The role of subleading power corrections to heavy quarkonium production

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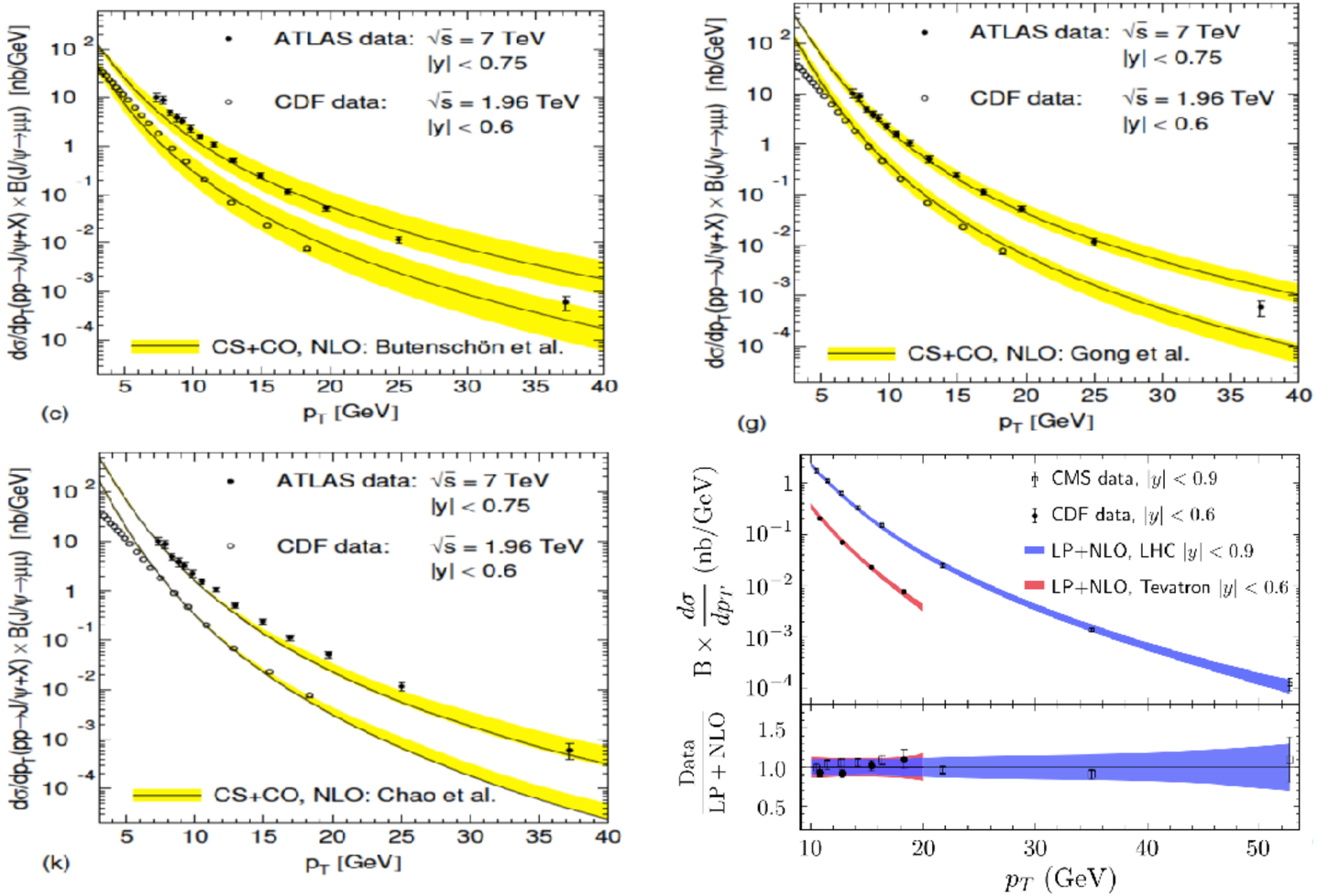
arXiv:2108.00305 [hep-ph] and in preparation

The XVth Quark Confinement and the Hadron Spectrum
@University of Stavanger, Aug 2, 2022

NRQCD vs. Experimental data



	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
Set II (Chao *et al.*)	1.16	8.9	0.30	1.26
Set III (Gong *et al.*)	1.16	9.7	−0.46	−2.14
Set IV (Bodwin *et al.*)	-	9.9	1.1	1.1

Fits in NRQCD

Butenschoen, Kniehl, PRD84, 051501 (2011).
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).
…

Fits in pNRQCD

Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

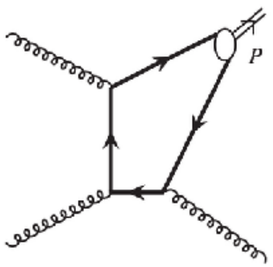
LDMEs should be universality, however:

- Numbers are not the same.
- Not even the sign.

More work is needed!

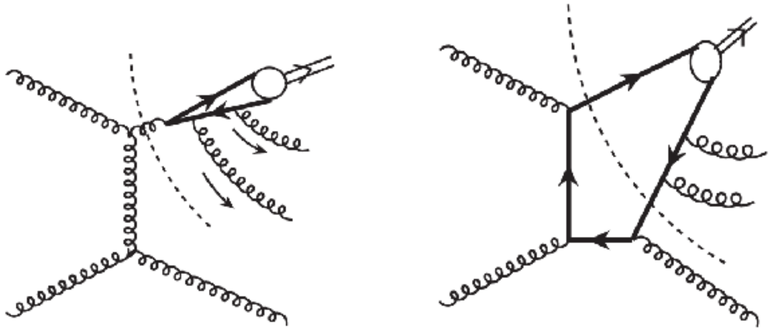
Heavy quarkonium production of high p_T

- High p_T quarkonium production ($p_T^2 \gg (2m)^2 \gg \Lambda_{\rm QCD}^2$): separation between the short distance part for $Q\bar{Q}$ production and the bound state formation part.
- NRQCD factorization: Long-Distance Matrix Elements (LDMEs) are organized by the power of the quark velocity $v^2 \sim q_T^2/m^2 < 1$ with q_T the relative momentum between a quark pair.



$$\text{LO}: \quad d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \alpha_s^3 m^4/p_T^8$$

- At high p_T, higher order corrections must be essential:
$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \alpha_s^3 m^4/p_T^8 \times \alpha_s p_T^2/m^2 = \alpha_s^4 m^2/p_T^6$
- More importantly, the gluon jet fragmentation gives $d\sigma \propto \alpha_s^5/p_T^4$ as well as $d\sigma \propto \alpha_s^2/p_\perp^4 \times \alpha_s^3 \ln(p_T^2/m^2)$. The later is enhanced even if $\alpha_s \ll 1$.
- We may not obtain reliable predictions by considering only diagrams in the naive α_s expansion as well as v expansion.

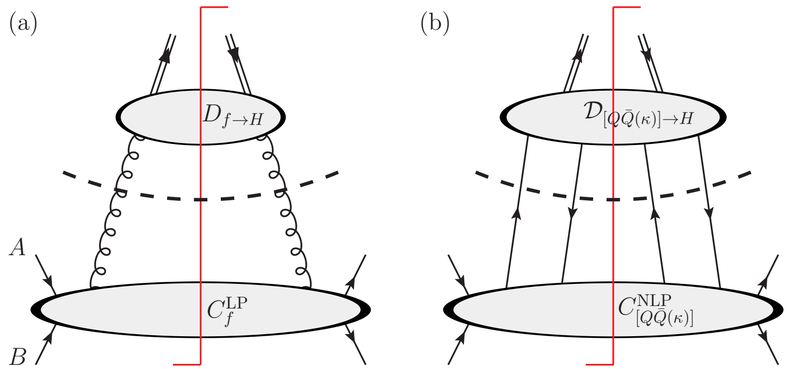


- In this talk, we consider QCD factorization with fragmentation functions (FFs).

QCD factorization approach

Nayak, Qiu, Sterman, PRD72 (2005) 114012
Kang, Qiu, Sterman, PRL108 (2012) 102002
Kang, Ma, Qiu, Sterman, PRD90 (2014) 3, 034006, PRD91 (2015) 1, 014030

Leading power (LP)



Subleading power (NLP): critical at moderate $p_T \gtrsim \mathcal{O}(2m)$

$$d\sigma^{\rm QCD-Res}_{A+B\to[f,Q\bar{Q}]\to H+X}(\mu) = \sum_{f=q,\bar{q},g} C^{\rm LP}_{A+B\to[f]+X}(\mu)\otimes D_{[f]\to H}(\mu) + \frac{1}{p_\perp^2}\left[\sum_n C^{\rm NLP}_{A+B\to[Q\bar{Q}(n)]+X}(\mu)\otimes \mathcal{D}_{[Q\bar{Q}(n)]\to H}(\mu)\right]$$

- In hadronic collisions, short distance coefficients (SDCs) $C^{\rm LP}$ and $C^{\rm NLP}$ are available up to NLO and LO in α_s expansion, respectively. $C^{\rm NLP}$ at NLO has been calculated only in e^+e^- collisions. Lee, Sterman, JHEP 09 (2020) 046
- D_f and $\mathcal{D}_{Q\bar{Q}}$: Twist-2 single parton (SP) and Twist-4 double parton (DP) fragmentation functions.
- Matching condition:

$$d\sigma_{A+B\to H+X}(m\neq 0) = d\sigma^{\rm QCD-Evol}_{A+B\to H+X}(m=0) - d\sigma^{\rm QCD-(n)}_{A+B\to H+X}(m=0) + d\sigma^{\rm NRQCD-(n)}_{A+B\to H+X}(m\neq 0)$$

$$\Rightarrow \begin{cases} d\sigma^{\rm QCD-Evol}_{A+B\to H+X} & \text{when} \quad p_\perp \gg m;\ d\sigma^{\rm NRQCD-(n)} \approx d\sigma^{\rm QCD-(n)} \\ d\sigma^{\rm NRQCD-(n)}_{A+B\to H+X} & \text{when} \quad p_\perp \to m;\ d\sigma^{\rm QCD-Evol} \approx d\sigma^{\rm QCD-(n)} \end{cases}$$

Renormalization group improvement

- Twist-2 evolution equation: DGLAP + **nonlinear quark pair corrections**

Kang, Ma, Qiu, Sterman, PRD 90, 3, 034006 (2014)

$$\frac{\partial D_{[f]\to H}}{\partial \ln \mu^2} = \gamma_{[f]\to[f']} \otimes D_{[f']\to H} + \frac{1}{\mu^2}\gamma_{[f]\to[Q\bar{Q}(\kappa)]} \otimes \boxed{\mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}}$$

The inhomogeneous term is added to the **slope**, not to the FF itself.

v: vector
a: axial vector
t: tensor

$$f, f' = q, Q, g$$

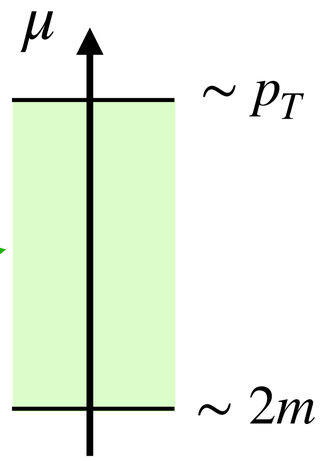
$$\kappa, n = v^{[8]}, v^{[1]}, a^{[8]}, a^{[1]}, t^{[8]}, t^{[1]}$$

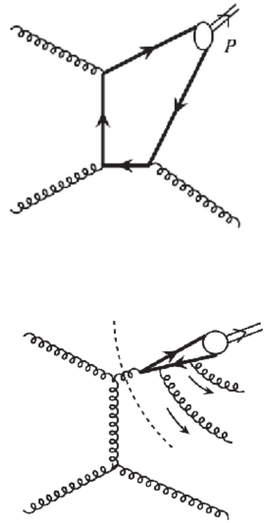
- Twist-4 "DGLAP like" evolution equation:

$$\boxed{\frac{\partial \mathscr{D}_{[Q\bar{Q}(n)]\to H}}{\partial \ln \mu^2}} = \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}$$

The RG improved factorized cross section covers all events in which the heavy quark pair can be produced:

1. at the short-distance: early stage (**NLP**)
2. at the input scale: later stage (**LP**)
3. in-between (**Nonlinear quark pair correction**)





Evolution of DP FFs in u, v-space

Lee, Qiu, Sterman, **KW**, in preparation.

Consider the derivative of a test function:

$$D'_{\kappa\to n}(z,u,v) \equiv \frac{2\pi}{\alpha_s}\frac{dD_{\kappa\to n}(z,u,v)}{d\ln\mu^2},$$

$$D(z,u,v) \to D_z(z)D_u(u)D_v(v),$$

$$D_z(z,\alpha) = \frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]},$$

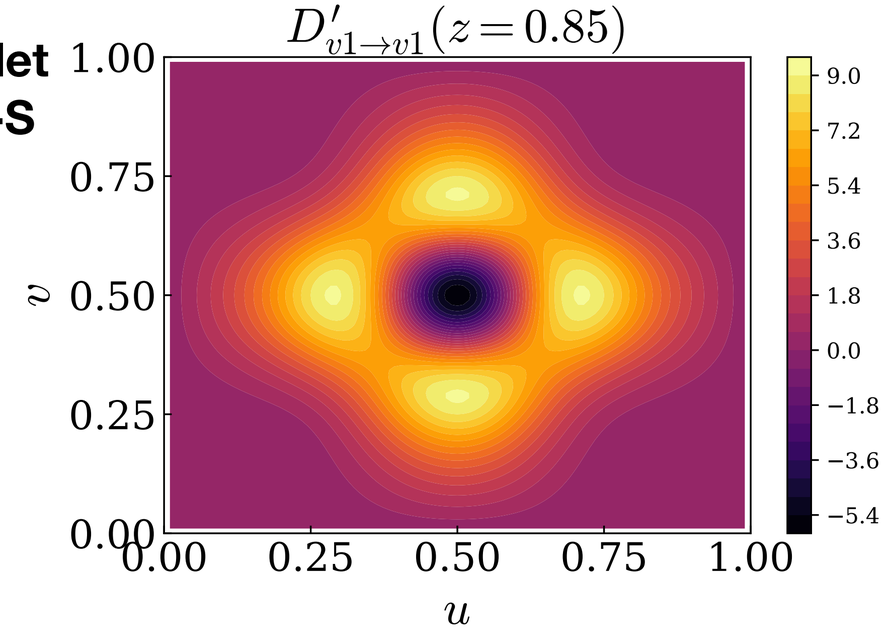
$$D_{u,v}(x,\gamma) = \frac{x^\gamma(1-x)^\gamma}{B[1+\gamma,1+\gamma]},$$

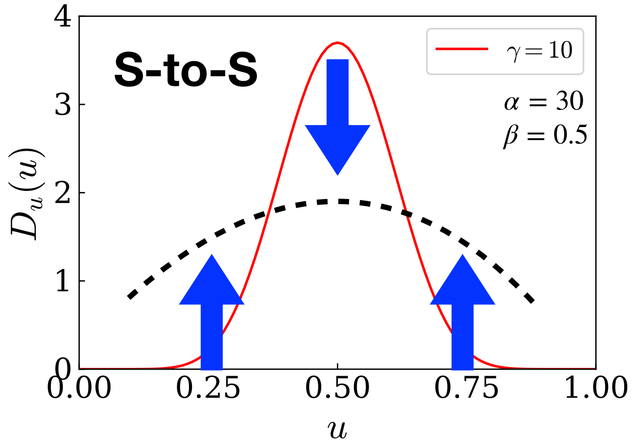
amplitude : $p_Q = up_c,\ p_{\bar{Q}} = (1-u)p_c$

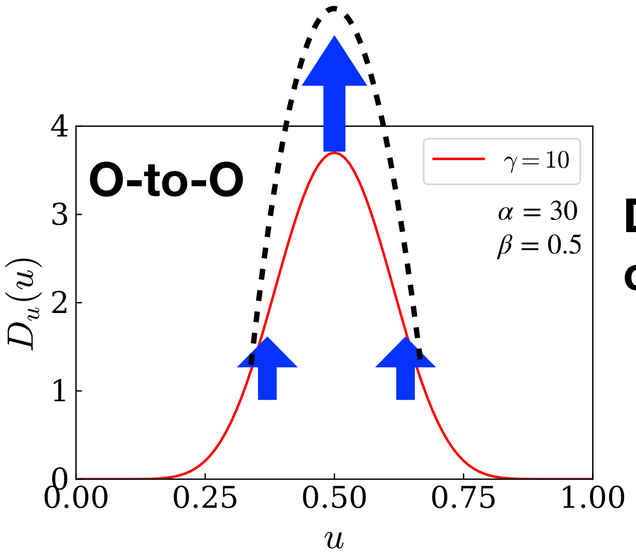
c . c . amplitude : $p_Q = vp_c,\ p_{\bar{Q}} = (1-v)p_c$

with $zp_c^+ = p^+$

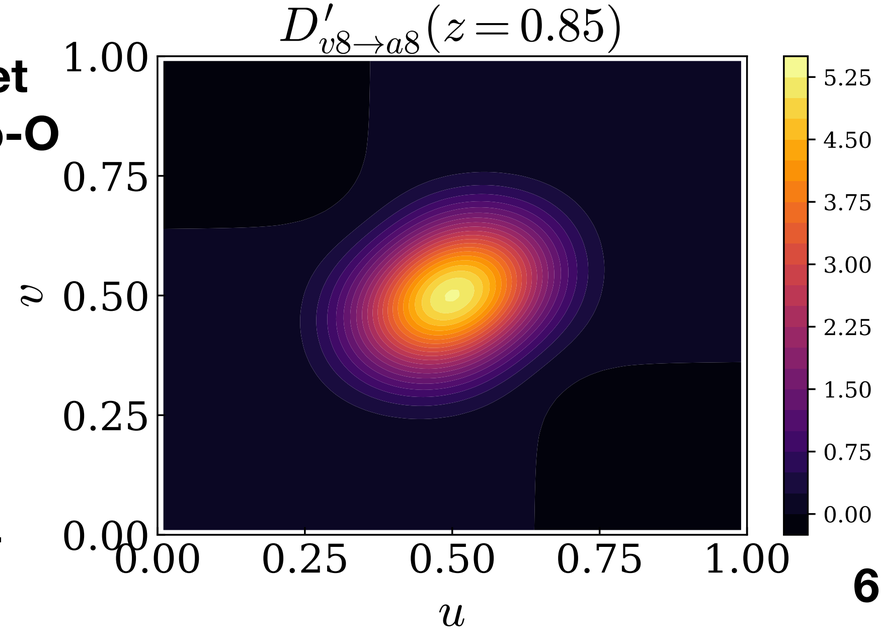
Diagonal singlet channel: S-to-S





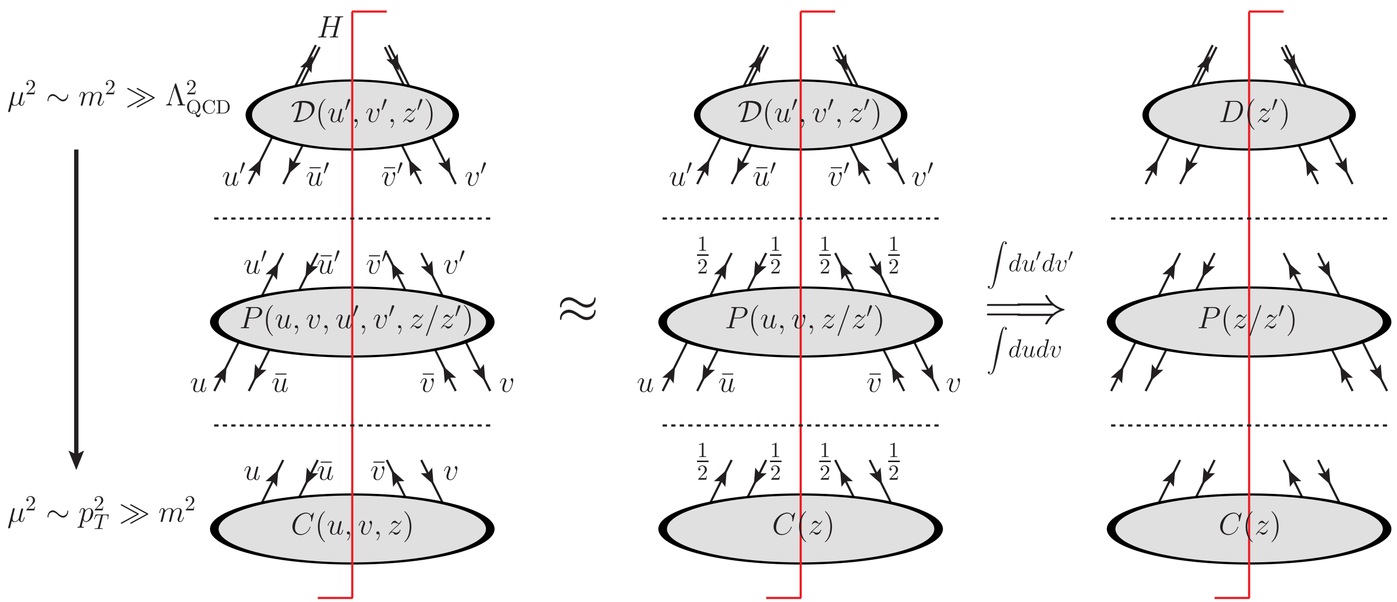


Diagonal octet channel: O-to-O



- S-to-S DP FFs get **broader** in u, v-space after evolution.
- O-to-O DP FFs become **narrower** with a large peak around $u = v = 1/2$.
- Off-diagonal channels: similar to O-to-O.

Evolution equations in a simplified situation



- The produced heavy quark pair is dominated by its on-shell state at high p_T.
- We may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around $u = v = 1/2$.
- This can be a reasonable approximation suggested by the evolution of DP FFs in u, v-space. S-to-S channels are not dominant at high p_T.

$$\frac{d\sigma^H_{\rm NLP}}{dyd^2p_T} = \int dzdudv\, C_{[Q\bar{Q}]}(p_Q, p_{\bar{Q}}, \mu)\, \mathscr{D}_{[Q\bar{Q}]\to H}(u, v, z, \mu) \approx \int dz\, C_{[Q\bar{Q}]}(\hat{p}^+_Q = \frac{1}{2}p^+_c, \hat{p}^+_{\bar{Q}} = \frac{1}{2}p^+_c, \mu) \underbrace{\int dudv\, \mathscr{D}_{[Q\bar{Q}]\to H}(u, v, z, \mu)}_{\equiv D_{[Q\bar{Q}]\to H}(z,\mu)}$$

$$\frac{\partial D_{[Q\bar{Q}(\kappa)]\to H}(z, \mu)}{\partial \ln \mu^2} \approx \sum_n \int_z^1 \frac{dz'}{z'} \int_0^1 du \int_0^1 dv\, \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z', \mu\right),$$

$$\frac{\partial D_{f\to H}(z, \mu)}{\partial \ln \mu^2} \approx \frac{\alpha_s}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}(z/z') D_{f'\to H}(z') + \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z', \mu\right)$$

Input FFs

Ma, Qiu, Zhang, PRD89, no.9, 094029, ibid. 094030 (2014)
Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale $\qquad \kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}$

Perturbative SDCs of input FFs in α_s and v expansion in the NRQCD are reliable only when SDCs $\ll \mathcal{O}(1)$. Indeed, the NRQCD factorization is not reliable as $z \to 1$ where SDCs $\hat{d}(z)$ include the following terms:

1. $\delta(1-z)$ at LO in α_s expansion
2. $f(z)\ln(1-z)$ with $f(z)$ being a regular function
3. $\dfrac{f(z)}{[1-z]_+}$, $f(z)\left[\dfrac{\ln(1-z)}{1-z}\right]_+$ due to the perturbative cancelation of IR divergences

In our current analysis, we use analytic results if those vanish as $z \to 1$, otherwise, singular or negative input FFs are cast into

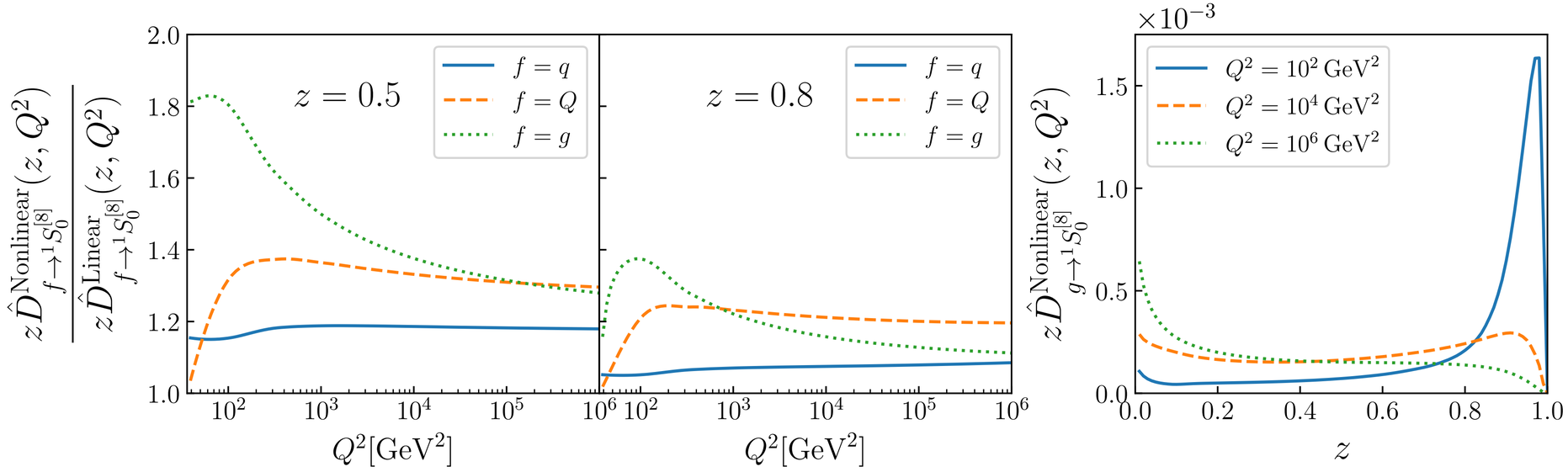
$$D_{[Q\bar{Q}(n)]}(z) = C_{[Q\bar{Q}(n)]}(\alpha_s)\frac{z^\alpha(1-z)^\beta}{B[1+\alpha,1+\beta]} \qquad (\alpha \gg 1,\ 1 > \beta > 0)$$

$\to$ to be tuned, imitating δ-function at LO.

$C_{[Q\bar{Q}(n)]}$: abs. value of the first moment

Quark pair corrections to SP FFs

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)
Lee, Qiu, Sterman, **KW**, in preparation.



$\alpha = 30, \beta = 0.5$

$\mu_0 = 4m_c = 6\,\text{GeV}$

$\mu_\Lambda = m_c = 1.5\,\text{GeV}$

The nonlinear quark pair corrections remain significant even at high $Q^2 = \mu^2 \sim p_T^2$.

$$\frac{\partial D_{f\to H}}{\partial \ln \mu^2} = \gamma_{f\to f'} \otimes D_{f'\to H} + \cancel{\frac{1}{\mu^2}\gamma_{f\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to H}}$$

$$\frac{\partial D^{\text{Nonlinear}}_{f\to H}}{\partial \ln \mu^2} \sim \frac{\partial D^{\text{Linear}}_{f\to H}}{\partial \ln \mu^2}$$

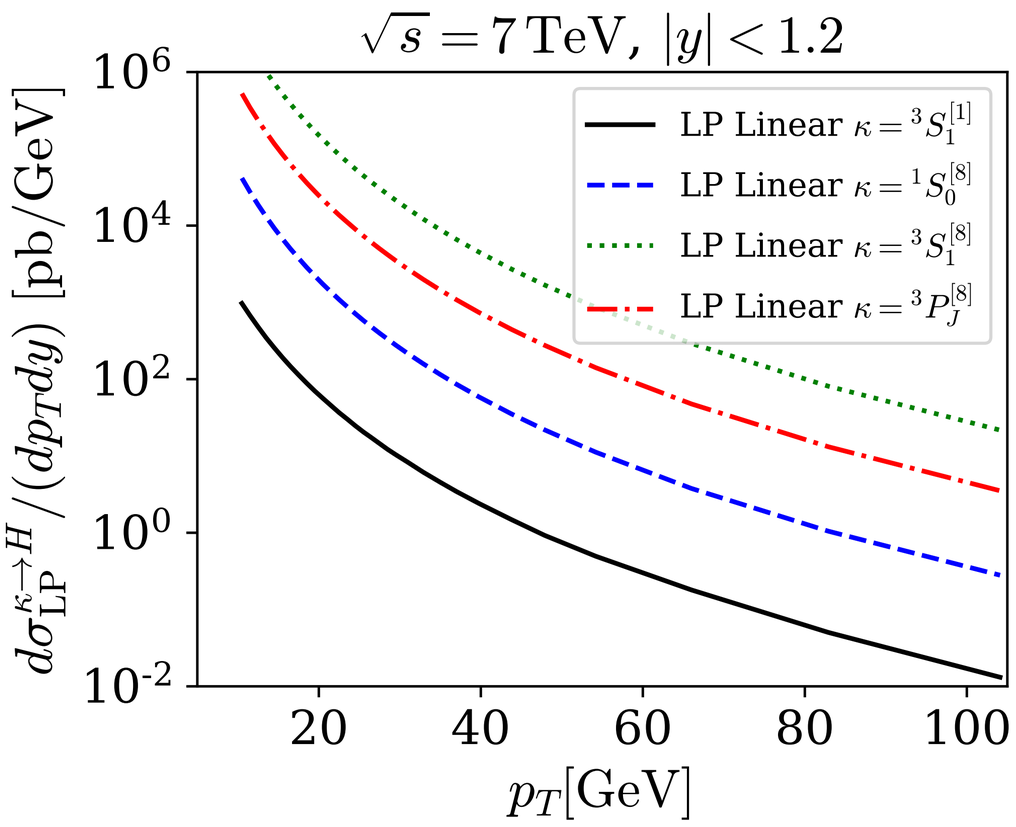
$\mu^2 \to \infty$: the slope of $D_{f\to H}$ is the same as LP DGLAP.

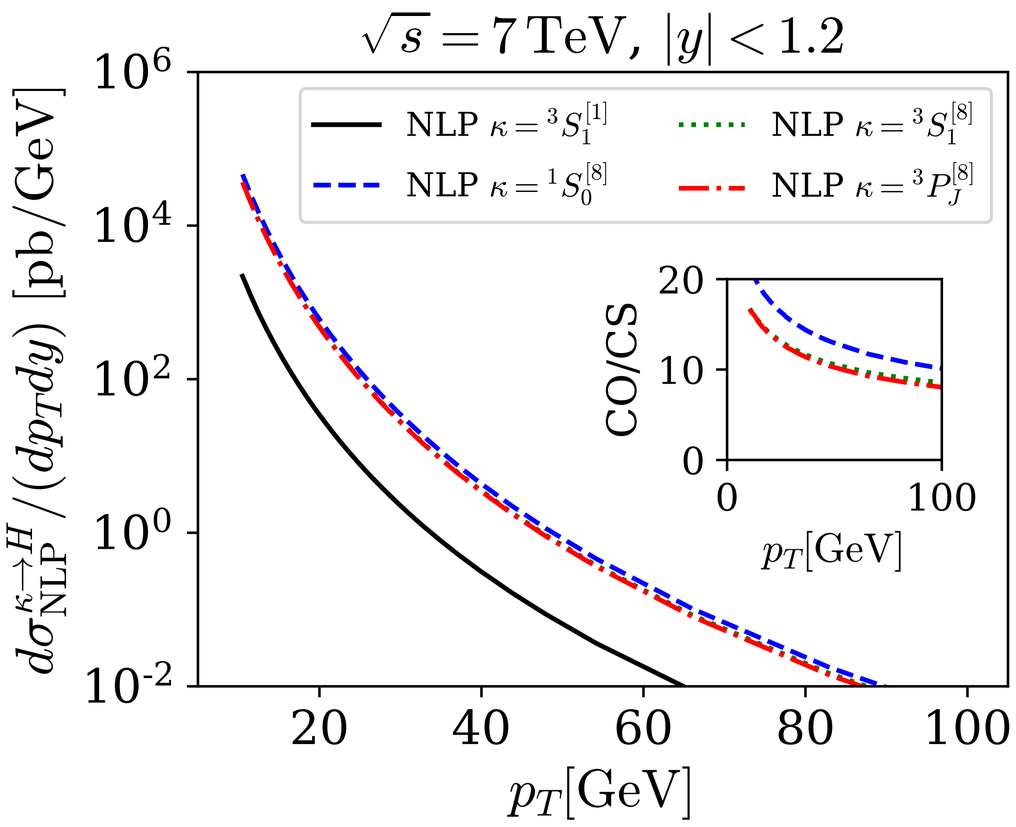
The power corrections effect at low μ^2 does not go away fast: **analogous to nonlinear gluon recombination effects to gluon PDF at small-x and large μ^2.**

Mueller and Qiu, NPB268, 427 (1986)
Qiu, NPB291, 746 (1987)
Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

Phenomenology (1/4)

Lee, Qiu, Sterman, **KW**, in preparation.



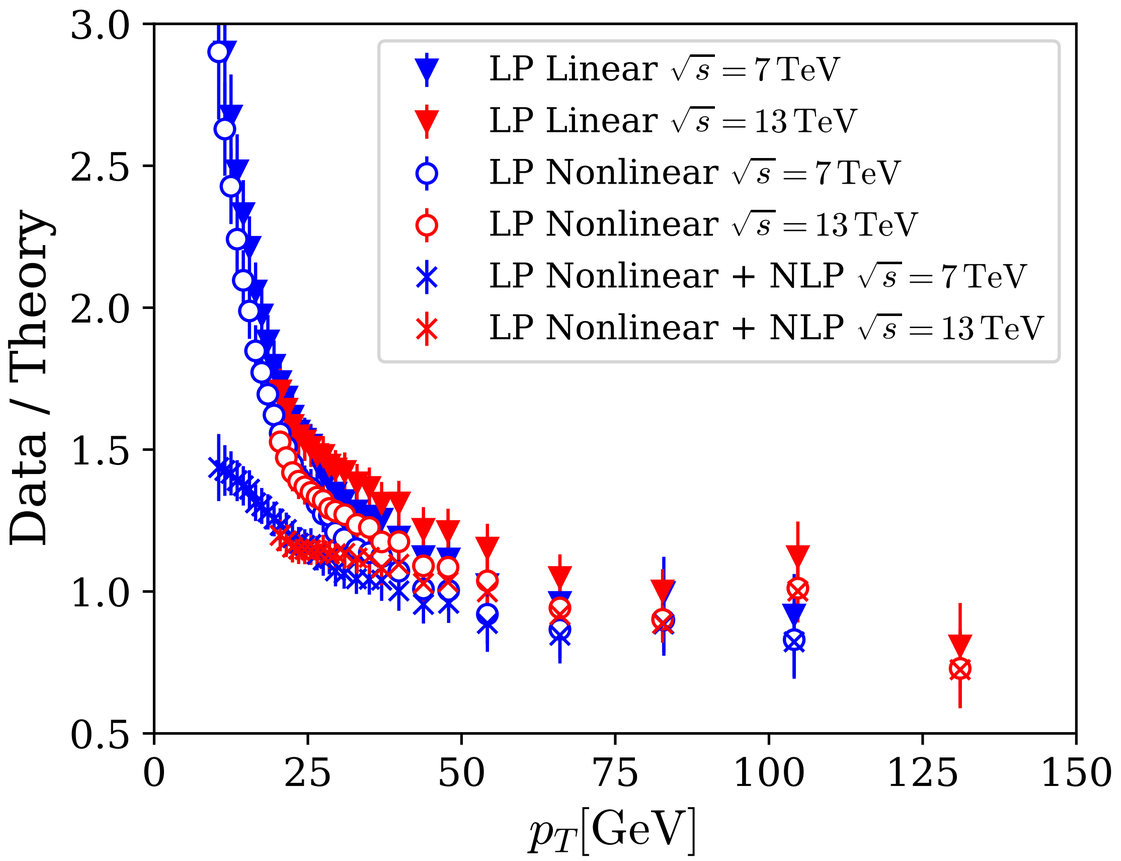


- Unweighted results: $\langle\mathcal{O}({}^3S_1^{[1]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^1S_0^{[8]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^3S_1^{[8]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^3P_0^{[8]})\rangle/\mathrm{GeV}^5 = 1.$
- $\alpha = 30, \beta = 0.5$ are fixed for both SP and DP FFs.
- ${}^1S_0^{[8]}$ is two orders of magnitude smaller than ${}^3S_1^{[8]}$ at LP.
- Three color octet channels at NLP provide similar contributions, steeply falling with p_T.

Phenomenology (2/4)

- Fitting the LP formalism with the linear evolution eq. to CMS data on high p_T prompt J/ψ at $\sqrt{s} = 7, 13\,\mathrm{TeV}$ in the bin, $|y| < 1.2$.
- # of data points in a fit: 3@7TeV + 4@13TeV = 7 for $p_T \geq 60\,\mathrm{GeV}$.
- Only the ${}^1S_0^{[8]}$ channel is considered, yielding unpolarized J/ψ. The other two color octet channels could overshoot data by combining LP and NLP.
- $\langle \mathcal{O}({}^1S_0^{[8]})\rangle/\mathrm{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ fitted by high p_T data is similar to the one extracted using fixed order NRQCD at NLO. Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)
- Global data fitting is useful to pin down LDMEs and the shape of input FFs.

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

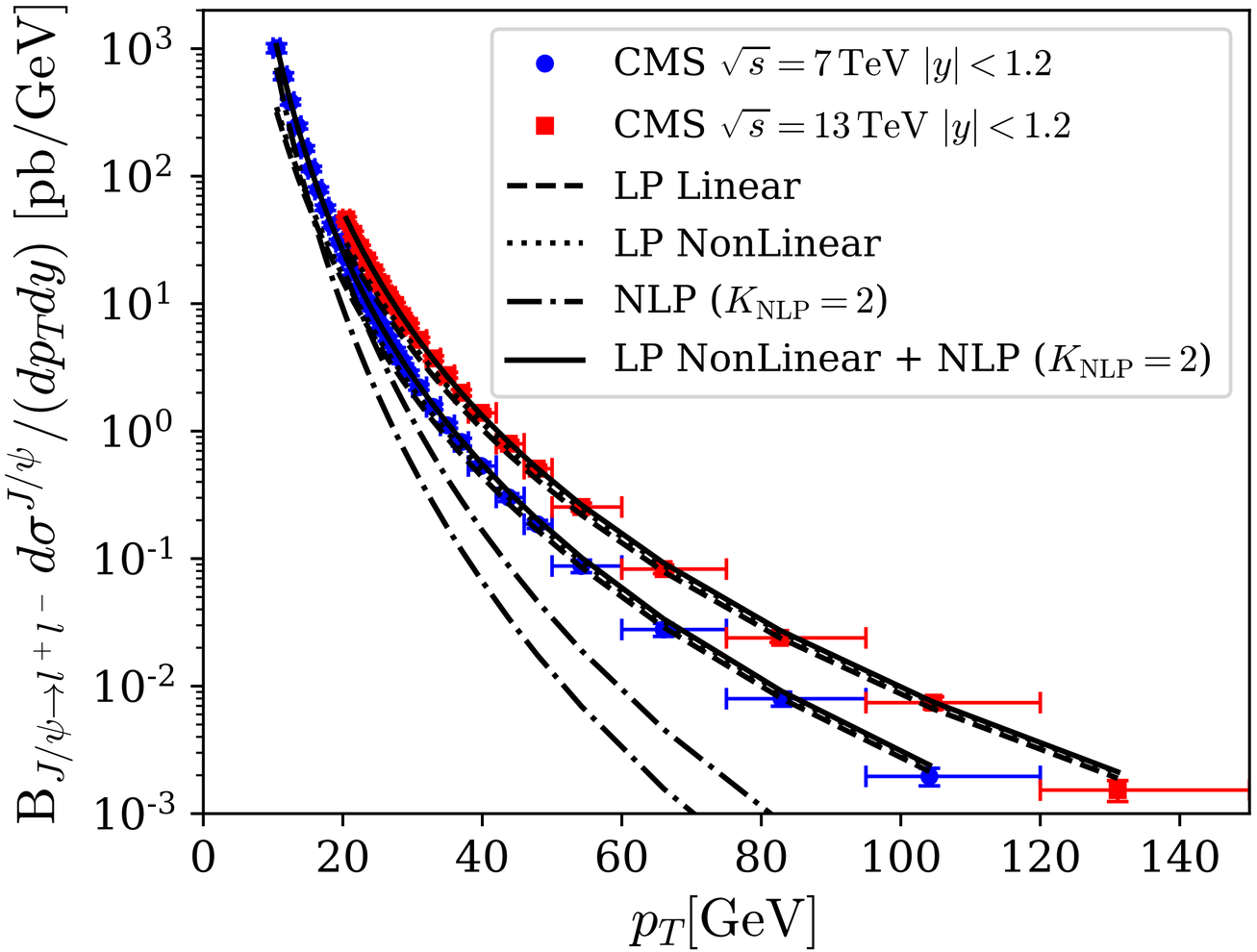


The power corrections do not vanish even at the highest p_T , giving **10-30% corrections**.
At $p_T = 30\,\mathrm{GeV}$ and below, the NLP corrections become significant.

Phenomenology (3/4)

Lee, Qiu, Sterman, **KW**, SciPost Phys. Proc.8, 143 (2022)

- Putting $\alpha = 30, \beta = 0.5$ at $\mu_0 = 4m_c$ and $\mu_\Lambda = m_c$, $\langle \mathcal{O}(^1S_0^{[8]})\rangle/\mathrm{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$ is obtained.
- *K*-factor is included to account for higher order corrections of the NLP partonic cross section. We simply fix $K_{\mathrm{NLP}} = 2$.



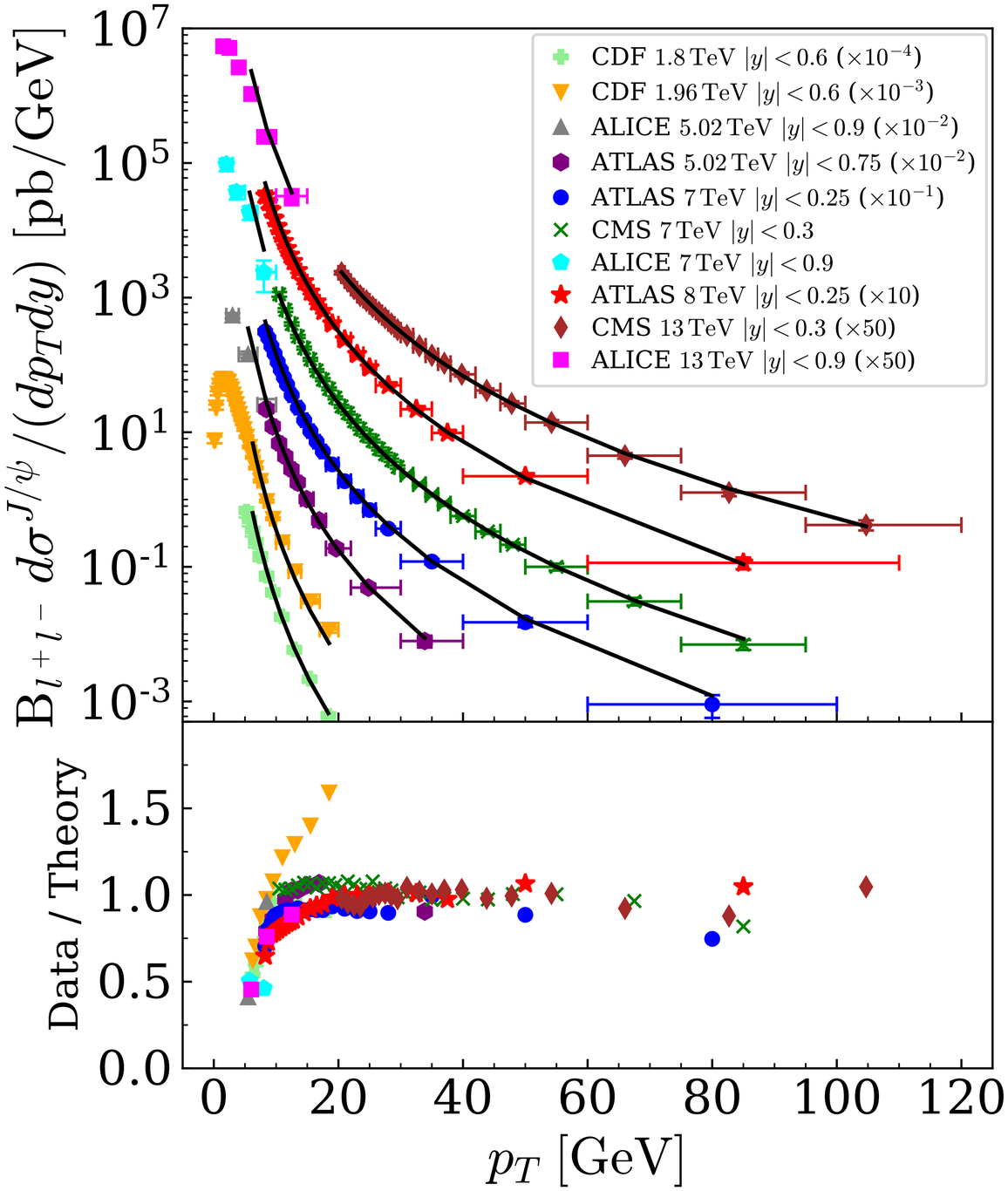
Phenomenology (4/4)

Lee, Qiu, Sterman, **KW**, in preparation.

- Given that the overall normalization factor is fixed, QCD factorization approach describes LHC data on prompt J/ψ production in hadronic collisions.

→ **QCD global data analysis is possible.**

- We could modify $K_{\rm NLP}$ at Tevatron energies, but $K_{\rm NLP} = 2$ is fixed here.



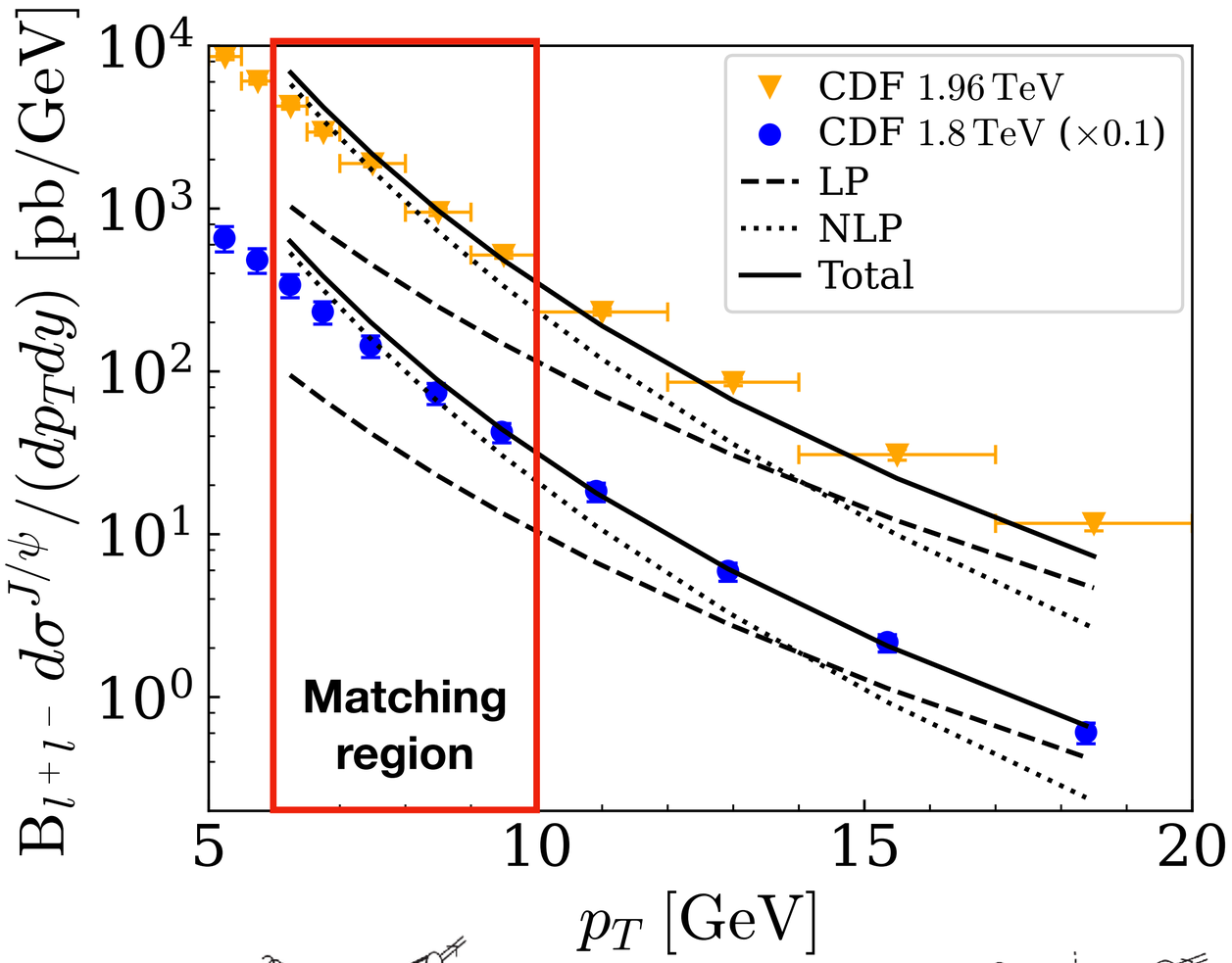
Discussion

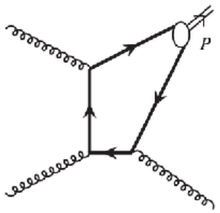
1. $\ln(p_T^2/m^2)$-type logarithmically enhanced contributions start to dominate when $p_T \gtrsim 5\,(\text{or}\,7) \times (2m_c) \sim 15 - 20\,\text{GeV}$, where the LP is significant, power corrections are small.

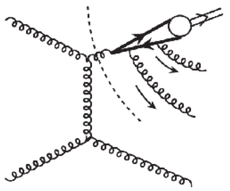
2. The NLP contribution is important at $p_T \lesssim 10\,\text{GeV} = \mathcal{O}(2m_c)$, where matching between QCD factorization and NRQCD factorization can be made.

3. Further exploration of the shape of the FFs at large-z would help us understand the quarkonium production mechanism.

Lee, Qiu, Sterman, **KW**, in preparation.







Summary

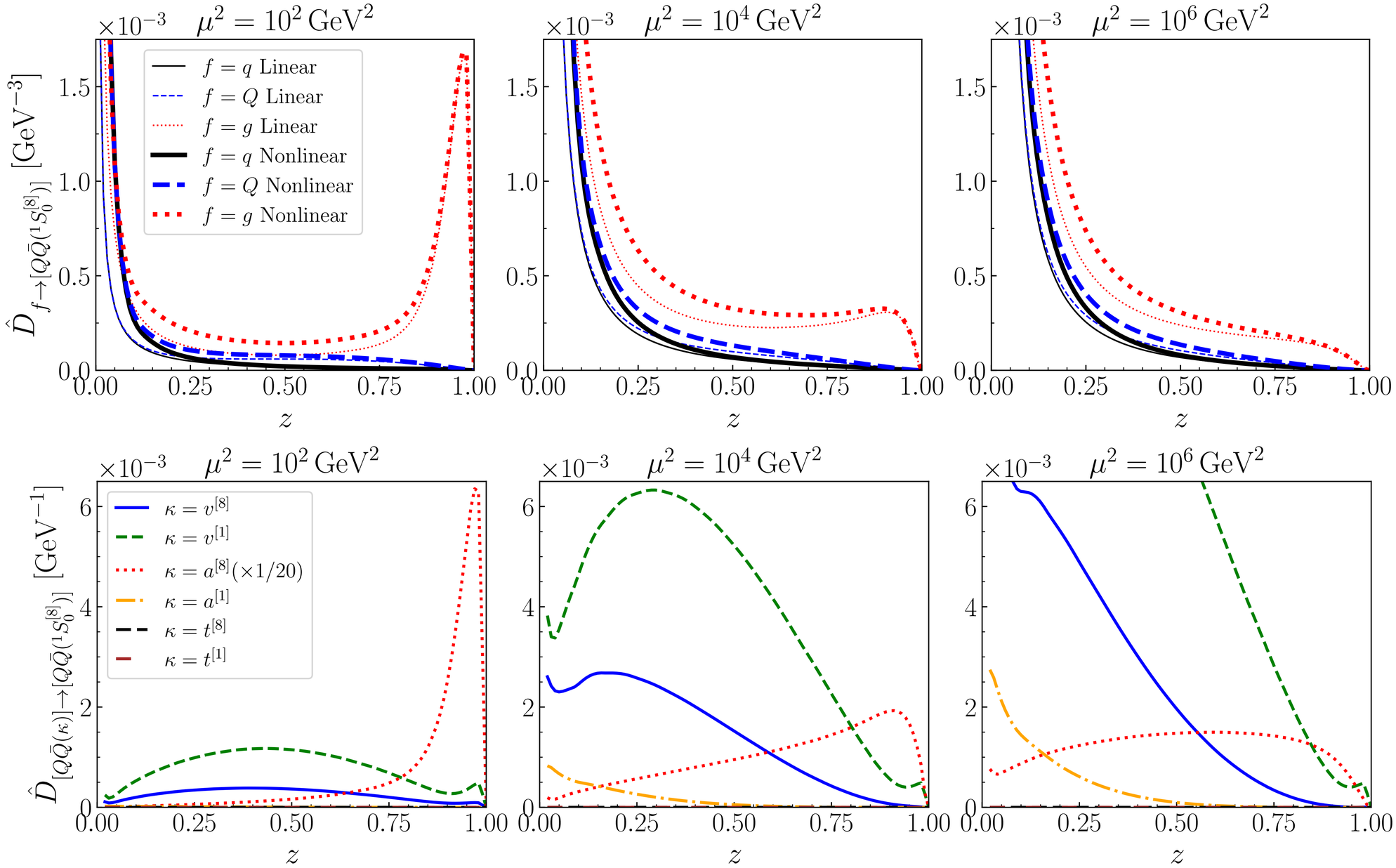
- We have studied the QCD factorization for hadronic quarkonium production at high p_T.
- We demonstrated that the LP contributions are significant for hadronic quarkonium production at high $p_\perp$ while the NLP contributions are sizable at lower p_T but different in shape.
- The power corrections to the quantum evolution of the SP FFs are not suppressed even at high p_T, enhancing high p_T quarkonium cross-section.
- The QCD factorization formalism should make possible a new global data analysis. There is sufficient room to improve the input FFs.
- Matching between the QCD factorization and fixed order NRQCD factorization should enable us to describe quarkonium production not only in hadronic collisions but also in other scattering processes in a broader p_T region.

Thank you!

backup

Sample of SP, DP FFs after evolution

Lee, Qiu, Sterman, **KW**, in preparation.



Off-diagonal channels: DP FFs in u, v-space

Lee, Qiu, Sterman, **KW**, in preparation.