New results for Reggeons field using FRG

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Outline

- Introduction Pomeron and Odderon
- ERG approach to Study critical properties of the Reggeon model
- Numerical results and Running parameters
- Summary and Outlook

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JHEP 1901(2019) 004
***PRD 104* (2021)**

Introduction

- Scattering process: description in terms of QCD (parton distributions) in the Regge Limit involving Strong Interaction

- In the Regge Limit becomes an effective 2+1 dimensional: transversal space and rapidity (Lipatov effective action)

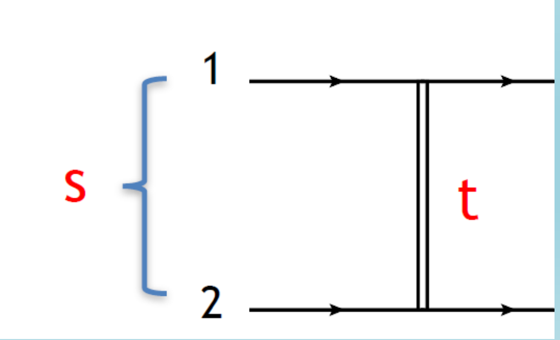
- At very small transverse distances: pQCD and BFKL Pomeron (1958)

- At very large transverse distances before QCD era, there was the Reggeon Field Theory description introduced by Gribov

- The Pomeron state of two gluon and C=1

- 1973 Odderon Lukaszuk and Nicolescu the partner of the Pomeron (C =-1) 3 gluons

There are another states with 4 gluons….

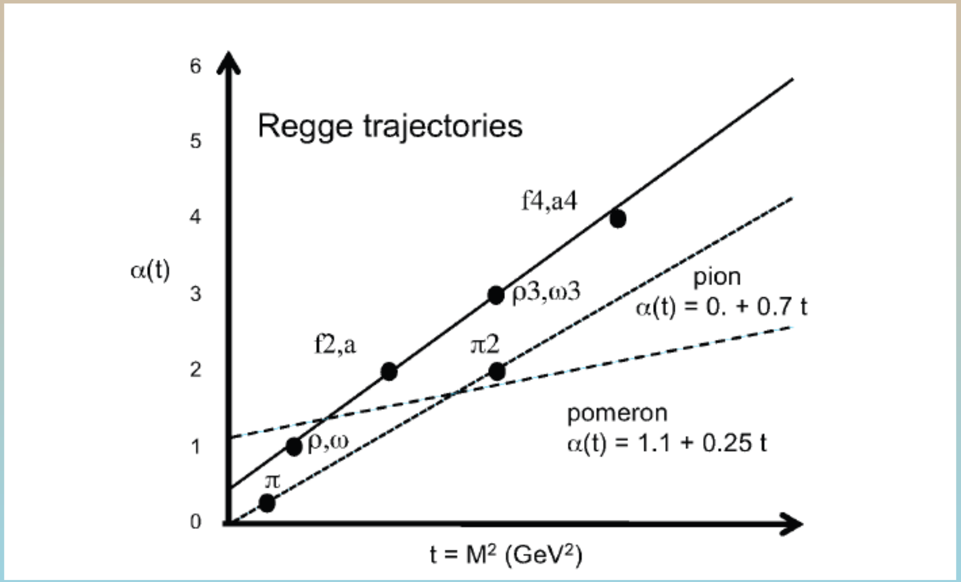


ZEUS Collaboration 1995
Result from HERA: evidence for the Pomeron

Reggeon Field Theory before QCD

P.D.B. Collins, *An introduction to Regge theory and high energy physics*, Cambridge University Press, Cambridge, 1977.

- V. N. Gribov introduce in the 60´s
- Scattering amplitude at high energies for hadrons is according Regge Theory: the exchange of "quasi particles" characterized by its Regge trajectories : $\alpha_i\,(t)$



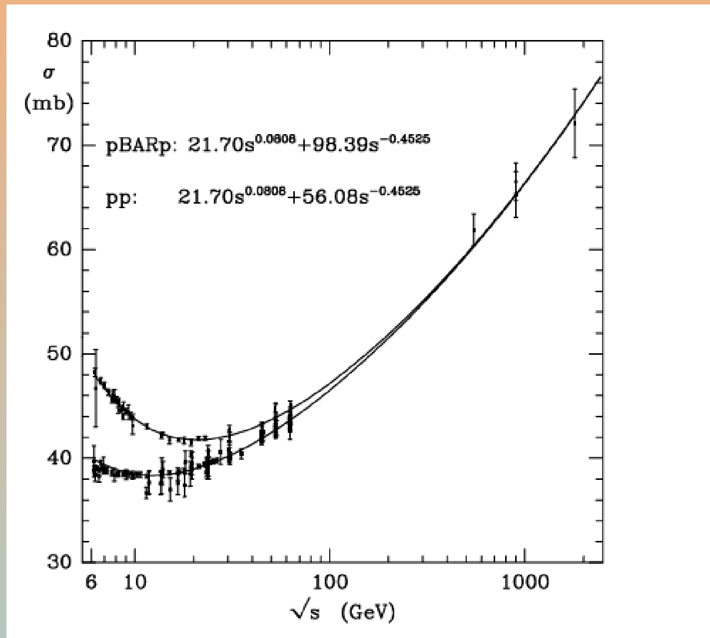
- the total Cross section, is given by: $\sigma_T = A_i s^{\alpha_i(0)-1}$
- Leading Pole: is Called Pomeron with vacuum quantum numbers

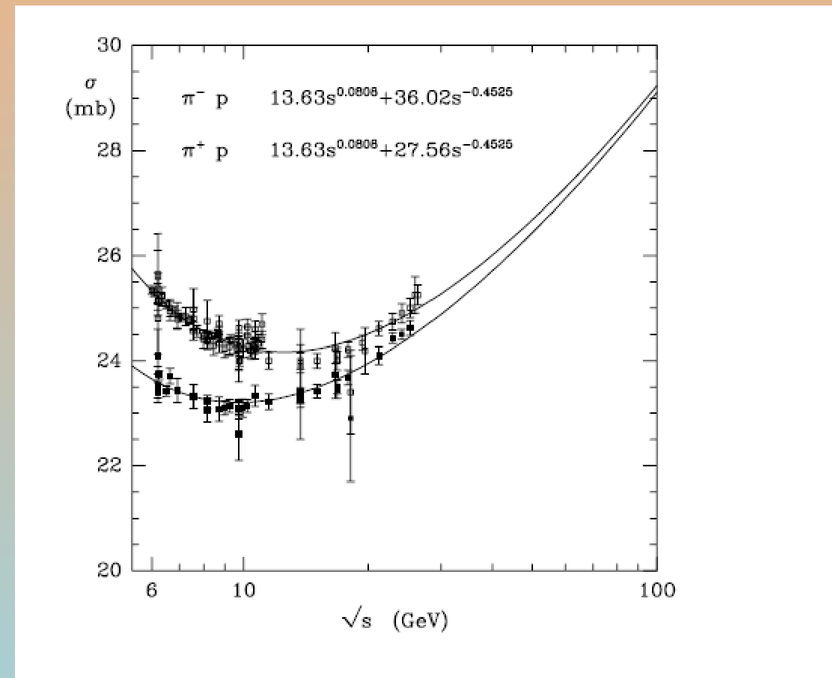
  $\alpha(t) = \alpha_0 + \alpha' t$ α_0 is the Pomeron intercept

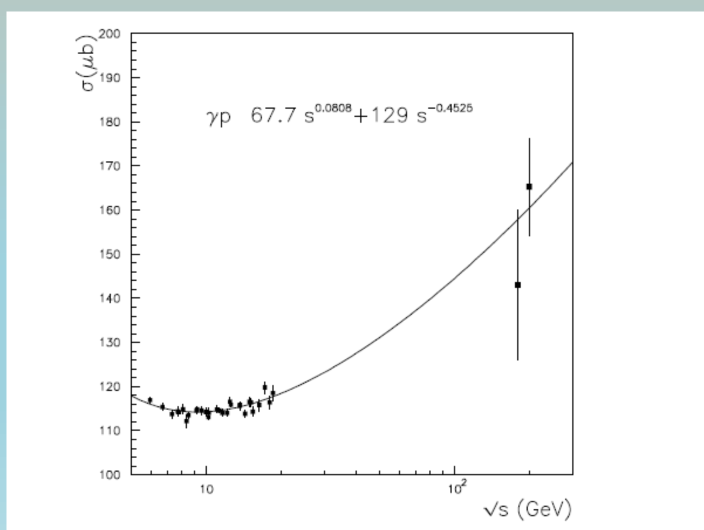
  and α' is the slope

A. Donnachie and Landshoff : arXiv 1309.1292 2013

$$\alpha_P(t) = 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$$







- cross section grows with energy (Soft Pomeron exchange)
- t-dependence of elastic cross section shows difference between pp and ppbar (evidence for existence of Odderon)

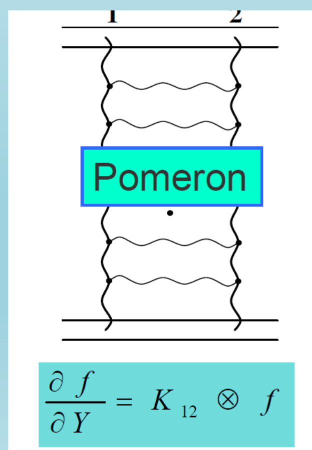
Hard Pomeron pQCD
BFKL Kernel

Balinsky, Fadin, Kuraev, Lipatov

- For Hard processes short Transversal distances $p + p \rightarrow p + p$

  we consider pQCD → Hard Pomeron dominate scattering
  Diffractive Scattering

$$\alpha_P(0) \approx 1 + \frac{\alpha_s N_c}{\pi} 4 \ln 2 \approx 1 + 0.5295 \quad Hard\ BFKL/QCD$$



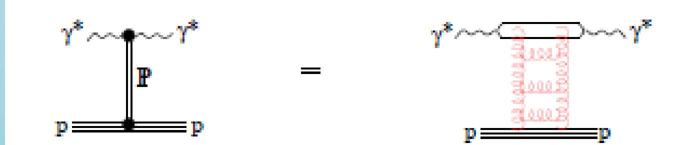


FIG. 4: The virtual photon interacts via its hadronic fluctuations which are $\bar{q}q$ dipoles and more complicated Fock states. The Pomeron exchange is illustrated as a perturbative ladder.

Soft Pomeron vs Hard Pomeron

$$\alpha_P(0) \approx 1 + 4\,\frac{\alpha_s N_c}{\pi}\ln 2 \qquad Hard\ BFKL/QCD$$

For Hard processes UV region
short Transversal distances
Large Momenta, large but finite energies

$$\alpha_P(0) \approx 1.08 \qquad soft$$

For soft processes
largest Transversal distances
small Momenta, large but finite energies

$\alpha_{P,k}(0)$ *can be considered as a variable which depends on the sizes of the projectiles.*

How we can connect regions of different sizes and different sorts of Pomerons

Use: *Functional Renormalization Group**

Phys.Lett.B 778 (2018) 414-418

21 Jan 2018

Did TOTEM experiment discover the Odderon?

Evgenij Martynov[a], Basarab Nicolescu[b]

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Abstract

The present study shows that the new TOTEM datum $\rho^{pp} = 0.098 \pm 0.01$ can be considered as the first experimental discovery of the Odderon, namely in its maximal form.

Keywords: Froissaron, Maximal Odderon, total cross sections, the phase of the forward amplitude.

1. Introduction

Very recently, the TOTEM experiment released the following values at $\sqrt{s} = 13$ TeV of pp total cross section σ^{pp} and ρ^{pp} parameter [1]

The Odderon is defined as a singularity in the complex j-plane, located at $j = 1$ when $t = 0$ and which contributes to the odd-under-crossing amplitude F_-. It was first introduced in 1973 on the theoretical basis of

Odderon and proton substructure from a model-independent Lévy imaging of elastic pp and p\bar{p}

T. Csörgö, R. Pasechnik and A. Ster *Eur. Phys. J. C* 79 (2019) 1, 62

Leading Pomeron Contributions and the TOTEM Data at 13 TeV

M. Broilo E.G.S. Luna M.J. Menon ***arXiv:1803.06560***

THE ODDERON DISCOVERY BY THE D0 AND TOTEM COLLABORATIONS 3

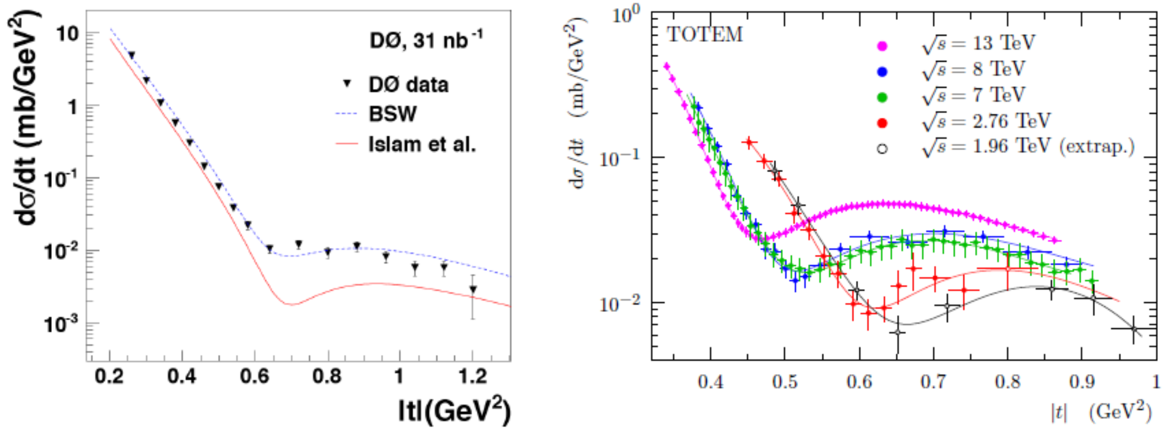
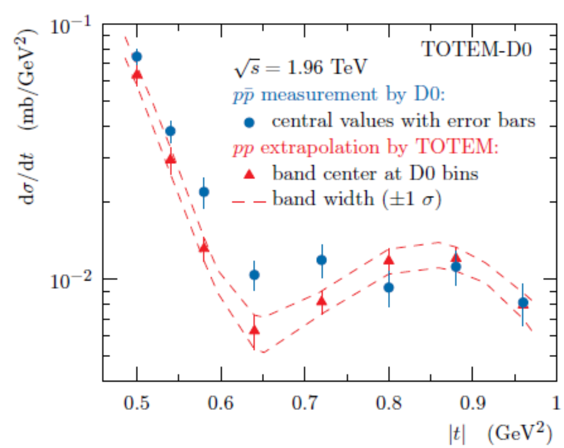


FIGURE 3. Left: $p\bar{p}$ elastic cross section as a function of $|t|$ at 1.96 TeV from the D0 collaboration at the Tevatron. Right: pp elastic cross sections as a function of $|t|$ at 2.76, 7, 8, and 13 TeV from the TOTEM collaboration at the LHC (full circles), and extrapolation to the Tevatron center-of-mass energy at 1.96 TeV (empty circles).

Comparison between D0 $p\bar{p}$ at 1.96 TeV and the extrapolated TOTEM pp cross section

$$\frac{d\sigma_{p\bar{p}}}{dt} - \frac{d\sigma_{pp}}{dt} = ?$$

CHRISTOPHE ROYON



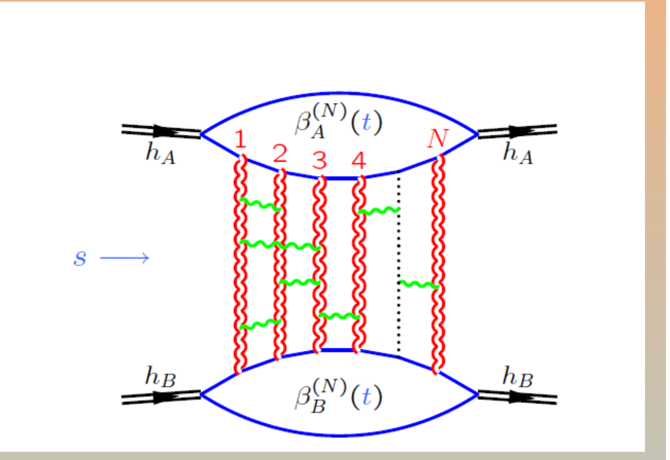
arXiv: 2203.02931

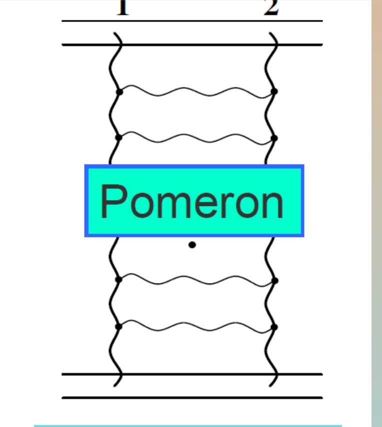
- This is evidence for the non-perturbative Odderon

How we can study another states with 3, 4 gluons ?

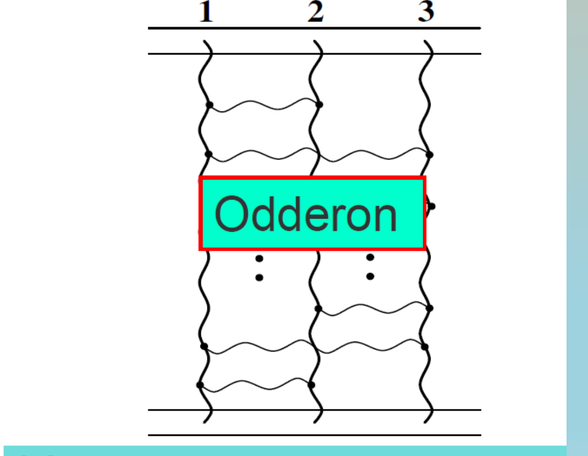
Multi-Reggeons equation BKP

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz





$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

What we can study:

- Solutions
- the Intercept and the Slope

Odderon is a crucial test of QCD

- pQCD the Odderon was studied and is bound state of three reggeized gluons

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

Solutions for the BKP equation:

Hard Odderon

- Janik - Wosiek 1999 with an intercept $\alpha_O = 0.96$
- Bartels, Lipatov, Vacca 2000 with an intercept exactly equal to one
- Variational calculations $\alpha_O > 1$ and $\alpha_O < 1$
- Lattice and Spectroscopy several calculations, all indicating a low intercept.

However, the way in which this intercept is identified in lattice calculations is questionable.

H. B. Meyer and M. J. Teper, Phys. Lett. B605 (2005) 344

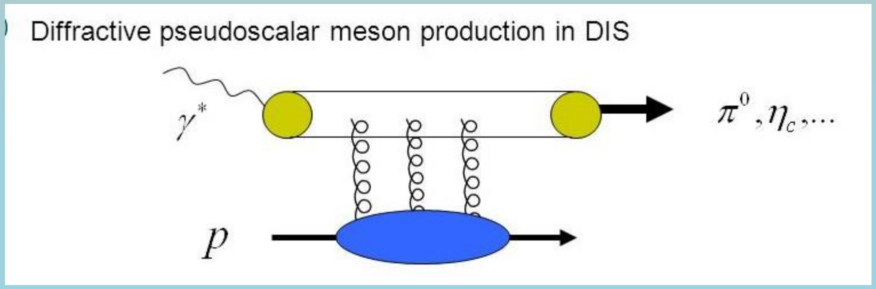
H. B. Meyer, PhD thesis at Oxford, hep-lat/0508002

Theory with Experimental data

- Only few experimental evidences for the non-perturbative Odderon
- We are looking till now only in channels where the Odderon is hidden by the huge Pomeron contribution
- We need experimental and theoretical evidence for the perturbative and non perturbative

Searching for the Odderon in Ultraperipheral Proton-Ion Collisions at the LHC non perturbative Odderon.

L.A. Harland-Lang, V.A. Khoze, A.D. Martinb and M.G. Ryskin arXiv:1811.12705

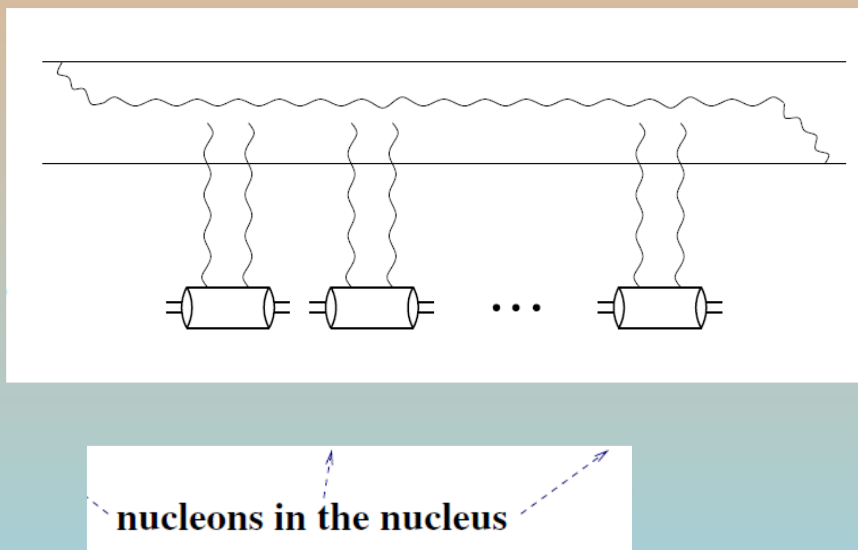


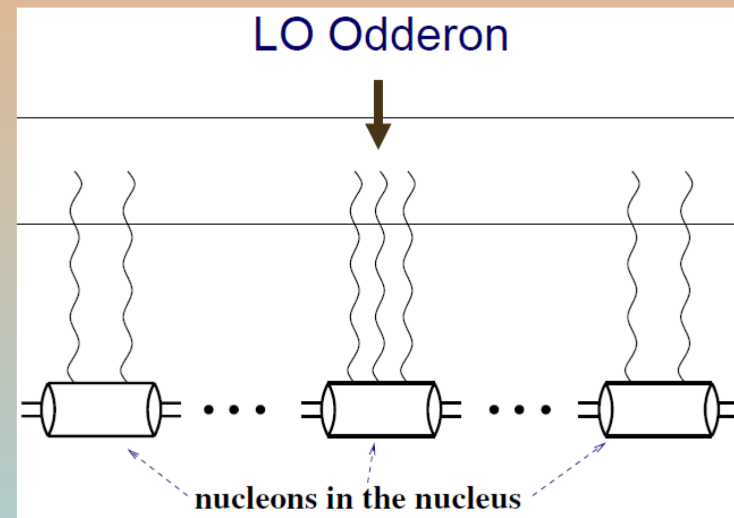
Chung I Tan
Pomeron/Odderon Ads/CFT

P. Lebiedowicz, O. Nachtmann A. Szczurek
arXiv 1901.11490

How we can see the Odderon!!

CGC: JIMWKL and dipole approximation: BFKL or BK equation





For a nuclear target, two-gluon multiple rescatterings bring in powers of $\alpha_S^2 A^{1/3} \sim 1$, so that single odderon exchange

makes the amplitude to be of the order $\alpha_S^3 A^{1/3} \sim \alpha_S << 1$

Soft Odderon parameters used in different papers

- C =- 1 partner of Pomeron Lukaszuk - Nicolescu ´73
- C. Ewerz report hep-th/0306137 discussion of intercept and slop for *P and O*
- Intercept: $\alpha_P > \alpha_O$ and Slope $\alpha'_P \sim \alpha'_O$
- Odderon Exchange: Ansatz for the propagator C = -1 Odderon is

$$i\Delta^{O}_{\mu\nu} = -\mathrm{i}g_{\mu\nu}\frac{\eta_O}{M_0^2}(s\,\alpha'_O)^{\alpha(t)-1}$$

$$\alpha(t) = \alpha_O(0) + \alpha'_O\,\mathrm{t}$$

And η_O = -1, M_0^2 = 1 GeV^2, $\alpha_O(0) = 1.05$ α'_O= 0.25 GeV^{-2}

P. Lebiedowicz, O. Nactmann A. Szczurek
arXiv 1901.11490

Odderon: Hard to Soft

We need more non perturbative information about:

- Intercept
- Slop
- Pomeron Vertices
- But we need from the Hard Odderon Information
- We have the idea that exist one ***P*** - ***O*** transition from soft to hard

Using FRG we can investigate if

Hard   Odderon and Pomeron in the UV region
are related
with Soft Odderon and Pomeron in the IR region

ERG ideas

Wilson 1976

Reuter and Wetterich 93

- The effective Wilson-Action is defined by Integrations of d.o.f in the UV $k < p < \Lambda$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\, \delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

Boundary Condition k = Λ

Γ_{Λ} with the condition

$\Gamma_{k\to\Lambda} \to S_{clas}$ and $\Gamma_{k\to 0} \to QT$

Berges, Tetradis and Wetterich: hep-ph/0005122

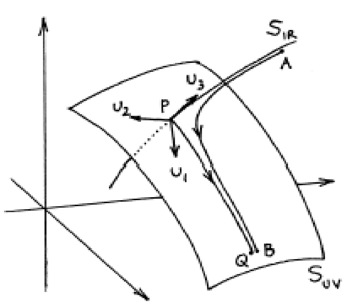
FRG Flows and Steps: $\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$

- $\Gamma_k(\phi, g_i; i, 1..M)$ define a **M dimensional Space of Coupling constants**
- **Local Expansion** of $\Gamma_k(\phi, g_i)) = \sum_i g_i(k)\, O_i(\phi)$ and $\{O_i\}$ operator basis.
- **Derivative expansion in term of RG time t** $= \ln(\frac{k}{\Lambda})$

$\partial_t \Gamma_k = \sum_i \partial_t g_i(k) * O_i(\phi) \rightarrow \beta_i(k)$ associated with operators $\{O_i\}$

- **Beta Funtions:** $\beta_i(k) = \partial_t g_i(k)$
- **Critical Properties:** Fixed Points Conditions $\rightarrow \partial_t \Gamma^*_{k\,FP} = \beta_i(k) = 0$
- **Critical Exponets**

$$g_i \simeq g_i^* + \sum_a c_a\, e^{\lambda_a t}\, v_i^a$$



Linealizations of the Flow equations close to a FP:
If $\lambda_a > 0$ define an IR – Critical Surface with relevant behaviour, where $\lambda_a < 0$ is an UV

$k < \Lambda \rightarrow IR$; $k > \Lambda \rightarrow UV$

Local effective RFT

$$\Gamma[\psi^\dagger,\psi,\chi^\dagger,\chi] = \int \mathrm{d}^D x\,\mathrm{d}\tau \Big(Z_P(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial}_\tau \psi - \alpha'_P \psi^\dagger \nabla^2 \psi) + Z_O(\frac{1}{2}\chi^\dagger \overleftrightarrow{\partial}_\tau \chi - \alpha'_O \chi^\dagger \nabla^2 \chi) + V_k[\psi,\psi^\dagger,\chi,\chi^\dagger] \Big). \quad (1)$$

Fourier Transformation

$$\Phi(\omega,q)^T = (\psi(\omega,q)\ \psi^\dagger(-\omega,q))$$

Regulator

$$\mathbb{R}_k = \begin{pmatrix} 0 & R_k \\ R_k & 0 \end{pmatrix}$$

$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi\psi^\dagger} \\ iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi^\dagger\psi} & V_{k\psi^\dagger\psi^\dagger} \end{pmatrix}$$

Flow Equations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k \right)^{-1} \partial_t R_k \right]$$

Optimized cutoff (Litim): $\mathcal{R}_k(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$.

LPA: Cubic potential

$$
\begin{aligned}
V_3 &= -\mu_P \psi^\dagger \psi + i\lambda \psi^\dagger (\psi + \psi^\dagger)\psi - \\
&\quad -\mu_O \chi^\dagger \chi + i\lambda_2 \chi^\dagger (\psi + \psi^\dagger)\chi + \lambda_3 (\psi^\dagger \chi^2 + {\chi^\dagger}^2 \psi).
\end{aligned}
$$

Quartic Potential

$$
\begin{aligned}
V_4 &= \lambda_{41} (\psi\psi^\dagger)^2 + \lambda_{42} \psi\psi^\dagger (\psi^2 + {\psi^\dagger}^2) + \lambda_{43} (\chi\chi^\dagger)^2 + i\lambda_{44} \chi\chi^\dagger (\chi^2 + {\chi^\dagger}^2) \\
&\quad + i\lambda_{45} \psi\psi^\dagger (\chi^2 + {\chi^\dagger}^2) + \lambda_{46} \psi\psi^\dagger \chi\chi^\dagger + \lambda_{47} \chi\chi^\dagger (\psi^2 + {\psi^\dagger}^2).
\end{aligned}
$$

High order Potential

$$
\begin{aligned}
V_5 &= i\left(\lambda_{51} (\psi\psi^\dagger)^2 (\psi + \psi^\dagger) + \lambda_{52}\, \psi\psi^\dagger (\psi^3 + {\psi^\dagger}^3) + \lambda_{53}\, \chi\chi^\dagger (\psi^3 + {\psi^\dagger}^3) + \lambda_{54}\, \psi\psi^\dagger \chi\chi^\dagger (\psi + \psi^\dagger)\right) \\
&\quad + \lambda_{55} (\chi^2 {\psi^\dagger}^3 + {\chi^\dagger}^2 \psi^3) + \lambda_{56}\, (\chi^2 {\psi^\dagger}^2 \psi + {\chi^\dagger}^2 \psi^\dagger \psi^2) + \lambda_{57}\, (\chi^2 \psi^\dagger \psi^2 + {\chi^\dagger}^2 {\psi^\dagger}^2 \psi) \\
&\quad + i\left(\lambda_{58}\, (\chi^4 \psi^\dagger + {\chi^\dagger}^4 \psi) + \lambda_{59}\, (\chi\chi^\dagger)^2 (\psi + \psi^\dagger)\right) \\
&\quad + \lambda_{510}\, \chi\chi^\dagger (\chi^2 \psi + {\chi^\dagger}^2 \psi^\dagger) + \lambda_{511}\, \chi\chi^\dagger (\chi^2 \psi^\dagger + {\chi^\dagger}^2 \psi).
\end{aligned}
$$

Dimensionless variables:

$$\tilde{\mu}_P = \frac{\mu_P}{Z_P \alpha'_P k^2}, \quad \tilde{\mu}_O = \frac{\mu_O}{Z_O \alpha'_P k^2},$$
$$\tilde{\lambda} = \frac{\lambda}{Z_P^{3/2} \alpha'_P k^2} k^{D/2}, \quad \tilde{\lambda}_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2} \alpha'_P k^2} k^{D/2}.$$

Anomalous dimensions

$$\eta_P = -\frac{1}{Z_P}\partial_t Z_P, \quad \eta_O = -\frac{1}{Z_O}\partial_t Z_O$$

$$\zeta_P = -\frac{1}{\alpha'_P}\partial_t \alpha'_P, \quad \zeta_O = -\frac{1}{\alpha'_O}\partial_t \alpha'_O.$$

$$r = \frac{\alpha'_O}{\alpha'_P},$$

$$\alpha'_O = r\,\alpha'_P.$$

$$\dot{r} = r\left(-\zeta_O + \zeta_P\right),$$

Flow Equations:

$$
\begin{aligned}
\dot{\mu}_P &= (-2+\eta_P+\zeta_P)\mu_P + 2A_P\frac{\lambda^2}{(1-\mu_P)^2} - 2A_O r\frac{\lambda_3^2}{(r-\mu_O)^2} \\
\dot{\mu}_O &= (-2+\eta_O+\zeta_P)\mu_O + 2(A_P+A_O r)\frac{\lambda_2^2}{(1+r-\mu_P-\mu_O)^2} \\
\dot{\lambda} &= (-2+D/2+\zeta_P+\frac{3}{2}\eta_P)\lambda + 8A_P\frac{\lambda^3}{(1-\mu_P)^3} - 4A_O r\frac{\lambda_2\lambda_3^2}{(r-\mu_O)^3} \\
\dot{\lambda}_2 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_2 \\
&\quad +\frac{2\lambda\lambda_2^2(6A_P+5A_O r)+4\lambda_2^3(A_P+A_O r)-4\lambda_2\lambda_3^2(A_P+2A_O r)}{(1+r-\mu_P-\mu_O)^3} \\
&\quad +\frac{2A_P\lambda\lambda_2^2(r-\mu_O)^2}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^3} - \frac{4A_O r\lambda_2\lambda_3^2(1-\mu_P)^2}{(1-\mu_O)^2(1+r-\mu_P-\mu_O)^3} \\
&\quad +\frac{2\lambda\lambda_2^2(3A_P+A_O r)(r-\mu_O)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^3} - \frac{4\lambda_2\lambda_3^2(A_P+3A_O r)(1-\mu_P)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^3} \\
\dot{\lambda}_3 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_3 \\
&\quad +\frac{2\lambda_2^2\lambda_3(A_P+2A_O r)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3(2A_P+A_O r)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^2} \\
&\quad +\frac{2\lambda_2^2\lambda_3 A_O r(1-\mu_P)}{(r-\mu_O)^2(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3 A_P(r-\mu_O)}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^2}.
\end{aligned}
$$

Fixed Points:

$$\mu_P = 0.274381, \ \mu_O = 0.26979$$

$$r = 0.88018$$

$$\mu = \alpha_0 - 1$$

$$\lambda = 1.34738, \ \lambda_2 = 1.79401, \ \lambda_3 = 0.$$

Anomalous dimensions :

$$\eta_P \simeq -0.33 \ , \eta_O \simeq -0.35$$

$$\zeta_P = \zeta_O \simeq +0.17.$$

The result is ok with the ε-expansion **arXiv 2001. 0599 M. Braum and G. P. Vacca**

$$\mu_P = \frac{\epsilon}{12}, \quad \lambda^2 = \frac{8\pi^2}{3}\epsilon, \quad \eta_P = -\frac{\epsilon}{6}, \quad \zeta_P = \zeta_O = \frac{\epsilon}{12},$$

$$\mu_O = \frac{95+17\sqrt{33}}{2304}\epsilon, \quad \lambda_2^2 = \frac{23\sqrt{6}+11\sqrt{22}}{48}\epsilon, \quad \lambda_3 = 0, \quad \eta_O = -\frac{7+\sqrt{33}}{72}\epsilon, \quad r = \frac{3}{16}(\sqrt{33}-1).$$

Results II:

The convergence is under control with the increasing the local truncation

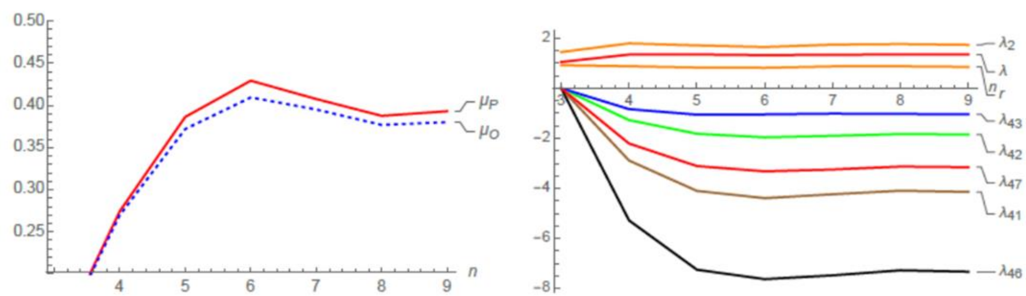
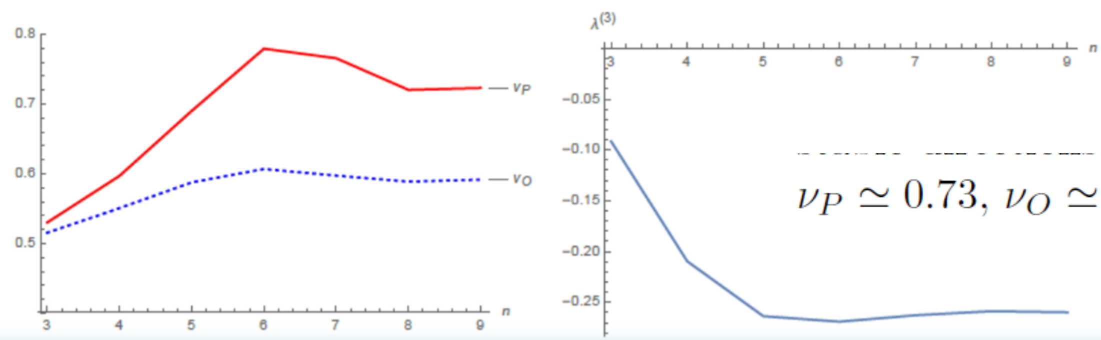


Figure 1: Values of the parameters of the fixed point solution of the LPA truncations for different orders n of the polynomial ($3 \leq n \leq 9$). The masses (which equal intercept minus one) μ_P (red curve) and μ_O (blue dotted curve) for the Pomeron and Odderon fields are in the left panel. The first non zero couplings $\lambda, \lambda_2, \lambda_{41}, \lambda_{42}, \lambda_{43}, \lambda_{46}, \lambda_{47}, r$ are reported on the right panel.



$$\nu_P \simeq 0.73, \; \nu_O \simeq 0.6.$$

ν = - 1/(most negative eigenvalue)

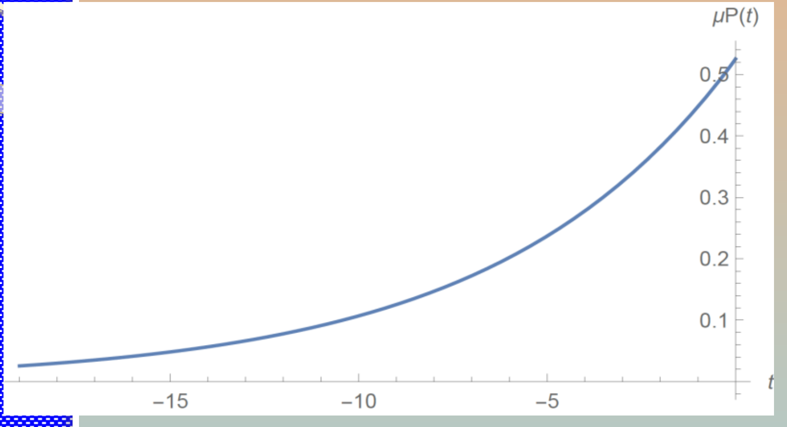
Results III

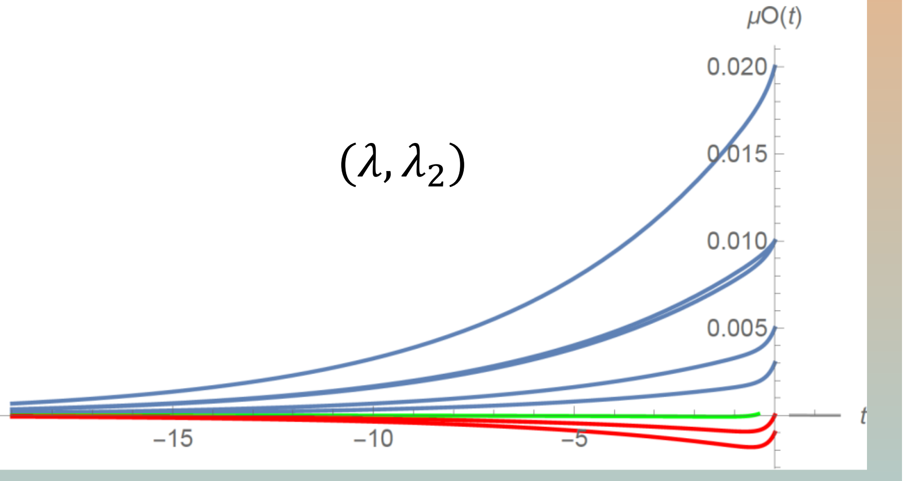
Renormalization Trajectories

$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$

Intercepts:

$$\mu O = \alpha_0 - 1$$





Slops

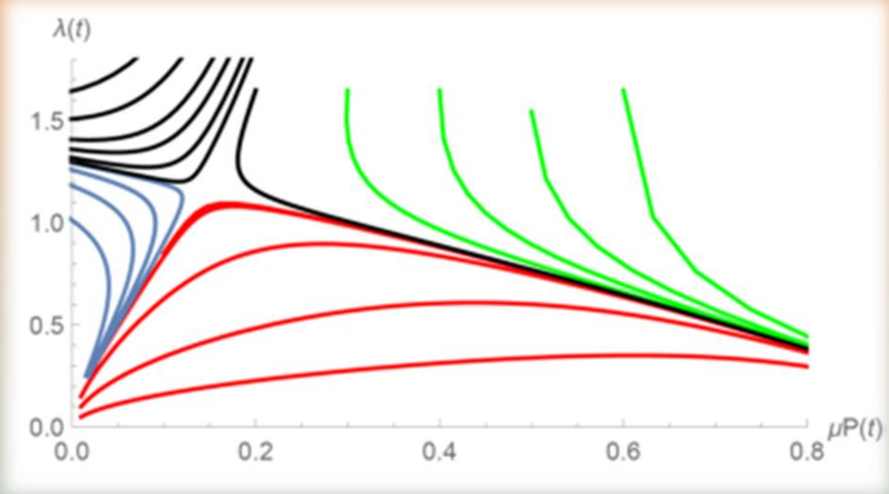
$$\dot{r} = r\left(-\zeta_O + \zeta_P\right),$$

$$\alpha'_O = r\,\alpha'_P.$$

$$\zeta_P = \zeta_O \simeq +0.17.$$

$$r = 0.88018$$

R IV: Renormalization Trajectories **Flow for only Pomeron**



1980 Cardy y Sugar found that the RFT is in the same Universality class of "Percolation"

Percolation and Monte Carlo Simulation:
The critical Exponent $\nu = 0.73$ with is related with our
ν = - 1/(most negative eigenvalue)

$$\nu_3 = 0.52\ ; \nu_4 = 0.59\ ; \nu_5 = 0.69\ ; \nu_6 = 0.78\ ; \nu_7 = 0.76\ , \ldots$$

Summary

Using Exact Renormalization Group approach critical properties of the Pomeron - Odderon Model are studied:

- We found a IR fixed Point

- Our results indicate that the running Odderon Intercept $\alpha_O[t]$ is consistent with the BLV solution: Intercept is One

- Only in special value of the running coupling of P O (λ, λ_2), the Intercept $\alpha_O < 1$

- The Odderon Slopes is relative constant at the IR fixed* and $\alpha'_P > \alpha'_O$

*M. Braun and G. P. Vacca similar result using bootstrap (private communication)

- We need more Information about the value of the UV coupling of POO to find with Renormalization Group Trajectory could describe the Physical data.

Thank You