Short-distance constraints on the hadronic light-by-light

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Vetenskapsrådet

The anomalous magnetic moment of the muon

- Anomalous magnetic moment: sensitive to "light" new physics

$$a_\mu = \frac{(g-2)_\mu}{2} \sim \frac{m_\mu^2}{M_{NP}^2}$$

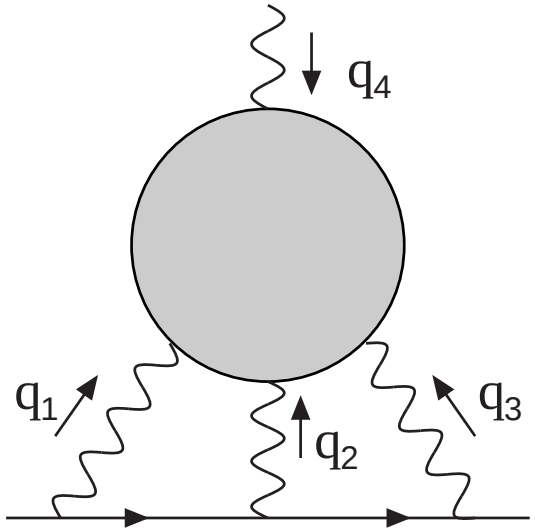
- Standard Model (SM) [White Paper, 2020] vs. Experimental value Brookhaven/Fermilab

$$a_\mu^{\mathrm{SM}} = 116\,591\,810(43) \times 10^{-11}\,,$$
$$a_\mu^{\mathrm{exp}} = 116\,592\,061(41) \times 10^{-11}\,,$$
$$\Delta a_\mu = 251(59) \times 10^{-11}$$

- 4.2σ deviation between experiment and theory [White Paper 2020]

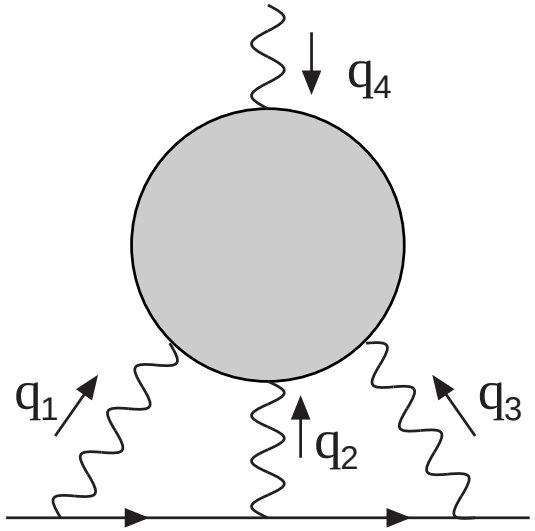
- SM uncertainty: Hadronic corrections (lattice and dispersion theory)
  - Hadronic vacuum polarisation: Tension [White Paper 2020; BMW, 2021; Mainz, 2022]
  - Hadronic light-by-light (HLbL): $\leftarrow$ reduce uncertainty further

The HLbL



- Can be calculated on the lattice or from a dispersive approach
- Problematic since several momentum scales involved in the diagram
- $q_4 \to 0$ (static limit) and $q_{1,2,3}$ integrated over
- Integral has different regions ($Q_i^2 = -q_i^2$):
  - $Q_i^2 \gg \Lambda_{\mathrm{QCD}}^2$ all large: short-distance region
  - $Q_i^2 \sim Q_j^2 \gg Q_k^2, \Lambda_{\mathrm{QCD}}^2$: Melnikov-Vainshtein limit
  - $Q_i^2 \ll \Lambda_{\mathrm{QCD}}^2$: low-energy limit

The HLbL



- The dispersive approach requires experimental input, and one has to insert sums over different states [Colangelo, Hoferichter et al.]
- Not possible to include everything → Models
- Short-distance constraints (SDCs) on the HLbL amplitude useful to constrain models and control errors
- $Q_i^2 \gg \Lambda_{\mathrm{QCD}}^2$ all large: short-distance region 1 soft photon limit
- $Q_i^2 \sim Q_j^2 \gg Q_k^2, \Lambda_{\mathrm{QCD}}^2$: Melnikov-Vainshtein 2 soft photons limit

Our work

- Motivation:
  - Wanted to make a systematic OPE to derive SDCs
  - Can reduce systematic uncertainties in the HLbL
  - One soft photon: Always *assumed* that pQCD quark loop is first term in an OPE and sufficiently good
    - → Which OPE is it?
    - → What about the higher order terms in the OPE?
    - → Paper I: [Bijnens, NHT, Rodríguez–Sánchez 2019]
      Paper II: [Bijnens, NHT, Laub, Rodríguez–Sánchez 2020]
      Paper III: [Bijnens, NHT, Laub, Rodríguez–Sánchez 2021]
  - Two soft photons: Leading-order OPE known [Melnikov, Vainshtein 2004]
    - → What about higher-order corrections?
    - → In preparation: [Bijnens, NHT, Rodríguez–Sánchez]

Some generalities

$$\Pi^{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3) = -\,i\int\frac{d^4q_4}{(2\pi)^4}\left(\prod_{i=1}^{4}\int d^4x_i\, e^{-iq_ix_i}\right)$$
$$\times\,\langle 0|\,T\Big\{J^{\mu_1}(x_1)J^{\mu_2}(x_2)J^{\mu_3}(x_3)J^{\mu_4}(x_4)\Big\}|0\rangle$$
$$= q_{4\,\nu_4}\frac{\partial\,\Pi^{\mu_1\mu_2\mu_3\nu_4}}{\partial q_4^{\mu_4}}$$

- For $(g-2)_\mu$ we want to obtain 19 scalar functions $\hat{\Pi}_i$

$$\lim_{q_4\to 0}\frac{\partial\Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4\nu_4}} = \lim_{q_4\to 0}\sum_{i=1}^{54}\frac{\partial\,\hat{T}_i^{\,\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4\nu_4}}\,\hat{\Pi}_i \quad \leftarrow \text{Projection}$$

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1\int_0^\infty dQ_2\int_{-1}^{1}d\tau\,\sqrt{1-\tau^2}\,Q_1^3Q_2^3\sum_{i=1}^{12}f_i\left(\{\hat{\Pi}_j\}\right)$$

- We can get a_μ^{HLbL} in the SD regime by putting restrictions in the integration consistent with $\hat{\Pi}_i$ from an OPE

An OPE for one soft photon

- HLbL tensor is a 4-point function: $\Pi^{\mu_1\mu_2\mu_3\mu_4}(q_1, q_2, q_3)$
- Problem because of soft photon for $(g-2)_\mu$: Do an OPE in an external EM field:

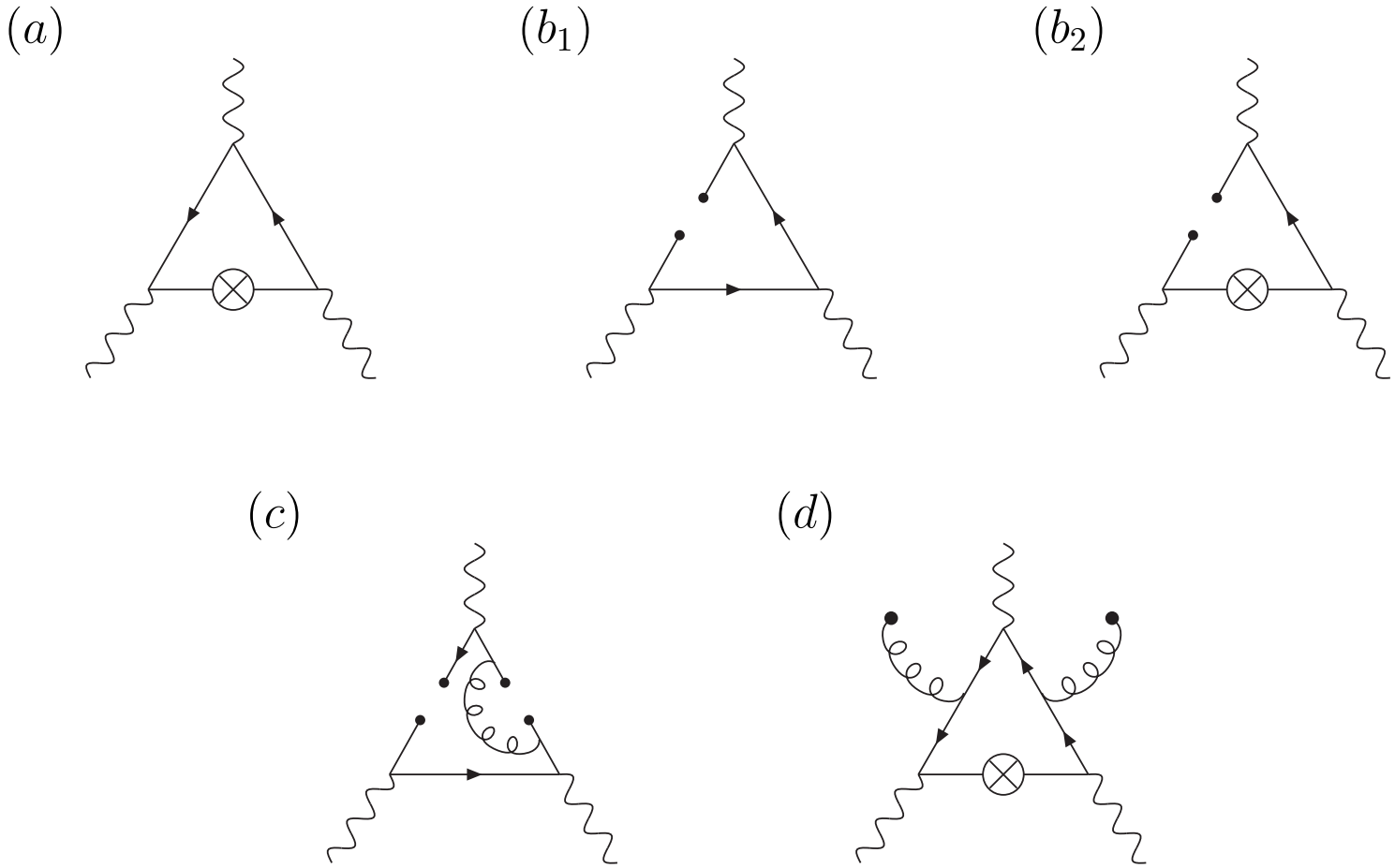
$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = -\frac{1}{e}\int \frac{d^4 q_3}{(2\pi)^4}\left(\prod_{i=1}^{3}\int d^4 x_i\, e^{-iq_i x_i}\right) \times \langle 0| T\left(J^{\mu_1}(x_1) J^{\mu_2}(x_2) J^{\mu_3}(x_3)\right)|\gamma(q_4)\rangle$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = i \lim_{q_4\to 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_4^{\nu_4}} \times \langle 0| e_q F_{\nu_4\mu_4}|\gamma(q_4)\rangle\,.$$

- We can thus obtain a_μ^{HLbL} from $\Pi^{\mu_1\mu_2\mu_3}$
- Such an OPE introduced for baryon magnetic moment sum rules [Balitsky,Yung, 1983], [Ioffe,Smilga, 1984], and later for the EW $g-2$ contributions as well [Czarnecki,Marciano,Vainshtein, 2003]
- Different from a usual vacuum OPE [Shifman, Vainshtein, Zakharov, 1979]

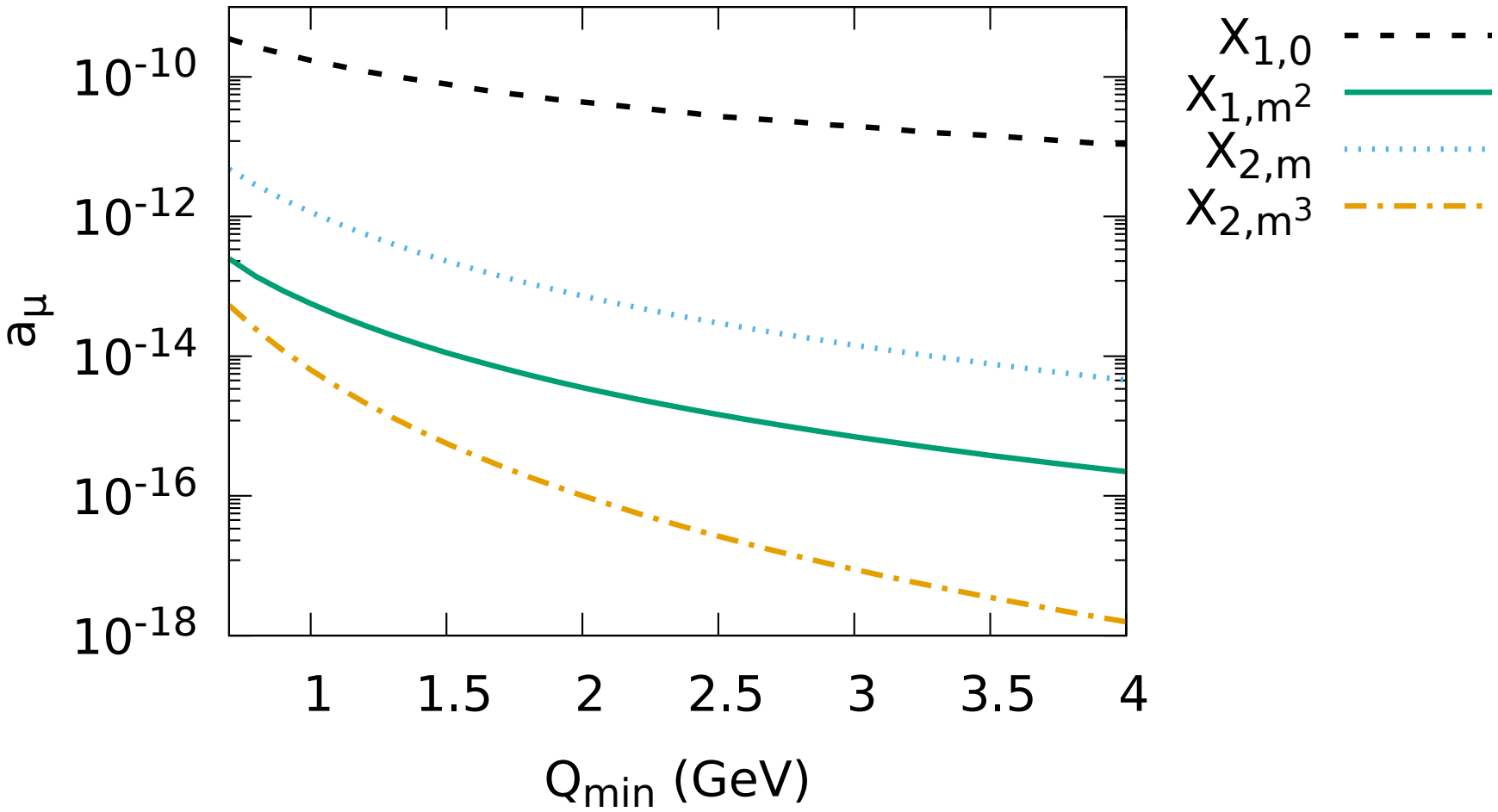
Terms of the OPE up to NNLO

- Papers I and II: Perturbative: (a) quark loop $1/Q^2$
- Non-perturbative: $1/Q^4$, $1/Q^6$:
  (b$_1$) $\langle \bar{q}\sigma_{\mu\nu}q\rangle$, (b$_2$) $\langle \bar{q}q\rangle$, (c) $\langle \bar{q}\,\Gamma_1 q\,\bar{q}\,\Gamma_2 q\rangle$, (d) $\langle \alpha_s GG\rangle$



Numerical results of papers I and II

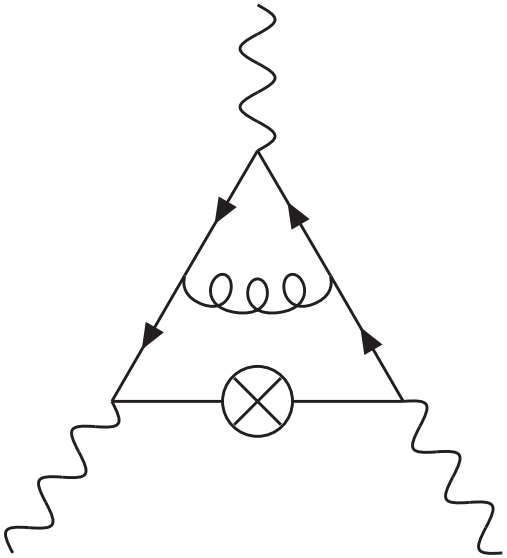
- Numerically studied these as well, for $Q_i^2 > Q_{\min}^2$

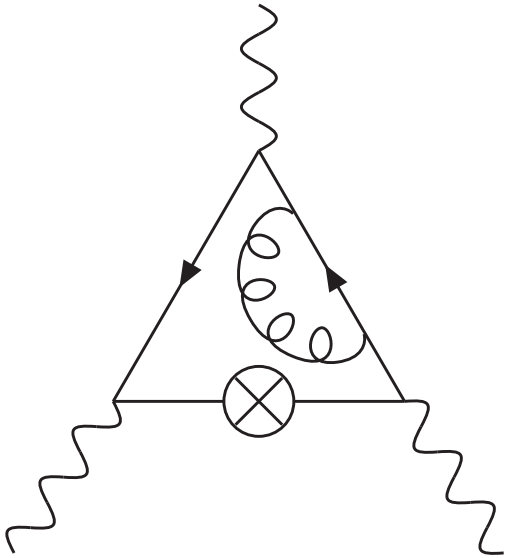


- Non-perturbative condensates suppressed by two orders of magnitude in general

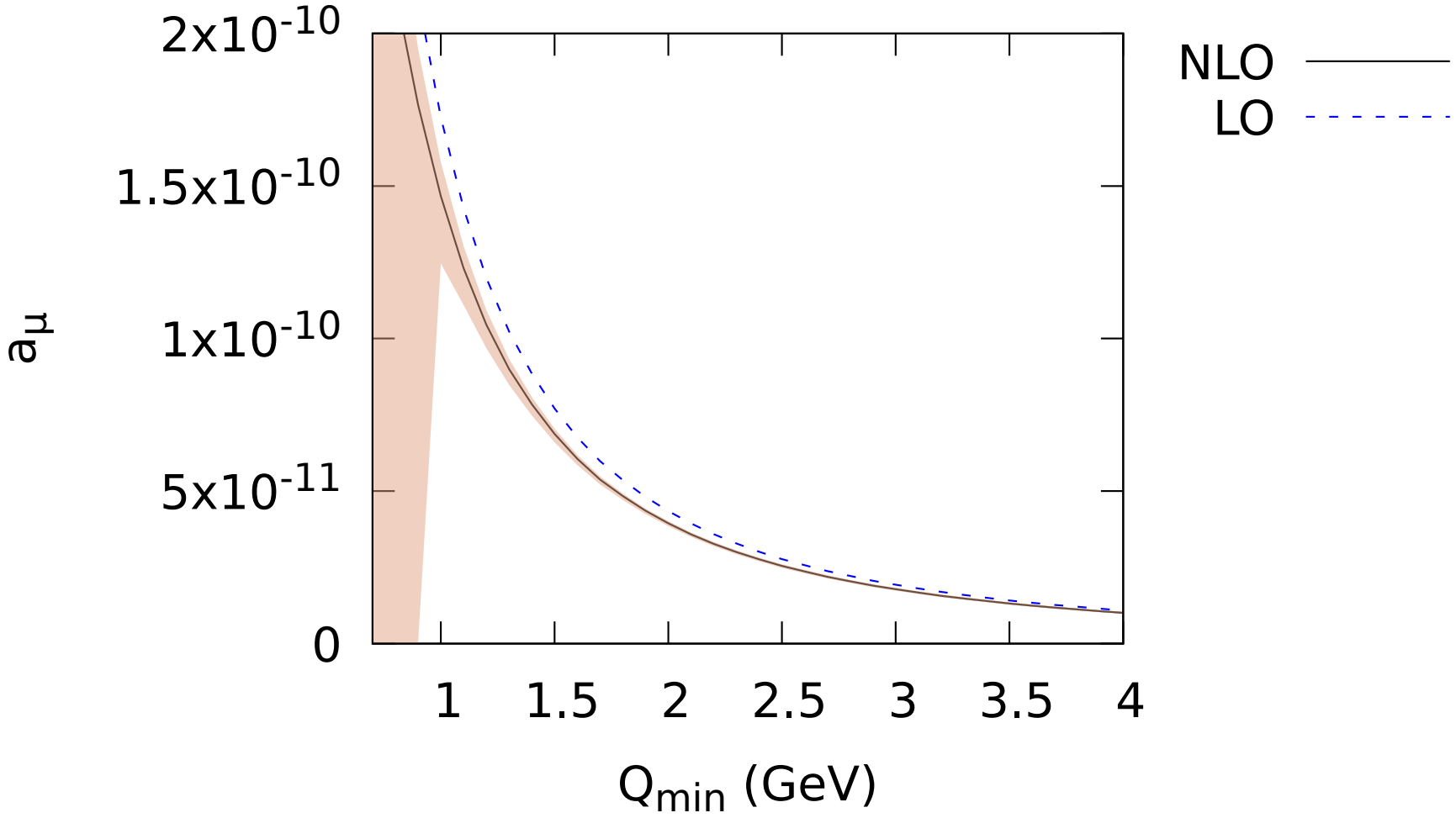
2-loop perturbative piece

- We thus see that non-perturbative corrections to the massless quark loop are very small
- Paper III: What about the massless perturbative $\mathcal{O}(\alpha_s)$ correction to the quark loop? 2 loops





Varying $Q_{\min}$



- Main uncertainty: $\alpha_s(\mu)$
- In general see about -10% of the quark-loop

Conclusions so far

- The massless quark-loop is the leading term in an OPE and a decent representation of the short-distance behaviour with $Q_1^2 \sim Q_2^2 \sim Q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$
- We have shown that higher-order terms in the OPE are numerically small (suppressed by m_q and small condensates)
- 2-loop correction is about -10% of the quark-loop
- This can now be used by the dispersive/model studies [Colangelo et al., 2020]
- Currently: $Q_1^2 \sim Q_2^2 \gg Q_3^2, \Lambda^2_{\mathrm{QCD}}$ very important to constrain models. Only leading order known $\rightarrow$ higher order terms and α_s needed

OPE overview

- Recall for one soft photon we study

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) \sim \int \frac{d^4q_3}{(2\pi)^4}\left(\prod_{i=1}^{3}\int d^4x_i\, e^{-iq_ix_i}\right) \times \langle 0|\, T\left(J^{\mu_1}(x_1)J^{\mu_2}(x_2)J^{\mu_3}(x_3)\right)|\gamma(q_4)\rangle$$

$Q_{1,2,3} \gg \Lambda_{\mathrm{QCD}}$: Keep $F_{\mu\nu}$–like operators

- Melnikov-Vainshtein limit: For two soft photons we instead consider

$$\Pi^{\mu_1\mu_2}(q_1) \sim \int \frac{d^4q_3}{(2\pi)^4}\left(\prod_{i=1}^{2}\int d^4x_i\, e^{-iq_ix_i}\right) \times \langle 0|\, T\left(J^{\mu_1}(x_1)J^{\mu_2}(x_2)\right)|\gamma^*(q_3)\gamma(q_4)\rangle$$

$Q_{1,2} \gg Q_3, \Lambda_{\mathrm{QCD}}$: Keep operators with the right quantum numbers
Can relate it to the HLbL as well

OPE in the Melnikov-Vainshtein limit

- OPE variables and definitions

$$\hat{q} \equiv \frac{q_1 - q_2}{2}$$
$$\langle \ldots \rangle^{3,4} = \langle 0 | \ldots | \gamma^*(q_3)\gamma(q_4) \rangle$$
$$\Gamma^{\mu\nu}(k) = \gamma^\mu S(k) \gamma^\nu$$

- OPE starts at dimension $D = 3$

$$\Pi^{\mu_1\mu_2} \approx -\frac{e_q^2}{e^2} \left\langle \bar{q}(0)[\Gamma^{\mu_1\mu_2}(-\hat{q}) - \Gamma^{\mu_2\mu_1}(-\hat{q})]q(0) \right\rangle^{3,4}$$
$$- \frac{ie_q^2}{e^2\hat{q}^2}(g^{\mu_1\delta}g_\beta^{\mu_2} + g^{\mu_2\delta}g_\beta^{\mu_1} - g^{\mu_1\mu_2}g_\beta^\delta)\left(g_{\alpha\delta} - 2\frac{\hat{q}_\delta\hat{q}_\alpha}{\hat{q}^2}\right)\left\langle \bar{q}(0)(\overrightarrow{\partial}^\alpha - \overleftarrow{\partial}^\alpha)\gamma^\beta q(0) \right\rangle^{3,4}$$
$$- \frac{1}{4}F_{\nu_3\mu_3}F_{\nu_4\mu_4} \left.\frac{\partial}{\partial q_{3\,\nu_3}}\right|_{q_3\to 0} \left.\frac{\partial}{\partial q_{4\,\nu_4}}\right|_{q_4\to 0} \Pi^{\mu_1\mu_2\mu_3\mu_4}_{\text{quark-loop}}$$

- $D = 4$: $F_{\mu\nu}F_{\alpha\beta}$ contributions appear and all three lines above needed

OPE in the Melnikov-Vainshtein limit

- $Q_1, Q_2 \gg Q_3 \gg \Lambda_{\mathrm{QCD}}$: limiting case of our previous work
- Through $D = 4$: Old quark loop agrees
- Argued in [Lüdtke, Procura 2020] that gluonic corrections to D=3 are

$$\Pi^{\mu_1\mu_2}_{D=3,\,\mathrm{NLO}} \approx -\frac{e_q^2}{e^2}\left(1 - \frac{\alpha_s}{\pi}\right)\left\langle \bar{q}(0)[\Gamma^{\mu_1\mu_2}(-\hat{q}) - \Gamma^{\mu_2\mu_1}(-\hat{q})]q(0)\right\rangle^{3,4}$$

- Take our new OPE and compare to the limit of our two-loop paper
- Agreement with the statement in [Lüdtke, Procura 2020]!

Conclusions and outlook

- OPEs to derive short-distance constraints for the HLbL

  One soft photon limit

  Two soft photons limit

- For $Q_1, Q_2, Q_3 \gg \Lambda_{\mathrm{QCD}}$:

  Quark loop is the leading term

  Non-perturbative corrections small

  Gluon corrections: -10% on the quark loop

- For $Q_1, Q_2 \gg Q_3, \Lambda_{\mathrm{QCD}}$:

  Limit $Q_3 \gg \Lambda_{\mathrm{QCD}}$: Agreement with the quark loop $\tilde{\Pi}_i$ through $D = 4$

  Need also $Q_3 \sim \Lambda_{\mathrm{QCD}}$: Non-perturbative extrapolations

  Impact of the short-distance constraints?