Search for critical point of strongly interacting matter

(Intermittency analysis by NA61/SHINE at CERN SPS)

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The XVth Quark Confinement and Hadron Spectrum, Stavanger, Norway

Outline

The goal of this study is to locate the critical point of the strongly interacting matter

\- - - - - - by - - - - - -

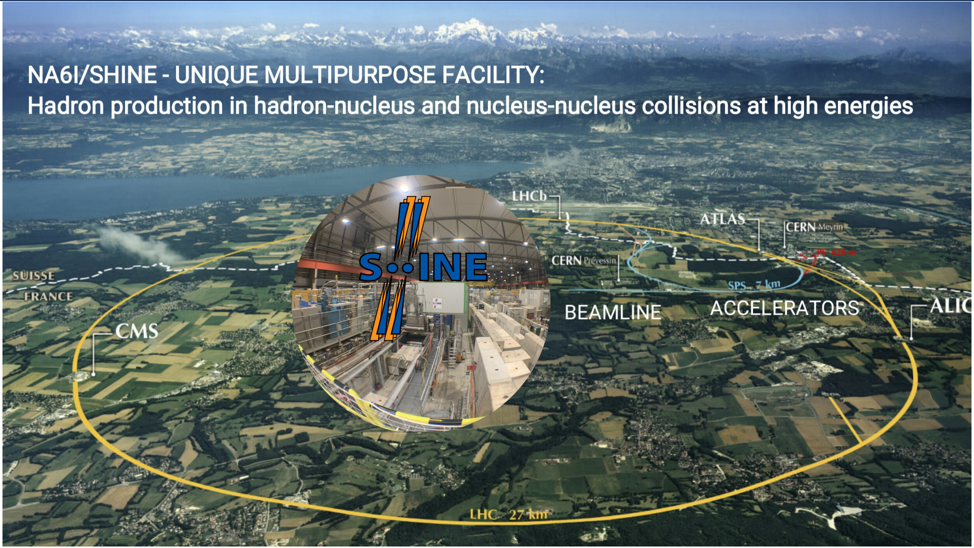
measuring scaled factorial moments of multiplicity distribution from a selection of

Pb+Pb at $13A$ GeV/c ($\sqrt{s_{NN}} \approx 5.1$ GeV)
Pb+Pb at $30A$ GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)
Ar+Sc at $150A$ GeV/c ($\sqrt{s_{NN}} \approx 17$ GeV)

using statistically independent data points and cumulative variables

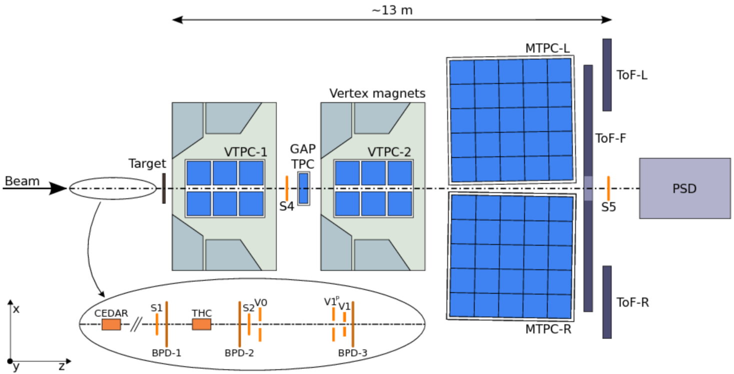
- Introduction
- Critical point of strongly interacting matter
- Experimental measures to search QCD critical point
  - Fluctuations in large momentum bins
    - Multiplicity fluctuations
    - Multiplicity-transverse momentum fluctuations
  - Fluctuation as a function of momentum bin size
    - Proton intermittency anlysis
    - h^- intermittency analysis
- Exclusion plot and intermittency analysis result
- Summary

NA61/SHINE at CERN SPS



NA61/SHINE (**S**PS **H**eavy **I**on and **N**eutrino **E**xperiment) is a particle physics fixed-target experiment at CERN SPS

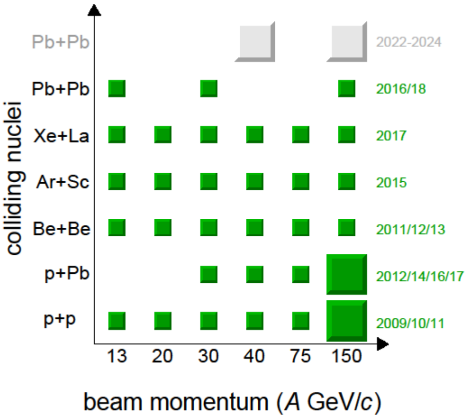
NA61/SHINE detector



- TPC system: track reconstruction and particle identification based on specific energy loss
- VTPC-1, VTPC-2 are placed in the magnetic field: used for tracks momentum reconstruction
- Projectile Spectator Detector (PSD): hadronic calorimeter, measures projectile spectators energy

N. Abgrall et al., [NA61/SHINE Collab.] arXiv:1401.4699 [physics.ins-det]

NA61/SHINE Physics program



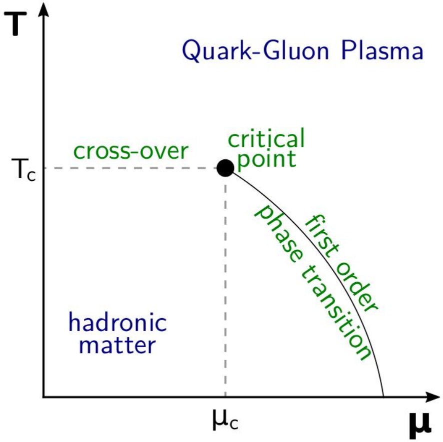
NA61/SHINE performs scan in beam momenta ($13A$ - $150A$ GeV/c) and mass of colliding nuclei (p+p, p+Pb, Be+Be, Ar+Sc, Xe+La, Pb+Pb)

- **One of the main goals of NA61/SHINE is to search for the QCD critical point**
- Study of the properties of the onset of deconfinement

  Talk by Oleksandra Panova: System size and energy dependence of proton production from NA61/SHINE at the CERN SPS
- Heavy quarks: direct measurement of open charm at SPS energies
- Measurements for the J-PARC and Fermilab neutrino programs
- Measurements of nuclear fragmentation cross sections for cosmic rays physics

Tobiasz Czopowicz, arXiv:1201.5829 [hep-ex]

Critical point of strongly interacting matter



Talk: Diagram of high-energy nuclear collisions from NA61/SHINE by Prof. Marek Gazdzicki on Friday

- Critical Point (CP): a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition
- Second order phase transition→the correlation length diverges → scale invariance → power-law form of correlation function
- These expectations are for fluctuations and correlations in the configuration space
- They are expected to be *projected* to the momentum space via quantum effect and/or collective flow
- However predictions on the CP existence, its location and what and how should fluctuate are model-dependent

Asakawa, Yazaki NPA 504 (1989) 668
Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

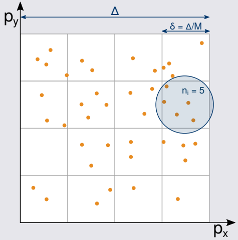
Scaled factorial moments of order r

In NA61/SHINE , intermittency analysis is performed at mid-rapidity and particle fluctuations are studied in transverse momentum plane to locate the QCD critical point by measuring scaled factorial moments of multiplicity distribution

$$F_r(\delta) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i-1)\ldots(n_i-r+1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^r}$$

δ: size of each of the M sub-division intervals of Δ

n_i : number of particles in i-th bin



When the system is a simple fractal and $F_r(\delta)$ follows a power law dependences:

$$F_r(\delta) = F_r(\Delta) \cdot (\frac{\Delta}{\delta})^{D\cdot\varphi_r}$$

where D is the embedding dimension (for transverse plane, $D = 2$) and exponent, φ_r obeys the relation:

$$D \cdot \varphi_r = (r-1) \cdot d_r$$

Where the anomalous fractal dimension d_r is independent of r

Wosiek, APPB 19 (1988) 863
Bialas, Hwa, PLB 253 (1991) 436
Bialas,Peschanski, NPB 273(1986) 703
Antoniou, Diakonos, Kapoyannis, Kousouris, PRL 97 (2006) 032002

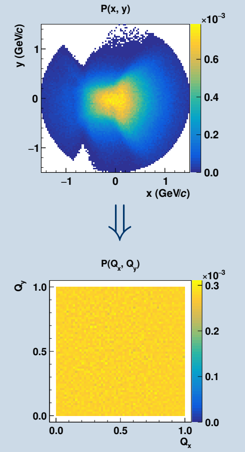
Cumulative variables

Instead of using p_x and p_y, one can use cumulative quantities:

$$Q_x = \int_{x_{min}}^{x} \rho(x)dx \bigg/ \int_{x_{min}}^{x_{max}} \rho(x)dx$$

$$Q_y = \int_{x_{min}}^{y} \rho(x)dx \bigg/ \int_{x_{min}}^{x_{max}} \rho(x,y)dy$$

- transform any distribution into uniform distribution (0,1)
- remove the dependence of F_r on the shape of the single-particle distribution
- intermittency index of an ideal power-law correlation function system described in two dimensions in momentum space was proven to remain approximately invariant after the transformation



(example for 0-5% Ar+Sc at 150A GeV/c)

Bialas, Gazdzicki, PLB 252 (1990) 483
Antoniou, Diakonos, https://indico.cern.ch/event/818624/

Preliminary results on intermittency analysis

Pb+Pb at 13A GeV/c ($\sqrt{s_{NN}} \approx 5.1$ GeV)

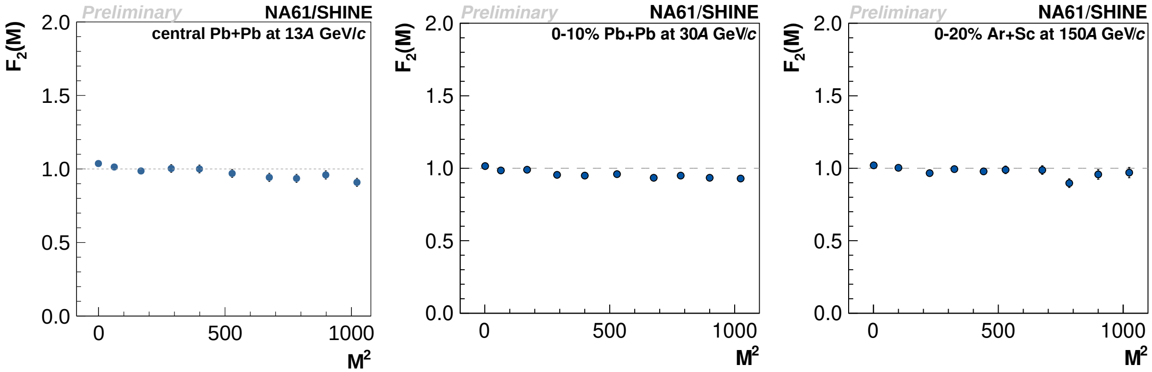
Pb+Pb at 30A GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)

Ar+Sc at 150A GeV/c ($\sqrt{s_{NN}} \approx 17$ GeV)

Cumulative variables
Independent sub-sample for each M points

Proton intermittency analysis result

(Experimental result on fluctuations as a function of momentum bin size)



CPOD 2021: https://indico.cern.ch/event/985460

ICNFP 2021: https://indico.cern.ch/event/1025480/

ISNP ERICE 2021: http://crunch.ikp.physik.tu-darmstadt.de/erice/2021

Quark Matter 2022 https://indico.cern.ch/event/895086/contributions/4555252/

No indication for power-law increase with bin size

Simple power-law model

A simple model that generates momentum of particles for a given number of events with a given multiplicity and transverse momentum distributions from data

It has two main parameters:

- ratio of correlated to uncorrelated particles
- power-law exponent

Correlated pairs (signal)

$$\rho(|\Delta\vec{p}_T|) = (|\Delta\vec{p}_T|)^{-\varphi}$$

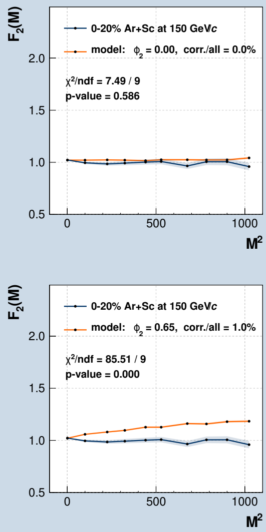
Lots of model data sets are generated:

- correlated-to-all ratio: vary from 0.0 to 4.0% (with 0.2 steps)
- power-law-exponent: vary from 0.0 to 1.0 (with 0.05 steps)

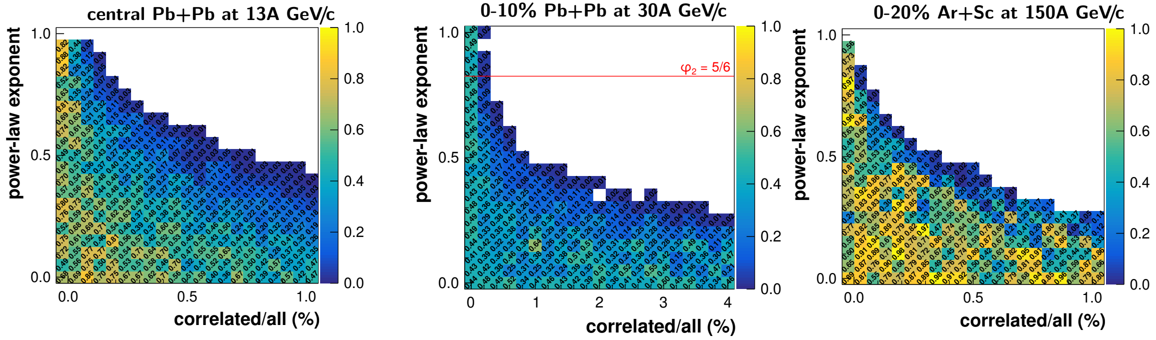
and compared with the experimental data

For the construction of exclusion plots, statistical uncertainties were calculated using model with statistics corresponding to the data.

M = 1 ... 32



Exclusion plot



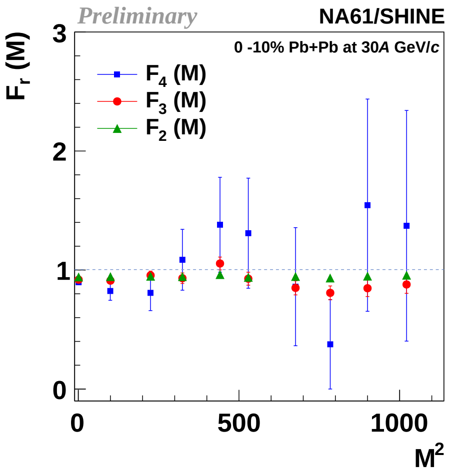
white area: p-value < 0.01

exclusion plots for parameters of simple power-law model

The intermittency index φ_2 for a system freezing out at the critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that belongs to the 3-D Ising universality class

Negatively charged hadron intermittency analysis

(Higher order moments in Pb+Pb at 30*A* GeV/*c* data)



Higher order scaled factorial moments

$$F_r(M) = \frac{r!(M^2)^{r-1}}{<N>^r}\left\langle \sum_{m=1}^{M^2}\binom{n_m}{r}\right\rangle$$

$$F_2(M) = \frac{2M^2}{\langle N\rangle^2}\langle N_2\rangle$$

$$F_3(M) = \frac{2M^4}{\langle N\rangle^4}\langle N_3\rangle$$

$$F_4(M) = \frac{2M^2}{\langle N\rangle^6}\langle N_4\rangle$$

M: number of bins in p_x and p_y

N: event multiplicity

n_m: numbers of particles in ith bin

$< \ldots >$: averaging over events

$N_2(M)$: number of pairs of particles

$N_3(M)$: number of triplets of particles

$N_4(M)$: number of quadruplets of particles

Expected relationship between intermittency indices for higher order moments:

$$\varphi_2 = \frac{\varphi_3}{2} = \frac{\varphi_4}{3}$$

No indication for power-law increase with bin size

Summary

- Results on the dependence of scaled factorial moments of multiplicity distribution on cumulative momentum bin size, analyzed using independent data points for:
  - protons in Pb+Pb at 13A GeV/c ($\sqrt{s_{NN}} \approx 5.1$ GeV)
  - protons in Pb+Pb at 30A GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)
  - protons in Ar+Sc at 150A GeV/c ($\sqrt{s_{NN}} \approx 17$ GeV)
  - negatively charged hadrons in Pb+Pb at 30A GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)

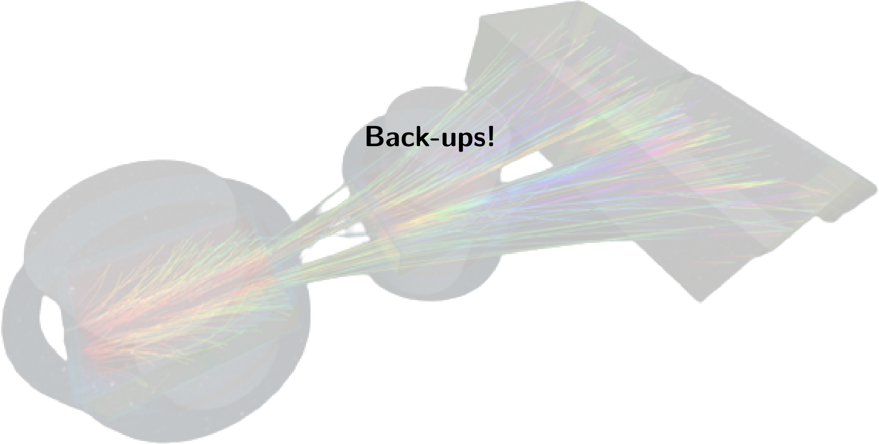
  show no indication of a power-law increase

- Exclusion plots for parameters of a simple model (ratio of correlated to background particles and power-law exponent) are shown

- We are continuing negatively charged hadron intermittency analysis for Pb+Pb at 13A GeV/c and proton intermittency analysis for Ar+Sc at 13A - 75A GeV/c data

Acknowledgement

My work is supported by the National Science Centre, Poland under grant no. 2018/30/A/ST2/0026 & the NA61/SHINE SPS CERN Collaboration

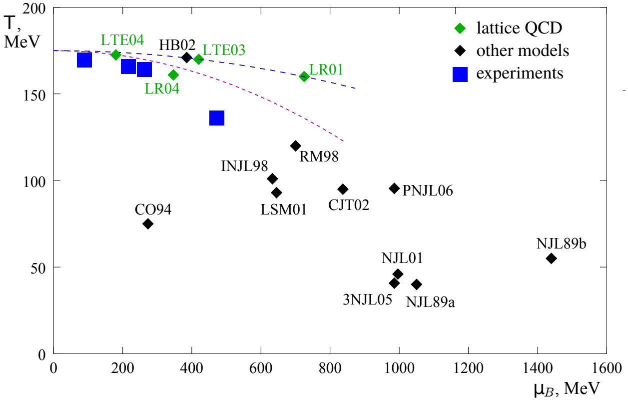


Back-ups!

QCD critical point

critical point search strategies

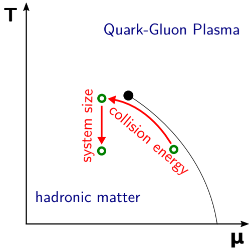
The main signal of the CP is anomaly in fluctuations in a narrow domain of the phase diagram.



However predictions on the CP existence, its location and what and how should fluctuate are model-dependent.

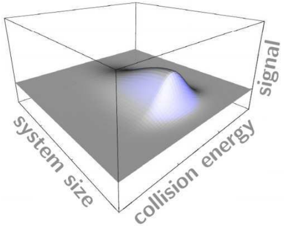
Exploring the phase diagram with heavy-ion collisions

critical point search strategies



Search for the critical point in heavy-ion collisions is performed by scan in the parameters controlled in laboratory (collision energy and nuclear mass number).

By changing them, we change freeze-out parameters (T,μ_B)



Sketch of the **critical hill** expected in the search for the critical point in the two dimensional plane of system size and collision energy.

Extensive quantities

A quantity proportional to W (WNM) or V in (IB-GCE) is called an extensive quantity. The most popular are particle number (multiplicity) distribution P(N) cumulants:

- $\kappa_1 = \langle N \rangle$
- $\kappa_2 = \left\langle (\delta N)^2 \right\rangle = \sigma^2$
- $\kappa_3 = \left\langle (\delta N)^3 \right\rangle = S\sigma^3$
- $\kappa_4 = \left\langle (\delta N)^4 \right\rangle - 3 \left\langle (\delta N)^2 \right\rangle^2 = \kappa\sigma^4$

These multiplicity cumulants characterize the shape of multiplicity distribution and quantify fluctuations.

WNM – Wounded Nucleon Model ($\langle N_{A+B} \rangle = \langle W_{A+B} \rangle /2 \cdot \langle N_{N+N} \rangle$)

IB-GCE – Ideal Boltzmann Grand Canonical Ensamble

Bialas, Bleszynski Czyz, NPB 111 461 (1976)

Intensive quantities

Fluctuations in large momentum bins

Ratio of any two extensive quantities is independent of W (WNM) or V (IB-GCE). It is an intensive quantity.
For example:

$$\langle A\rangle \,/\, \langle B\rangle = W \cdot \langle a\rangle \,/ W \cdot \langle b\rangle = \langle a\rangle \,/\, \langle b\rangle$$

where A and B are any extensive event quantities, i.e. $\langle \mathrm{A}\rangle \sim \mathrm{W}$, $\langle \mathrm{B}\rangle \sim \mathrm{W}$ and $\langle \mathrm{a}\rangle = \langle \mathrm{A}\rangle$ and $\langle \mathrm{b}\rangle = \langle \mathrm{B}\rangle$ for $\mathrm{W} = 1$.

Popular examples:

- $\frac{\kappa_2}{\kappa_1} = \omega[\mathrm{N}] = \frac{\sigma^2[\mathrm{N}]}{\langle \mathrm{N}\rangle} = \frac{\mathrm{W}\cdot\sigma^2[\mathrm{n}]}{\mathrm{W}\cdot\langle \mathrm{n}\rangle} = \omega[\mathrm{n}]$ (scaled variance)
- $\frac{\kappa_3}{\kappa_2} = \mathrm{S}\sigma$
- $\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$

Strongly intensive quantities

Fluctuations in large momentum bins

For an event sample with varying W, cumulants are not extensive quantities anymore. For example:

$$\kappa_2 = \sigma^2[N] = \sigma^2[n] \langle W \rangle + \langle n \rangle^2 \sigma^2[W]$$

But having two extensive event quantities, one can construct quantities that are independent of P(W)!

Popular example:

$$\Sigma[N, P_T] = \frac{1}{C}(\omega[N] \langle P_T \rangle + \omega[B] \langle N \rangle - 2(\langle NP_T \rangle - \langle P_T \rangle \langle N \rangle))$$

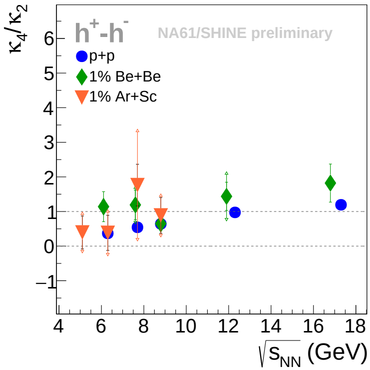
Where $P_T = \sum_{i=1}^{N} p_{T,i}$ and C is any extensive quantity (e.g. $\langle N \rangle$)

Gazdzicki, Gorenstein, PRC 84(2011) 014904

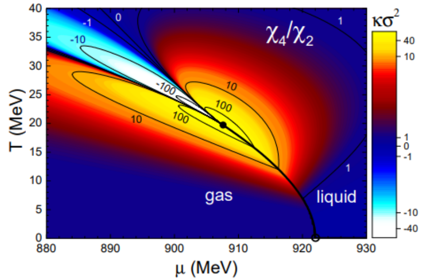
Gazdzicki, Gorenstein, Mackowiak-Pawlowska PRC 88(2013) 024907

Multiplicity fluctuations

Experimental results on Fluctuations in large momentum bins



Critical fluctuations in models with van der Waals interactions



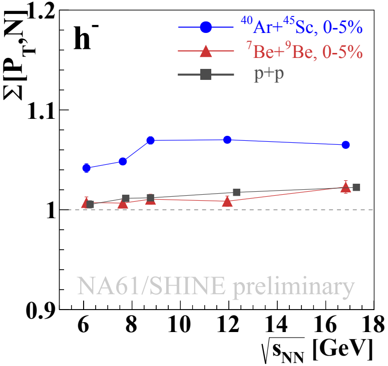
No prominent structures that could be related to the critical point are observed so far...

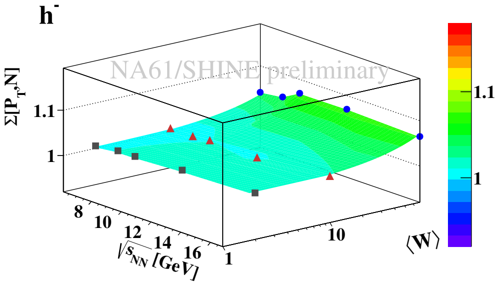
NA61/SHINE: PoS CPOD2017 (2018) 012

V. Vovchenko, D. V. Anchishkin, and M. I. Gorenstein, J. Phys. A48,305001 (2015)

Multiplicity-transverse momentum fluctuations

Experimental results on Fluctuations in large momentum bins



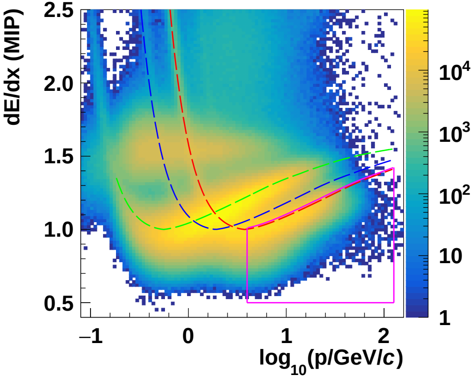


No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: Acta Phys.Polon.Supp. 10(2017) 449

Particle identification

Example: proton candidates selection
(Pb+Pb at 30A GeV/c)



- - - - - : theoretical BB function for protons
- - - - - : theoretical BB function for kaons
- - - - - : theoretical BB function for pions

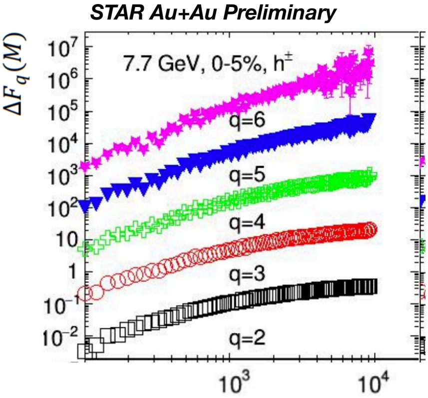
selection of protons is based on dEdx measurements in TPCs

- $0.60 < log_{10}(p/GeV/c) < 2.10$
- $0.5 \leq$ dEdx $\leq BB_{proton} + 0.15(\ BB_{kaon}$ - $BB_{proton})$

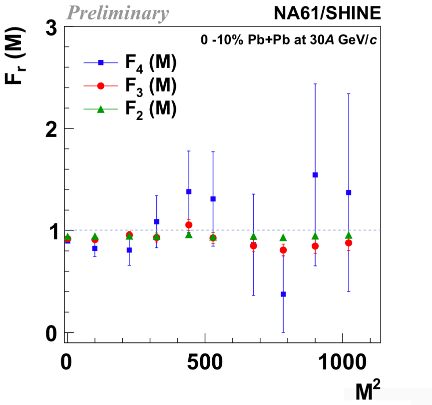
around 60% protons are selected with few percent kaon contamination

BB: theoretical Bethe-Bloch function

STAR vs NA61/SHINE preliminary results on ΔFq(M)



$$\Delta F_q(M) = F_q^{data}(M) - F_q^{mix}(M)$$



calculated for cumulative variables

h-

This seems to be in tension with corresponding results by the STAR Collaboration

Intermittency analysis

In NA61/SHINE , intermittency analysis is performed at mid-rapidity and particle fluctuations are studied in transverse momentum plane to locate the critical point of the strongly interacting matter measuring scaled factorial moments of multiplicity distribution

modified equivalent formula

$$F_r(M) = \frac{r!(M^2)^{r-1}}{< N >^r} \left\langle \sum_{m=1}^{M^2} \binom{n_m}{r} \right\rangle$$

for r =2

$$F_2(M) = \frac{2M^2}{\langle N \rangle^2} \langle N_{pp} \rangle$$

M: number of bins in p_x and p_y

N: event multiplicity

n_i: numbers of particles in ith bin

$< .... >$: averaging over events

N_{pp}: total number of pairs in M^2 bins in an event

Error propagation:

$$\frac{\sigma_{F_r}}{|F_r|} = \sqrt{\frac{Var(N_r)}{\langle N_r \rangle^2} + r^2 \frac{Var(N)}{\langle N \rangle^2} - 2r \frac{Cov(N_r, N)}{\langle N \rangle \langle N_r \rangle}}$$

Scaled factorial moments in WNM

In WNM: $N = \sum_{i=1}^{W} n_i$; W: constant number of wounded nucleon

$$\langle N \rangle = W \cdot \langle n \rangle \text{ and } \omega[N] = \omega[n]$$

Second scaled factorial moments in WNM:

$$F_2[N] = \frac{1}{W} F_2[n] + 1 - \frac{1}{W}$$

F_2[N] is neither extensive ($\approx$ W), nor intensive (const(W))

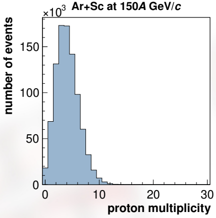
For F_2[n] $>>$ 1, W $>>$ 1 limit,

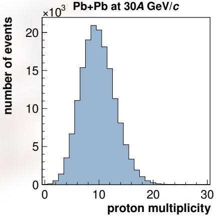
$$F_2[N] = \frac{1}{W} F_2[n]$$

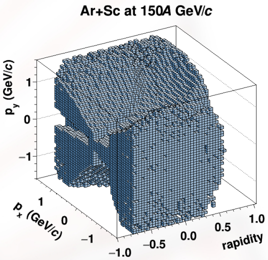
Scaled factorial moments is inversely extensive quantities

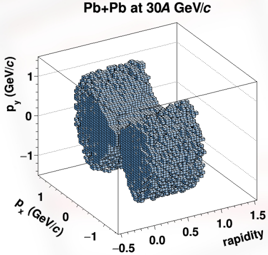
Analysis acceptance

$|p_x| < 1.5$ GeV/c
$|p_y| < 1.5$ GeV/c
Ar+Sc: -0.75 $<$ rapidity $<$ 0.75
Pb+Pb: 0.00 $<$ rapidity $<$ 0.75









mTTD cut

- Time Projection Chambers (TPCs) do not allow to reconstruct tracks too close to each other
- To reject those close tracks momentum-based two-track distance cut introduced for Pb+Pb at 30A GeV/c and Ar+Sc at 150A GeV/c data

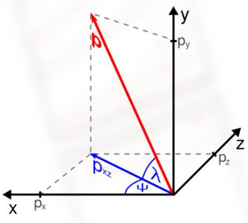
Momentum based two-tracks distance cut

Coordinates of momentum will be used:

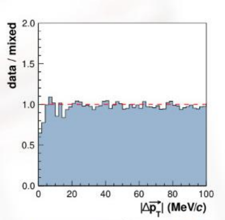
$$s_x = \frac{p_x}{p_{xz}} = cos(\Psi)$$
$$s_y = \frac{p_y}{p_{xz}} = sin(\lambda)$$
$$\rho = \frac{1}{p_{xz}}$$

For pairs:

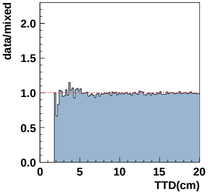
$$\Delta s_x = s_{x_2} - s_{x_1}$$
$$\Delta s_y = s_{y_2} - s_{y_1}$$
$$\Delta \rho = \rho_2 - \rho_1$$



Effect of mTTD cut:



Effect seen for $|\Delta \vec{p_T}| < 10$ MeV/c



Link: momentum based two track distance cut note

Intermittency analysis in high energy physics

- Analysis of particle number fluctuations as a function of momentum bin size
- Optimized for detection of scale invariant properties of particle production
- Introduced by Bialas and Peschanski in 1985

Moments of Rapidity Distributions as a Measure of Short Range Fluctuations in High-Energy Collisions

A. Bialas (Orsay, LPT and Jagiellonian U.), Robert B. Peschanski (Saclay) (Sep 1, 1985)

Published in: *Nucl.Phys.B* 273 (1986) 703-718

DOI cite 1,033 cit

*Intermittency basically means random deviations from smooth or regular behavior, concept of intermittency originally developed in the study of turbulent flow

Why intermittency analysis to search for the QCD critical point?

INTERMITTENCY AND CRITICAL BEHAVIOUR

Helmut SATZ

Theory Division, CERN, CH-1211 Geneva 23, Switzerland
and
Fakultät für Physik, Universität Bielefeld, D-48 Bielefeld, FRG

Received 9 March 1989

Following a brief introduction to intermittent behaviour, we show that the Ising model leads to intermittency at the critical point. The intermittency indices are given in terms of the critical exponents. This result is expected to hold for second-order phase transitions in general; for finite temperature SU(2) gauge theory, it follows from universality.

- J. Wosiek has found a hints for intermittent behaviour in the critical region of the two-dimentional Ising model
- This suggested the general question: what, if any, relation is there between intermittency and critical behaviour?
- The answer is of the interest also for the study of high energy nuclear collisions, in which one hopes to find evidence for the transition from HM to QGP

Why proton intermittency analysis to search for QCD critical point?

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Proton-Number Fluctuation as a Signal of the QCD Critical End Point

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We argue that the event-by-event fluctuation of the proton number is a meaningful and promising observable for the purpose of detecting the QCD critical end point in heavy-ion collision experiments. The long range fluctuation of the order parameter induces a characteristic correlation between protons which can be measured. The proton fluctuation also manifests itself as anomalous enhancement of charge fluctuations near the end point, which might be already seen in existing data.

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- Protons are carry both the baryon and electric charges. They are sensitive to the fluctuation of the order parameter
- At the critical end-point, the singularity of the baryon number susceptibility is completely reflected in the proton numbers fluctuation

Why intermittency analysis in transverse momentum space

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Critical Opalescence in Baryonic QCD Matter

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We show that critical opalescence, a clear signature of second-order phase transition in conventional matter, manifests itself as critical intermittency in QCD matter produced in experiments with nuclei. This behavior is revealed in transverse momentum spectra as a pattern of power laws in factorial moments, to all orders, associated with baryon production. This phenomenon together with a similar effect in the isoscalar sector of pions (sigma mode) provide us with a set of observables associated with the search for the QCD critical point in experiments with nuclei at high energies.

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- Intermittency attempts to detect a geometrical fractal structure in the fireball created by the collision; evolution of the system in time degrades this structure, and the deformation is much more severe in the longitudinal direction than in the transverse one
- In case of intermittency analysis in rapidity distribution, in the longitudinal direction, there is possibility of string branching, which can gives possible power-law signal

Why analysis in mid-rapidity region?

Search for critical fluctuations of the proton density in central A+A collisions at maximum SPS energy

Research Article

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Abstract: We performed an intermittency analysis of the proton density fluctuations in transverse momentum space for the collisions Si+A (A=Al,Si,P) and C+A (A=C,N) at maximum SPS energy ($\sqrt{s_{NN}} \approx 17$ GeV). In our analysis we used exclusively proton tracks in the midrapidity region ($|y_{CM}| \leq 0.75$). For the Si+A system we find signature of power-law distributed density fluctuations quantified by the intermittency index ϕ_2 which approaches in size the predictions of critical QCD [Phys. Rev. Lett. 97, 032002 (2006)]. This result supports further the recent findings of power-law fluctuations in the density of (π^+, π^-) pairs with invariant mass close to their production threshold for the Si+Si at the same energy, reported in [Phys. Rev. C 81, 064907 (2010)].

The critical intermittency index is determined from the first principle (universality class arguments) and it is valid for protons produced in the mid-rapidity region which necessary for avoiding spectators in the considered data-sets