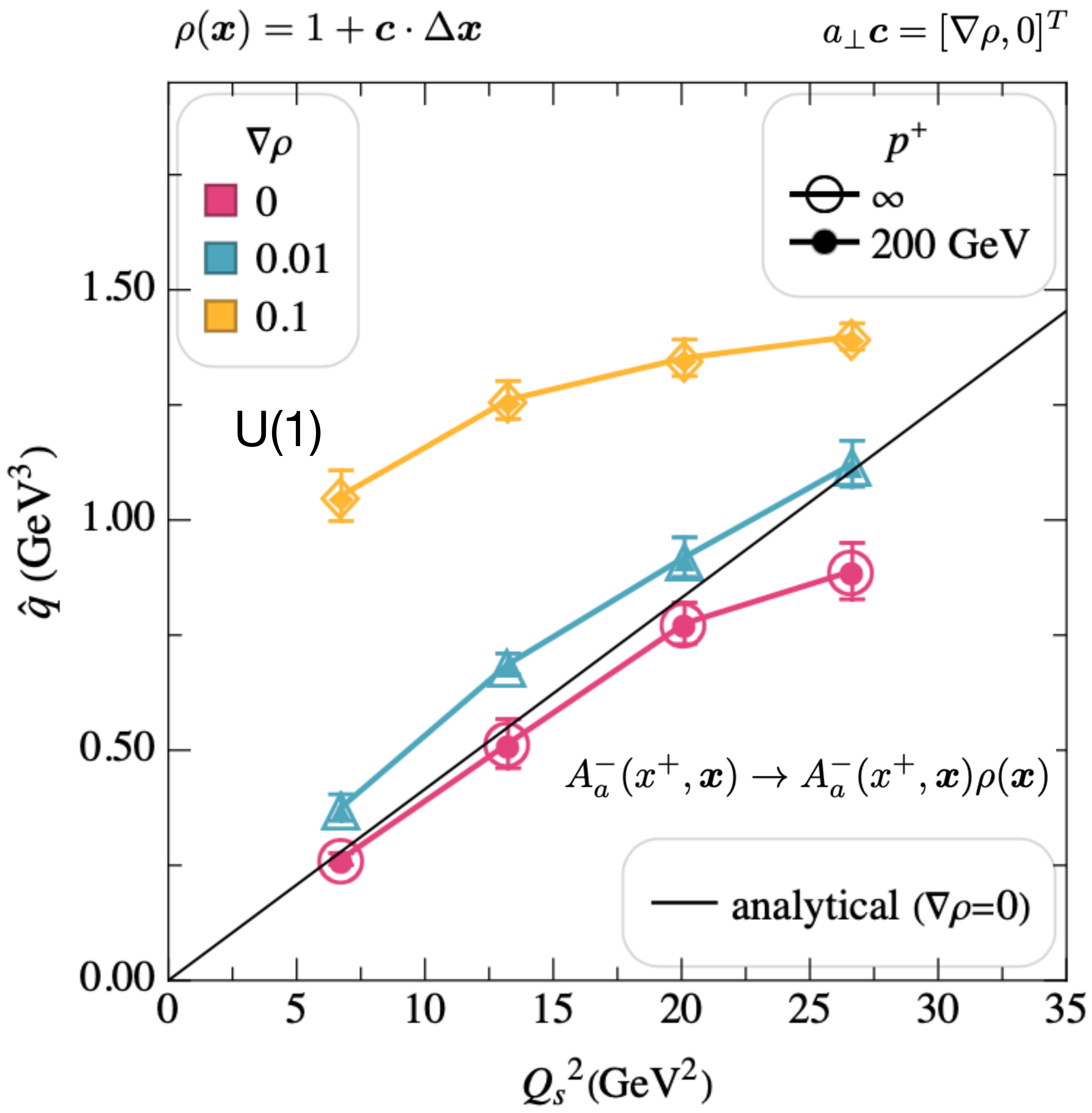
also works by JB, Sadofyev, Salgado; Fu, Casalderrey-Solana, Wang; Sadofyev, Sievert, Vitev; Sadofyev, Dominguez, Andrés; …

See also talk by A. Sadofyev





Energy independent terms might give sizable contribution !

Numerical results

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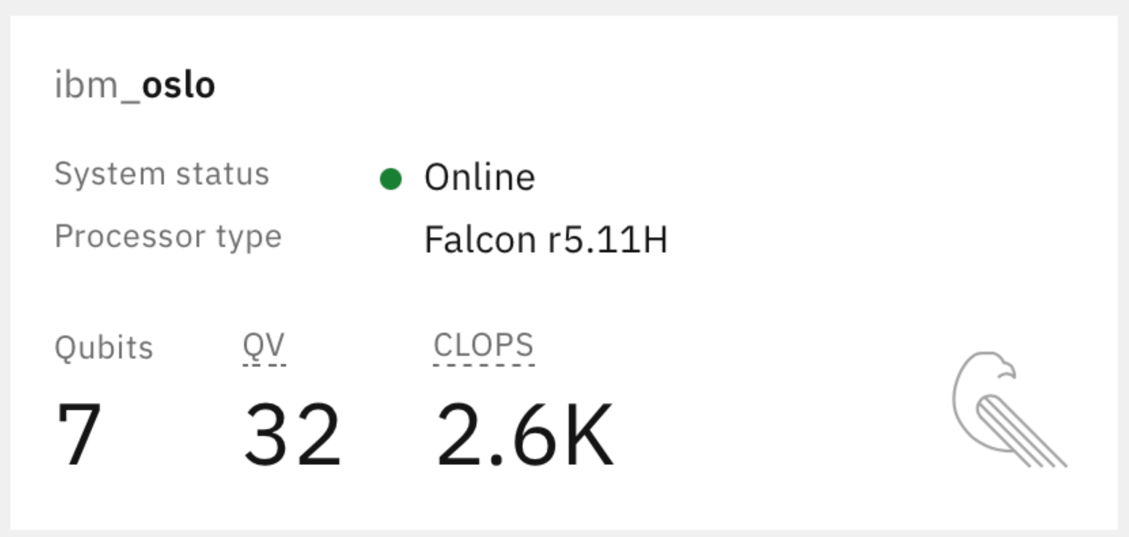

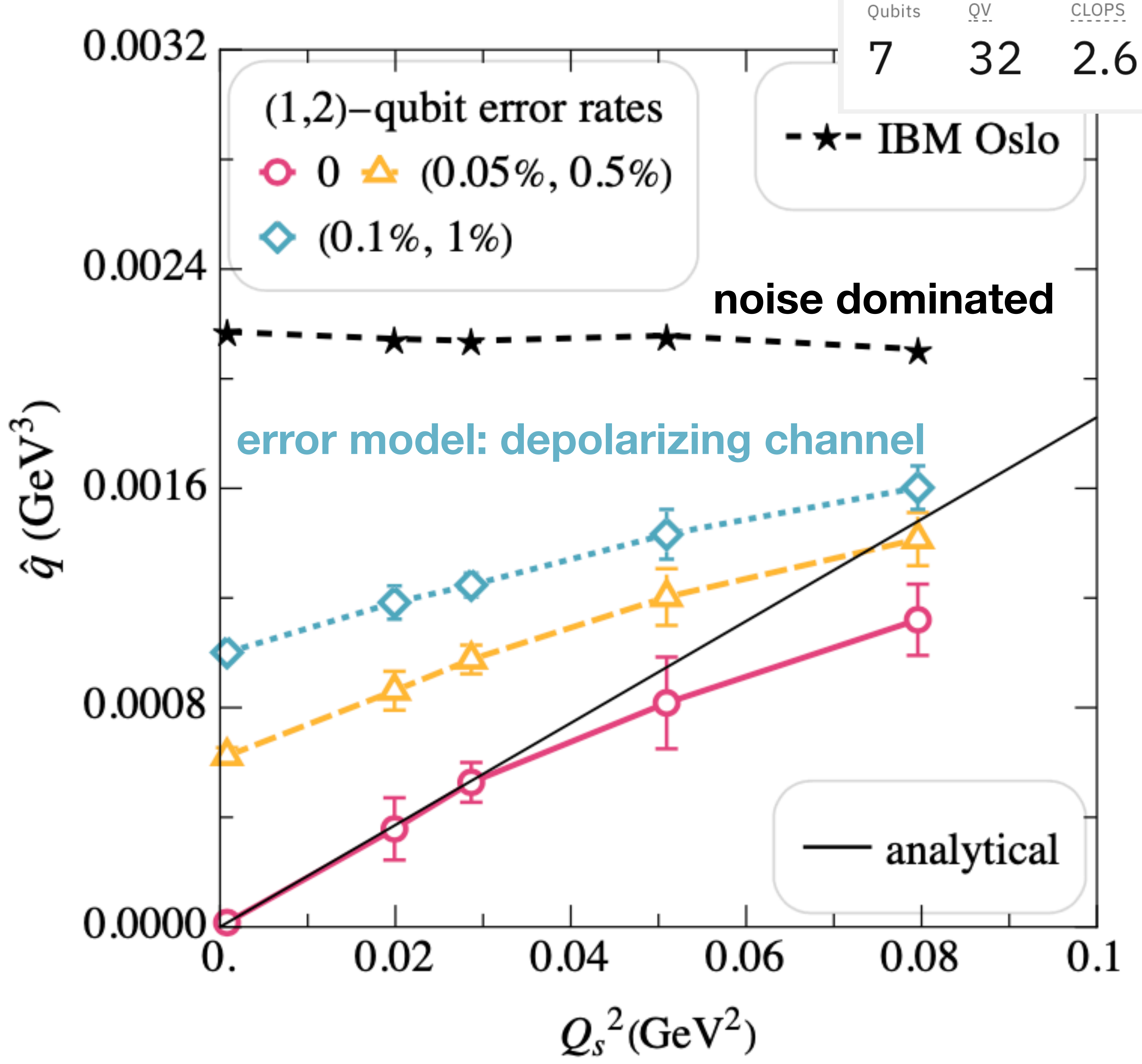

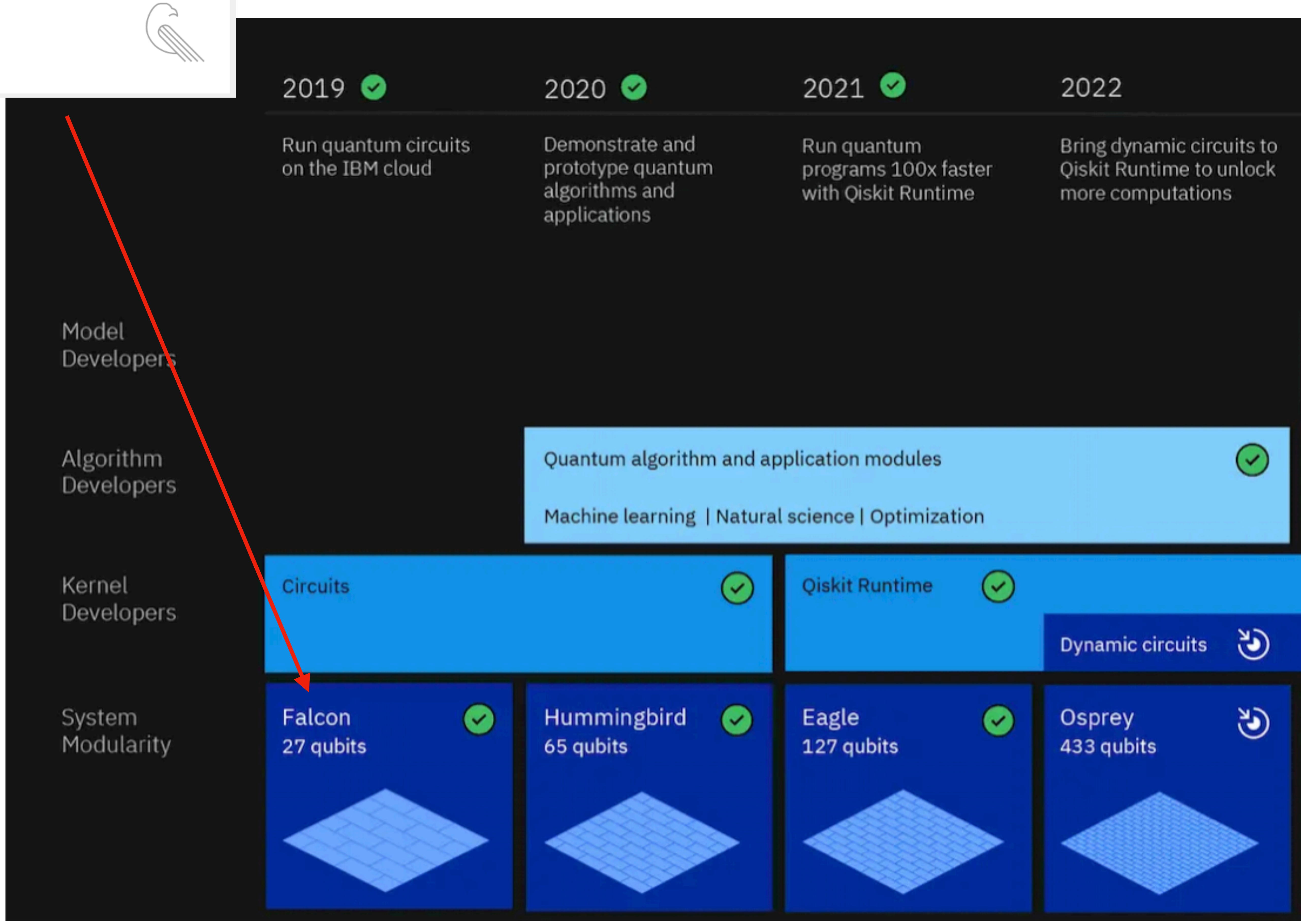

Available but not free

Prototype

Conclusion and Outlook

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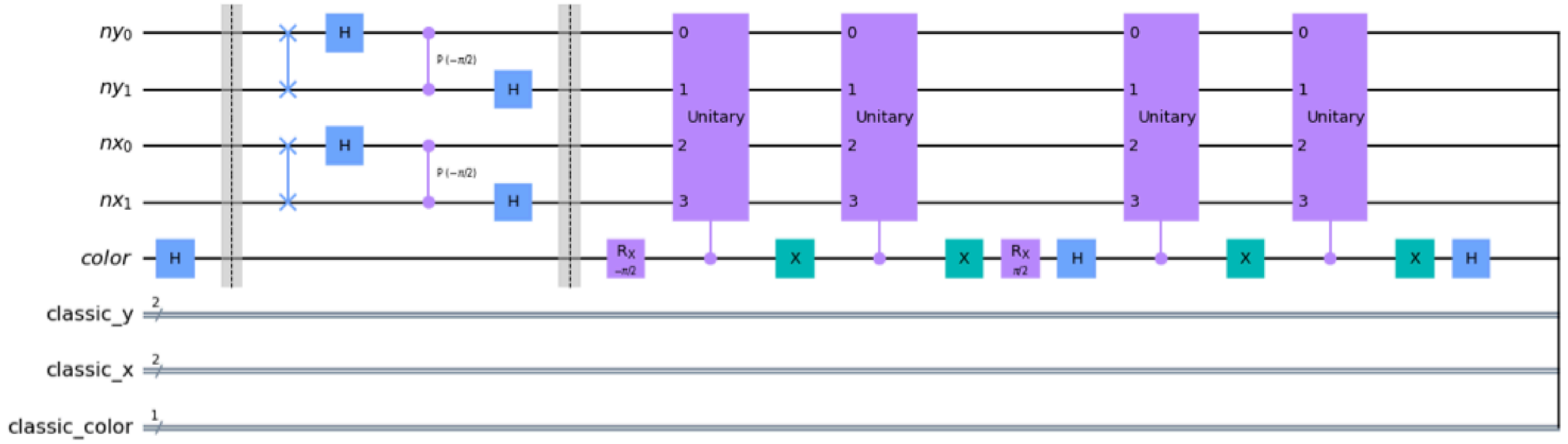
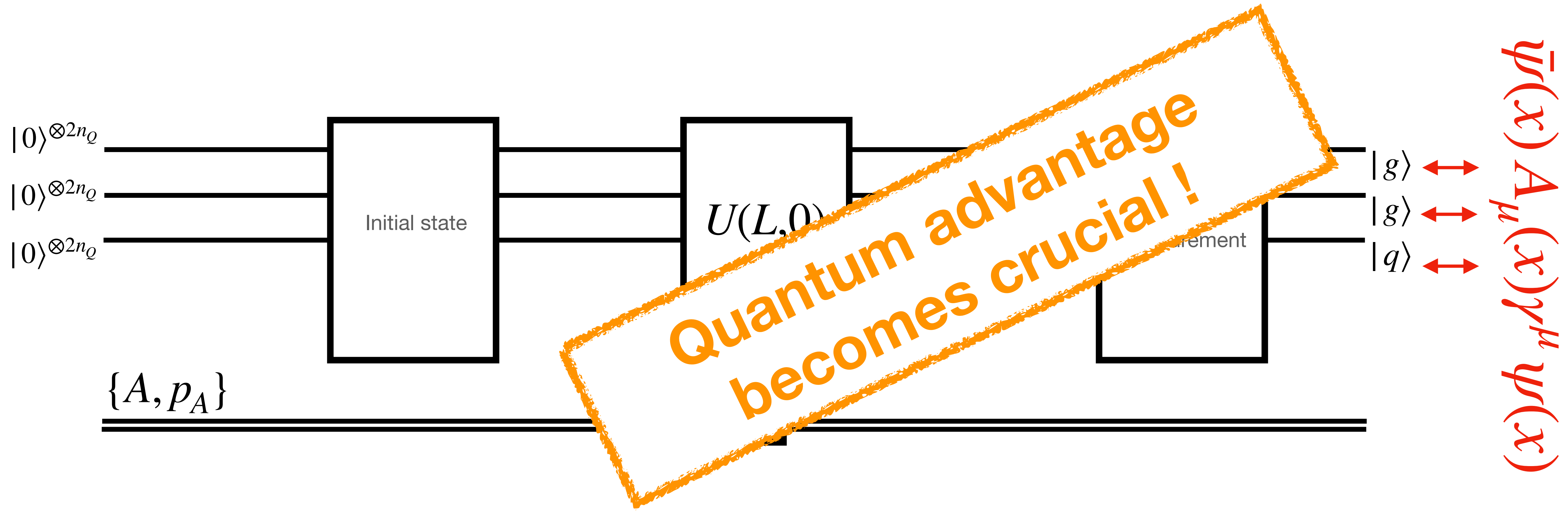
- We proposed and implemented a circuit to benchmark jet broadening in a Qcomputer
- Numerics agree with analytics + give new insight to more realistic medium model



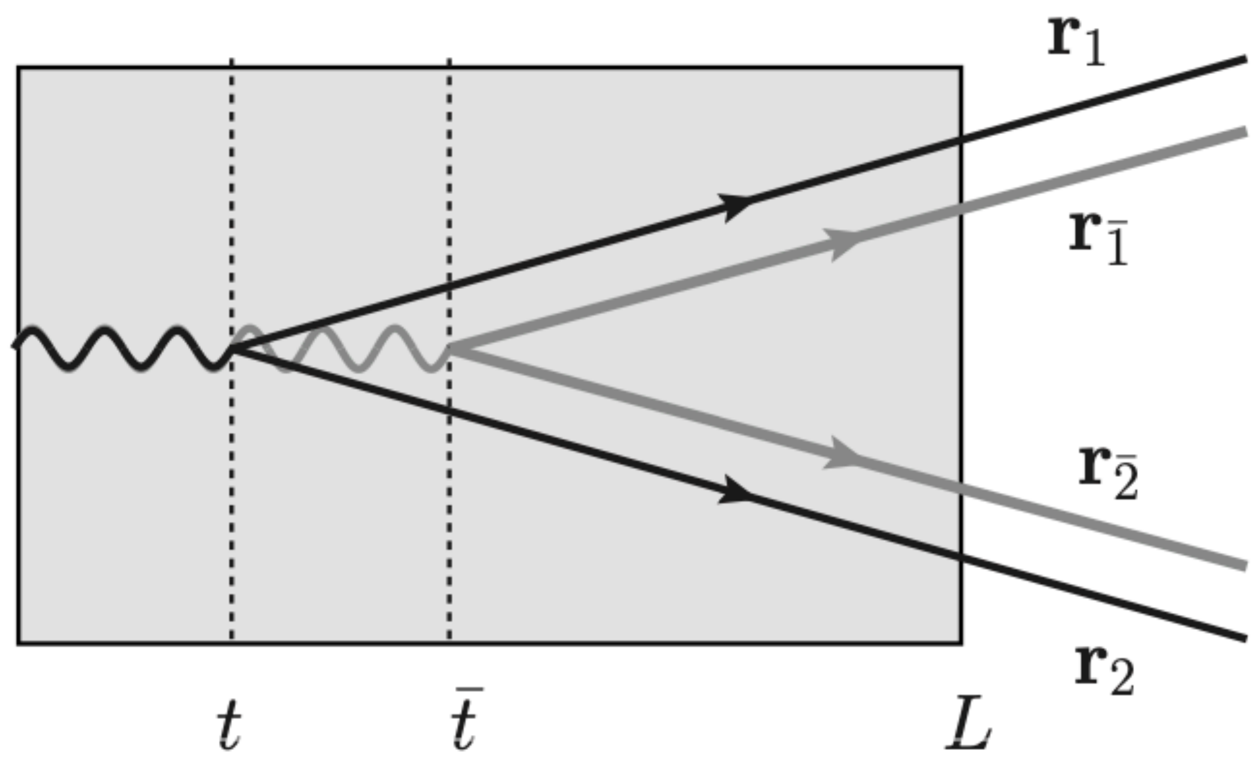
Implementation for $SU(2)$

- Future plans:

$$|\psi\rangle = c_1|q\rangle + c_2|qg\rangle + c_3|qgg\rangle + \cdots$$



Multiple particle quantum effects already accessible



See e.g. : 1907.03653 F. Dominguez, J. G. Milhano, C. Salgado, K. Tywoniuk, V. Vila