New results for Reggeons field using FRG

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Outline

- Introduction  Pomeran and Odderon
- ERG approach to  Study critical properties of the Reggeon model
- Numerical results and Running parameters
- Summary and Outlook
Introduction

- Scattering process: description in terms of QCD (parton distributions) in the Regge Limit involving Strong Interaction

- In the Regge Limit becomes an effective 2+1 dimensional: transversal space and rapidity (Lipatov effective action)

- At very small transverse distances: pQCD and BFKL Pomeron (1958)

- At very large transverse distances before QCD era, there was the Reggeon Field Theory description introduced by Gribov

- The Pomeron state of two gluon and $C=1$

- 1973 Odderon Lukaszuk and Nicolescu the partner of the Pomeron ($C=-1$) 3 gluons

There are another states with 4 gluons….
V. N. Gribov introduce in the 60’s

Scattering amplitude at high energies for hadrons is according Regge Theory: the exchange of “quasi particles” characterized by its Regge trajectories: \( \alpha_i(t) \)

The total cross section is given by:

\[
\sigma_T = A_i S^{\alpha_i(0) - 1}
\]

Leading Pole: is Called Pomeron with vacuum quantum numbers

\[
\alpha(t) = \alpha_0 + \alpha' t \quad \alpha_0 \text{ is the Pomeron intercept}
\]

and \( \alpha' \) is the slope
• cross section grows with energy (Soft Pomeron exchange)

• $t$-dependence of elastic cross section shows difference between pp and ppbar (evidence for existence of Odderon)
Hard Pomeron \( pQCD \)

BFKL Kernel

Balinsky, Fadin, Kuraev, Lipatov

- For Hard processes short Transversal distances \( p + p \rightarrow p + p \)

we consider \( pQCD \rightarrow \text{Hard Pomeron dominate scattering} \)

Diffractive Scattering

\[
\alpha_p(0) \approx 1 + \frac{\alpha_s N_c}{\pi} 4 \ln 2 \approx 1 + 0.5295 \quad \text{Hard BFKL/QCD}
\]
\[ \alpha_P(0) \approx 1 + 4 \frac{\alpha_s N_c}{\pi} \ln 2 \quad \text{Hard BFKL/QCD} \]

\[ \alpha_P(0) \approx 1.08 \quad \text{soft} \]

\( \alpha_{P,k}(0) \) can be considered as a variable which depends on the sizes of the projectiles.

How we can connect regions of different sizes and different sorts of Pomerons

Use: Functional Renormalization Group*
Did TOTEM experiment discover the Odderon?

Evgenij Martynov, Basarab Nicolescu

Abstract

The present study shows that the new TOTEM datum $p\bar{p} = 0.098 \pm 0.01$ can be considered as the first experimental discovery of the Odderon, namely in its maximal form.

Keywords: Froissard, Maximal Odderon, total cross sections, the phase of the forward amplitude.

1. Introduction

Very recently, the TOTEM experiment released the following values at $\sqrt{s} = 13$ TeV of $pp$ total cross section $\sigma_{pp}$ and $\sigma_{p\bar{p}}$ parameter $[1]$. The Odderon is defined as a singularity in the complex $j$-plane, located at $j = 1$ when $t = 0$ and which contributes to the odd-under-crossing amplitude $F_{-}$. It was first introduced in 1973 on the theoretical basis of the Pomeron. According to Ref. [1], the collinear Pomeron is given by $[1]$

Odderon and proton substructure from a model-independent Lévy imaging of elastic $pp$ and $p\bar{p}$

T. Csörgő, R. Pasechnik and A. Ster  

Leading Pomeron Contributions and the TOTEM Data at 13 TeV

M. Broilo  E.G.S. Luna  M.J. Menon  
arXiv:1803.06560
Comparison between D0 $p\bar{p}$ at 1.96 TeV and the extrapolated TOTEM $pp$ cross section

$$\frac{d\sigma_{p\bar{p}}}{dt} - \frac{d\sigma_{pp}}{dt} = ?$$
How we can study another states with 3, 4 gluons?

**Multi-Reggeons equation BKP**

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

What we can study:

- Solutions
- the Intercept and the Slope

\[
\frac{\partial f}{\partial Y} = K_{12} \otimes f
\]

\[
\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O
\]
Odderon is a crucial test of QCD

- pQCD the Odderon was studied and is bound state of three reggeized gluons

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

Solutions for the BKP equation:

**Hard Odderon**

- Janik - Wosiek 1999 with an intercept $\alpha_0 = 0.96$
- Bartels, Lipatov, Vacca 2000 with an intercept exactly equal to one
- Variational calculations $\alpha_0 > 1$ and $\alpha_0 < 1$
- Lattice and Spectroscopy several calculations, all indicating a low intercept. However, the way in which this intercept is identified in lattice calculations is questionable.

H. B. Meyer, PhD thesis at Oxford, hep-lat/0508002
Only few experimental evidences for the non-perturbative Odderon

We are looking till now only in channels where the Odderon is hidden by the huge Pomeron contribution

We need experimental and theoretical evidence for the perturbative and non perturbative Odderon.

Searching for the Odderon in Ultraperipheral Proton-Ion Collisions at the LHC non perturbative Odderon.


Chung I Tan
Pomeron/Odderon Ads/CFT

P. Lebiedowicz, O. Nachtmann A. Szczurek arXiv 1901.11490
How we can see the Odderon!!

CGC: JIMWKL and dipole approximation: BFKL or BK equation

For a nuclear target, two-gluon multiple rescatterings bring in powers of \( \alpha_s^2 A^{1/3} \sim 1 \), so that single odderon exchange makes the amplitude to be of the order \( \alpha_s^3 A^{1/3} \sim \alpha_s \ll 1 \)
Soft Odderon parameters used in different papers

- $C = -1$ partner of Pomeron Lukaszuk - Nicolescu ’73
- C. Ewerz report hep-th/0306137 discussion of intercept and slop for $P$ and $O$
- Intercept: $\alpha_P > \alpha_O$ and Slope $\alpha'_P \sim \alpha'_O$
- Odderon Exchange: Ansatz for the propagator $C = -1$ Odderon is
  \[
i\Delta_{\mu\nu}^O = -ig_{\mu\nu} \frac{\eta_O}{M_0^2} (s \alpha'_O)^{\alpha(t)-1}
  \]
  \[
  \alpha(t) = \alpha_O(0) + \alpha'_O t
  \]
  And $\eta_O = -1, \ M_0^2 = 1 \text{ GeV}^2, \ \alpha_O(0) = 1.05 \ \alpha'_O = 0.25 \text{ GeV}^{-2}$

P. Lebiedowicz, O. Nactmann A. Szczurek
arXiv 1901.11490
Odderon: Hard to Soft

We need more non-perturbative information about:

- Intercept
- Slope
- Pomeron Vertices
- But we need from the Hard Odderon Information
- We have the idea that exist one $P \to O$ transition from soft to hard

Using FRG we can investigate if

Hard Odderon and Pomeron in the UV region are related
with Soft Odderon and Pomeron in the IR region
ERG ideas

Wilson 1976
Reuter and Wetterich 93

- The effective Wilson-Action is defined by Integrations of d.o.f in the UV $k < p < \Lambda$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R_k \right)^{-1} \partial_t R_k \right]$$

Boundary Condition $k = \Lambda$

$$\Gamma_{\Lambda} \text{ with the condition}$$

$$\Gamma_{k \rightarrow \Lambda} \rightarrow S_{\text{clas}} \text{ and } \Gamma_{k \rightarrow 0} \rightarrow QT$$

Berges, Tetradis and Wetterich: hep-ph/0005122
\( \Gamma_k(\phi, g_i; i, 1..M) \) define a \( M \) dimensional Space of Coupling constants

**Local Expansion** of \( \Gamma_k(\phi, g_i) = \sum_i g_i(k) O_i(\phi) \) and \( \{O_i\} \) operator basis.

**Derivative expansion in term of RG time** \( t = \ln(\frac{k}{\Lambda}) \)

\[ \partial_t \Gamma_k = \sum_i \partial_t g_i(k) * O_i(\phi) \rightarrow \beta_i(k) \] associated with operators \( \{O_i\} \)

**Beta Funtions:** \( \beta_i(k) = \partial_t g_i(k) \)

**Critical Properties:** Fixed Points Conditions \( \rightarrow \partial_t \Gamma_k^{FP} = \beta_i(k) = 0 \)

**Critical Exponets**

\[ g_i \sim g_i^* + \sum_a c_a e^{\lambda_a t} v_i^a \]

Linealizations of the Flow equations close to a FP:

If \( \lambda_a > 0 \) define an IR – Critical Surface with relevant behaviour, where \( \lambda_a < 0 \) is an UV

\[ k < \Lambda \rightarrow IR; \quad k > \Lambda \rightarrow UV \]
**Local effective RFT**

\[ \Gamma[\psi^\dagger, \psi, \chi^\dagger, \chi] = \int d^D x \, d\tau \left( Z_P \left( \frac{1}{2} \psi^\dagger \partial_\tau \psi - \alpha_P^\dagger \psi^\dagger \nabla^2 \psi \right) + Z_O \left( \frac{1}{2} \chi^\dagger \partial_\tau \chi - \alpha_O^\dagger \chi^\dagger \nabla^2 \chi \right) + V_k[\psi, \psi^\dagger, \chi, \chi^\dagger] \right). \]

**Fourier Transformation**

\[ \Phi(\omega, q)^T = (\psi(\omega, q) \, \psi^\dagger(-\omega, q)) \]

\[ R_k = \begin{pmatrix} 0 & R_k \\ R_k & 0 \end{pmatrix} \]

**Regulator**

\[ \Gamma_k^{(2)} + R = \begin{pmatrix} V_k \psi \psi & -iZ_k \omega + Z_k \alpha_k^\dagger q^2 + R_k + V_k \psi \psi^\dagger \\ iZ_k \omega + Z_k \alpha_k q^2 + R_k + V_k \psi^\dagger \psi & V_k \psi^\dagger \psi^\dagger \end{pmatrix} \]

**Flow Equations**

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi^\dagger \delta \phi} + R_k \right)^{-1} \partial_t R_k \right] \]

Optimized cutoff (in time): \( R_k(p^2) = (k^2 - p^2)\theta(k^2 - p^2). \)
LPA: Cubic potential

\[ V_3 = -\mu P \psi^\dagger \psi + i \lambda \psi^\dagger (\psi + \psi^\dagger) \psi - \mu O \chi^\dagger \chi + i \lambda_2 \chi^\dagger (\psi + \psi^\dagger) \chi + \lambda_3 (\psi^\dagger \chi^2 + \chi^\dagger^2 \psi). \]

Quartic Potential

\[ V_4 = \lambda_{41} (\psi \psi^\dagger)^2 + \lambda_{42} \psi \psi^\dagger (\psi^2 + \psi^\dagger^2) + \lambda_{43} (\chi \chi^\dagger)^2 + i \lambda_{44} \chi \chi^\dagger (\chi^2 + \chi^\dagger^2) + i \lambda_{45} \psi \psi^\dagger (\chi^2 + \chi^\dagger^2) + \lambda_{46} \psi \psi^\dagger \chi \chi^\dagger + \lambda_{47} \chi \chi^\dagger (\psi^2 + \psi^\dagger^2). \]

High order Potential

\[ V_5 = i \left( \lambda_{51} (\psi \psi^\dagger)^2 (\psi + \psi^\dagger) + \lambda_{52} \psi \psi^\dagger (\psi^3 + \psi^\dagger^3) + \lambda_{53} \chi \chi^\dagger (\psi^3 + \psi^\dagger^3) + \lambda_{54} \psi \psi^\dagger \chi \chi^\dagger (\psi + \psi^\dagger) \right) + \lambda_{55} (\chi^2 \psi^\dagger^3 + \chi^\dagger^2 \psi^3) + \lambda_{56} (\chi^2 \psi^\dagger^2 \psi + \chi^\dagger^2 \psi^\dagger \psi^2) + \lambda_{57} (\chi^2 \psi^\dagger \psi^2 + \chi^\dagger^2 \psi^\dagger^2 \psi) + i \left( \lambda_{58} (\chi^4 \psi^\dagger + \chi^\dagger^4 \psi) + \lambda_{59} (\chi \chi^\dagger)^2 (\psi + \psi^\dagger) \right) + \lambda_{510} \chi \chi^\dagger (\chi^2 \psi + \chi^\dagger^2 \psi^\dagger) + \lambda_{511} \chi \chi^\dagger (\chi^2 \psi^\dagger + \chi^\dagger^2 \psi). \]
Dimensionless variables:

\[ \tilde{\mu}_P = \frac{\mu_P}{Z_P \alpha'_P k^2}, \quad \tilde{\mu}_O = \frac{\mu_O}{Z_O \alpha'_P k^2}, \]

\[ \tilde{\lambda} = \frac{\lambda}{Z_P^{3/2} \alpha'_P k^2} k^{D/2}, \quad \tilde{\lambda}_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2} \alpha'_P k^2} k^{D/2}. \]

Anomalous dimensions

\[ \eta_P = -\frac{1}{Z_P} \partial_t Z_P, \quad \eta_O = -\frac{1}{Z_O} \partial_t Z_O \]

\[ \zeta_P = -\frac{1}{\alpha'_P} \partial_t \alpha'_P, \quad \zeta_O = -\frac{1}{\alpha'_O} \partial_t \alpha'_O. \]

\[ r = \frac{\alpha'_O}{\alpha'_P}, \]

\[ \alpha'_O = r \alpha'_P. \]

\[ \dot{r} = r \left(-\zeta_O + \zeta_P\right), \]
Flow Equations:

\[
\begin{align*}
\dot{\mu}_P &= (-2 + \eta_P + \zeta_P)\mu_P + 2A_P \frac{\lambda^2}{(1 - \mu_P)^2} - 2A_{OR} \frac{\lambda^2_3}{(r - \mu_O)^2} \\
\dot{\mu}_O &= (-2 + \eta_O + \zeta_P)\mu_O + 2(A_P + A_{OR}) \frac{\lambda^2}{(1 + r - \mu_P - \mu_O)^2} \\
\dot{\lambda} &= (-2 + D/2 + \zeta_P + \frac{3}{2} \eta_P)\lambda + 8A_P \frac{\lambda^3}{(1 - \mu_P)^3} - 4A_{OR} \frac{\lambda^2_3}{(r - \mu_O)^3} \\
\dot{\lambda}_2 &= (-2 + D/2 + \zeta_P + \frac{1}{2} \eta_P + \eta_O)\lambda_2 \\
&\quad + \frac{2\lambda\lambda^2_2(6A_P + 5A_{OR}) + 4\lambda^3(2A_P + A_{OR}) - 4\lambda_2\lambda^2_3(A_P + 2A_{OR})}{(1 + r - \mu_P - \mu_O)^3} \\
&\quad + \frac{2A_P\lambda\lambda^2_2(r - \mu_O)^2}{(1 - \mu_P)^2(1 + r - \mu_P - \mu_O)^3} - \frac{4A_{OR}\lambda_2\lambda^2_3(1 - \mu_P)^2}{(1 - \mu_O)^2(1 + r - \mu_P - \mu_O)^3} \\
&\quad + \frac{2\lambda\lambda^2_2(3A_P + A_{OR})(r - \mu_O)}{(1 - \mu_P)(1 + r - \mu_P - \mu_O)^3} - \frac{4\lambda_2\lambda^2_3(A_P + 3A_{OR})(1 - \mu_P)}{(r - \mu_O)(1 + r - \mu_P - \mu_O)^3} \\
\dot{\lambda}_3 &= (-2 + D/2 + \zeta_P + \frac{1}{2} \eta_P + \eta_O)\lambda_3 \\
&\quad + \frac{2\lambda\lambda^2_3(A_P + 2A_{OR})}{(r - \mu_O)(1 + r - \mu_P - \mu_O)^2} + \frac{4\lambda\lambda_2\lambda^2_3(2A_P + A_{OR})}{(1 - \mu_P)(1 + r - \mu_P - \mu_O)^2} \\
&\quad + \frac{2\lambda\lambda^2_3A_{OR}(1 - \mu_P)}{(r - \mu_O)^2(1 + r - \mu_P - \mu_O)^2} + \frac{4\lambda\lambda_2\lambda^2_3A_P(r - \mu_O)}{(1 - \mu_P)^2(1 + r - \mu_P - \mu_O)^2}.\end{align*}
\]
Fixed Points:

\[ \mu = \alpha_0 - 1 \]

\[ \mu_\sigma = 0.274381, \mu_\sigma = 0.26979 \]

\[ r = 0.88018 \]

\[ \lambda = 1.34738, \lambda_2 = 1.79401, \lambda_3 = 0. \]

Anomalous dimensions:

\[ \eta_\sigma \approx -0.33, \eta_\sigma \approx -0.35 \]

\[ \zeta_\sigma = \zeta_\sigma \approx +0.17. \]

The result is ok with the \( \varepsilon \)-expansion

arXiv 2001.0599 M. Braum and G. P. Vacca
Results II:

The convergence is under control with the increasing the local truncation

\[ \nu = - \frac{1}{\text{most negative eigenvalue}} \]
Results III

Renormalization Trajectories

Intercepts:

\[ \mu O = \alpha_0 - 1 \]

Slopes:

\[ \dot{r} = r (-\zeta_O + \zeta_P) \]

\[ \zeta_P = \zeta_O \approx +0.17 \]

\[ r = 0.88018 \]
1980 Cardy and Sugar found that the RFT is in the same Universality class of “Percolation”

**Percolation and Monte Carlo Simulation:**

The critical Exponent $\nu = 0.73$ with is related with our $\nu = -1/(\text{most negative eigenvalue})$

$\nu_3 = 0.52; \nu_4 = 0.59; \nu_5 = 0.69; \nu_6 = 0.78; \nu_7 = 0.76, ...$
Summary

Using Exact Renormalization Group approach critical properties of the Pomeron - Odderon Model are studied:

- We found a IR fixed Point

- Our results indicate that the running Odderon Intercept $\alpha_0[t]$ is consistent with the BLV solution: Intercept is One

- Only in special value of the running coupling of P O $(\lambda_1, \lambda_2)$, the Intercept $\alpha_0 < 1$

- The Odderon Slopes is relative constant at the IR fixed* and $\alpha'_p > \alpha'_0$

*M. Braun and G. P. Vacca similar result using bootstrap (private communication)

- We need more Information about the value of the UV coupling of POO to find with Renormalization Group Trajectory could describe the Physical data.
THANK YOU