Decay parameters and the axial-vector form factors for strangeness changing as well as strangeness conserving semileptonic decays of the hyperons

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Overview

Internal Structure of the Hadrons

- Quantum Chromodynamics (QCD) provides fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- **Internal Structure:** The knowledge of internal structure of hadrons in terms of quark and gluon degrees of freedom in QCD provide a basis for understanding more complex, strongly interacting matter. Facility is needed to investigate, with precision, the dynamics of gluons and sea quarks and their role in the structure of hadrons.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.

Fundamental Quantities

Electromagnetic Dirac and Pauli form factors: further related to the static low-energy observables

- **Structure:** Magnetic moments
  Dirac theory (1.0 μ_N) and experiment (2.5 μ_N).
  Proton is not an elementary Dirac particle but has an inner structure.
- **Spin:** Quantum quantity.
  Proton's spin is sum of the spins of its three constituent quarks.
- **Size:** Spatial extension.
  Proton charge distribution given by charge radius r_p.
- **Shape:** Nonspherical charge distribution.
  Quadrupole moment of the transition $N \rightarrow \Delta$.

Fundamental Questions

- How are the static observable related to each other and how do they emerge?
- How are the sea quarks and their spins, distributed in space and momentum inside the nucleon?
- Role of orbital angular momentum of the quarks and gluons in the non-perturbative regime of QCD.
- The role played by non-valence flavors in understanding the nucleon internal structure.
- How do the quarks and gluons interact with a nuclear medium?

Quantum chromodynamics (QCD): Present Theory of Strong Interactions

- At high energies, (α_s is small), QCD can be used perturbatively.
- At low energies, (α_s becomes large), one has to use other methods such as effective Lagrangian models to describe physics.
- Wide range of applications ranging from the dynamics and structure of hadrons and nuclei to the properties and phases of hadronic matter at the earliest stages of the universe.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory, for example the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering etc..
- Many fundamental questions have not been resolved. The most challenging nonperturbative problem in QCD is to determine the structure and spectrum of hadrons in terms of their quark and gluon degrees of freedom.

Spin Structure

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin).
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis".
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Flavor Structure

- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Recently confirmed by E866 and HERMES
- Measured quark sea asymmetry established that the study of the structure of the nucleon is intrinsically a nonperturbative phenomena.
- Sum Rules
  - Bjorken Sum Rule: $\Delta_3 = \Delta u - \Delta d$
  - Ellis-Jaffe Sum Rule: $\Delta_8 = \Delta u + \Delta d - 2\Delta s$
    (Reduces to $\Delta_8 = \Delta\Sigma$ when $\Delta s = 0$)
  - Strange quark fraction: $f_s \simeq 0.10$
  - Gottfried Sum Rule: $I_G = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]dx = 0.254 \pm 0.026$

Quark Sea

- Recently, a wide variety of accurately measured data have been accumulated for static properties of hadrons: masses, electromagnetic moments, charge radii etc. low energy dynamical properties: scattering lengths and decay rates etc.
- These lie in the non perturbative range of QCD.
- Flavor and spin structure of the nucleon is not limited to u and d quarks only.
- Non-perturbative effects explained only through the generation of “quark sea”

Nonperturbative Regime

- The direct calculations of these quantities form the first principle of QCD are extremely difficult, because they require non-perturbative methods.
- **Naive Quark Model** is able to provide a intuitive picture and successfully accounts for many of the low-energy properties of the hadrons in terms of the valence quarks.
- Techniques such as lattice gauge theory, QCD sum rules, and a wide variety of models have been developed to study this extremely interesting energy regime.

Chiral Constituent Quark Model

- χCQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NQM.
- The fluctuation process describing the effective Lagrangian is

$$q^{\uparrow\downarrow} \to \mathrm{GB} + q^{'\downarrow\uparrow} \to (q\bar{q}^{'}) + q^{'\downarrow\uparrow}$$

  $q\bar{q}^{'} + q^{'}$ constitute the sea quarks.
- Incorporates *confinement* and *chiral symmetry breaking.*
- "Justifies" the idea of constituent quarks.

χCQM

- The GB field can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} P_\pi \frac{\pi^0}{\sqrt{2}} + P_\eta \frac{\eta}{\sqrt{6}} + P_{\eta'} \frac{\eta^{'}}{\sqrt{3}} & P_\pi \pi^+ & P_K K^+ \\ P_\pi \pi^- & -P_\pi \frac{\pi^0}{\sqrt{2}} + P_\eta \frac{\eta}{\sqrt{6}} + P_{\eta'} \frac{\eta^{'}}{\sqrt{3}} & P_K K^o \\ P_K K^- & P_K \bar{K}^0 & -P_\eta \frac{2\eta}{\sqrt{6}} + P_{\eta'} \frac{\eta^{'}}{\sqrt{3}} \end{pmatrix}.$$

- The chiral fluctuations $u(d) \to d(u) + \pi^{+(-)}$, $u(d) \to s + K^{+(0)}$, $u(d,s) \to u(d,s) + \eta$, and $u(d,s) \to u(d,s) + \eta^{'}$ are given in terms of the transition probabilities P_π, P_K, P_η and $P_{\eta'}$ respectively.

Pion Cloud Mechanism

- Quark sea is believed to originate from process such as virtual pion production.
- It is suggested that in the deep inelastic lepton-nucleon scattering, the lepton probe also scatters off the pion cloud surrounding the target proton. The $\pi^+(\bar{d}u)$ cloud, dominant in the process $p \to \pi^+ n$, leads to an excess of $\bar{d}$ sea.
- However, this effect should be significantly reduced by the emissions such as $p \to \Delta^{++} + \pi^-$ with $\pi^-(\bar{u}d)$ cloud. Therefore, the pion cloud idea is not able to explain the significant $\bar{d} > \bar{u}$ asymmetry.
- This approach can be improved upon by adopting a mechanism which operates in the *interior* of the hadron.

Chiral Symmetry Breaking

- The dynamics of light quarks (u, d, and s) and gluons can be described by the QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + i\bar{\psi}_R D\psi_R + i\bar{\psi}_L D\psi_L - \bar{\psi}_R M\psi_L - \bar{\psi}_L M\psi_R \,,$$

  $G^a_{\mu\nu}$ is the gluonic gauge field strength tensor, D^μ is the gauge-covariant derivative, M is the quark mass matrix and ψ_L and ψ_R are the left and right handed quark fields
- Mass terms change sign as $\psi_R \to \psi_R$ and $\psi_L \to -\psi_L$ under the chiral transformation ($\psi \to \gamma^5\psi$), the Lagrangian no longer remains invariant. If neglected, the Lagrangian will have global chiral symmetry of the $SU(3)_L \times SU(3)_R$ group. Hadrons do not display parity doublets $\to$ the chiral symmetry is believed to be spontaneously broken around a scale of 1 GeV as

$$SU(3)_L \times SU(3)_R \to SU(3)_{L+R} \,.$$

- As a consequence, there exists a set of massless particles, referred to as the Goldstone bosons (GBs), which are identified with the observed (π, K, η mesons).
- Within the region of QCD confinement scale ($\Lambda_{QCD} \simeq 0.1 - 0.3$ GeV) and the chiral symmetry breaking scale $\Lambda_{\chi SB}$, the constituent quarks, the octet of GBs (π, K, η mesons), and the *weakly* interacting gluons are the appropriate degrees of freedom.
- The effective interaction Lagrangian in this region can be expressed as

$$\mathcal{L}_{\rm int} = \bar{\psi}(iD + V)\psi + ig_A\bar{\psi}A\gamma^5\psi + \cdots ,$$

where g_A is the axial-vector coupling constant. The gluonic degrees of freedom can be neglected owing to small effect in the effective quark model at low energy scale. The vector and axial-vector currents V_μ and A_μ are defined as

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2}(\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger),$$

where $\xi = \exp(2i\Phi/f_\pi)$, f_π is the pseudoscalar pion decay constant ($\simeq 93$ MeV).

- The field Φ describes the dynamics of GBs as

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} \end{pmatrix} .$$

Expanding V_μ and A_μ in the powers of Φ/f_π, we get

$$\begin{aligned} V_\mu &= 0 + O\left((\Phi/f_\pi)^2\right) , \\ A_\mu &= \frac{i}{f_\pi}\partial_\mu \Phi + O\left((\Phi/f_\pi)^2\right) . \end{aligned}$$

- The effective interaction Lagrangian between GBs and quarks from in the leading order can now be expressed as

$$\mathcal{L}_{\rm int} = -\frac{g_A}{f_\pi}\bar{\psi}\partial_\mu \Phi \gamma^\mu \gamma^5 \psi ,$$

which using the Dirac equation $(i\gamma^\mu\partial_\mu - m_q)q = 0$ can be reduced to

$$\mathcal{L}_{\rm int} \approx i \sum_{q=u,d,s} \frac{m_q + m_{q'}}{f_\pi} \bar{q}'\Phi\gamma^5 q = i \sum_{q=u,d,s} c_8 \bar{q}'\Phi\gamma^5 q .$$

- $c_8 \left(= \frac{m_q+m_{q'}}{f_\pi}\right)$ is the coupling constant for octet of GBs and m_q $(m_{q'})$ is the quark mass parameter. The Lagrangian of the quark-GB interaction, suppressing all the space-time structure to the lowest order, can now be expressed as

$$\mathcal{L}_{\rm int} = c_8 \bar{\psi} \Phi \psi \,.$$

- The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, which would imply the existence of ninth GB. This breaking symmetry picks the η' as the ninth GB.
- The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can now be expressed as

$$\mathcal{L}_{\rm int} = c_8 \bar{\psi} \Phi \psi + c_1 \bar{\psi} \frac{\eta'}{\sqrt{3}} \psi = c_8 \bar{\psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi = c_8 \bar{\psi} \left(\Phi' \right) \psi \,,$$

where $\zeta = c_1/c_8$, c_1 is the coupling constant for the singlet GB and I is the 3×3 identity matrix.

Successes of χCQM

- "Proton spin problem" including quark spin polarizations, orbital angular momentum of quarks etc.
- Quark flavor distributions, fraction of a particular quark (antiquark) present in a baryon, flavor structure functions, the Gottfried integral and the meson-baryon sigma terms
- Magnetic moments of octet and decuplet baryons including their transitions and the Coleman-Glashow sum rule
- Axial-vector form factors of the low lying octet baryons, singlet (g_0^A) and nonsinglet (g_3^A and g_8^A) axial-vector coupling constants
- The spin independent (F_1^N and F_2^N) and the spin dependent g_1^N structure functions, longitudinal spin asymmetries of nucleon (A_1^N)

Contd...

- Hyperon β decay parameters including the axial-vector coupling parameters F and D
- Magnetic moments of octet baryon resonances well as Λ resonances
- Charge radii and quadrupole moment of the baryons
- The model is successfully extended to predict the important role played by the small intrinsic charm content in the nucleon spin in the SU(4) χCQM and to calculate the magnetic moment and charge radii of charm baryons including their radiative decays

Axial-Vector Form Factors

- The transition matrix element for this process in the momentum space is given by

$$M = \frac{G_F^2}{\sqrt{2}} V_{q_i q_f} \langle B_f(p_f)|J_h^{\mu}|B_i(p_i)\rangle \times (\overline{\mathrm{u}}_l(p_l)\gamma_\mu(1-\gamma_5)u_\nu(p_{\nu_l})),$$

- where $u_{\bar{\nu}_l}(p_\nu)$ and $\bar{u}_l(p_l)$ are the Dirac spinors of the neutrino and the corresponding lepton, respectively. G_F represents the Fermi coupling constant, the CKM element $V_{q_i q_f}$ corresponds to V_{ud} for the strangeness conserving, $\Delta S = 0$ and V_{us} for the strangeness changing, $\Delta S = 1$ processes.

Axial-Vector Form Factors

- The weak hadronic current, J_h^μ can further be expressed in terms of the vector ($V^{\mu,a} = \bar{\mathbf{q}}\gamma^\mu \frac{\lambda^a}{2}\mathbf{q}$) and axial-vector ($A^{\mu,a} = \bar{\mathbf{q}}\gamma^\mu\gamma_5 \frac{\lambda^a}{2}\mathbf{q}$) currents as $J_h^\mu = J_V^\mu - J_A^\mu$ and we have

$$
\begin{aligned}
\langle B_f | J_h^\mu | B_i \rangle &= \langle B_f | V^{\mu,a} | B_i \rangle - \langle B_f | A^{\mu,a} | B_i \rangle \\
&= \langle B_f | \bar{\mathbf{q}}\gamma^\mu \frac{\lambda^a}{2}\mathbf{q} | B_i \rangle - \langle B_f | \bar{\mathbf{q}}\gamma^\mu\gamma_5 \frac{\lambda^a}{2}\mathbf{q} | B_i \rangle .
\end{aligned}
$$

- Here, λ^a are the Gell-Mann matrices of SU(3) relating to the flavor structure of the 3 light quarks. For the strangeness conserving, $\Delta S = 0$ transitions, we have $a = 1 \pm i2$ and for the strangeness changing, $\Delta S = 1$ transitions, we have $a = 4 \pm i5$.

Axial-Vector Form Factors

- The matrix elements for the vector current is given in terms of the dimensionless vector functions $f_i^{B_i B_f}(Q^2)$ $(i = 1, 2, 3)$

$$\langle B_f|\bar{\mathbf{q}}\gamma^\mu \frac{\lambda^a}{2}\mathbf{q}|B_i\rangle = \bar{u}_f(p_f)\left(f_1^{B_i B_f}(Q^2)\gamma^\mu + \frac{f_2^{B_i B_f}(Q^2) i\sigma^{\mu\nu}q_\nu}{M_{B_i}+M_{B_f}} + \frac{f_3^{B_i B_f}(Q^2)q^\mu}{M_{B_i}+M_{B_f}}\right)\frac{\lambda^a}{2}u_i(p_i)\,,$$

  f_1, f_2 and f_3 are the vector, induced tensor or weak magnetism and induced scalar form factors respectively.

- For the axial-vector current we have the dimensionless axial-vector functions $g_i^{B_i B_f}(Q^2)$ $(i = 1, 2, 3)$

$$\langle B_f|\bar{\mathbf{q}}\gamma^\mu\gamma_5 \frac{\lambda^a}{2}\mathbf{q}|B_i\rangle = \bar{u}_f(p_f)\left(g_1^{B_i B_f}(Q^2)\gamma^\mu + \frac{g_2^{B_i B_f}(Q^2) i\sigma^{\mu\nu}q_\nu}{M_{B_i}+M_{B_f}} + \frac{g_3^{B_i B_f}(Q^2)q^\mu}{M_{B_i}+M_{B_f}}\right)\gamma^5\frac{\lambda^a}{2}u_i(p_i)\,.$$

  g_1, g_2 and g_3 are the axial-vector, induced pseudotensor or weak electricity and the induced pseudoscalar scalar form factors respectively.

- M_{B_i} (M_{B_f}) are the masses of the initial (final) baryon states, respectively. The four momenta transfer is given as $Q^2 = -q^2$, where $q \equiv p_i - p_f$. At the quark level only the first class currents corresponding to the $d \to u$ transitions occur, the magnitude of second class currents is very small. f_3 also gets eliminated by Conserved Vector Current(CVC) hypothesis as its contribution to the decay rate is very small.

CKM Matrix Elements V_{ud} and V_{us}

- Using the transition amplitude, the matrix element $V_{q_i q_f}$ can be calculated from the total decay rate given as

$$R = G_F^2 \frac{\Delta M^5 |V_{us}|^2}{60\pi^3} \Big\{ \Big(1 - \frac{3}{2}E + \frac{6}{7}E^2\Big) f_1^2 + \frac{4}{7}E^2 f_2^2$$
$$+ \Big(3 - \frac{9}{2}E + \frac{12}{7}E^2\Big) g_1^2 + \frac{12}{7}E^2 g_2^2 + \frac{6}{7}E^2 f_1 f_2 + \Big(-4E + 6E^2\Big) g_1 g_2 \Big\},$$

  where $E = \frac{\Delta M}{\Sigma M}$, $\Sigma M = M_i + M_f$ and $\Delta M = M_i - M_f$ and $V_{q_i q_f}$ becomes V_{us} for $\Delta S = 1$ and V_{ud} for $\Delta S = 0$ decays.

- This expression includes contribution from four form factors instead of six, contributions from f_3 and g_3 are neglected as the q^μ contribution to the decay is proportional to m_l^2, m_l being the mass of the lepton in the final decay. Since our final state lepton is e and hence the effect of small electron mass can be neglected.

- Amongst all the possible hyperon semileptonic/beta decays, the required experimental data is available only for five of these strangeness changing decays and we calculate V_{us} from them. The decay $\Xi^- \to \Xi^0 e^- \bar{\nu}_e$ has not been observed yet and the upper limit on its branching ratio as of now is set to be $B(\Xi^- \to \Xi^0 e^- \bar{\nu}_e) < 2.59 \times 10^{-4}$ at 90% confidence level.

Decay	$M_i(GeV)$	$M_f(GeV)$	R(GeV)	f_1	f_2	g_1	g_2	V_{us}
$\Sigma^- \rightarrow ne^- \bar{\nu}_e$	1.197	0.939	4.531×10^{-18}	-1.0	1.813	0.314	0.017	0.22416
$\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	1.321	1.192	3.498×10^{-19}	0.707	2.029	0.898	0.310	0.22452
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$	1.321	1.116	2.264×10^{-18}	1.225	-0.450	0.262	0.047	0.23915
$\Lambda \rightarrow pe^- \bar{\nu}_e$	1.116	0.938	2.083×10^{-18}	-1.225	-1.037	-0.909	-0.170	0.21498
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$	1.315	1.189	5.726×10^{-19}	1.0	2.854	1.27	0.446	0.21526

Table: The decay constants $f_1(Q^2 = 0)$, $f_2(Q^2 = 0)$, $g_1(Q^2 = 0)$, $g_2(Q^2 = 0)$ and the CKM matrix V_{us} for the strangeness changing ($\Delta S = 1$) decays.

Decay	$M_i(GeV)$	$M_f(GeV)$	R(GeV)	$f_1(0)$	$f_2(0)$	$g_1(0)$	$g_2(0)$	V_{ud}
$n \to pe^- \bar{\nu}_e$	0.939	0.938	7.249×10^{-28}	1.00	2.612	1.270	-0.004	1.30355
$\Sigma^- \to \Sigma^0 e^- \bar{\nu}_e$	1.197	1.192	–	1.414	1.033	0.676	-0.010	–
$\Sigma^- \to \Lambda e^- \bar{\nu}_e$	1.197	1.116	2.553×10^{-19}	0	2.265	0.646	-0.152	0.91202
$\Sigma^+ \to \Lambda e^- \bar{\nu}_e$	1.189	1.116	1.644×10^{-19}	0	2.257	0.646	-0.136	0.94797
$\Xi^- \to \Xi^0 e^- \bar{\nu}_e$	1.322	1.315	–	-1.00	2.253	0.314	-0.007	–

Table: The decay constants $f_1(Q^2=0)$, $f_2(Q^2=0)$, $g_1(Q^2=0)$, $g_2(Q^2=0)$ and the CKM matrix V_{ud} for the strangeness conserving ($\Delta S = 0$) decays.

Decay	[exp]	$\frac{g_1(0)}{f_1(0)}$ in χCQM_{config}
$\Sigma^- \rightarrow n e^- \bar{\nu}_e$	-0.340 ± 0.017	-0.314
$\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	–	1.270
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$	0.25 ± 0.05	0.214
$\Lambda \rightarrow p e^- \bar{\nu}_e$	0.718 ± 0.015	0.742
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$	1.22 ± 0.55	1.27

Table: Ratio $\frac{g_A}{g_V} = \frac{g_1(0)}{f_1(0)}$ in our model and the corresponding latest experimental results for $\Delta S = 1$ decays.

Decay	[exp]	$\frac{g_1(0)}{f_1(0)}$ in χCQM_{config}
$n \to p e^- \bar{\nu}_e$	1.2756 ± 0.0013	1.270
$\Sigma^- \to \Sigma^0 e^- \bar{\nu}_e$	–	0.478
$\Sigma^- \to \Lambda e^- \bar{\nu}_e$	$\frac{f_1}{g_1} = 0.01 \pm 0.10$	–
$\Sigma^+ \to \Lambda e^- \bar{\nu}_e$	–	–
$\Xi^- \to \Xi^0 e^- \bar{\nu}_e$	–	-0.314

Table: Ratio $\frac{g_A}{g_V} = \frac{g_1(0)}{f_1(0)}$ in our model and the corresponding latest experimental results for $\Delta S = 0$ decays.

- For the case of vector form factor f_1 we can see that

$$f_1(\Lambda \to pe^-\bar{\nu}_e) < f_1(\Sigma^- \to ne^-\bar{\nu}_e) < f_1(\Xi^- \to \Sigma^0 e^-\bar{\nu}_e) < f_1(\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e) < f_1(\Xi^- \to \Lambda e^-\bar{\nu}_e),$$

and $f_1(\Lambda \to pe^-\bar{\nu}_e) = -f_1(\Xi^- \to \Lambda e^-\bar{\nu}_e)$ and $f_1(\Sigma^- \to ne^-\bar{\nu}_e) = -f_1(\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e)$.

$$f_1(\Xi^- \to \Xi^0 e^-\bar{\nu}_e) < f_1(\Sigma^+ \to \Lambda e^-\bar{\nu}_e) = f_1(\Sigma^- \to \Lambda e^-\bar{\nu}_e) < f_1(n \to pe^-\bar{\nu}_e) < f_1(\Sigma^- \to \Sigma^0 e^-\bar{\nu}_e),$$

and $f_1(n \to pe^-\bar{\nu}_e) = -f_1(\Xi^- \to \Xi^0 e^-\bar{\nu}_e)$ and $f_1(\Sigma^+ \to \Lambda e^-\bar{\nu}_e) = f_1(\Sigma^- \to \Lambda e^-\bar{\nu}_e)$.

- For the case of induced tensor or weak magnetism form factor f_2 we have

$$f_2(\Lambda \to pe^-\bar{\nu}_e) < f_2(\Xi^- \to \Lambda e^-\bar{\nu}_e) < f_2(\Sigma^- \to ne^-\bar{\nu}_e) < f_2(\Xi^- \to \Sigma^0 e^-\bar{\nu}_e) < f_2(\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e),$$

$$f_2(\Sigma^- \to \Sigma^0 e^-\bar{\nu}_e) < f_2(\Xi^- \to \Xi^0 e^-\bar{\nu}_e) < f_2(\Sigma^+ \to \Lambda e^-\bar{\nu}_e) < f_2(n \to pe^-\bar{\nu}_e) < f_2(\Sigma^- \to \Lambda e^-\bar{\nu}_e).$$

- For the case of axial-vector form factor g_1

$$g_1(\Lambda \to p e^- \bar{\nu}_e) < g_1(\Xi^- \to \Lambda e^- \bar{\nu}_e) < g_1(\Sigma^- \to n e^- \bar{\nu}_e) < g_1(\Xi^- \to \Sigma^0 e^- \bar{\nu}_e) < g_1(\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e),$$

$$g_1(\Xi^- \to \Xi^0 e^- \bar{\nu}_e) < g_1(\Sigma^+ \to \Lambda e^- \bar{\nu}_e) = g_1(\Sigma^- \to \Lambda e^- \bar{\nu}_e) < g_1(\Sigma^- \to \Sigma^0 e^- \bar{\nu}_e) < g_1(n \to p e^- \bar{\nu}_e).$$

- For the case of induced pseudotensor or weak electricity form factor g_2

$$g_2(\Lambda \to p e^- \bar{\nu}_e) < g_2(\Sigma^- \to n e^- \bar{\nu}_e) < g_2(\Xi^- \to \Lambda e^- \bar{\nu}_e) < g_2(\Xi^- \to \Sigma^0 e^- \bar{\nu}_e) < g_2(\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e),$$

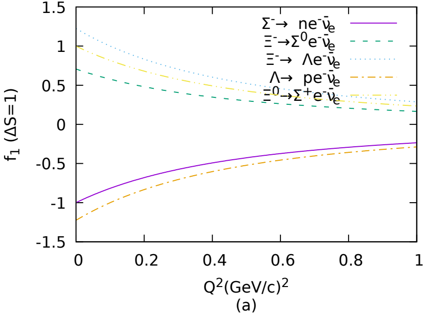
$$g_2(\Sigma^- \to \Lambda e^- \bar{\nu}_e) < g_2(\Sigma^+ \to \Lambda e^- \bar{\nu}_e) < g_2(\Sigma^- \to \Sigma^0 e^- \bar{\nu}_e) < g_2(\Xi^- \to \Xi^0 e^- \bar{\nu}_e) < g_2(n \to p e^- \bar{\nu}_e).$$

- For a low and moderate momentum transfer $Q^2 \leq 1$, the dipole form of parametrization has been conventionally used to analyse the vector and axial-vector form factors

$$f_i^{B_i B_f}(Q^2) = \frac{f_i^{B_i B_f}(0)}{\left(1 + \frac{Q^2}{M_{f_i}^2}\right)^2},$$
$$g_i^{B_i B_f}(Q^2) = \frac{g_i^{B_i B_f}(0)}{\left(1 + \frac{Q^2}{M_{g_i}^2}\right)^2},$$

where $f_i^{B_i B_f}(0)$ and $g_i^{B_i B_f}(0)$ are the vector and axial-vector coupling constants at zero momentum transfer.

- Here M_{f_i} and M_{g_i} are the dipole masses for vector and axial vector part respectively. This parameterization in addition to providing correct low energy behaviour also gives correct asymptomatic limit $G_A \propto \frac{1}{Q^4}$ and $G_P \propto \frac{1}{Q^6}$ at large momentum transfer.



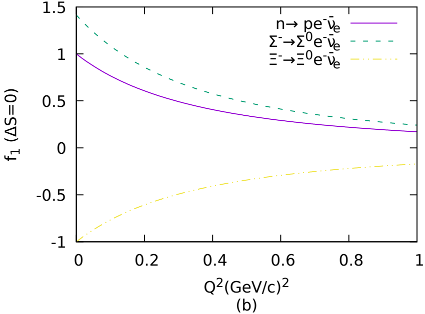


Figure: Variation of $f_1(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.

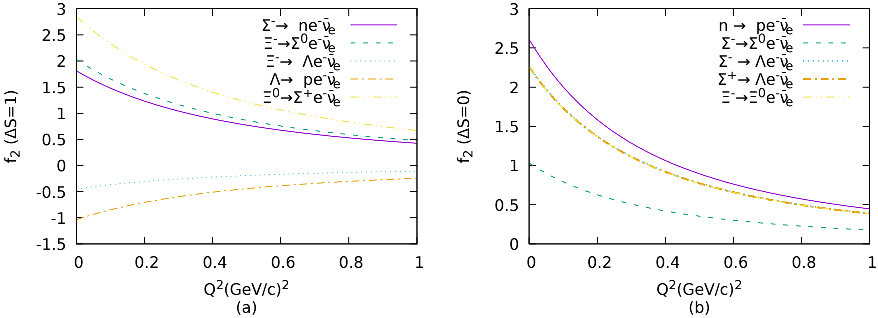


Figure: Variation of $f_2(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.

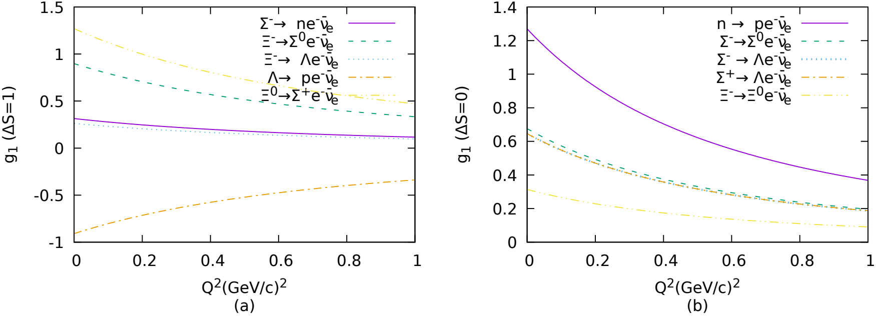


Figure: Variation of $g_1(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.

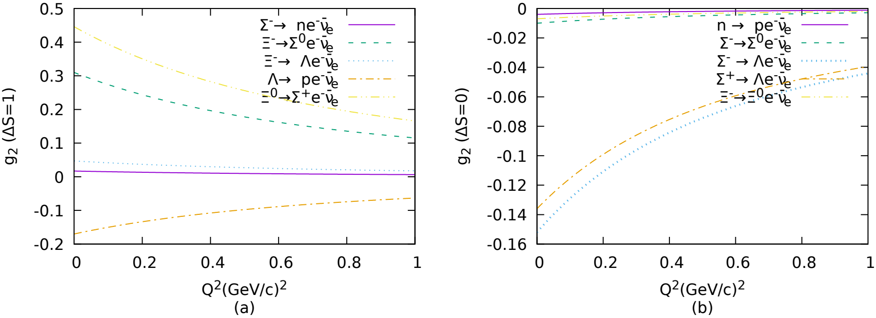


Figure: Variation of $g_2(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.

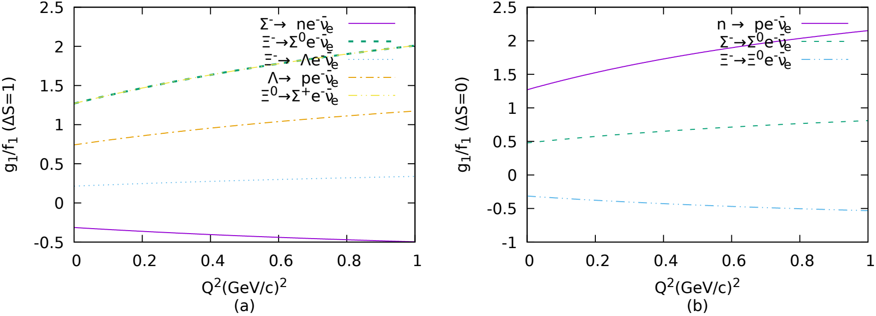


Figure: Variation of $\frac{g_1(Q^2)}{f_1(Q^2)}$ with Q^2 for $\Delta S = 1$ and for $\Delta S = 0$ decays.

Summary

Understanding the spin structure of the hadrons will help to resolve the most challenging problems facing subatomic physics which include

- What happens to the spin in the transition between current and constituent quarks in the low energy QCD?
- How can we distinguish between the *current quarks* and the *constituent quarks*?
- How is the spin of the proton and other hadrons built out from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents?
- What is the role played by non-valence flavors in understanding the hadronic internal structure?