Topological susceptibility in high temperature QCD: a new investigation with spectral projectors

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Outline

The θ term

$$\mathcal{L}_{QCD}^{\theta} = \mathcal{L}_{QCD} + \theta q(x); \qquad q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{a\,\mu\nu}F^{a\,\rho\sigma}$$

Main properties of the θ term:

1. θ is a dimensionless parameter in $[0, 2\pi)$
2. the θ term is a four-divergence: no effect on the classical equations of motion, purely quantistic and nonperturbative effects
3. on smooth configurations $Q = \int q(x)\mathrm{d}^4x \in \mathbb{Z}$
4. behaviour under $U(1)_A$: if $\psi_j \to e^{i\alpha\gamma_5}\psi_j$ and $\bar{\psi}_j \to \bar{\psi}_j e^{i\alpha\gamma_5}$ then $\theta \to \theta - 2\alpha N_f$ and $m_j \to m_j e^{2i\alpha}$
5. it breaks explicitly P and CP simmetry

The general form of the free-energy density $F(\theta, T)$

$$F(\theta, T) = -\frac{1}{V_4}\log\int[\mathscr{D}A][\mathscr{D}\bar{\psi}][\mathscr{D}\psi]\exp\left(-\int_0^{1/T}\mathrm{d}t\int\mathrm{d}^3x\,\mathcal{L}_\theta^E\right)$$

From analiticity at $\theta = 0$ we can parametrize $F(\theta, T)$ as:

$$F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2\left[1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right],$$

and it is easy to see that $\chi = \frac{1}{V_4}\langle Q^2\rangle_0$ and $b_2 = -\frac{\langle Q^4\rangle_0 - 3\langle Q^2\rangle_0^2}{12\langle Q^2\rangle_0}$, where $\langle\quad\rangle_0$ denotes the average at $\theta = 0$.

The study of θ-dependence at high T is important both on his own (to better understand QCD) and for his consequences in axion physics.

BSM physics: if $\theta \leftrightarrow a/f_a$ ("a"=axion) then $m_a^2 = \chi/f_a^2$,
$\chi b_2/f_a^4 \sim$ quartic coupling and so on...

Analytical expectations

For $T \ll \Lambda_{QCD}$ we can trade θ-dependence for m-dependence by using a $U(1)_A$ transofrmation, and perform computations with ChPT (see e.g. Grilli di Cortona, Hardy, Pardo Vega, Villadoro 1511.02867 for NLO).

For $T \gg \Lambda_{QCD}$ semiclassical (DIGA) *and* perturbative computations become reliable and (Gross, Pisarski, Yaffe 1981)

$$\begin{aligned} \mathrm{DIGA}: &\quad F(\theta, T) - F(0, T) \simeq \chi(T)(1 - \cos\theta) \\ \mathrm{PT}: &\quad \chi(T) \propto m^{N_f} T^{4 - \frac{11}{3}N - \frac{1}{3}N_f} \end{aligned}$$

(see also Boccaletti, Nogradi 2001.03383).

The value of $\chi(T)$ for T of the order of GeVs is relevant for axion phenomenology, and Lattice QCD appears to be the only first principle method available to reliably investigate this range of temperatures (Berkowitz et al. 1505.07455).

The numerical problems

Lattice QCD simulations with physical light quarks are notoriously complicated from the computational point of view.

The study of $\chi(T)$ in the high temperature phase is much worst, since

1. topological observables are extremely sensitive to the explicit chiral symmetry breaking of the lattice fermion discretization ("huge lattice artifacts" problem)
2. at high T we have $\chi(T) \to 0$, the probability $P(Q)$ of observing a configuration with charge Q gets strongly peaked at $Q = 0$ ("small box" problem)
3. as the continuum limit is approached autocorrelation times grows exponentially fast with the inverse lattice spacing, and simulations get stuck in a fixed topological sector ("freezing" problem)

The combined effect of all these problems almost results in a no go theorem.

A possible way to go

To solve the small box problem we use the multicanonical approach as previously done in Bonati et al. 1807.07954: a modified distribution is sampled and the results are a posteriori re-weighted (in a stochastically exact way) to extract expectation values for the original distribution.

To reduce lattice artefacts we use the spectral projectors definition of the topological charge. This definition was introduced in Giusti, Lüscher 0812.3638 for Wilson-type fermion discretizations and it was extended to the staggered case in Bonanno, Clemente, D'Elia, Sanfilippo 1908.11832.

At $T = 0$ the spectral projector definition was shown in Alexandrou et al. 1709.06596 to have much smaller discretization errors than the other commonly adopted discretizations. Possible explanation (still to be better understood): since the same Dirac operator is used in the weight of the configuration and in the measure, some discretization effects get cancelled.

The spectral projector definition

In the continuum $Q = \mathrm{Tr}(\gamma_5) = \sum_{\mathrm{zero\ modes}} u_0^\dagger \gamma_5 u_0 = n_+ - n_-$

On the lattice ($\Gamma_5 = \gamma_5^{(\mathrm{stag})}$, $\mathbb{P}_M = \sum_{|\lambda|\le M} u_\lambda u_\lambda^\dagger$, $iD_{\mathrm{stag}} u_\lambda = \lambda u_\lambda$)

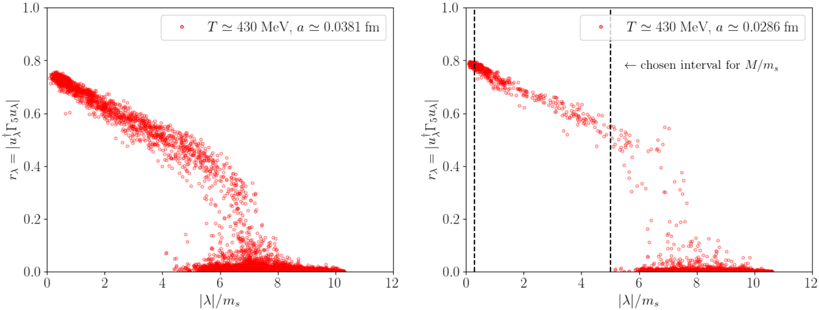
$$Q^{(stag)}_{\mathrm{SP,bare}} = \frac{1}{2^{d/2}} \mathrm{Tr}(\Gamma_5 \mathbb{P}_M) = \frac{1}{2^{d/2}} \sum_{|\lambda|\le M} u_\lambda^\dagger \Gamma_5 u_\lambda$$

where M is a cut-off parameter. $Q^{(stag)}_{\mathrm{SP,bare}}$ renormalizes multiplicatively and χ can be computed as (Bonanno, Clemente, D'Elia, Sanfilippo 1908.11832)

$$\chi = \frac{1}{2^d} \frac{\langle \mathrm{Tr}(\mathbb{P}_M)\rangle}{\langle \mathrm{Tr}(\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M)\rangle} \frac{\mathrm{Tr}(\Gamma_5 \mathbb{P}_M)^2\rangle}{V_4}$$

To have standard $O(a^2)$ corrections M has to be rescaled with the lattice spacing as a quark mass. We kept fixed M/m_s while moving on a LCF.

The spectrum at $T \simeq 430$ MeV

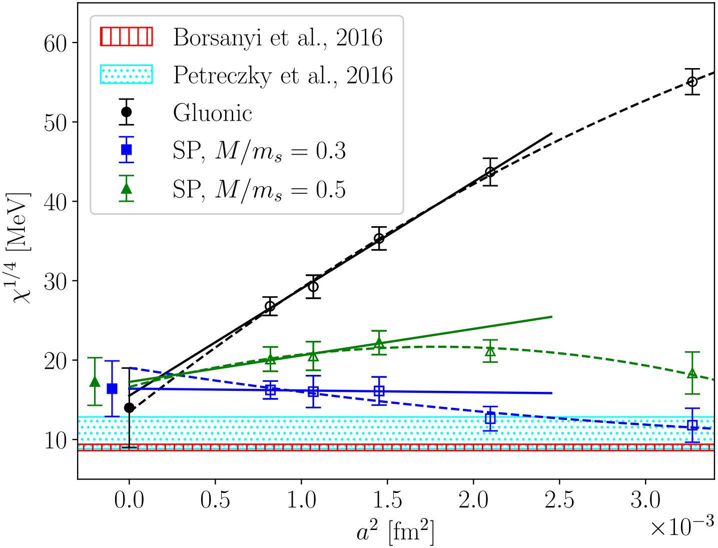


r_λ=chirality of the mode. Continuum: $r_\lambda = 1$ iff $\lambda = 0$.

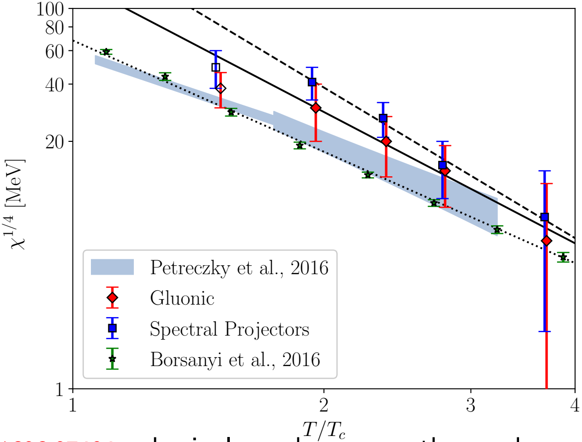
On the lattice (with non chiral quark discretization) zero modes becomes "would be zero modes": $\lambda \to 0$ only when $a \to 0$.

The M-interval is chosen to include all the likely "would be zero modes".

$\chi(T)$ at $T \simeq 430$ MeV



All results and comparison with others groups



Borsanyi et al. 1606.07494: physical quark masses, thermodynamical integration at fixed Q and a posteriori near zero mode reweighting (note: no isospin symmetry breaking for all the data above).
Petreczky et al. 1606.03145: $m_\pi \simeq 160$ MeV, χ rescaled using DIGA expectations $\chi \propto m_\ell^2 \propto m_\pi^4$.

Conclusions and perspectives

We presented results for $\chi(T)$ in the high temperature regime of QCD, obtained by using the spectral projector discretization and the multicanonical algorithm.

pro: much better control on the systematics of the continuum extrapolation due to smaller lattice artifacts and to the presence of a new parameter.

con: errors still bigger than we hoped for...

A precise unbiased first-principle calculation of $\chi(T)$ is still an extremely challenging task. Larger statistics and smaller lattice spacings are required to settle this problem.

New algorithms to cope with the exponential critical slowing down of topological modes are being developed/tested.

Thank you for your attention!

Backup with something more

Possible solutions of the strong CP problem

1. At least a massless quark ($m_u = 0$).
2. Assume a CP invariant lagrangian for the standard model and explain CP violation by CP SSB.
3. "Dynamical" θ angle.

Realization of mechanism 3: add to SM a pseudoscalar field a with coupling $\frac{a}{f_a} F\tilde{F}$ and only derivative interactions. Since the free energy has a minimum at $\theta = 0$, a will acquire a VEV such that $\theta + \frac{\langle a \rangle}{f_a} = 0$.

Goldstone bosons have only derivatives coupling, so the simplest possibility is to think of a as the GB of some $U(1)$ axial symmetry (Peccei-Quinn symmetry). The effective low-energy lagrangian is thus

$$\mathcal{L} = \mathcal{L}_{QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a + \left(\theta + \frac{a(x)}{f_a}\right) q(x) + \frac{1}{f_a}\left(\begin{smallmatrix} model \ dependent \\ terms \end{smallmatrix}\right)$$

Axions as dark matter

Cosmological sources of axions: 1) thermal production 2) decay of topological objects 3) misalignment mechanism

Idea of the misalignment mechanism: the EoM of the axion is

$$\ddot{a}(t) + 3H(t)\dot{a}(t) + m_a^2(T)a(t) = 0$$

at $T \gg \Lambda_{QCD}$ the second term dominates and we have $a(t) \sim \mathrm{const}$ (assuming $\dot{a} \ll H$ initially); when $m_a \sim H$ the field start oscillating around the minimum. When $m_a \gg H$ a WKB-like approx. can be used

$$a(t) \sim A(t) \cos \int^t m_a(\tilde{t}) \mathrm{d}\tilde{t}; \qquad \frac{\mathrm{d}}{\mathrm{d}t}(m_a A^2) = -3H(t)(m_a A^2)$$

and thus the number of axions in the comoving frame $N_a = m_a A^2/R^3$ is conserved.

Overclosure bound: axion density $\leq$ dark matter density