# The complex heavy-quark potential in an anisotropic quark-gluon plasma - Statics and dynamics

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(arXiv 2205.10349, May 20, 2022)

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**15th Quark Confinement and Hadron Spectrum Conference**

**4.20 pm, August 5, 2022**

# What we do?

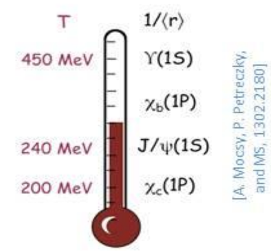
We study the dynamics of heavy quarkonium states ($q\bar{q}$) inside the Quark Gluon Plasma (QGP)

**Bottomonium: $b\bar{b}$**

| $n^{2s+1}L_J$ | $J^{PC}$ | Particle | Mass (MeV) |
|---|---|---|---|
| $1^3S_1$ | $1^{--}$ | $\Upsilon(1S)$ | 9460.30 |
| $1^3P_0$ | $0^{++}$ | $\chi_{b0}\,(1P)$ | 9859.44 |
| $1^3P_1$ | $1^{++}$ | $\chi_{b1}\,(1P)$ | 9892.78 |
| $1^3P_2$ | $2^{++}$ | $\chi_{b2}\,(1P)$ | 9912.21 |
| $2^3S_1$ | $1^{--}$ | $\Upsilon(2S)$ | 10023.26 |
| $2^3P_0$ | $0^{++}$ | $\chi_{b0}\,(2P)$ | 10232.50 |
| $2^3P_1$ | $1^{++}$ | $\chi_{b1}\,(2P)$ | 10255.46 |
| $2^3P_2$ | $2^{++}$ | $\chi_{b2}\,(2P)$ | 10268.65 |
| $3^3S_1$ | $1^{--}$ | $\Upsilon(3S)$ | 10355.20 |

**Charmonium: $c\bar{c}$**

| $n^{2s+1}L_J$ | $J^{PC}$ | Particle | Mass (MeV) |
|---|---|---|---|
| $1^3S_1$ | $1^{--}$ | $J/\psi$ | 3096.90 |
| $1^3P_0$ | $0^{++}$ | $\chi_{c0}\,(1P)$ | 3414.71 |
| $1^3P_1$ | $1^{++}$ | $\chi_{c1}\,(1P)$ | 3510.67 |
| $1^3P_2$ | $2^{++}$ | $\chi_{c2}\,(1P)$ | 3556.17 |




Suppression factor,

$$
\begin{aligned}
R_{AA} &= \frac{\text{number produced in } AA}{\langle N_{\rm bin}\rangle_b \,(\text{number produced in } pp)}\,, \quad \langle N_{\rm bin}\rangle_b = \text{number of nucleonic collisions} \\
&= 1\,,\ \text{No effect} \\
&> 1\,,\ \text{Enhancement} \\
&< 1\,,\ \text{Suppression}
\end{aligned}
$$

Full complex potential $\Rightarrow$ Solve SWE $\Rightarrow$ Obtain observables such as $R_{AA}$ and elliptic flow ($v_2$) $\Rightarrow$ Compare with experimental data available from the ATLAS, ALICE, and CMS collaborations.

# What we did in our current work?

L. Dong, Y. Gua, A. Islam, A. Rothkopf, and M. Strickland : arXiv 2205.10349, May 20, 2022

3D Anisotropic Potential Model

↓

1D Effective Potential Model

*easier to solve
*includes the splitting of different p-wave polarizations

# Isotropic Potential

L. Dong, Y. Gua, A. Islam, A. Rothkopf, and M. Strickland : arXiv 2205.10349, May 20, 2022

- **Perturbative contribution:**

The complex HQ potential model in an isotropic QCD plasma is defined by a Fourier transform of the real time resummed gluon propagator in the static limit. Such a gluon propagator includes a perturbative contribution, which is calculable in the HTL resummed perturbation theory.

$$V(\lambda, r) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1) D^{00}(p_0 = 0, \mathbf{p}, \lambda)$$

$$\mathrm{Re}\, V_{\mathrm{pt}}(\lambda, r) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\left(\frac{1}{p^2 + m_D^2} - \frac{1}{p^2}\right) \equiv \alpha m_D (\mathcal{I}_1(\hat{r}) - 1)$$

$$\mathrm{Im}\, V_{\mathrm{pt}}(\lambda, r) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\frac{-\pi\lambda m_D^2}{p(p^2 + m_D^2)^2} \equiv \alpha\lambda(\mathcal{I}_2(\hat{r}) - 1)$$

Where,

$$\mathcal{I}_1(\hat{r}) = 4\pi \int \frac{d^3\hat{\mathbf{p}}}{(2\pi)^3} e^{i\hat{\mathbf{p}}\cdot\hat{\mathbf{r}}} \frac{1}{\hat{p}^2(\hat{p}^2+1)} = \frac{1 - e^{-\hat{r}}}{\hat{r}}, \quad \mathcal{I}_2(\hat{r}) = 4\pi^2 \int \frac{d^3\hat{\mathbf{p}}}{(2\pi)^3} e^{i\hat{\mathbf{p}}\cdot\hat{\mathbf{r}}} \frac{1}{\hat{p}(\hat{p}^2+1)^2} = \phi_2(\hat{r})$$

$$\phi_n(\hat{r}) = 2\int_0^\infty dz \frac{\sin(z\hat{r})}{z\hat{r}} \frac{z}{(z^2+1)^n}\,.$$

# Isotropic Potential (continued)

L. Dong, Y. Gua, A. Islam, A. Rothkopf, and M. Strickland : arXiv 2205.10349, May 20, 2022

- **Non-perturbative contribution:**

The gluon propagator also includes a non-perturbative string contribution originating from the dimension two gluon condensate.

$$\mathrm{Re}\, V_{\mathrm{npt}}(\lambda, r) = -g^2 C_F m_G^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\frac{p^2 + 5m_D^2}{(p^2 + m_D^2)^3} \equiv -\frac{2\sigma}{m_D}(\mathcal{I}_3(\hat{r}) - 1)$$

$$\mathrm{Im}\, V_{\mathrm{npt}}(\lambda, r) = -g^2 C_F m_G^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)\frac{4\pi\lambda m_D^2(p^2 - 2m_D^2)}{p(p^2 + m_D^2)^4} \equiv \frac{4\sigma\lambda}{m_D^2}(\mathcal{I}_4(\hat{r}) - 1)$$

Where,

$$\mathcal{I}_3(\hat{r}) = 4\pi \int \frac{d^3\hat{\mathbf{p}}}{(2\pi)^3} e^{i\hat{\mathbf{p}}\cdot\hat{\mathbf{r}}} \frac{\hat{p}^2 + 5}{(\hat{p}^2 + 1)^3} = (1 + \hat{r}/2)e^{-\hat{r}}\,,$$

$$\mathcal{I}_4(\hat{r}) = 8\pi^2 \int \frac{d^3\hat{\mathbf{p}}}{(2\pi)^3} e^{i\hat{\mathbf{p}}\cdot\hat{\mathbf{r}}} \frac{2 - \hat{p}^2}{\hat{p}(\hat{p}^2 + 1)^4} = -2\phi_3(\hat{r}) + 6\phi_4(\hat{r})\,.$$

# Isotropic potential (continued...)

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Total potential

= perturbative contribution + non-perturbative contribution

$$\mathrm{Re}\, V_{\mathrm{Iso}}(r) = \alpha m_D \left( \frac{1 - e^{-rm_D}}{rm_D} \right) - \alpha m_D - \frac{\sigma}{m_D} (2 + rm_D)\, e^{-rm_D} + \frac{2\sigma}{m_D} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r}$$

$$\mathrm{Im}\, V_{\mathrm{Iso}}(r) = \alpha\lambda\phi_2\,(rm_D) - \alpha\lambda - \frac{8\sigma\lambda}{m_D^2}\phi_3(rm_D) + \frac{24\sigma\lambda}{m_D^2}\phi_4(rm_D) - \frac{4\sigma\lambda}{m_D^2}$$

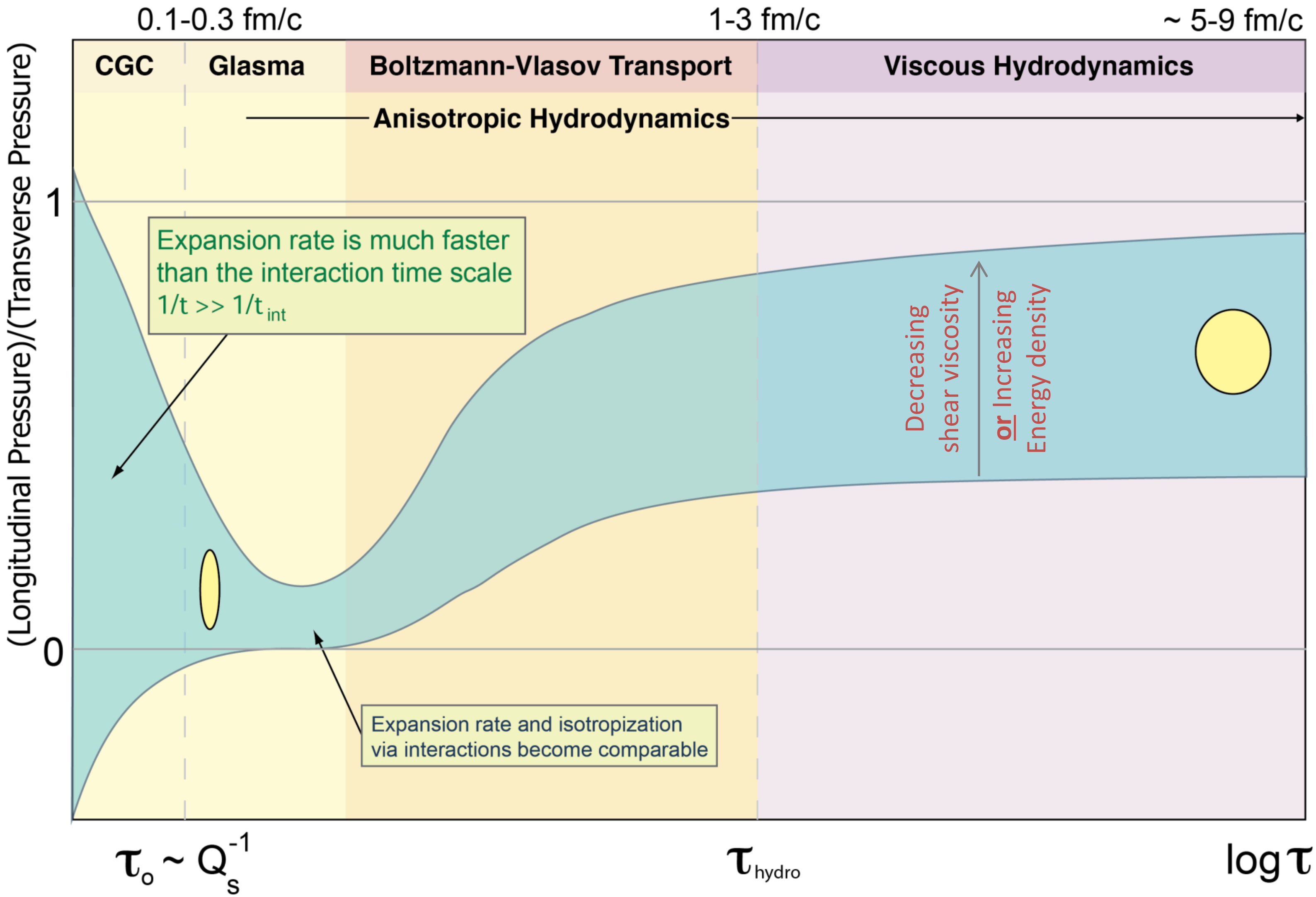
$$m_D = Ag\lambda\sqrt{\frac{N_C}{3} + \frac{N_F}{6}}$$

$\lambda$ is a temperature-like scale which, only in the thermal equilibrium limit, should be understood as the temperature T of the system.

$$\alpha_s = \tfrac{g^2}{4\pi}, \quad \alpha = \alpha_s\, C_F, \quad C_F = \tfrac{N_C^2 - 1}{2N_C}. \quad N_C = 3, \quad N_F = 2, \quad C_F = \tfrac{4}{3}$$

$$A = 1.4, \quad g = 1.72, \quad \alpha = 0.272, \quad \sigma = 0.215\ GeV^2, \quad m_b = 4.7\ GeV, \quad m_c = 1.3\ GeV$$

# QGP momentum anisotropy cartoon



# Anisotropic Distribution Function

P. Romatschke and M. Strickland : Phys. Rev. D 68, 036004 (2004), Phys. Rev. D 70, 116006 (2004)

- In order to model the physical processes occurring in heavy-ion experiments, the following anisotropic distribution function of Romatschke-Strickland form is used extensively

$$f_{\rm aniso}^{\rm LRF}(\mathbf{k}) \equiv f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{\mathbf{k}^2+\xi(\mathbf{k}\cdot\mathbf{n})^2}\right)$$

- It takes into account the rapid longitudinal expansion of the QGP, which results in momentum anisotropy in the local rest frame (LRF)

An adjustable parameter $\xi$ in the range $-1 < \xi < \infty$ is used to quantify the degree of momentum-space anisotropy

$$\xi = \frac{1}{2}\frac{\langle \mathbf{k}_\perp^2\rangle}{\langle k_z^2\rangle} - 1\,,$$

where $k_z \equiv \mathbf{k}\cdot\mathbf{n}$ and $\mathbf{k}_\perp \equiv \mathbf{k} - \mathbf{n}(\mathbf{k}\cdot\mathbf{n})$ correspond to the particle momenta along and perpendicular to the direction of anisotropy, respectively.

# 3D Anisotropic Potential

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$$\mathrm{Re}\, V_{\mathrm{Aniso}}(r,\theta,\xi) = \alpha m_D^A \left( \frac{1 - e^{-r m_D^R}}{r m_D^R} \right) - \alpha m_D^A - \frac{\sigma}{m_D^A} \left( 2 + r m_D^R \right) e^{-r m_D^R} + \frac{2\sigma}{m_D^A} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r}$$

$$\mathrm{Im}\, V_{\mathrm{Aniso}}(r,\theta,\xi) = \alpha \lambda^A \phi_2 \left( r m_D^I \right) - \alpha \lambda^A - \frac{8\sigma\lambda^A}{(m_D^A)^2} \phi_3 \left( r m_D^I \right) + \frac{24\sigma\lambda^A}{(m_D^A)^2} \phi_4 \left( r m_D^I \right) - \frac{4\sigma\lambda^A}{(m_D^A)^2}$$

where,

$$m_D^A = m_D \left( 1 - \frac{\xi}{6} \right), \quad \lambda^A = \lambda \left( 1 - \frac{\xi}{6} \right)$$

$$m_D^R = m_D \left[ 1 + \xi \left( 0.108 \cos 2\theta - 0.131 \right) \right], \quad m_D^I = m_D \left[ 1 + \xi \left( 0.026 \cos 2\theta - 0.158 \right) \right]$$

**Disadvantages:**

(1) The angular dependence in the potential model requires solving a 3D Schrodinger eq.

(2) Time consuming and much more complicated, compared to the case where a spherically symmetric HQ potential can be used.

(3) This is indeed the main obstacle to developing phenomenological applications which include momentum-anisotropy effects.

# 1D Effective Potential

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As proposed in our work, one possible solution to this difficulty is to employ an angle-averaged effective screening mass

$$\begin{aligned}\mathcal{M}_{lm}(\lambda,\xi) &= \langle \mathrm{Y}_{lm}(\theta,\phi)|\tilde{m}_D(\lambda,\xi,\theta)|\mathrm{Y}_{lm}(\theta,\phi)\rangle\,,\\ &= \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi \mathrm{Y}_{lm}(\theta,\phi)\tilde{m}_D(\lambda,\xi,\theta)\mathrm{Y}^*_{lm}(\theta,\phi)\end{aligned}$$

and where $\mathrm{Y}_{lm}(\theta,\phi)$ refers to the spherical harmonics with azimuthal quantum number $l$ and magnetic quantum number $m$. The idea is to replace the anisotropic screening mass with $\mathcal{M}_{lm}(\lambda,\xi)$ which recovers the spherical symmetry in the potential model, thus significantly simplifying the numerics.

$$\mathrm{Im}\, V_{\mathrm{Eff}}(r,\xi) = \alpha m_D^A \left(\frac{1-e^{-r\mathcal{M}^R_{lm}}}{r\mathcal{M}^R_{lm}}\right) - \alpha m_D^A - \frac{\sigma}{m_D^A}\left(2+r\mathcal{M}^R_{lm}\right)e^{-r\mathcal{M}^R_{lm}} + \frac{2\sigma}{m_D^A} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r}$$

$$\mathrm{Im}\, V_{\mathrm{Eff}}(r,\xi) = \alpha\lambda^A\phi_2\left(r\mathcal{M}^I_{lm}\right) - \alpha\lambda^A - \frac{8\sigma\lambda^A}{(m_D^A)^2}\phi_3\left(r\mathcal{M}^I_{lm}\right) + \frac{24\sigma\lambda^A}{(m_D^A)^2}\phi_4\left(r\mathcal{M}^I_{lm}\right) - \frac{4\sigma\lambda^A}{(m_D^A)^2}$$

where,

$$K_{lm} = \frac{2l(l+1)-2m^2-1}{4l(l+1)-3}$$

$$\mathcal{M}^R_{lm} = m_D\Big[1+\xi\left(0.216K_{lm}-0.239\right)\Big]\,,\quad \mathcal{M}^I_{lm} = m_D\Big[1+\xi\left(0.052K_{lm}-0.184\right)\Big]$$

# Static Results: Bottomonium and Charmonium

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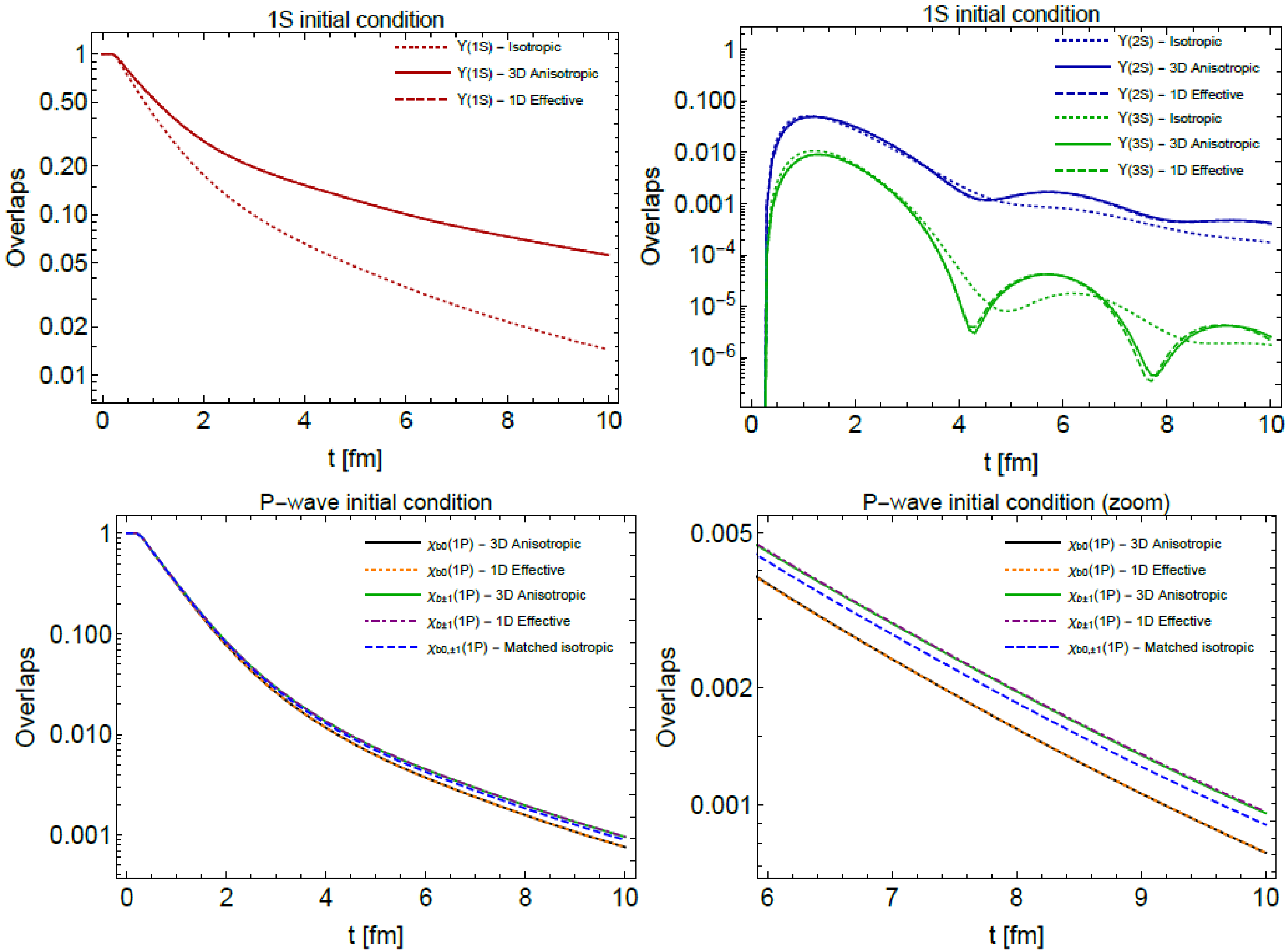
| $\Upsilon(1S)$ | Re$E$ | $\delta$Re$E$ | $E_{\rm bind}$ | Im$E$ | $\delta$Im$E$ |
|---|---|---|---|---|---|
| $T_o$ | 182.869 | 0.611 | −662.669 | 11.838 | 0.027 |
| $1.1T_o$ | 174.957 | 0.593 | −570.612 | 14.830 | 0.031 |
| $1.2T_o$ | 166.556 | 0.573 | −493.689 | 18.190 | 0.034 |
| $1.4T_o$ | 148.439 | 0.531 | −372.540 | 26.004 | 0.039 |

| $J/\Psi$ | Re$E$ | $\delta$Re$E$ | $E_{\rm bind}$ | Im$E$ | $\delta$Im$E$ |
|---|---|---|---|---|---|
| $T_o$ | 439.336 | 1.230 | −406.202 | 41.980 | 0.107 |
| $1.1T_o$ | 422.207 | 1.163 | −323.362 | 51.467 | 0.105 |
| $1.2T_o$ | 404.597 | 1.095 | −255.648 | 61.698 | 0.098 |
| $1.3T_o$ | 386.604 | 1.028 | −199.583 | 72.564 | 0.086 |
| $1.4T_o$ | 368.301 | 0.963 | −152.678 | 83.958 | 0.070 |

The exact results of the complex eigenenergies ($E$) and binding energies ($E_{\rm bind}$) for different quarkonium states at various temperatures with $\xi = 1$. Comparing with the results obtained based on the 1D potential model with effective screening masses, the corresponding differences as denoted by $\delta E$ are also listed. The reference temperature $T_o$ is 192 MeV and all the results are given in the units of MeV.

# Dynamic Results: Bottomonium

L. Dong, Y. Gua, A. Islam, A. Rothkopf, and M. Strickland : arXiv 2205.10349, May 20, 2022



# Dynamic Results: Charmonium

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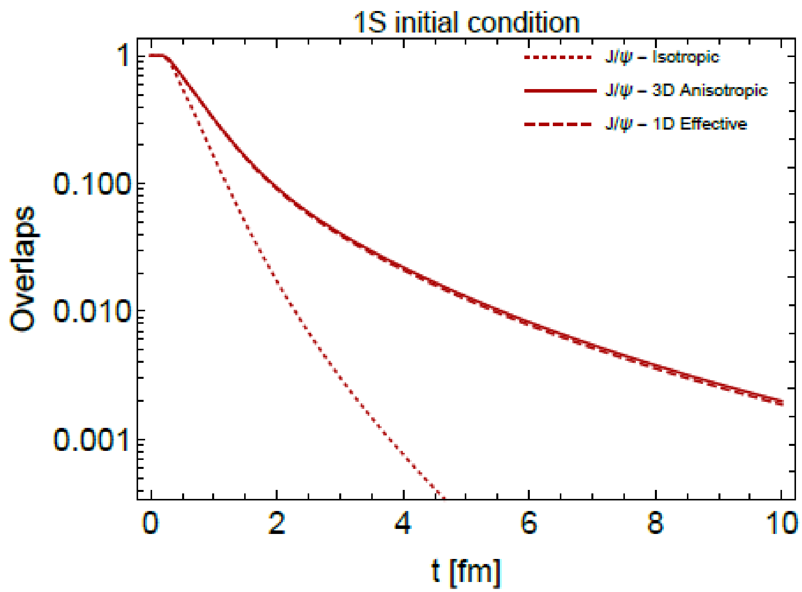


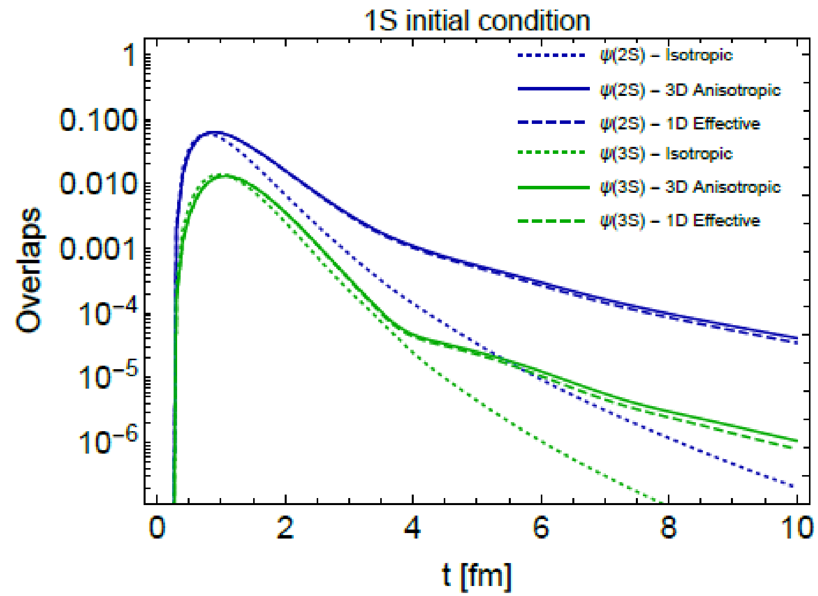


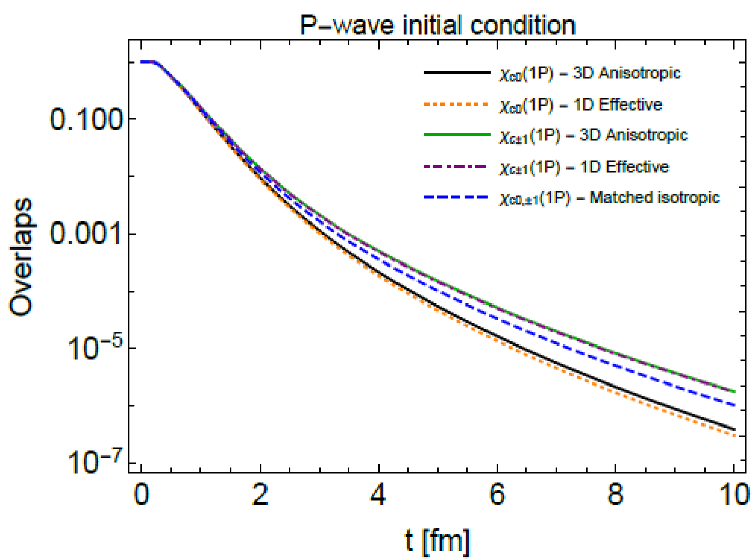


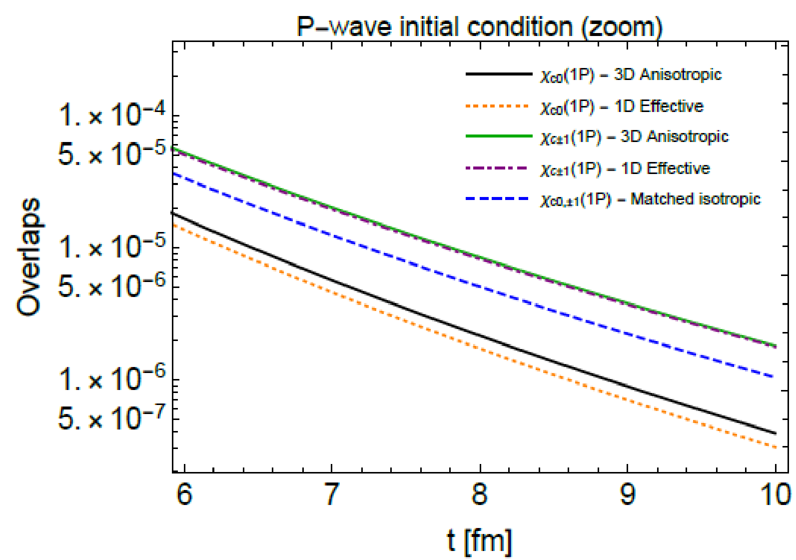

# Summary

- We demonstrate that, using the resulting 1D effective potential model, one can solve a 1D Schrödinger equation and reproduce the full 3D results for the energies and binding energies of low-lying heavy-quarkonium bound states to relatively high accuracy.
- The 1D effective isotropic potential is much easier to solve as compared to the 3D anisotropic potential.
- The resulting 1D effective isotropic potential model includes the splitting of different p-wave polarizations.

# Future Directions

- To extend the model for large anisotropic parameter ($\xi$)
- Phenomenological studies
- The resulting 1D effective isotropic potential model could provide an efficient method for including momentum anisotropy effects in open quantum system simulations of heavy-quarkonium dynamics in the quark gluon plasma.

N. Brambilla, M. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo, and P. Vander Griend : arXiv 2205.10289, May 20, 2022