

The phenomenology of the nonet of the $\pi_1(1600)$ and $\eta_1(1855)$

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History of unconventional states

- Non-relativistic quark model → $0^{-+}(\pi)$, $0^{++}(a_0(1450))$, $1^{--}(\rho)$,
 $1^{++}(a_1(1260))$, $1^{+-}(b_1(1235))$, etc
- Additional degrees of freedom ⇒ states with “exotic” quantum numbers.
- $\bar{q}qg$, tetraquarks, meson molecules, etc.
- Exotic states: 0^{+-} , 0^{--} , $1^{-+}(\pi_1(1600), \eta_1(1855))$, etc.
- Hybrids:
 - Jaffe and Johnson ([Phys.Lett.B 60 \(1976\) 201-204](#)), Vainshtein and Okun ([Yad.Fiz. 23 \(1976\) 1347-1348](#)).
 - Quark models, effective theories, sum rules, lattice, . . .
 - Lightest hybrid has mass $\sim 1.7 - 2.2$ GeV.
- Experimental efforts:
 - Two (three) hybrids: $\pi_1(1400/1600)$, and $\eta_1(1855)$.
 - First confirmed results ($\pi_1(1400/1600)$) - IHEP-Brussels-Los Alamos-Annecy(LAPP) Collaboration ([Phys.Lett.B 205 \(1988\) 397](#)) and Brookhaven E852 experiment ([Phys.Rev.Lett. 81 \(1998\) 5760-5763](#)).
 - Most recently $\eta_1(1855)$ - BESIII ([hep-ex/2202.00621](#))

History of unconventional states

- Non-relativistic quark model → $0^{-+}(\pi)$, $0^{++}(a_0(1450))$, $1^{--}(\rho)$,
 $1^{++}(a_1(1260))$, $1^{+-}(b_1(1235))$, etc
- Additional degrees of freedom ⇒ states with “exotic” quantum numbers.
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For more: Friday 14:20, Track B

$\pi_1(1400)$ or $\pi_1(1600)$?

- PDG lists: $\pi_1(1400)$ and $\pi_1(1600)$
- Masses: 1354 MeV and 1661 MeV.
- Widths: 330 MeV and 240 MeV.
- Decay Channels:
 - $\pi_1(1400)$: $\eta\pi$
 - $\pi_1(1600)$: $\eta'\pi$, $\rho\pi$, $b_1(1235)\pi$, and $f_1\pi$.
- Lattice: $\pi_1(1564)$ (Phys.Rev.D 103 (2021) 5, 054502)
- Mass: 1564 MeV; Width: 139 – 590 MeV. (Mass same as JPAC (Phys.Rev.Lett. 122 (2019) 4, 042002), Width within range)
- Decay Channels: $\eta\pi$, $\eta'\pi$, $\rho\pi$, K^*K , $b_1(1235)\pi$, $f_1\pi$, $f'_1\pi$, and $\rho\omega$.

Final state interactions? Background effects? (more later)

Lagrangian

Model Lagrangian reduced to the $SU(2)$ limit:

$$\begin{aligned} \mathcal{L}_{hyb}^{\pi} = & g_{b_1\pi}^c \langle \pi_{1,\mu} b_1^\mu \pi \rangle + g_{b_1\pi}^d \langle \pi_{1,\mu\nu} b_1^{\mu\nu} \pi \rangle + g_{f_1\pi} \langle \pi_{1,\mu} f_{1,N}^{\mu\nu} \partial_\nu \pi + \pi_{1,\mu} f_{1,S}^{\mu\nu} \partial_\nu \pi \rangle \\ & + g_{\eta\pi} \langle \pi_{1,\mu} (\eta_N \partial^\mu \pi + \eta_S \partial^\mu \pi) \rangle + g_{\rho\pi} \langle \tilde{\pi}_{1,\mu\nu} \rho^{\mu\nu} \pi \rangle + g_{\rho\omega} \langle \pi_{1,\mu} (\rho^{\mu\nu} \omega_\nu + \omega^{\mu\nu} \rho_\nu) \rangle \end{aligned} \quad (1)$$

- Invariant under $SU(3)$ flavor symmetry and C and P invariance.
- $\Pi_{1,\mu}^{hyb} = \bar{q}_j G_{\mu\nu} \gamma^\nu q_i \rightarrow$ “Homo-chiral” \Rightarrow small mixing between isoscalars.
- Partial wave analysis, axial symmetry, etc

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- Invariant under $SU(3)$ flavor symmetry and C and P invariance.
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Partial Waves

- The angular information is lost when calculating the decay width.
- Decompose the *amplitude* into partial waves.
- Helicity formalism (*Annals Phys.* 7 (1959) 404-428); Tensor formalism (*Phys.Rev.* 140 (1965) B97-B108); Covariant helicity formalism (*Phys.Rev.D*.48.1225).
- Larger 3-momentum \Rightarrow Larger contribution from higher partial waves (*mostly*).
- Eg: $b_1(1235) \rightarrow \omega\pi$ ($D/S = 0.277 \pm 0.027$),
 $\pi_2(1670) \rightarrow \rho\pi$ ($F/P = -0.72 \pm 0.16$),
 $\pi_2(1670) \rightarrow f_2(1270)\pi$ ($D/S = -0.18 \pm 0.06$).
- Hence two interaction terms for the $b_1(1235)\pi$ channel.

More details and application to the decays of conventional mesons in
[Phys.Rev.D.105.054022](#); [Rev.Mex.Fis.Suppl.3.0308055](#)

Symmetries, etc . . .

Model Lagrangian reduced to the $SU(2)$ limit:

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- ⊗ Axial-vector decay channel ($f_1(1285)\pi$) arises from the purely axial-vector coupling and subsequent field redefinition: $A_\mu \rightarrow A_\mu + \# \partial_\mu P$ (Phys.Rev.D 87 (2013) 1, 014011).
- ⊗ Also leads to the $\eta_1(1855) \rightarrow f_1(1285)\eta$ channel (in a moment).
- ⊗ Predictions: $\eta_L \rightarrow a_1(1260)\pi$, $K_1^{hyb} \rightarrow a_1(1260)K$, $K_1(1270/1400)\pi$, $\eta_1(1855) \rightarrow K_1(1270)K$.
- ⊕ Axial anomaly induced decay channel: $\eta\pi$, $\eta'\pi$ (Eur.Phys.J.Plus 135 (2020) 12, 945)
- ⊕ $\eta_1(1855)$ observed in $\eta\eta'$ channel.
- ⊕ Predictions: $\eta_L \rightarrow \eta\eta'$, $K_1^{hyb} \rightarrow \eta^{(\prime)}K$

Available Data

Expt:

- $m_{\pi_1} = 1661^{+15}_{-11}$ MeV • $\frac{BR(\pi_1 \rightarrow b_1\pi)_D}{BR(\pi_1 \rightarrow b_1\pi)_S} = 0.3 \pm 0.1 = \frac{|G_2|^2}{|G_0|^2}$
- $\Gamma_{\text{tot}} = 240 \pm 50$ MeV • $\frac{\Gamma_{f_1\pi}}{\Gamma_{\eta'\pi}} = 3.8 \pm 0.78$

- + The D/S -ratio of the $b_1\pi$ channel is large \Rightarrow derivative interactions are necessary (Phys.Rev.D 105 (2022) 5, 054022).
- + Sign not specified. Two sets of parameters.
- + Set-1 ($D/S > 0$) and Set-2 ($D/S < 0$).

Available Data

Lattice data: (Phys.Rev.D 103 (2021) 5, 054502)

- | | |
|--|---|
| 1. $\Gamma_{b_1\pi} = 139\text{-}529 \text{ MeV},$ | 5. $\Gamma_{f'_1\pi} = 0\text{-}2 \text{ MeV},$ |
| 2. $\Gamma_{\rho\pi} = 0\text{-}20 \text{ MeV},$ | 6. $\Gamma_{\rho\omega} \leq 0.15 \text{ MeV},$ |
| 3. $\Gamma_{K^*K} = 0\text{-}2 \text{ MeV},$ | 7. $\Gamma_{\eta\pi} = 0\text{-}1 \text{ MeV},$ |
| 4. $\Gamma_{f_1\pi} = 0\text{-}24 \text{ MeV},$ | 8. $\Gamma_{\eta'\pi} = 0\text{-}12 \text{ MeV}.$ |

- Take the central value as the partial width; 50% error.

Flavor input:

- $f_1\pi$ and $f'_1\pi$ channels must have the same coupling constants (except for the mixing angle) $\Rightarrow \Gamma_{f'_1\pi}/\Gamma_{f_1\pi} = 0.0512$
- Also true for $\eta\pi$ and $\eta'\pi$ $\Rightarrow \Gamma_{\eta'\pi}/\Gamma_{\eta\pi} = 12.72$
- $\rho\pi$ and K^*K widths differ only in isospin factor and 3-momenta
 $\Rightarrow \Gamma_{K^*K}/\Gamma_{\rho\pi} = 0.178$
- The flavor symmetry inputs assumed to have $\sim 30\%$ errors.

Fitted Parameters

Parameter	Value	
	Set-1 ($D/S > 0$)	Set-2 ($D/S < 0$)
m_{π_1}	1.663 ± 0.01 GeV	1.662 ± 0.01
$g_{b_1\pi}^c$	88 ± 23 GeV	$-(119 \pm 22)$
$g_{b_1\pi}^d$	$-(23.3 \pm 5.60)$ GeV $^{-1}$	26.7 ± 5.3
$g_{\rho\pi}$	0.35 ± 0.05 GeV	0.35 ± 0.05
$g_{f_1\pi}$	8.02 ± 0.83 GeV	8.12 ± 0.83
$g_{\rho\omega}$	$-(0.37 \pm 0.07)$	$-(0.38 \pm 0.07)$
$g_{\eta\pi}$	4.91 ± 0.56	4.94 ± 0.55
$\chi^2/d.o.f$	0.35	0.28

Table 1: The values of the mass of π_1 and the coupling constants along with the uncertainties when the D/S -ratio for the $b_1\pi$ decay channel is positive, and negative .

The $\pi_1(1600)$

Channel i	Width (Γ_i , MeV)	
	Set-1 ($D/S > 0$)	Set-2 ($D/S < 0$)
$b_1\pi$	220 ± 34	208 ± 35
$f_1\pi$	16.2 ± 3.1	16.0 ± 3.1
$f'_1\pi$	0.83 ± 0.16	0.83 ± 0.16
$\rho\pi$	7.1 ± 1.8	7.1 ± 1.8
K^*K	1.2 ± 0.3	1.2 ± 0.3
$\eta\pi$	0.37 ± 0.08	0.36 ± 0.08
$\eta'\pi$	4.6 ± 1.0	4.5 ± 1.0
$\rho\omega$	0.08 ± 0.03	0.08 ± 0.03
Γ_{tot}	250 ± 34	238 ± 35

Table 2: The partial widths and branching ratios of various decay channels and the total width.

Kaons and Isoscalars

Mass relations: (leading terms [Eur.Phys.J.Plus 135 \(2020\) 12, 945](#))

$$m_{\eta_{1,N}}^2 = m_{\pi_1}^2 \quad (2)$$

$$m_{K_1}^2 = m_{\pi_1}^2 + \delta_s^{hyb} \quad (3)$$

$$m_{\eta_{1,S}}^2 = m_{\pi_1}^2 + 2\delta_S^{hyb} \quad (4)$$

Isoscalar mixing:

$$\begin{pmatrix} |\eta_1^L\rangle \\ |\eta_1^H\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_h & \sin \theta_h \\ -\sin \theta_h & \cos \theta_h \end{pmatrix} \begin{pmatrix} |\bar{n}n\rangle_h \\ |\bar{s}s\rangle_h \end{pmatrix} \quad (5)$$

Isoscalar masses:

$$m_{\eta_1^L}^2 = m_{\pi_1}^2 + \delta_S^{hyb} (1 - \sec(2\theta_h)) \quad (6)$$

$$m_{\eta_1^H}^2 = m_{\pi_1}^2 + \delta_S^{hyb} (1 + \sec(2\theta_h)) \quad (7)$$

Scenarios

	m_{K_1} (GeV)	$m_{\eta_1^L}$ (GeV)	$m_{\eta_1^H}$ (GeV)	θ_h	δ_S^{hyb} (GeV 2)
Scenario-1	1.707	1.542	1.855	36.7°	0.151
Scenario-2	1.761	1.661	1.855	0°	0.341
Scenario-3	1.754	1.646	1.855	15°	0.317

Table 3: The masses of the kaons and the isoscalars, and isoscalar mixing angle for the three scenarios.

State	Scenario-1		Scenario-2		Scenario-3		Parameter Set
	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
K_1^{hyb}	1707	142 ± 46	1761	312 ± 97	1754	286 ± 88	1
		74 ± 30		170 ± 65		155 ± 59	2
η_1^L	1540	21 ± 4	1661	81 ± 16	1646	69 ± 13	1
		22 ± 4		83 ± 16		71 ± 13	2
η_1^H	1855	608 ± 158	1855	258 ± 92	1855	410 ± 130	1
		249 ± 80		156 ± 68		191 ± 80	2

Table 4: The masses and widths of the kaons and the isoscalars in the three scenarios.

The “hybrid” Kaons: $K_1(1750)$

- “Scenario-2”: $M = 1760$ MeV.
- Possible mixing with the $1^- K^*(1410)$ and $K^*(1680)$ not considered.
(Significant difference in decay channels)
- Width sensitive to $h_1 - h'_1$ mixing angle.
- The D/S -ratio for the decay into axial kaons is > 1 for Set-2 and < 1 for Set-1.

	$K_1(1270)\pi$		$K_1(1400)\pi$	
	Set-1	Set-2	Set-1	Set-2
Scenario-1	0.58 ± 0.09	19 ± 82	0.16 ± 0.01	0.25 ± 0.02
Scenario-2	0.57 ± 0.07	1.76 ± 0.47	0.19 ± 0.01	0.28 ± 0.02
Scenario-3	0.57 ± 0.07	1.93 ± 0.59	0.18 ± 0.01	0.28 ± 0.02

Table 5: D/S -ratios of the decay of the hybrid kaon into $K_1(1270)\pi$ and $K_1(1400)\pi$.

The “hybrid” Kaons: $K_1(1750)$

Channel i	Width (Γ_i , MeV)		Channel i	Width (Γ_i , MeV)	
	Set-1	Set-2		Set-1	Set-2
$K_1(1270)\pi$	125 ± 42	48 ± 25	ρK	2.18 ± 0.56	2.19 ± 0.57
$K_1(1400)\pi$	103 ± 45	98 ± 43	ωK	0.82 ± 0.21	0.82 ± 0.21
$h_1(1170)K$	1.53 ± 0.28	1.37 ± 0.24	ϕK	0.49 ± 0.12	0.49 ± 0.13
ηK	0.29 ± 0.07	0.29 ± 0.07	$K^*\pi$	0.67 ± 0.17	0.67 ± 0.17
$\eta' K$	2.77 ± 0.62	2.81 ± 0.62	$K^*\eta$	0.30 ± 0.08	0.30 ± 0.08
ρK^*	0.045 ± 0.016	0.047 ± 0.016	ωK^*	0.011 ± 0.004	0.012 ± 0.004
$a_1 K$	11.0 ± 2.32	11.3 ± 2.35	$b_1 K$	64 ± 14	3.11 ± 2.88
			Γ_{tot}	312 ± 97	170 ± 65

Table 6: The partial widths and branching ratios of various decay channels and the total width for the hybrid kaon $K_1^{hyb}(1750)$.

The isoscalars: $\eta_1(1660)$, $\eta_1(1855)$

Channel i	Width (Γ_i , MeV)		Channel i	Width (Γ_i , MeV)	
	Set-1	Set-2		Set-1	Set-2
$a_1\pi$	80 ± 15	82 ± 16	$K_1(1270)K$	253 ± 92	151 ± 67
K^*K	0.29 ± 0.075	0.29 ± 0.075	K^*K	1.45 ± 0.37	1.46 ± 0.38
$\eta'\eta$	0.77 ± 0.18	0.79 ± 0.18	$\eta'\eta$	1.08 ± 0.24	1.10 ± 0.24
$K_1(1270)K$	0	0	$a_1\pi$	0	0
$\rho\rho$	0.081 ± 0.028	0.082 ± 0.029	$\rho\rho$	0	0
K^*K^*	0	0	K^*K^*	0.075 ± 0.027	0.077 ± 0.028
$\omega\phi$	0	0	$\omega\phi$	$\sim 10^{-4}$	$\sim 10^{-4}$
$f_1\eta$	0	0	$f_1\eta$	2.15 ± 0.56	2.21 ± 0.57
Γ_{tot}	81 ± 16	83 ± 16	Γ_{tot}	258 ± 92	156 ± 68

Table 7: The partial widths of various decay channels and the total width of the $\eta_1(1660)$ (left) and the $\eta_1(1855)$ (right) for $\theta_h = 0^\circ$.

Short Summary

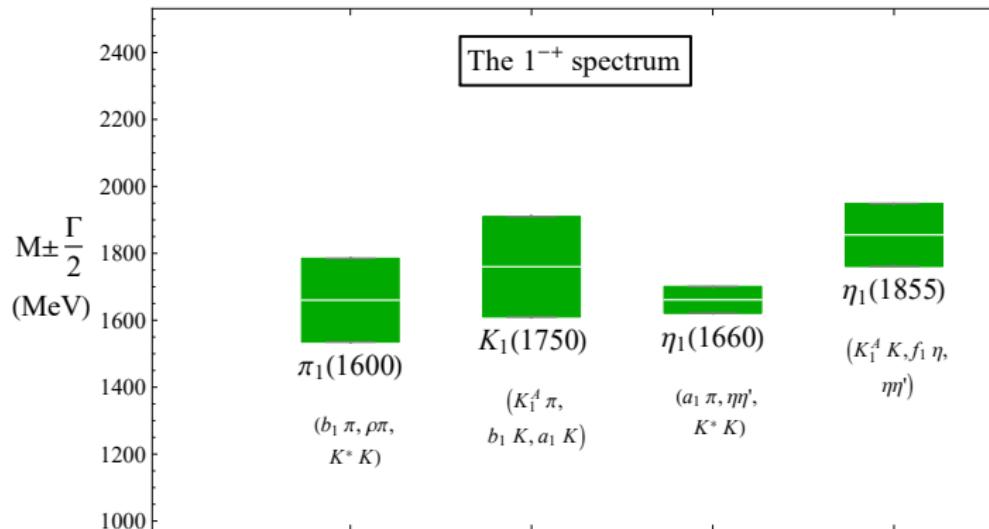


Figure 1: The 1⁻⁺ hybrid spectrum.

- * More info needed to fine tune the parameters and predictions.

Summary

- Analysed the spectrum and decays of the 1^{-+} hybrid states
- Model Lagrangian invariant under $SU(3)$ flavor symmetry, parity reversal and charge conjugation
- Statistical fit using the available experimental and lattice data
- Fit results provide partial widths with improved accuracy compared to lattice data.
- Masses and total widths of the isovector and isoscalar states reproduced.
- Predictions for new decay channels and two new states - kaon and light isoscalar.
- Kaon is broad and decays mostly to $K_1(1270/1400)\pi$, ρK , $h_1(1170)K$.
- Light isoscalar is the narrowest of all and decays mostly into $a_1(1260)\pi$.
- Isoscalars do not mix, true to their homo-chiral nature.

Outlook and take aways

- Tree-level calculations - loop effects to be studied (relatively broad states).
- Radiative decays (to be attempted)
- $\eta_1(1660)$ can be found in the 4π , $2K\pi$, and $2K2\pi$ channels
- Kaons in $K3\pi$, $K2\pi$, $K4\pi$, $K\eta$, $K\pi\eta$ channels (also, $3K$, $K\eta2\pi$)
- $\eta_1(1855)$ in the $2K2\pi$, $2\eta2\pi$, and $2K\pi$ channels

More details: 2203.04327 [hep-ph]

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More details: 2203.04327 [hep-ph]

Thank you!

Lagrangian

$SU(3)$ Lagrangian:

$$\begin{aligned} \mathcal{L}_{hyb}^{\pi} = & g_{b_1\pi}^c \text{Tr}\left[\Pi_{\mu}^{hyb}[P, B^{\mu}]\right] + g_{b_1\pi}^d \text{Tr}\left[\Pi_{\mu\nu}^{hyb}[P, B^{\mu\nu}]\right] \\ & - g_{\eta\pi\eta N} \text{Tr}\left[\Pi_{\mu}^{hyb} \partial^{\mu} P\right] + g_{\rho\pi} \text{Tr}\left[\tilde{\Pi}_{\mu\nu}[P, V^{\mu\nu}]\right] \\ & + g_{\rho\omega} \text{Tr}\left[\Pi_{\mu}^{hyb} \{V^{\mu\nu}, V_{\nu}\}\right] + g_{f_1\pi} \text{Tr}\left[\Pi_{\mu}^{hyb} \{A^{\mu\nu}, \partial_{\nu} P\}\right] \end{aligned} \quad (8)$$

- $SU_L(3) \times SU_R(3)$ flavor symmetry, P and C invariance.
- Dilaton symmetry.
- Partial wave amplitude data available for the $\pi_1(1600) \rightarrow b_1(1235)\pi$ decay channel!

Fixed Parameters

The following values were used for the masses and mixing angles of the product states:

Meson	Mass (GeV)	Meson	Mass (GeV)	Mixing	Angle
π	0.135	b_1	1.23	$\eta - \eta'$	-44.5°
K	0.494	$K_1(1270)$	1.253	$\phi - \omega$	0°
η	0.548	$h_1(1170)$	1.17	$f_1 - f'_1$	24°
η'	0.958	a_1	1.23	$h_1 - h'_1$	25°
ρ	0.775	$K_1(1400)$	1.403	$K_1^A - K_1^B$	56°
ω	0.782	f_1	1.285		
K^*	0.892	f'_1	1.426		
ϕ	1.02				

Table 8: Masses and mixing angles of the decay products.

Which ℓ values are valid?

$$A(J^P, k_0) \rightarrow B(s^\pi, k_1) + C(\sigma^\kappa, k_2)$$

Spin states: $|J, M\rangle$ (parent), $|s, s_3\rangle$, and $|\sigma, \sigma_3\rangle$ (decay products).

Parity: P (parent), π, κ (decay products)

Relative angular momentum: $|\ell, m_\ell\rangle$

$$|J, M\rangle = |\ell, m_\ell\rangle \oplus |S, \delta\rangle, \quad |S, \delta\rangle = |s, s_3\rangle \oplus |\sigma, \sigma_3\rangle, \quad \delta = s_3 - \sigma_3 \quad (9)$$

Thus, $\ell \in [|J - S|, J + S]$. But, are all these values allowed?

$$P = \pi \otimes \kappa \otimes (-1)^\ell \quad (10)$$

Ex: $a_1(1260) \rightarrow \rho\pi$: $J^P = 1^+$, $s^\pi = 1^-$, $\sigma^\kappa = 0^-$. So, $\ell \in [0, 2]$. But,
 $+1 = (-1)(-1)(-1)^\ell \Rightarrow \ell \in \text{even}$

Helicity Amplitude

The amplitude,

$$i\mathcal{M}^J(\theta, \phi; J_3) \propto D_{J_3 S_3}^{J*}(\phi, \theta, 0) F_{S_3 0}^J; \quad (11)$$

In the frame of reference where momenta: $k_{0,\mu} = (M_p, \vec{0})$ (parent), $k_{1,\mu} = (E_{d,1}, 0, 0, -k)$, and $k_{2,\mu} = (E_{d,2}, 0, 0, k)$ (daughters) ($\theta = \phi = 0$),

$$i\mathcal{M}^J(0, 0; J_3) \propto F_{S_3 0}^J \quad (12)$$

The helicity amplitudes ($F_{S_3 0}^J$) are related to the ℓS coupling amplitudes ($G_{\ell S}^J$) as,

$$F_{S_3 0}^J = \sum_{\ell S} \sqrt{\frac{2\ell + 1}{2J + 1}} \langle \ell 0 S S_3 | J S_3 \rangle \langle S S_3 0 0 | S S_3 \rangle G_{\ell S}^J \quad (13)$$

$$F_{S_3 \sigma_3}^J = \sum_{\ell S} \sqrt{\frac{2\ell + 1}{2J + 1}} \langle \ell 0 S \delta | J \delta \rangle \langle s s_3 \sigma \sigma_3 | S \delta \rangle G_{\ell S}^J \quad (14)$$

Decay width:

$$\Gamma_{A \rightarrow BC} = \# \frac{k}{8\pi M_A} \sum_{spin} |i\mathcal{M}|^2 = \frac{k}{8\pi M_A} \sum_{\ell S} |G_{\ell S}^J|^2 \quad (15)$$

Chiral Symmetry and Meson Phenomenology

- $SU_L(3) \times SU_R(3)$ flavor symmetry, P and C invariance.
- Chiral symmetry and its (spontaneous and explicit) breaking dominates low-energy QCD.
- Flavor multiplets transform in two possible ways under $SU_L(3) \times SU_R(3)$ transformations (Phys. Rev. D 97 (2018) 9, 091901):
 - “Hetero-chiral” multiplets: $M \rightarrow U_L M U_R^\dagger$; (Large mixing)
 - “Homo-chiral” multiplets: $M \rightarrow U_R M U_R^\dagger$; (Small mixing)
- Chiral partners have to transform in the same fashion.

Chiral multiplets

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

$J^{PC}, \frac{2S+1}{2}L_J$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_L \times SU(3)_R \times U(1)_A$
$0^-, {}^1S_0$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ ($\Phi^{ij} = \bar{q}_R^j q_L^i$)	$\Phi \rightarrow e^{-2ia} U_L \Phi U_R^\dagger$
$0^+, {}^3P_0$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
$1^-, {}^1S_1$	$V_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ ($L_\mu^{ij} = \bar{q}_L^j \gamma_\mu q_L^i$)	$L_\mu \rightarrow U_L L_\mu U_L^\dagger$
$1^+, {}^3P_1$	$A_\mu^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ ($R_\mu^{ij} = \bar{q}_R^j \gamma_\mu q_R^i$)	$R_\mu \rightarrow U_R R_\mu U_R^\dagger$
$1^-, {}^1P_1$	$P_\mu^{ij} = -\frac{1}{2}\bar{q}^j \gamma^5 \overset{\leftrightarrow}{D}_\mu q^i$	$\Phi_\mu = S_\mu + iP_\mu$	$\Phi_\mu \rightarrow e^{-2ia} U_L \Phi_\mu U_R^\dagger$
$1^-, {}^3D_1$	$S_\mu^{ij} = \frac{1}{2}\bar{q}^j (\overset{\leftrightarrow}{D}_\mu \overset{\leftrightarrow}{D}_\nu q^i)$		
$2^+, {}^3P_2$	$V_{jk}^{ij} = \frac{1}{2}\bar{q}^j (\gamma_\mu i\overset{\leftrightarrow}{D}_\mu + \dots) q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ ($L_{\mu\nu}^{ij} = \bar{q}_L^j (\gamma_\mu i\overset{\leftrightarrow}{D}_\nu + \dots) q_L^i$)	$L_{\mu\nu} \rightarrow U_L L_{\mu\nu} U_L^\dagger$
$2^-, {}^3D_2$	$A_{jk}^{ij} = \frac{1}{2}\bar{q}^j (i\gamma^5 \gamma_\mu i\overset{\leftrightarrow}{D}_\nu + \dots) q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ ($R_{\mu\nu}^{ij} = \bar{q}_R^j (\gamma_\mu i\overset{\leftrightarrow}{D}_\nu + \dots) q_R^i$)	$R_{\mu\nu} \rightarrow U_R R_{\mu\nu} U_R^\dagger$
$2^-, {}^1D_2$	$P_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j (i\gamma^5 \overset{\leftrightarrow}{D}_\mu \overset{\leftrightarrow}{D}_\nu + \dots) q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$	$\Phi_{\mu\nu} \rightarrow e^{-2ia} U_L \Phi_{\mu\nu} U_R^\dagger$
$2^+, {}^3F_2$	$S_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j (\overset{\leftrightarrow}{D}_\mu \overset{\leftrightarrow}{D}_\nu + \dots) q^i$		

Figure 2: List of chiral multiplets (taken from PRD.97.091901)

Chiral Multiplets

Nonets	J^{PC}	States
Pseudoscalar (P)	0^{-+}	$\pi^{\pm,0}, K^{\pm,0,\bar{0}}, \eta_N, \eta_S (\eta, \eta')$
Vector (V_μ)	1^{--}	$\rho^{\pm,0}, K^{*\pm,0,\bar{0}}, \omega_N, \omega_S (\omega, \phi)$.
Axial-vector (A_μ)	1^{++}	$a_1^{\pm,0}, K_{1,A}^{\pm,0,\bar{0}}, f_{1,A}^N, f_{1,A}^S (f_1, f'_1)$
Pseudovector (B_μ)	1^{+-}	$b_1^{\pm,0}, K_{1,B}^{\pm,0,\bar{0}}, f_{1,B}^N, f_{1,B}^S (h_1, h'_1)$
Tensor ($X_{\mu\nu}$)	2^{++}	$a_2^{\pm,0}, K_2^{*\pm,0,\bar{0}}, f_2^N, f_2^S (f_2, f'_2)$
Axial-tensor ($W_{\mu\nu}$)	2^{--}	$--, K_2^{\pm,0,\bar{0}}, --$.
Pseudotensor ($T_{\mu\nu}$)	2^{-+}	$\Pi_2^{\pm,0}, K_2^{\pm,0,\bar{0}}, \eta_2^N, \eta_2^S (\eta_2, \eta'_2)$
Higher spins	3^{--}	$\rho_3^{\pm,0}, K_3^{*\pm,0,\bar{0}}, \omega_3, \phi_3 (\omega_3, \phi_3)$
Hybrid	1^{-+}	$\pi_1(1600), ??, ??, \eta_1(1855)$
More ??		

Table 9: Some conventional and exotic meson nonets, their spin and parity, and the observed members.

Mixing between states

“Physical states are admixtures of flavor states.”

- Kaons and isoscalars exhibit mixing.
- Iso-singlet mixing due to $U(1)_A$ breaking (mixing between singlet and octet states or $|\bar{n}n\rangle$ and $|\bar{s}s\rangle$ states).
- Large mixing angle ($\theta \sim -40^\circ$) among the pseudoscalars (η, η'); Small mixing angle ($\theta \sim -3^\circ$) among the vectors (ω, ϕ)
- Similar mixing observed among other iso-singlets:
 $f_1 - f'_1$ ($\theta \sim +24^\circ$ (LHCb, 2013)), $h_1 - h'_1$ ($\theta \sim 0^\circ$ * (BESIII, 2017)), etc.

“Kaons no C ”!

- Axial kaons ($K_1(1270)$, $K_1(1400)$) are mixtures of the $K_{1,A}$ and $K_{1,B}$ states ($\theta \in [-35^\circ, +72^\circ]$ * (Tayduganov, 2012)).
- Kaonic mixing and isoscalar mixing are related (according to the Gell-Mann-Okubo relations).

Partial wave analysis

Width (MeV)		
Decay	Theory	PDG
$f_1(1285) \rightarrow K^*K$	4.78 ± 0.57	not seen
$h_1(1170) \rightarrow \rho\pi$	146 ± 14	seen
D/S -ratio		
Decay	Theory	PDG
$f_1(1285) \rightarrow K^*K$	$-(0.436 \pm 0.87) \times 10^{-3}$	---
$f'_1(1420) \rightarrow K^*K$	-0.0116 ± 0.005	---
$h_1(1170) \rightarrow \rho\pi$	0.281 ± 0.035	---
$h'_1(1415) \rightarrow K^*K$	0.021 ± 0.001	---

Table 10: Predictions for the ratios of PWA for some $J = 1$ mesons. See PRD.105.054022 for details.

Partial wave analysis

Decay	Width (MeV)	D/S	G/S	F/P
$\pi_2(1670) \rightarrow f_2(1270)\pi$	Input	Input	0.0042 ± 0.0014	$\times \times \times$
$\pi_2(1670) \rightarrow f'_2(1520)\pi$	0.43 ± 0.21	$(9.25 \pm 3.10)m$	$-(7.49 \pm 2.7)\mu$	$\times \times \times$
$\pi_2(1670) \rightarrow K^*K$	5.11 ± 1.4	$\times \times \times$	$\times \times \times$	-0.447 ± 0.099

Table 11: Predictions for the ratios of PWA for some $J = 2$ mesons. See PRD.105.054022 for details. $m : 10^{-3}$, $\mu : 10^{-6}$.

Pseudotensor isoscalars

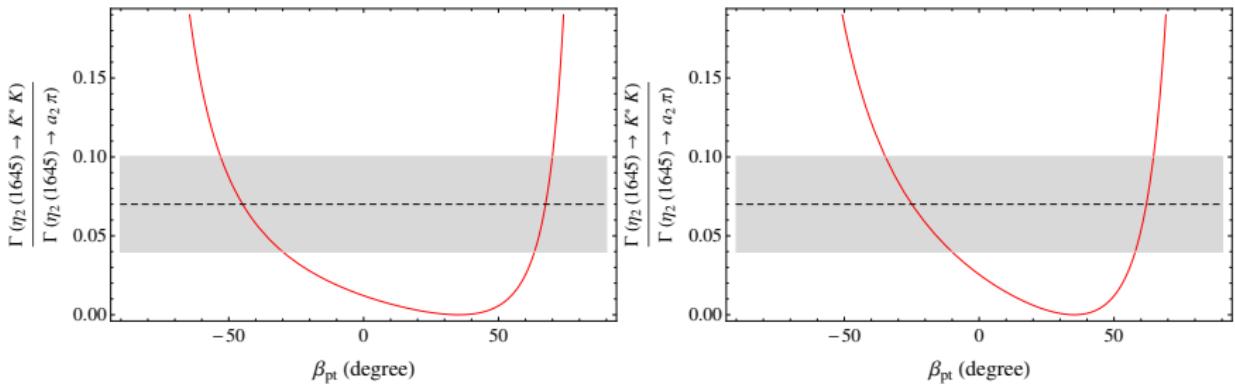


Figure 3: Left: with higher derivatives, Right: Only contact terms.