Are there nuclear pastas in neutron stars?

Nicolas Chamel
Institute of Astronomy and Astrophysics
Université Libre de Bruxelles, Belgium

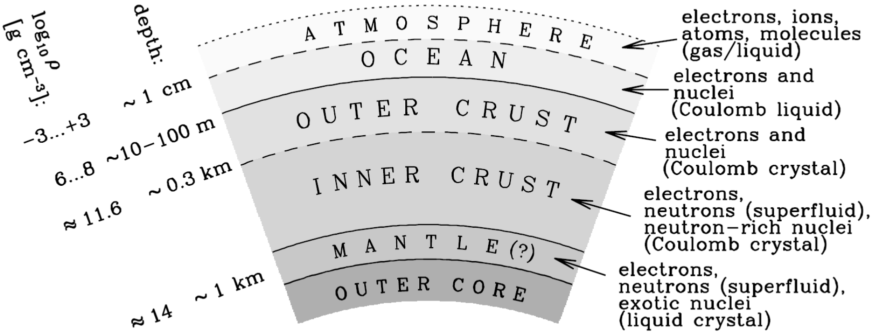
in collaboration with
John Michael Pearson (UMontréal, Canada)

LA LIBERTÉ DE CHERCHER

EOS
THE EXCELLENCE
OF SCIENCE

XVth Quark Confinement and the Hadron Spectrum 2022

Motivation



picture taken from Haensel, Potekhin, Yakovlev, "Neutron Stars" (Springer, 2007)

The mantle may consist of very exotic configurations so called nuclear "pastas": nuclear clusters with extreme shapes in a neutron liquid.

Blaschke&Chamel, Astrophys. Space Sci. Lib. 457, eds L. Rezzolla, P. Pizzochero, D. I. Jones, N. Rea, I. Vidaña p. 337-400 (Springer, 2018), arXiv:1803.01836

Nuclei in dense stellar environments

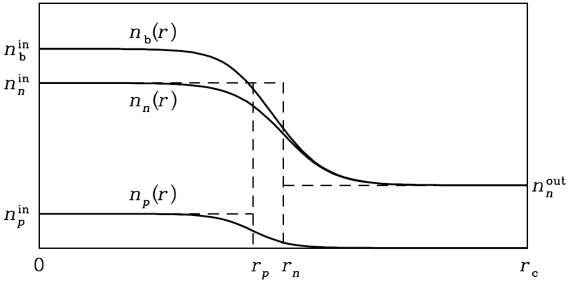
Back in 1971, Baym, Bethe and Pethick anticipated within the **liquid drop picture** that spherical nuclei in the inner crust of neutron stars would become unstable at high enough densities:

> The fraction of space, u, occupied by nuclei is seen in table 2 to increase monotonically until the nuclei begin to touch. For a bcc lattice the value $u = \sqrt{3}\pi/8 = 0.68$ corresponds to the nuclei just touching. The picture of nuclei as spherical drops is certainly not valid beyond this point. The nuclear parameters A, Z and n_N, given for $\rho = 2.26 \times 10^{14}$ and 2.39×10^{14} g/cm^3 should be regarded as highly tentative since they are particularly sensitive to the precise way in which the surface energy tends to zero as $n_n/n \to 1$. Furthermore, our model has neglected deformations of the nuclei, which become important here. In fact, it might be more favorable, beyond $u = 0.5$, for the nuclei to "turn inside out", that is, for the neutron gas to exist as a lattice of droplets in a sea of nuclear matter.

Baym, Bethe, Pethick, Nucl. Phys. A175, 225 (1971)

Compressible liquid-drop models

Nuclear clusters and the neutron liquid are treated **classically as two distinct coexisting homogeneous phases**.



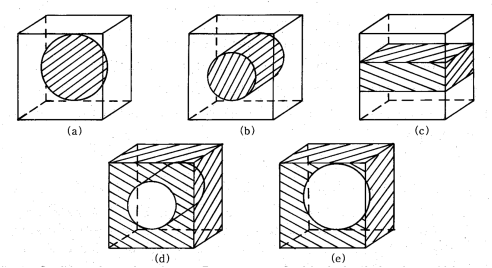
picture taken from Haensel, Potekhin, Yakovlev, "Neutron Stars" (Springer, 2007)

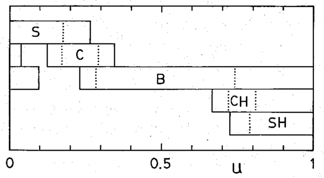
- Liquid-drop models are **computationally very fast** and therefore allow for systematic studies.
- They were used to show that the formation of clusters arise from a **detailed balance between surface and Coulomb effects**.

Nuclear "pastas": first predictions

Nuclear "pastas" were first studied based on **liquid-drop models**.

Ravenhall et al., PRL50, 2066 (1983); Hashimoto et al., PTP71, 320 (1984)





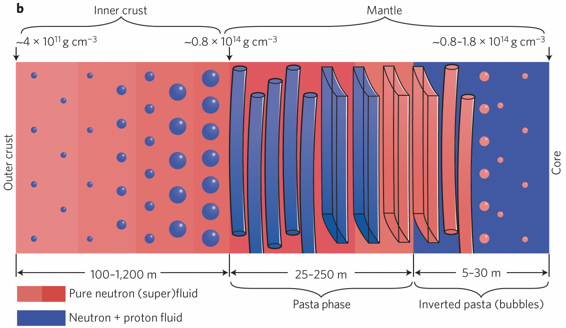
With increasing filling fraction *u*: (a) spheres (S), (b) cylinders (C) ’spaghetti’, (c) boards (B) ’lasagna’, (d) tubes (CH) ’bucatini’, and (e) spherical bubbles (SH) ’Swiss cheese’.

Predicted in 1983-84, the presence of nuclear pastas remains uncertain and model-dependent (surface tension, curvature).

Nuclear pastas in neutron stars

Liquid-drop models allow for statistical analyses. According to recent studies, pastas could represent about **50% of the crust mass.**

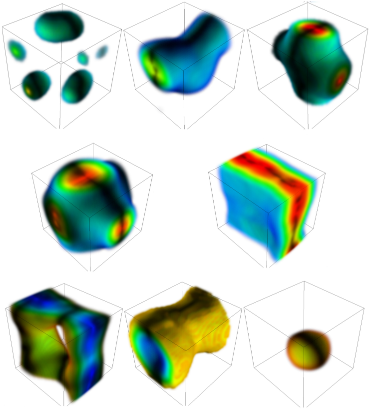
e.g. *Newton et al. EpJA58, 69 (2022); Dinh Thi et al., AA 654, A114 (2021)*



W. G. Newton, Nature Phys. 9, 396 (2013)

Pastas could have implications for thermal and dynamical evolution of neutron stars, gravitational-wave emission.

Models of nuclear pastas



Schuetrumpf et al., PRC87, 055805 (2013)

Pastas have been studied using

- liquid-drop models,
- semiclassical models,
- mean-field models (but limited due to computational cost).

Brief review:

BlaschkeChamel, Astrophys. Space Sci. Lib. 457, eds L. Rezzolla, P. Pizzochero, D. I. Jones, N. Rea, I. Vidaña p. 337-400 (Springer, 2018), arXiv:1803.01836

In all these approaches, only a few specific nuclear shapes are usually considered, or some symmetries are assumed.

Formation of nuclear pastas

The formation of nuclear pastas has been explored using large scale molecular dynamics in a box with periodic boundary conditions.

Dorso et al., in Neutron Star Crust, Eds Bertulani&Piekarewicz (Nova Science Publishers, New York, 2012), pp. 151-169.

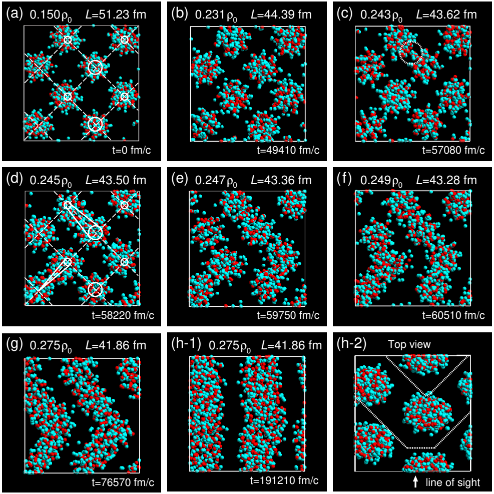
- **classical molecular dynamics** ($N \sim 10^3 - 10^5$)
  pointlike particles interacting through a two-body potential
  Berry et al., PRC94,055801(2016); Dorso et al., PRC86,055805(2012)
- **"quantum" molecular dynamics** ($N \sim 10^3$)
  Gaussian wave packets moving (classically) in a mean field.
  Phenomenological antisymmetrisation (Pauli potential).
  Maruyama et al., Prog.Theor.Exp.Phys. 2012,01A201(2012)
- **fermionic molecular dynamics** (very costly, scale as N^4 vs N^2)
  Slater determinants. Few attempts so far.
  Vantournhout et al.,Prog.Part.Nucl.Phys.66,271(2011); Vantournhout&Feldmeier, J.Phys.Conf.Ser.342,012011(2012)

Results could be influenced by the geometry of the box and the treatment of Coulomb interactions.

Giménez Molinelli&Dorso,NPA933,306 (2015); Alcain et al., PRC89,055801(2014)

Onset of nuclear pastas

As clusters get closer with increasing density, they eventually fuse and connect into herringbone structures turning into "spaghettis".



Watanabe et al., PRL103, 121101 (2009)

Quantum physics of nuclear pastas

To which extent do shell effects and nuclear pairing impact the existence of nuclear pastas?

We use the 4th order Extended Thomas-Fermi+Strutinsky Integral (ETFSI) approach:

- **semiclassical expansion in powers of $\hbar$**: the energy becomes a functional of $n_n(\boldsymbol{r})$ and $n_p(\boldsymbol{r})$ and their derivatives only.
- **proton shell and pairing effects** are added perturbatively.
- allowance for different (fixed) pasta shapes,
- to speed-up the computations, $n_n(\boldsymbol{r}), n_p(\boldsymbol{r})$ are parametrized.

Pearson et al., Phys. Rev. C 105, 015803 (2022); Phys. Rev. C 101, 015802 (2020)
Pearson et al., Phys.Rev.C91, 018801 (2015); Phys.Rev.C85,065803 (2012)
Onsi et al., Phys.Rev.C77,065805 (2008)

The ETFSI method is a fairly accurate and computationally very fast approximation to mean-field equations (Hartree-Fock-Bogoliubov)
Shelley&Pastore, Universe 6, 206 (2020)

Brussels-Montreal Skyrme functionals (BSk)

We have fitted a series of nuclear energy-density functionals with full HFB calculations using extended Skyrme functionals

Experimental data/constraints:

- $\sim$ 2300 atomic masses (rms $\sim 0.5 - 0.6$ MeV$/c^2$)
- $\sim$ 900 nuclear charge radii (rms $\sim$ 0.03 fm)
- symmetry energy $29 \leq J \leq 32$ MeV (no good mass fit outside!)
- incompressibility $K_v = 240 \pm 10$ MeV (giant resonances in nuclei)

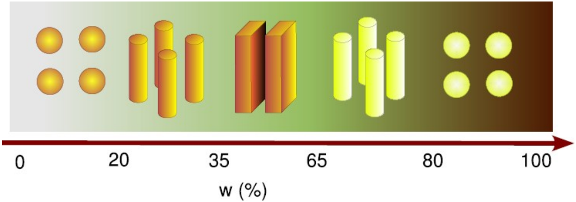
Many-body ab initio calculations:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter (+giant resonances in nuclei)
- stability against spin and spin-isospin fluctuations

Nuclear cooking without microscopic corrections

Ignoring SI correction (pure ETF calculations with BSk24):

- The sequence of pastas follows more or less liquid-drop model predictions with increasing filling fraction w



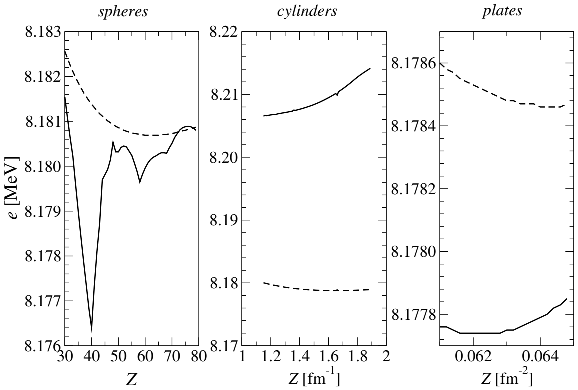
- Absence of holes explained by filling fractions w<55%
- Results differ slightly from those of Noel&Martin with the same functional but nucleon profiles are different
  Phys. Rev. C 92, 015803 (2015)
- Proton drip is disfavored by pastas
- Negligible impact ($< 1\%$) on the equation of state

Pearson et al., Phys. Rev. C 101, 015802 (2020)

Nuclear cooking with microscopic corrections

With SI correction, pastas disappear almost entirely!

Pearson&Chamel, Phys. Rev. C 105, 015803 (2022)

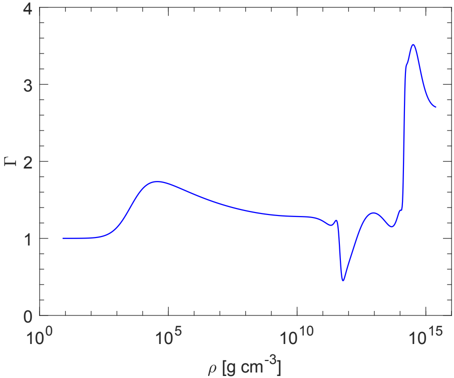


- No true shell effects in pastas (no fluctuations)
- Origin of SI correction: overbinding of ETF method vs HFB, underbinding due to parametrized nucleon profiles.

Unified equations of state

We have constructed thermodynamically consistent equations of state for all regions (outer+inner crust, core) of neutron stars.
Pearson et al.,MNRAS 481, 2994 (2018)

Available on CompOSE: `https://compose.obspm.fr/`



Unified equations of state can hardly be parametrized by polytropes!

$$\Gamma \equiv \left(1 + \frac{P}{\rho c^2}\right) \frac{d \log P}{d \log \rho}$$

Conclusions & Perspectives

We have have studied the existence of nuclear pastas in neutron-star crusts based on precision-fitted nuclear functionals:

- purely semiclassical calculations lead to usual pasta shapes
- with microscopic corrections, pastas are strongly disfavored.

Similar conclusions might hold for pastas at the quark-hadron phase transition in the inner core of neutron stars.

Perspectives:

- Development of new functionals with improved treatment of nuclear deformations (Guilherme Grams, Wouter Ryssen, Stephane Goriely)
- Sensitivity with respect to parametrization of nucleon profiles
- Extension to finite temperatures (Guilherme Grams)
- Full 3D HFB calculations of nuclear pastas (Nikolai Shchechilin)