

TWO GLUEBALL SCATTERING AND THE GLUEBALLONIUM

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XVTH QUARK CONFINEMENT AND THE HADRON SPECTRUM CONFERENCE

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OVERVIEW

- Mesons and glueballs
- Scattering of two scalar glueballs
- Bound state of two glueballs: glueballonium
- Effect of other glueballs on the scattering
- Conclusions

CLASSIFICATION OF MESONS

Hadrons

<i>Baryons</i>	$[qqq]$
<i>Mesons</i>	$[\bar{q}q]$

$q\bar{q}$ nonets	J^{PC}	Flavor states
Scalar (S)	0^{++}	$a_0^{\pm, o}, K_0^{*\pm, o, \bar{o}}, \sigma_N, \sigma_S (f_0^L, f_0^H)$
Pseudoscalar (P)	0^{-+}	$\pi^{\pm, o}, K^{\pm, o, \bar{o}}, \eta_N, \eta_S (\eta, \eta')$
Vector (V_μ)	1^{--}	$\rho^{\pm, o}, K^{*\pm, o, \bar{o}}, \omega_N, \omega_S (\omega, \phi)$.
Axial-vector (A_μ)	1^{++}	$a_1^{\pm, o}, K_{1,A}^{\pm, o, \bar{o}}, f_{1,A}^N, f_{1,A}^S (f_1, f'_1)$.
Pseudovector (B_μ)	1^{+-}	$b_1^{\pm, o}, K_{1,B}^{\pm, o, \bar{o}}, f_{1,B}^N, f_{1,B}^S (h_1, h'_1)$.

Table: Examples of nonets of conventional mesons, their spin and parity, and the flavor states. * means that the physical states are unknown

- QCD predicts also exotic mesons as tetraquarks ($\bar{q}\bar{q}qq$) and glueballs

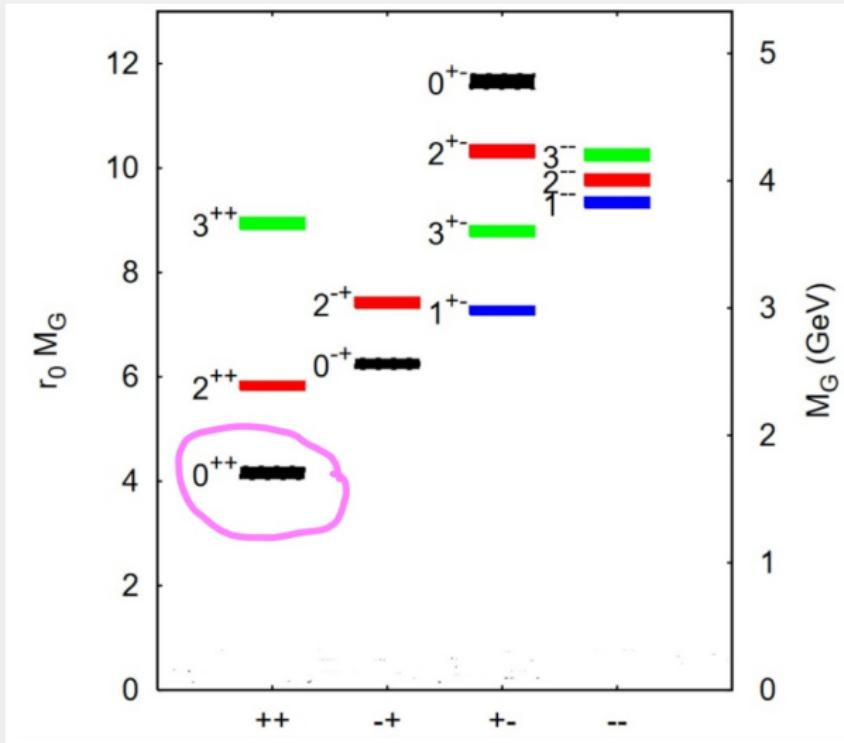
GLUEBALLS

- Gluons carry color charge and interact strongly with each other
- Several lattice QCD simulations of glueballs [arXiv:nucl-th/0309068, Morningstar,Peardon].
- Possible state that is predominantly a glueball state is the resonance $f_0(1710)$ [arXiv:1408.4921v1 [hep-ph], Janowski et al.]

$$\begin{aligned}\sigma_N &\approx (\bar{u}u + \bar{d}d)/\sqrt{2} \\ \sigma_S &\approx (\bar{s}s)\end{aligned}$$

$f_0(1370) :$	$83\% \sigma_N,$	$6\% \sigma_S,$	$11\% G ,$
$f_0(1500) :$	$9\% \sigma_N,$	$88\% \sigma_S,$	$3\% G ,$
$f_0(1710) :$	$8\% \sigma_N,$	$6\% \sigma_S,$	$86\% G .$

We worked with a single scalar glueball with $m_G = 1.7$ GeV.
Below: plot from Lattice QCD: a glueball spectrum is found.



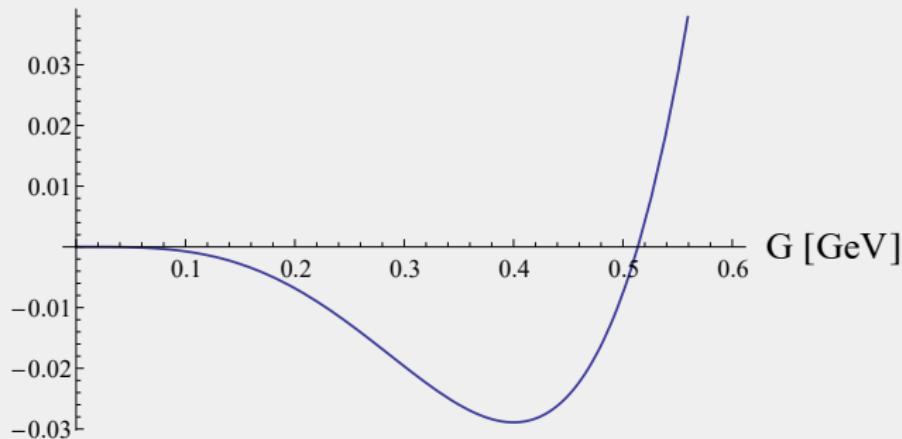
Y. Chen et al, Phys. Rev. D73, 014516 (2006)

SCATTERING OF TWO SCALAR GLUEBALLS: TREE-LEVEL

Now only the YM sector of QCD will be considered.

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V(G) = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left(G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right)$$

$V [GeV^4]$



A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

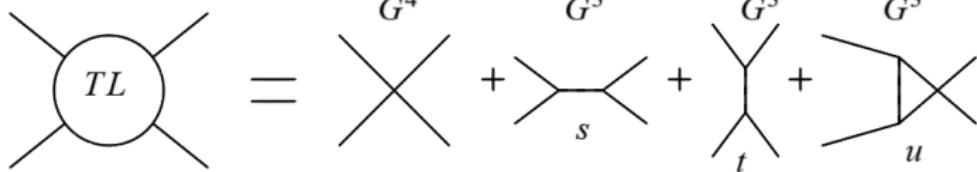
TL AMPLITUDE

Taylor expansion around $\min V(G = \Lambda_G \approx 0.4 \text{ GeV})$

$$V(G) = V(\Lambda_G) + \frac{1}{2} m_G^2 G^2 + \frac{1}{3!} \left(5 \frac{m_G^2}{\Lambda_G} \right) G^3 + \frac{1}{4!} \left(11 \frac{m_G^2}{\Lambda_G^2} \right) G^4 + \frac{1}{5!} \left(6 \frac{m_G^2}{\Lambda_G^3} \right) G^5 + \dots$$

$$A(s, t, u) = -11 \frac{m_G^2}{\Lambda_G^2} - \left(5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{s - m_G^2} - \left(5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{t - m_G^2} - \left(5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{u - m_G^2}$$

TREE LEVEL



l-th scattering length

$k = \frac{1}{2} \sqrt{s - 4m_G^2}$: 3-momentum of any particle in the center of mass. $s_{th} \equiv 4m_G^2$

$$A_l(s) = \frac{1}{2} \int_{-1}^{+1} d\cos\theta \ A(s, \cos\theta) P_l(\cos\theta),$$

$$A_0(s) = -11 \frac{m_G^2}{\Lambda_G^2} - 25 \frac{m_G^4}{\Lambda_G^2} \frac{1}{s - m_G^2} + 50 \frac{m_G^4}{\Lambda_G^2} \frac{\log\left(1 + \frac{s - 4m_G^2}{m_G^2}\right)}{s - 4m_G^2}$$

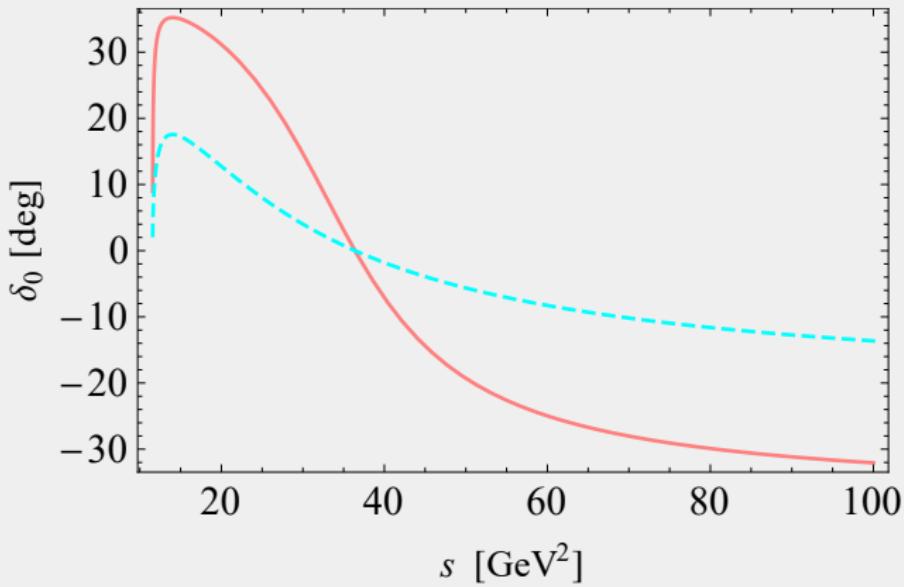
$$a_l = \frac{A_l(4m_G^2)}{32\pi m_G k^{2l}} \stackrel{l=0}{\equiv} \frac{1}{32\pi m_G} \frac{92m_G^2}{3\Lambda_G^2}$$

$$a_0(s_{th}) = \frac{23m_G}{24\pi\Lambda_G^2}$$

$$(a_l = \frac{25(l!)^2 4^{l-2}}{\pi (2l+1)(2l)!} \frac{1}{\Lambda_G^2 m_G^{2l-1}}, \text{ for } l \geq 2)$$

phase shift

$$\delta_l(s) = \frac{1}{2} \arg \left(1 + \frac{i\sqrt{s-4m_G^2}}{16\pi\sqrt{s}} A_l \right).$$



$\Lambda_G = 0.4$ (pink) and 0.8 (blue) GeV

$\Delta\delta_0 := \delta_0(s \rightarrow \infty) - \delta_0(4m_G^2) \neq n\pi \rightarrow$ unitarization

SCHEMATIC REPRESENTATION OF THE UNITARIZATION THROUGH LOOPS

TREE LEVEL

$$\text{TL} = G^4 + G^3_s + G^3_t + G^3_u$$

Unitarization

$$U = \text{TL} + \text{TL} \text{---} \text{TL} \text{---} \text{TL} + \dots$$

$$= \text{TL} \text{---} \text{TL} \text{---} \text{TL} \text{---} \text{TL} + \dots =$$

$$= \text{TL} + \text{TL} \text{---} \text{TL} \text{---} \text{U}$$

$$U = \text{TL} + (\text{TL}) \sum U$$

UNITARIZATION (S,D,G WAVE)

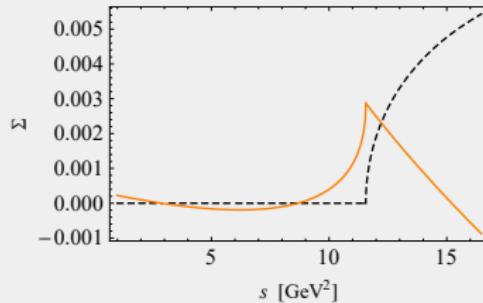
Loop function $\Sigma(s)$ in such a way to:

-preserve the pole corresponding to $s = m_G^2$

- $A_l^{unit}(s = 3m_G^2) = \infty$: the branch point $s = 3m_G^2$ is generated by the single particle pole for m_G^2 along the t and u channels.

It follows that:

$$\Sigma(s) = \frac{(s - m_G^2)(s - 3m_G^2)}{\pi} \int_{4m_G^2}^{\infty} \frac{\frac{s'}{4} - m_G^2}{(s' - s)(s' - 3m_G^2)(s' - m_G^2)} \frac{ds'}{16\pi\sqrt{s'}}$$



$Re\Sigma$ (orange) vs. $Im\Sigma$ (black)

The unitarized amplitude is so given by:

$$A_l^{unit}(s) = [A_l^{-1}(s) - \Sigma(s)]^{-1}$$

Now:

$$\delta_l^{unit}(s) = \frac{1}{2} \arg \left[1 + 2i \frac{\sqrt{\frac{s}{4} - m_G^2}}{16\pi\sqrt{s}} A_l(s)^{unit} \right]$$

and

$$a_o^U(s_{th}) \left(= \frac{1}{32\pi m_G} A_o^U(s) \right)_{s=s_{th}} = \frac{1}{32\pi m_G} \frac{1}{\frac{3\Lambda_G^2}{92m_G^2} - 0.0028715}.$$

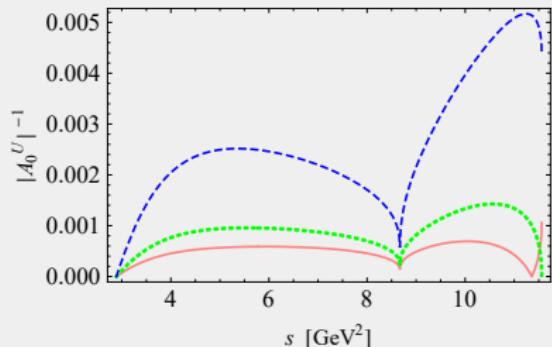
With our value of m_G :

$$a_o(s_{th}) = \infty \leftrightarrow \Lambda_G = \Lambda_{G,crit} \approx 0.504 \text{ GeV}$$

REMARKS ON THE UNITARIZATION SCHEME

- Twice-subtracted loop ($s = m_G^2, 3m_G^2$): single-particle pole and branch point fixed to the ones of TL amplitude.
Once-subtracted loop ($s = m_G^2$): ghost with negative norm
- Results depend not only on the chosen subtraction, but also on the applied unitarization scheme (overall phenomenology expected similar)
- Multiparticle states contribute to the imaginary part: $\text{Im}\Sigma$ not valid up to $\infty \rightarrow$ Cutoff function (form not known) interpretable as G wf overlap: this would introduce a model dependence on the result.

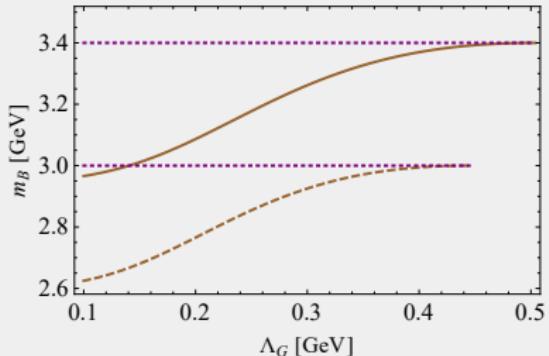
THE GLUEBALLONIUM



$|A_0^U|^{-1}(s)$ for:

- ▷ $\Lambda_G = 0.4 \text{ GeV}$ (pink)
- ▷ $\Lambda_G = \Lambda_{G,crit} \approx 0.504 \text{ GeV}$ (green)
- ▷ $\Lambda_G = 0.8 \text{ GeV}$ (blue).

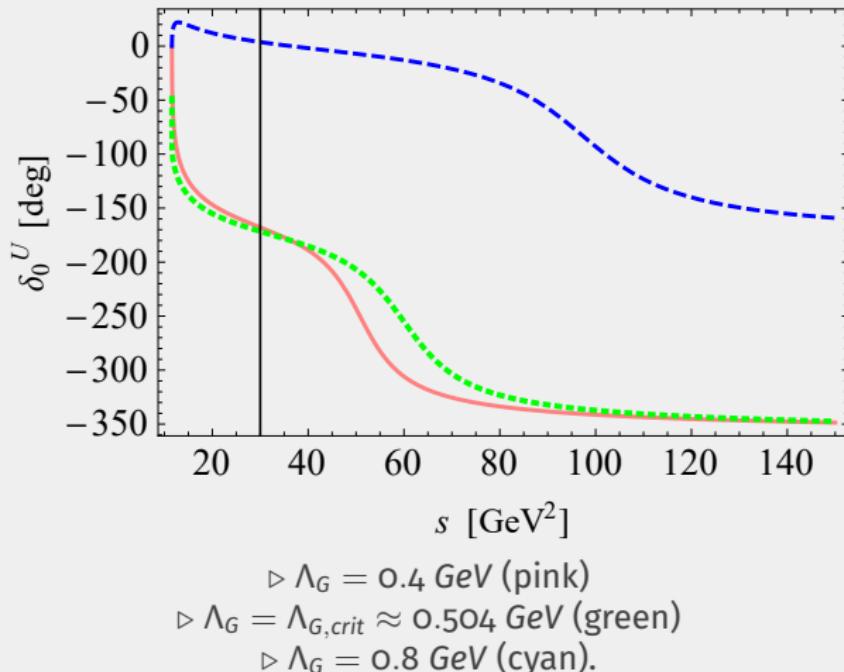
$\Lambda_{G,crit} : a_0^U(s = s_{th})|_{\Lambda_G=\Lambda_{G,crit}} = \infty$



Mass of the glueballonium as function of Λ_G

For $m_G = 1.5$ and 1.7 GeV :
 $\Lambda_{G,crit} \sim 0.445$ and 0.504 GeV .
 For $\Lambda_G > \Lambda_{G,crit}$, \nexists bound state.

PHASE SHIFT



$\Delta\delta_0^U \rightarrow -2\pi \rightarrow$ Levinson theorem fulfilled

OTHER GLUEBALLS EFFECT: HEAVY SCALAR

H: other glueball entering the s-channel (heavy scalar and non-scalar)



$$\mathcal{L} = \mathcal{L}_{\text{dil}} + \frac{1}{2}(\partial_\mu H)^2 - \frac{\alpha}{2}G^2H^2 - \beta H^4$$

No 3-leg interaction terms (H^2G, \dots): break dilatation invariance (only $\ln \left| \frac{G}{\Lambda_G} \right|$ does it in the proper way).

No $\ln \left| \frac{H}{\Lambda_H} \right|$: another scale would be needed (not done here).

The interaction term proportional to G^2H^2 affects the scattering via an intermediate HH loop, relevant at $(2m_H)^2 \sim 36 \text{ GeV}^2$, far from threshold.

NON SCALAR J^{PC}

No $J > 2$ for dilatation invariance.

	P	C
0^{-+}	\times	\checkmark
0^{+-}	\times	\times
0^{--}	\checkmark	\times
1^{++}	\times	\checkmark
1^{-+}	\checkmark	\checkmark
1^{+-}	\times	\times
1^{--}	\checkmark	\times

Table: Symmetries for XG^2 terms involving a heavy nonscalar glueball X and two scalar glueballs G .

The term in \mathcal{L} corresponding to the coupling XG^2 , $X \equiv 1^{-+}$, is the only acceptable. However, vector current for a neutral scalar vanishes:
 $G\partial^\mu G - (\partial^\mu G)G = 0$.

CONCLUSIONS

- TL is not enough.
- Unitarized amplitudes for the l-th waves ($l=0,2,4$).
- The evaluated scattering lengths and phase shifts could be simulated on the lattice, hence a comparison is possible.
- Emergence of a 2 scalar glueball bound state, that we named "glueballonium", could be found on the lattice and/or in experiments (PANDA).
- $m_B(\Lambda_G)$. \exists bound state if $\Lambda_G < \Lambda_{G,crit} \simeq 0.504$ GeV if $m_G = 1.7$ GeV
- Other glueballs are not expected to sizably affect the results.
- FOR FUTURE: scattering of other glueballs or using other unitarization schemes.

Thank you for the attention

Decay $G \rightarrow \pi\pi$ takes the form (inserting a nonzero pion mass):

$$\Gamma_{G \rightarrow \pi\pi} = 6 \frac{\sqrt{\frac{m_G^2}{4} - m_\pi^2}}{8\pi m_G^2} \left(\frac{m_\sigma^2}{2\Lambda_G} \right)^2.$$

For $m_G \simeq 1.7$ GeV, $m_\sigma \simeq 1.3$ GeV ($\simeq f_0(1370)$), $\Lambda_G \simeq 0.4$ GeV: $\Gamma_{G \rightarrow \pi\pi} \simeq 0.310$ GeV.
 $SU(3)$ extension:

$$\Gamma_{G \rightarrow KK} = 8 \frac{\sqrt{\frac{m_G^2}{4} - m_K^2}}{8\pi m_G^2} \left(\frac{m_\sigma^2}{2\Lambda_G} \right)^2, \quad \Gamma_{G \rightarrow \eta\eta} = 2 \frac{\sqrt{\frac{m_G^2}{4} - m_\eta^2}}{8\pi m_G^2} \left(\frac{m_\sigma^2}{2\Lambda_G} \right)^2.$$

$\Gamma_{G \rightarrow KK} \simeq 0.340$ GeV and $\Gamma_{G \rightarrow \eta\eta} \simeq 0.080$ GeV.

The sum of the 3 pseudoscalar channel: 0.729 GeV.

$\oplus G \rightarrow \rho\rho \rightarrow 4\pi$, expected sizable, cannot be determined within this simple approach (using $V(G, \sigma, \pi)$).

Such a glueball would have a total decay of about 1 GeV: would not be observable.

YM FIELD

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}, \quad G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c$$

gluonic quantum fluctuation: dilatation symmetry

$$g_0 \xrightarrow[\text{anomaly}]{\text{trace}} g(\mu)$$

$$\Rightarrow \partial_\mu J_{\text{dil}}^\mu = (T_\mu^\mu)_{\text{YM}} = \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0, \quad \beta(g) = \partial g / \partial \ln \mu$$

At one loop ($\beta(g) = -bg^3$ & $b = 11N_c/(48\pi^2)$):

$$g^2(\mu) = \frac{1}{2b \ln(\mu/\Lambda_{\text{YM}})}, \quad \Lambda_{\text{YM}} \approx 250 \text{ MeV}$$

Nonvanishing expectation value of the trace anomaly:

$$\langle T_\mu^\mu \rangle_{\text{YM}} = -\frac{11N_c}{24} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{24} C^4$$

GLUEBALL FIELD

Gluons $\xrightarrow[\text{confinement}]{} \text{not the asymptotic states of the theory}$

Scalar field G describing scalar glueball & trace anomaly at the composite level

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G)$$

$$\partial_\mu J_{\text{dil}}^\mu = 4V - G \partial_G V \propto G^4 \text{ only if } V(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left(G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right)$$

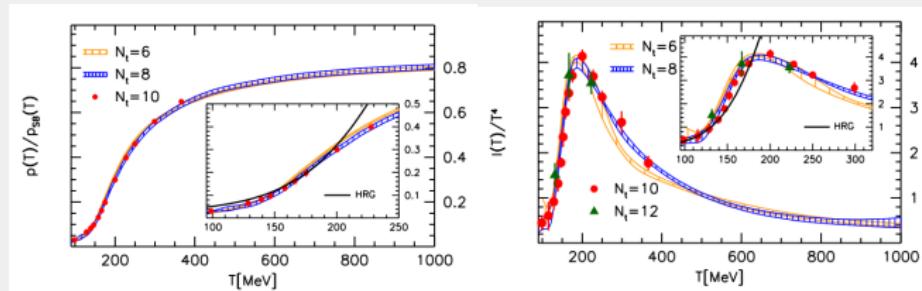
$\Lambda_G \simeq 0.4 \text{ GeV}$: $\left[\partial_\mu J_{\text{dil}}^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4 \right] \ominus \langle T_\mu^\mu \rangle_{YM}$: Dilaton field saturates the trace of the dilatation current.

■ Hadron Resonance Gas model:

$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3} \left(\sum_{i \in mes} \log \mathcal{Z}^M(T, V, \mu_{X^a}, m_i) + \sum_{i \in bar} \log \mathcal{Z}^B(T, V, \mu_{X^a}, m_i) \right)$$

(Fugacities: $z_i := \exp \left\{ \frac{1}{T} \sum_a X_i^a \mu_{X^a} \right\}$. X^a : all possible conserved charges.)

$$\log \mathcal{Z}^{M,B}(T, V, \mu_{X^a}, m_i) := \mp V d_i \int_0^\infty \frac{dk}{2\pi^2} k^2 \log \left(1 \mp z_i e^{-\frac{\sqrt{k^2 + m_i^2}}{T}} \right)$$

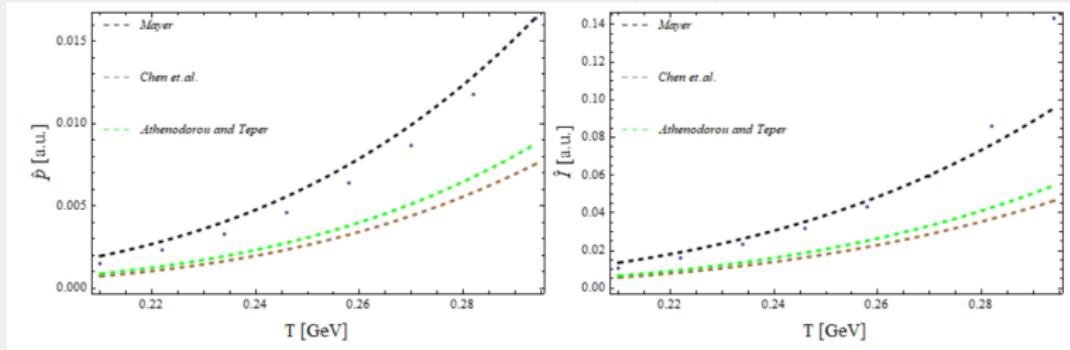
 ■ All baryons and mesons ($m < 2.5$ GeV) from PDG [Borsigny et al. JHEP11(2010)077]


■ Can we do in pure YM (GRG)?

Pressure and energy density

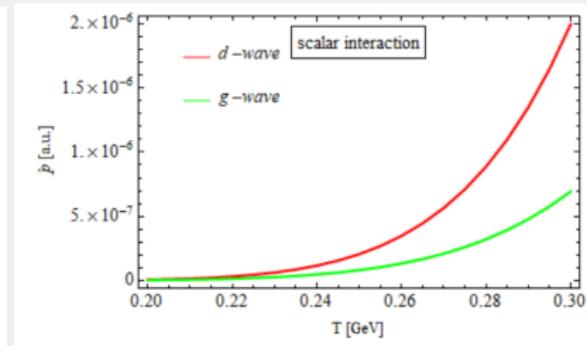
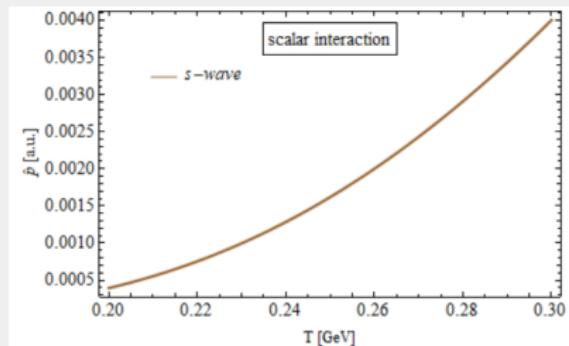
$$\hat{p} = -T \sum_J (2J+1) \int_0^\infty \frac{k^2}{2\pi^2} \ln \left(1 - e^{-\frac{\sqrt{k^2+m^2}}{T}} \right) dk$$

$$\hat{p} = (2J+1) \int_0^\infty \frac{k^2}{2\pi^2} \frac{\sqrt{k^2+m^2}}{\exp \left[\frac{\sqrt{k^2+m^2}}{T} \right] - 1} dk$$

Trace anomaly $\hat{l} = \hat{p} - 3\hat{\rho}$. $T_c \approx 0.3$ GeV [Braun et.al. Eur. Phys.J.C 70, 689 (2010).]

SCALAR GLUEBALL

$$P^{\text{int}} + P_B = -T \sum_{l=0}^{\infty} \int_0^{\infty} dx \frac{2l+1}{\pi} \frac{d\delta_l^U(s=x^2)}{dx} \int \frac{d^3k}{(2\pi)^3} \ln \left(1 - e^{-\beta \frac{\sqrt{k^2+x^2}}{T}} \right)$$



TENSOR $\rightarrow \dots$