Direct CP violation in $K \to \pi\pi$ decay at the physical point with periodic BCs

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K → ππ & Direct CPV

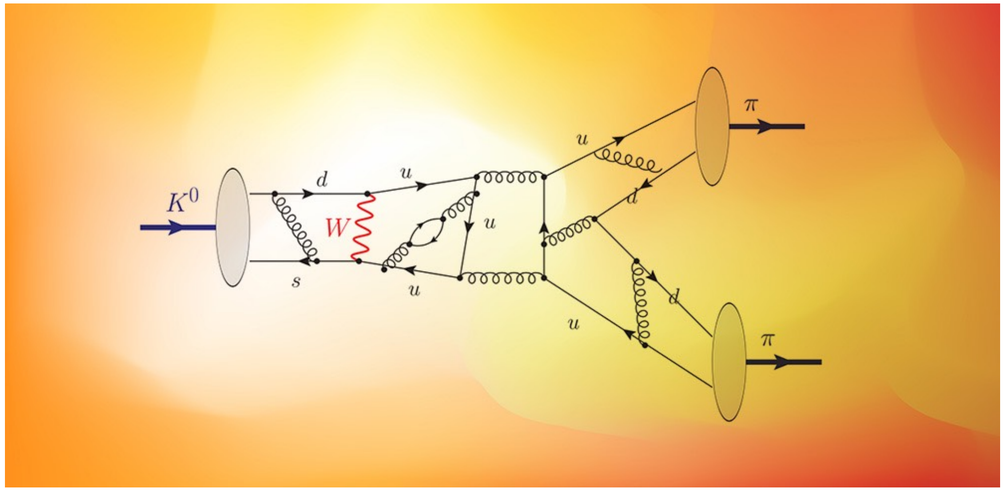
$$|K_L\rangle = |K_2\rangle + \varepsilon\,|K_1\rangle$$

CP odd — CP even

ε' direct CPV

ε indirect CPV

$|\pi\pi\rangle$ CP even

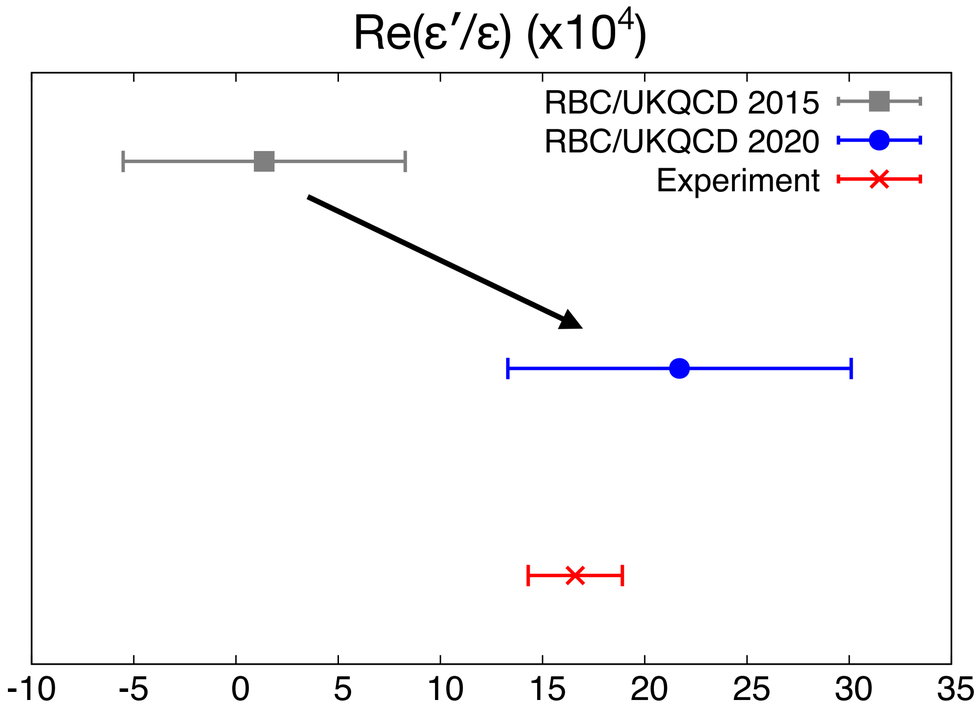


- ε' vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (KTeV, NA48)
  - Explained by SM?
- Key to understanding the nature of matter/anti-matter asymmetry

G-parity BC calculation done

- $E_{\pi\pi} = 2m_\pi \approx 280$ MeV state in Euclidean correlators forbidden
- Useful to extract $E_{\pi\pi} = m_K$ state at large time separations

PRD 102,054509



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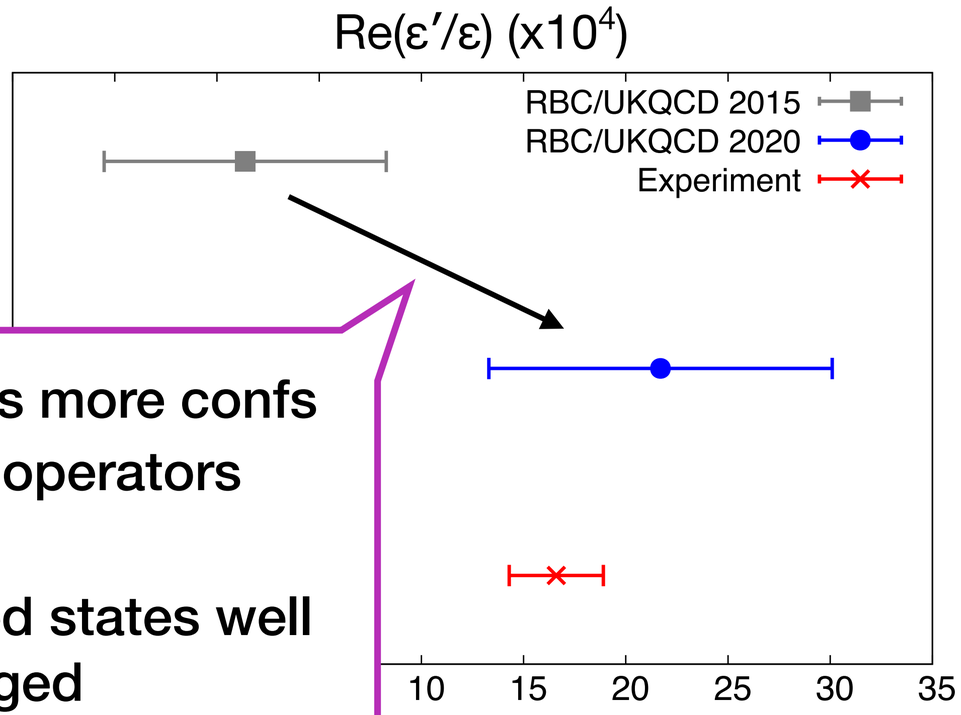

- 3+ times more confs
- # of ππ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

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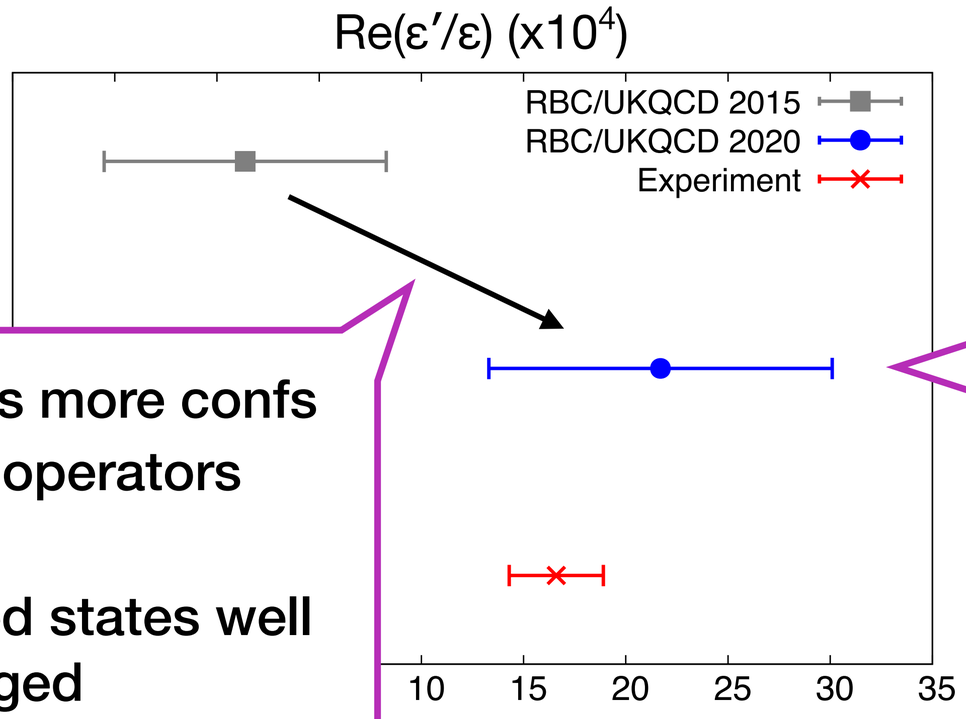

- 3+ times more confs
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- Step scaling in NPR

$$21.7(2.6)_{\text{stat}}(6.2)_{\text{sys}}(5.0)_{\text{EM/IB}} \times 10^{-4}$$

- Independent calculations desired
- Systematic errors
  - Isospin breaking effects
  - Finite lattice cutoff
  - Wilson coefficients

Why periodic BCs?

- Already have lattice ensembles with physical pion mass
  - 1.0 GeV, 24^3 x 64 & 1.4 GeV, 32^3x64 & …
  - Continuum limit possible

- Hope to introduce QED/IB effects near future
  - Difficult with G-parity BCs
  - Straightforward with periodic BCs

- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as in G-parity BCs: $\sim e^{-(m_K - 2m_\pi)t}$
  - interesting to see if it's possible to extract signal of excited state

Lattice setup

- RBC/UKQCD's 2+1-flavor ensembles with MDWFs at physical pion & kaon masses
  - $24^3 \times 64$, $a^{-1} = 1.0$ GeV, ~250 confs
  - $32^3 \times 64$, $a^{-1} = 1.4$ GeV, ~100 confs

- Chiral symmetry of DWFs → strong constraints on operator mixings
  - with lower-dimensional operators
  - among different representations WRT chiral symmetry (8,1), (8,8) & (27,1)

What to calculate

ππ phase shifts

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction

Renormalization matrix

$$A_I = F\frac{G_F}{2}V_{us}^* V_{ud}\sum_{i,j}\underbrace{[z_i(\mu)+\tau y_i(\mu)]}_{\text{Wilson coefs. pQCD}}\underbrace{Z_{ij}(\mu)}_{\text{LQCD (+pQCD)}}\underbrace{\langle(\pi\pi)_I|Q_j^{lat}|K\rangle}_{\text{LQCD}}$$

- δ_I, F being determined via π-π scattering work w/ GEVP & Lüscher formalism
- $A_I = \langle(\pi\pi)_I|H_W|K\rangle$ from 3pt correlation functions
- I = 0 challenging — disconnected diagrams, power divergences

Matrix elements

- For extraction of ground-state ME

$$M^{\rm eff}(t_2,t_1) = C^{(3)}(t_2,t_1)\left[\frac{e^{E^{\pi\pi}t_2}e^{E^K t_1}}{C^{\pi\pi}(t_2)C^K(t_1)}\right] \xrightarrow{\text{large } t_1 \ \& \ t_2} M$$

- Excited ππ state needed for on-shell kinematics with PBCs

$$\boxed{M_n^{\rm eff}(t_2,t_1) = C_n^{(3)}(t_2,t_1)\left[\frac{e^{E_n^{\pi\pi}t_2}e^{E^K t_1}}{C_n^{\pi\pi}(t_2)C^K(t_1)}\right]} \xrightarrow{\text{large } t_1 \ \& \ t_2} M_n$$

$C_n^{\pi\pi}$: 2-pt function of ππ operators that couple only with $|E_n^{\pi\pi}\rangle$ & $|E > E_{\rm max}^{\pi\pi}\rangle$

$C_n^{(3)}$: K → ππ 3-pt function with ππ operator used in $C_n^{\pi\pi}$

State decompositions

- 2pt functions of interpolation operators:

$$C_{ab}(t) = \langle O_a(t) O_b(0)^\dagger \rangle = \sum_n A_{n,a} A^*_{n,b} e^{-E_n t}$$

- Good combinations of interpolation operators:

$$O_a \to O'_n = \sum_a v^*_{n,a} O_a$$

$$C'_n(t) = \sum_{a,b} v^*_{n,a} C_{ab}(t) v_{n,b} = A'_n A'^*_n e^{-E_n t} + O(e^{-E_{N+1} t})$$

- $v_{n,a}$ well determined by solving GEVP (Generalized Eigenvalue Problem)

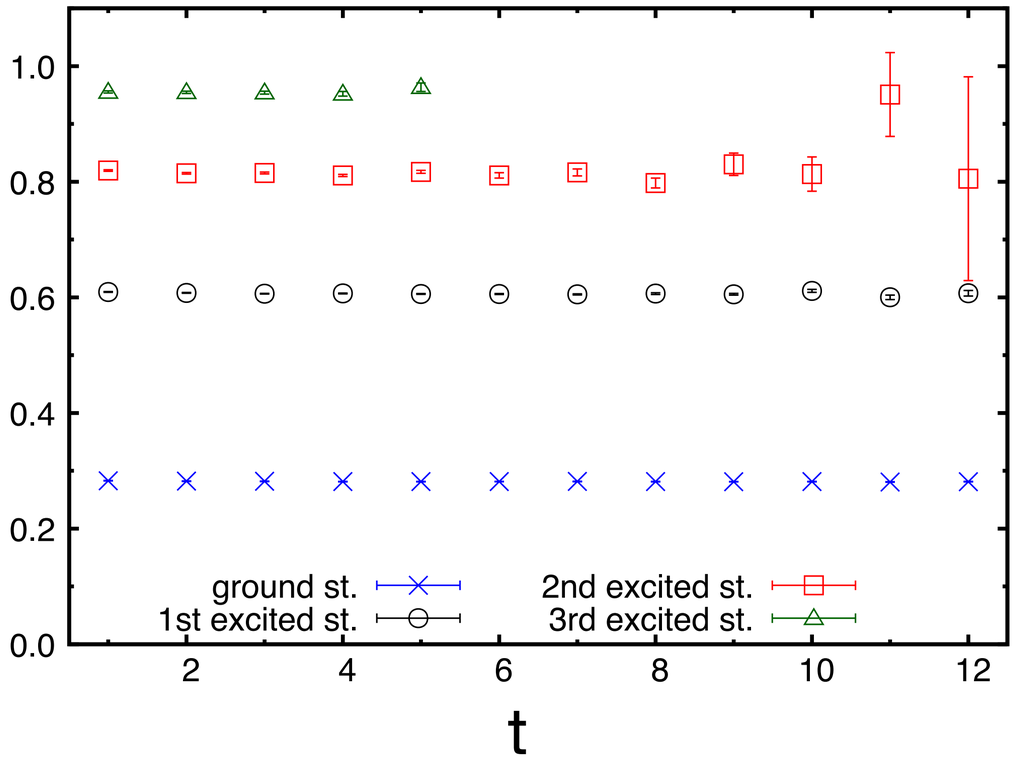
$$C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0)$$

ππ states

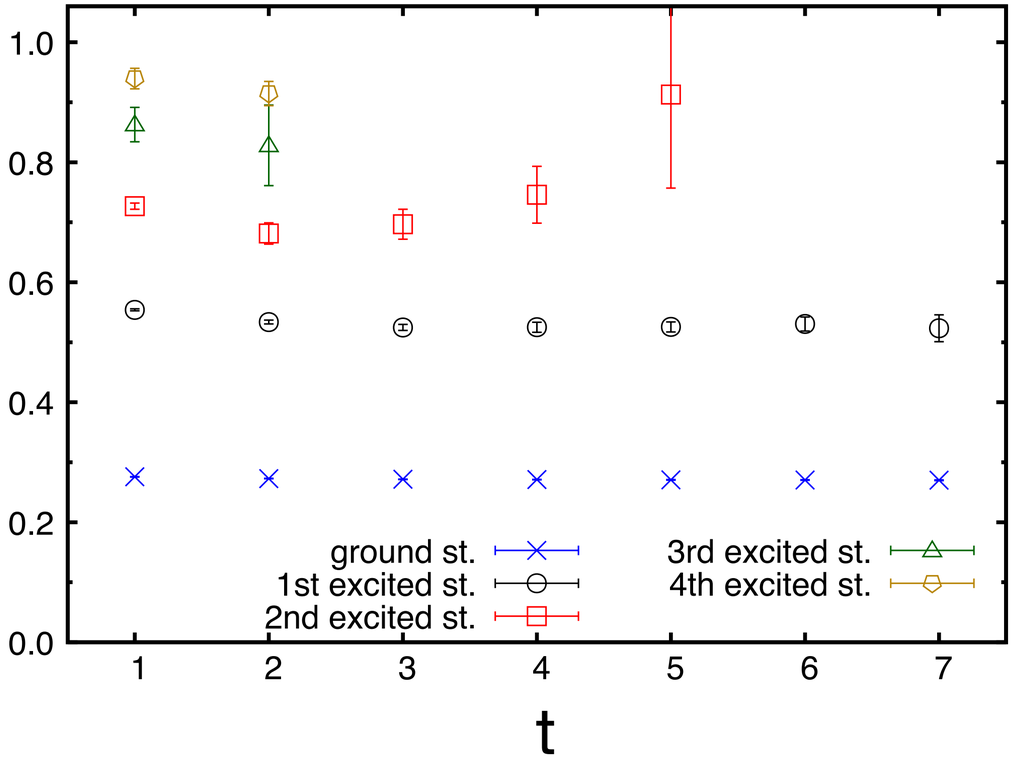
- Effective ππ energies from 2pt functions
- Used GEVP with some improvements

Paper on this topic in preparation

$aE_{\pi\pi}(I=2)$



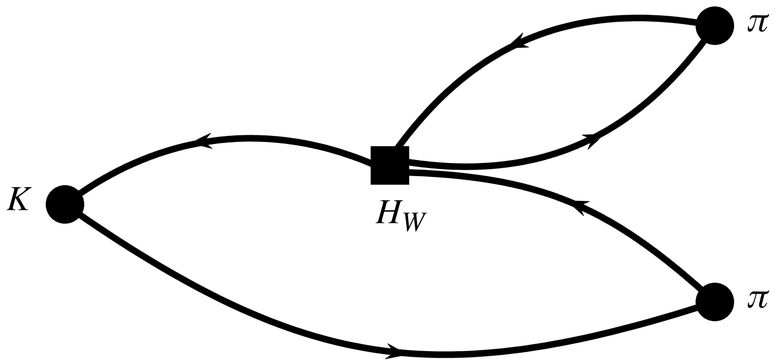
$aE_{\pi\pi}(I=0)$



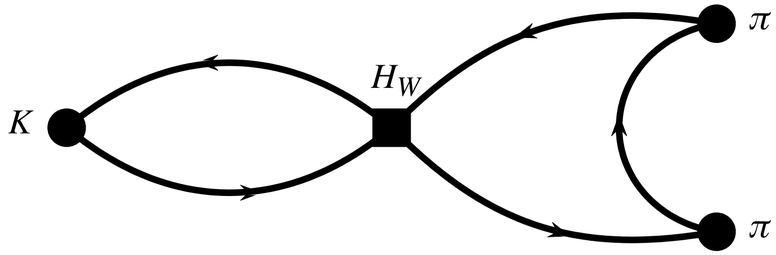
$K \to \pi\pi$ calculation

- 258 configurations (on 24^3x64), physical pion & kaon masses
- All-to-all quark propagators
  - 2,000 low modes for light quarks (no low mode for strange)
  - high-mode part: spin, color and time dilutions => 768 inversions
- 28 (5 independent) interpolation ππ operators
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$, $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$, $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$, $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$ & σ
  - to control effects from various states

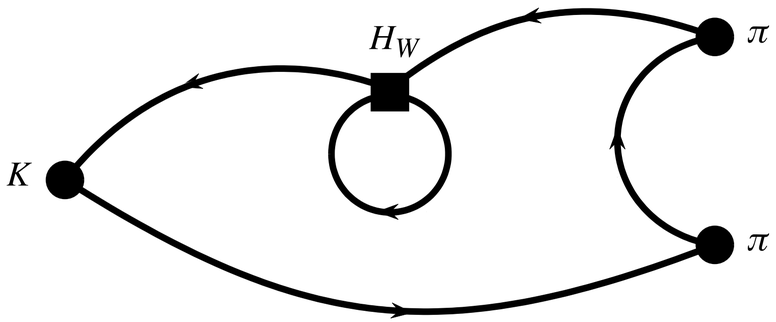
4 types of diagrams



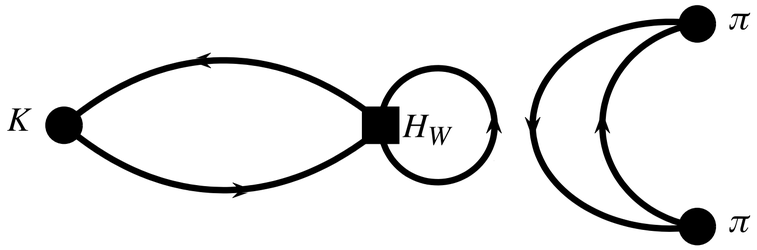
type 1



type 2



type 3

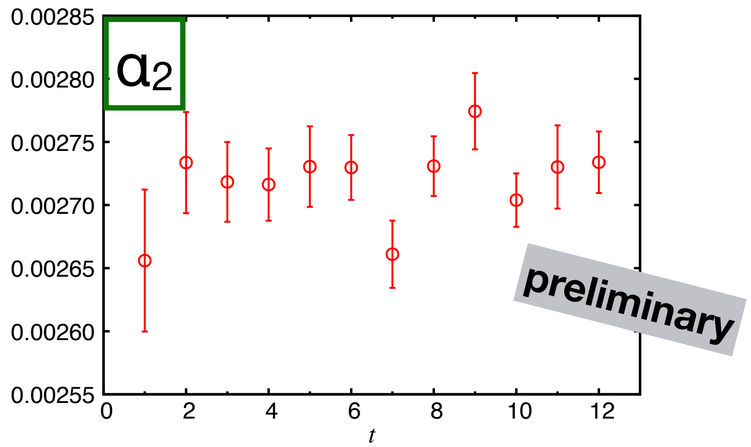


type 4

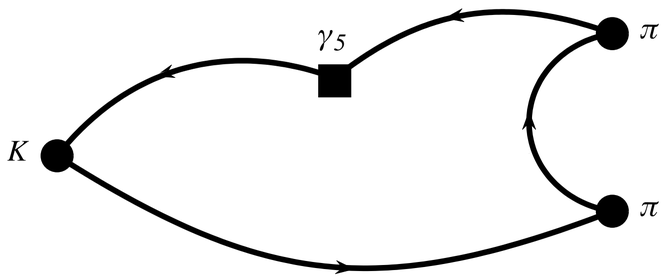
Subtraction of quadratic divergence

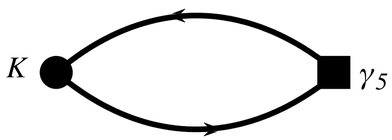
- Loop diagrams (types3,4) have a^{-2} divergence due to mixing with bilinear operators
- Subtraction
  - $Q_i' = Q_i - \alpha_i \bar{s}\gamma_5 d$
  - Condition: $\langle Q_i'(t)K(0)^\dagger\rangle = 0$

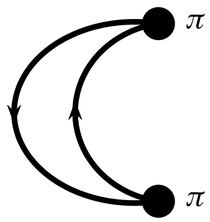
$$\alpha_i = \frac{\langle Q_i(t)K(0)^\dagger\rangle}{\langle \bar{s}\gamma_5 d(t)K(0)^\dagger\rangle}$$

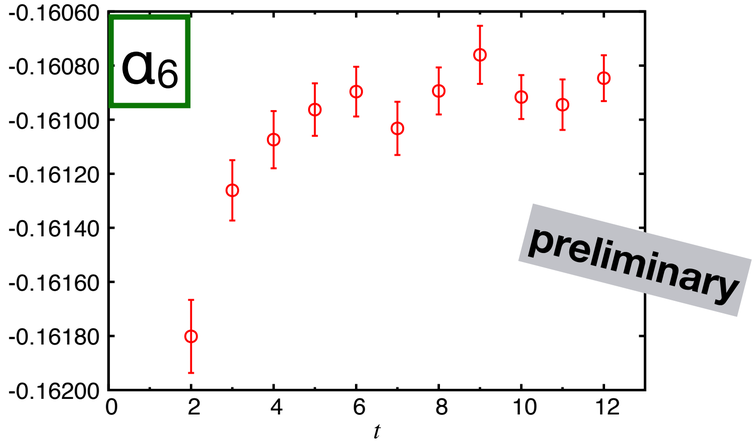


- Additional contractions



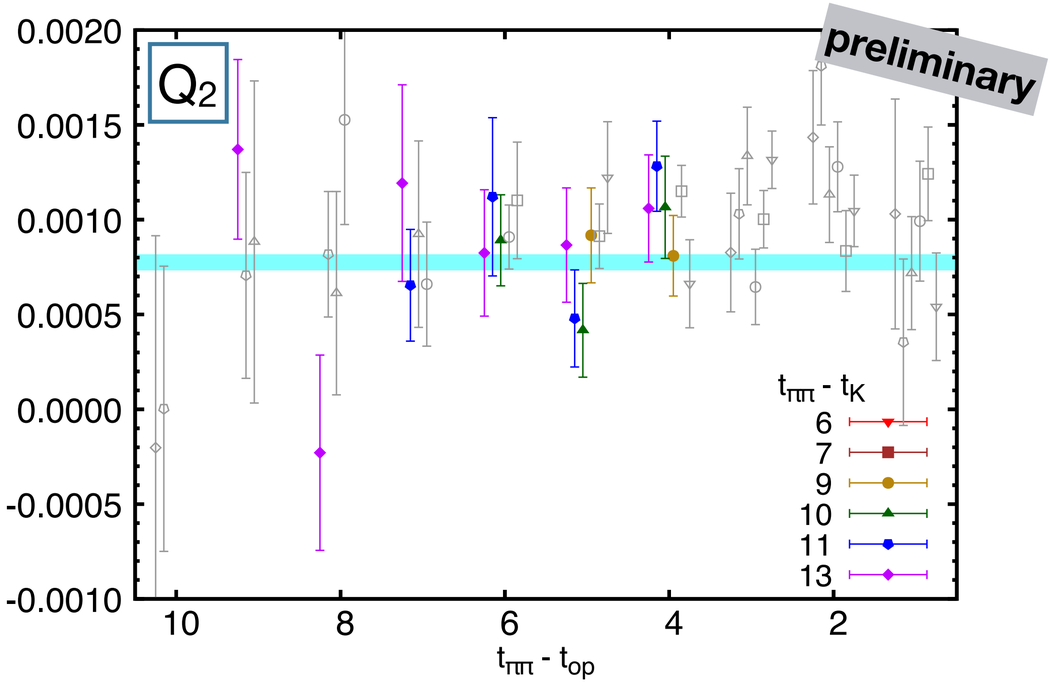


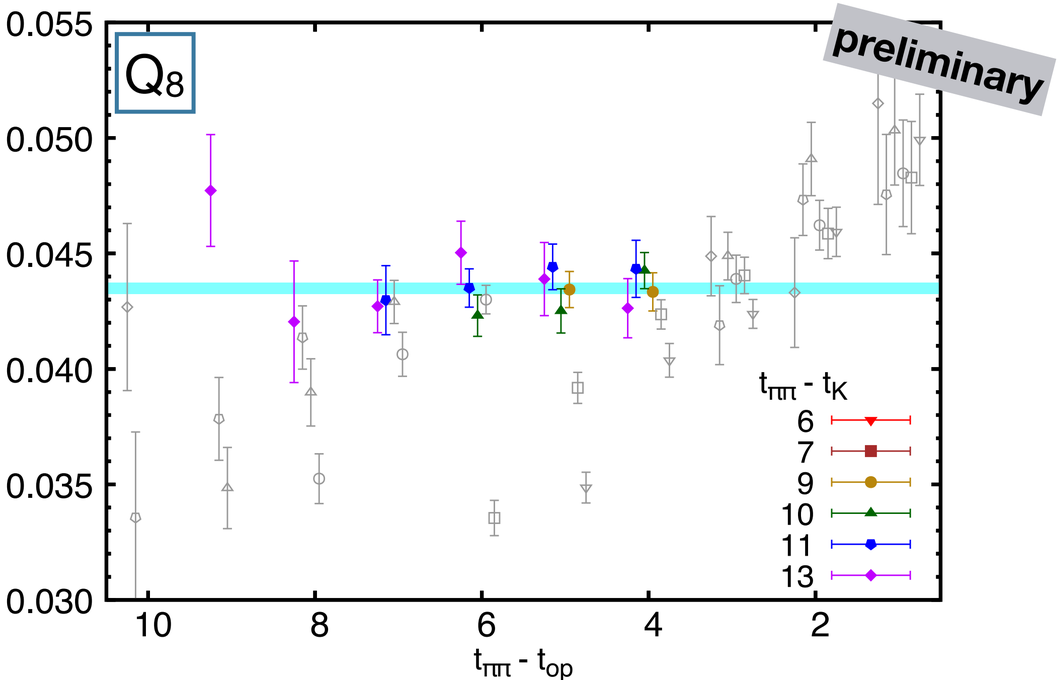




Effective matrix elements ($\Delta I = 3/2$)

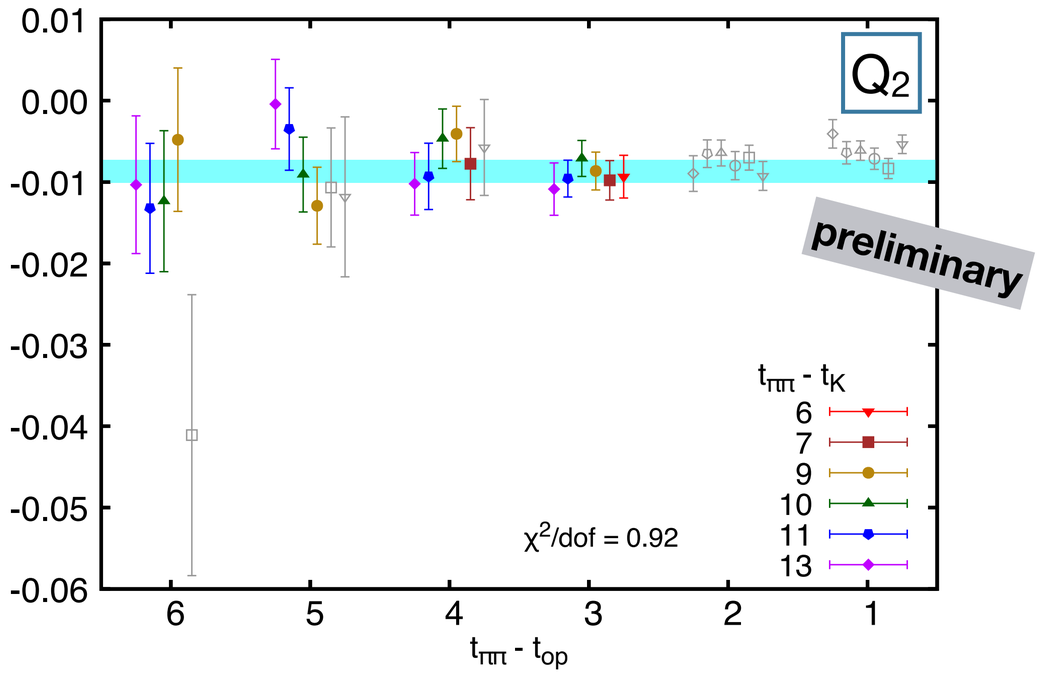
- Plateau appears
- Global fit with various $t_{\pi\pi}-t_K$, $t_{\pi\pi}-t_{op}$

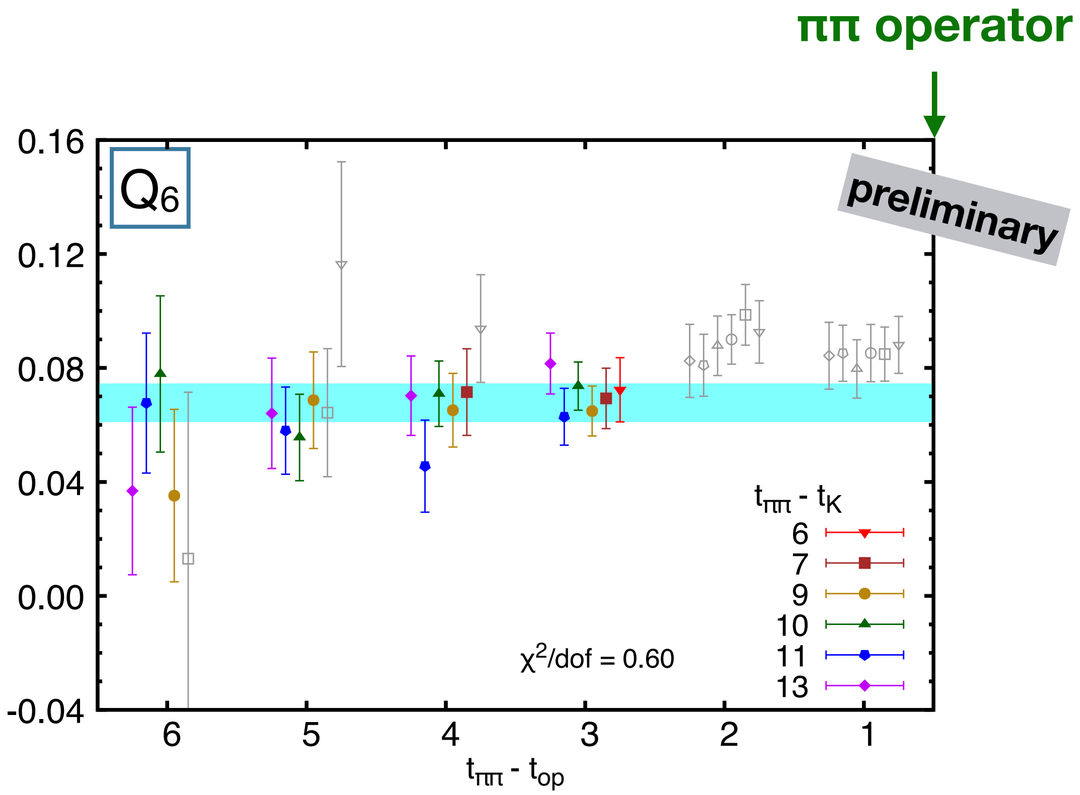




Effective matrix elements ($\Delta I = 1/2$)

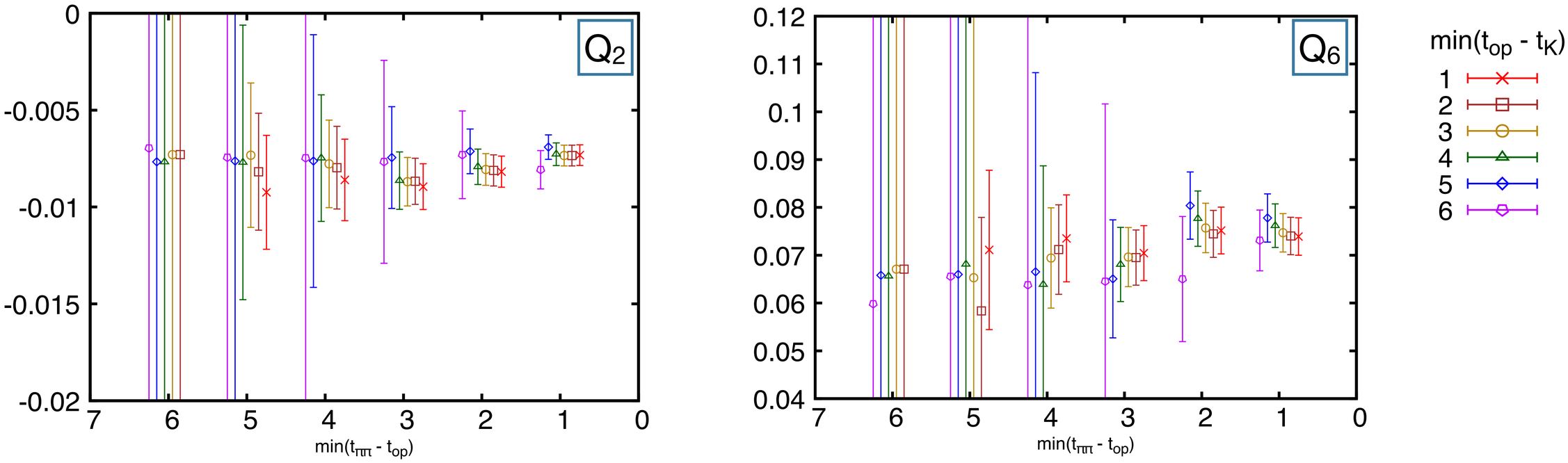
- Plateau appears
- Global fit with various $t_{\pi\pi}-t_K$, $t_{\pi\pi}-t_{op}$





Fit results ($\Delta I = 1/2$)

- **Fit range-dependence**



- No obvious dependence on fit range for $\min(t_{\pi\pi}-t_K)$, $\min(t_{\pi\pi}-t_{op}) \geq 3$

Precision performance

	32^3 G-parity BCs (previous work)	24^3 Periodic BCs	32^3 Periodic BCs (w/o AMA correction)
# configurations	741	258	107
Q_2^{lat}	10%	14%	14%
Q_6^{lat}	6.5%	8.9%	11%
Re A_0	11%	13%	14%

SN ratio

Preliminary

- Fascinating precision performance compared to G-parity calculation

Summary

- Purpose
  - New independent calculation of $K \to \pi\pi$ decays
  - to get measure of direct CPV
  - Periodic-BC study gives prospect of introducing QED/IB effects
- On-shell final state = 1st excited state
  - Interesting challenge to extract signal of excited state
  - GEVP works
  - Perhaps more efficient than previous G-parity calculation
- First results being summarized