Charm mass effects in the static energy computed in $2+1+1$ flavor lattice QCD

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For the TUMQCD collaboration
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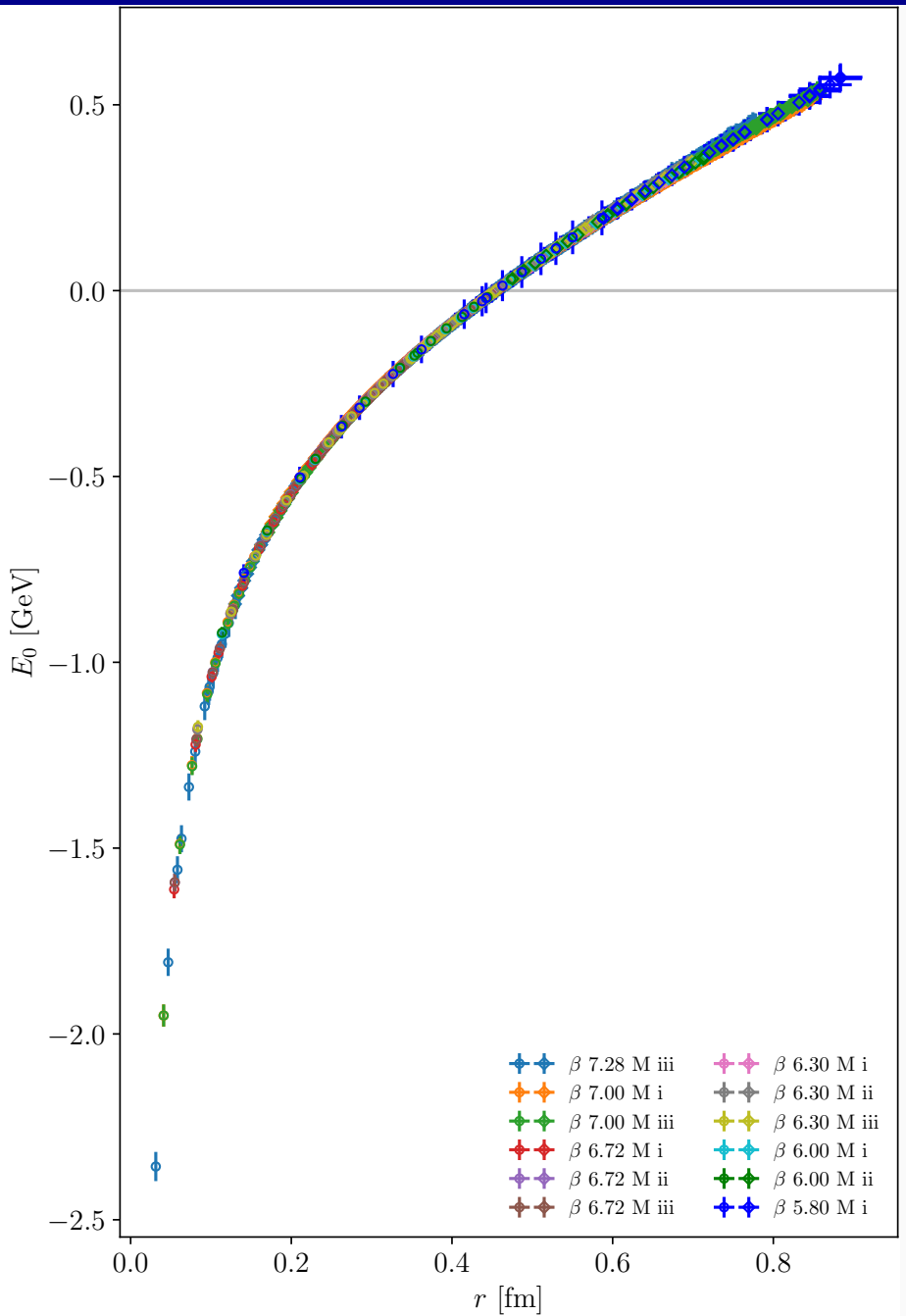
Static energy



- The QCD static energy of a quark-antiquark pair is a physical observable both in continuum and on the lattice
- Calculated on the lattice with high precision up to 2+1+1 flavors
- Perturbatively known up to N^{3}LL accuracy
- Can be used to set the scale, extract strong coupling constant α_s

Static energy in perturbation theory

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E_0(r) = \Lambda_{\mathrm{s}} - \frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{r}\left(1 + \#\alpha_{\mathrm{s}} + \#\alpha_{\mathrm{s}}^2 + \#\alpha_{\mathrm{s}}^3 + \#\alpha_{\mathrm{s}}^3 \ln\alpha_{\mathrm{s}} + \#\alpha_{\mathrm{s}}^4 \ln^2\alpha_{\mathrm{s}} + \#\alpha_{\mathrm{s}}^4 \ln\alpha_{\mathrm{s}} \ldots\right)$$

- Determined from the large-time behavior of Wilson loops

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}, \qquad W_{r\times T} = P\left\{\exp\left(i\oint_{r\times T} dz_\mu\, gA_\mu\right)\right\}$$

- Can be calculated nonperturbatively on the lattice
- Set scale as $\mu = 1/r$, other scales (e.g. ultrasoft$\sim \alpha_{\mathrm{s}}/r$) also involved
- In the dimensional regularization requires a renormalon subtraction
- In the lattice regularization diverges as $1/a$ towards continuum limit
- Both regularization problems can be absorbed into a constant term
- Alternatively, consider the static force $F(r) = \partial_r E_0(r)$

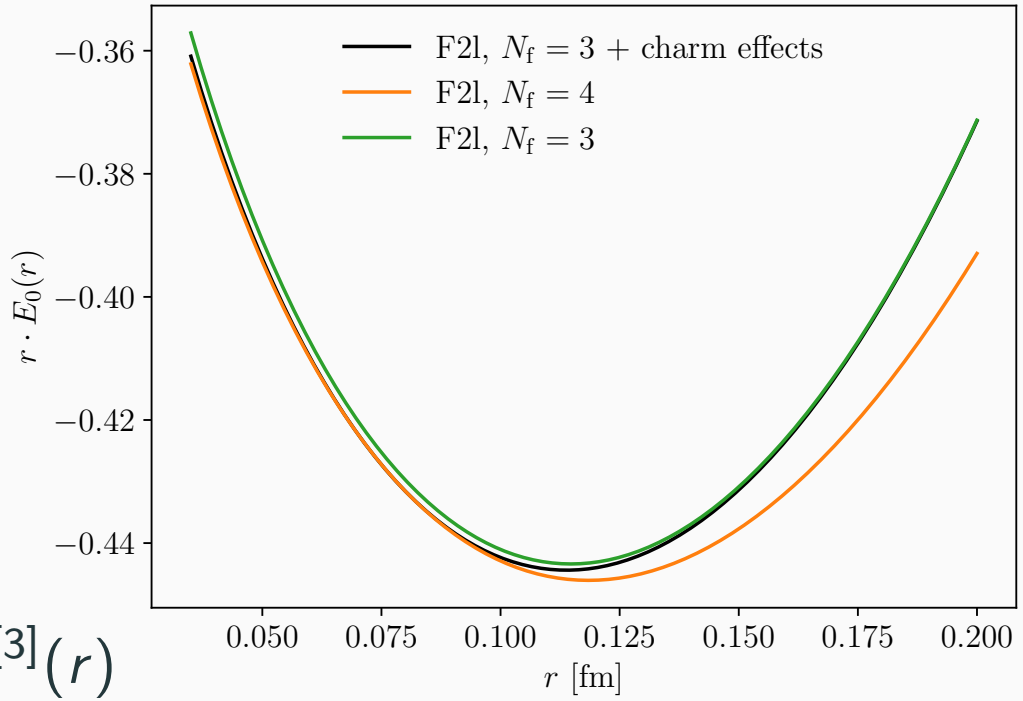
[1] For review of perturbative results, see: X. Tormo Mod. Phys. Lett. A28 (2013)

Charm quark mass effects in perturbation theory

- Effects due to finite mass of a heavy quark gives correction $\delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r)$

$$E_{0,m}^{(N_{\mathrm{f}})}(r) = \int_{r^*}^{r} \mathrm{d}r' F^{(N_{\mathrm{f}})}(r') + \delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r) + \mathrm{const}$$

$$E_{0,m}^{(N_{\mathrm{f}})} \to E_0^{(N_{\mathrm{f}})} \quad \text{for } r \gg 1/m$$
$$E_{0,m}^{(N_{\mathrm{f}})} \to E_0^{(N_{\mathrm{f}}+1)} \quad \text{for } r \ll 1/m$$



- Finite mass corrections known up to two loops[1]
$\delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r) = \delta V_{\mathrm{m}}^{(N_{\mathrm{f}}),[2]}(r) + \delta V_{\mathrm{m}}^{(N_{\mathrm{f}}),[3]}(r)$

[1] D. Eiras, J. Soto, PRD61 (2000); M. Melles PRD62 (2000); A. H. Hoang hep-ph/0008102 (2000)

Lattice simulations

our naming	$N_\sigma^3 \times N_\tau$	β	$a_{f_{p4s}}$ (fm)	u_0	am_l	am_s	am_c	m_l/m_s	$(am_s)_{\text{tuned}}$	M_π (MeV)	#conf.
β 5.80 M i	$32^3 \times 48$	5.80	0.15294	0.85535	0.00235	0.0647	0.831	phys	0.06852	131	1041
β 6.00 M ii	$32^3 \times 64$	6.00	0.12224	0.86372	0.00507	0.0507	0.628	1/10	0.05296	217	1000
β 6.00 M i	$48^3 \times 64$				0.00184			phys		132	709
β 6.30 M iii	$32^3 \times 96$				0.0074	0.037	0.44	1/5		316	1008
β 6.30 M ii	$48^3 \times 96$	6.30	0.08786	0.874164	0.00363	0.0363	0.43	1/10	0.03627	221	1031
β 6.30 M i	$64^3 \times 96$				0.0012		0.432	phys		129	1074
β 6.72 M iii	$48^3 \times 144$				0.0048	0.024	0.286	1/5		329	1017
β 6.72 M ii	$64^3 \times 144$	6.72	0.05662	0.885773	0.0024			1/10	0.02176	234	1103
β 6.72 M i	$96^3 \times 192$				0.0008	0.022	0.26	phys		135	1268
β 7.00 M iii	$64^3 \times 192$	7.00	0.0426	0.892186	0.00316	0.0158	0.188	1/5	0.01564	315	1165
β 7.00 M i	$144^3 \times 288$				0.000569	0.01555	0.1827	phys		134	478
β 7.28 M iii	$96^3 \times 288$	7.28	0.03216	0.89779	0.00223	0.01115	0.1316	1/5	0.01129	309	821

- Measure static energy as the ground state of the Wilson line correlation function in Coulomb gauge
- Use state of the art 2+1+1 HISQ[1] ensembles from MILC[2]
- Three different light quark masses, physical strange and sea
- Six lattice spacings via f_{p4s} scale

[1] E. Follana, *et.al.*, PRD75 (200/); [2] A. Bazavov, *et.al.*, PRD98 7 (2018)

Static energy on the lattice

- E_0 as correlation of Wilson lines $W(\mathrm{r},\tau,a) = \prod_{u=0}^{\tau/a-1} U_4(\mathrm{r}, ua, a)$

$$C(\mathrm{r},\tau,a) = \left\langle \frac{1}{N_\sigma^3} \sum_x \sum_{y=R(r)} \frac{1}{N_\mathrm{c} N_\mathrm{r}} \mathrm{Tr}\, W^\dagger(x+y,\tau,a) W(x,\tau,a) \right\rangle$$

$$C(\mathrm{r},\tau,a) = \mathrm{e}^{-\tau E_0(\mathrm{r},a)} \left(C_0(\mathrm{r},a) + \sum_{n=1}^{N_\mathrm{st}-1} C_n(\mathrm{r},a) \prod_{m=1}^{n} \mathrm{e}^{-\tau \Delta_m(\mathrm{r},a)} \right) + \cdots$$

- Vary the fit range with N_st and $|\mathrm{r}|$, Bayesian fits:

$$|\mathrm{r}| + 0.2\ \mathrm{fm} \leq \tau_{\mathrm{min},1} \leq 0.3\ \mathrm{fm} \qquad \text{for } N_\mathrm{st} = 1, \quad \Rightarrow \text{ prior values}$$

$$\tfrac{2}{3}|\mathrm{r}| + 0.1\ \mathrm{fm} \leq \tau_{\mathrm{min},2} \leq \tau_{\mathrm{min},1} - 2a \qquad \text{for } N_\mathrm{st} = 2, \quad \Rightarrow \text{ Our pick}$$

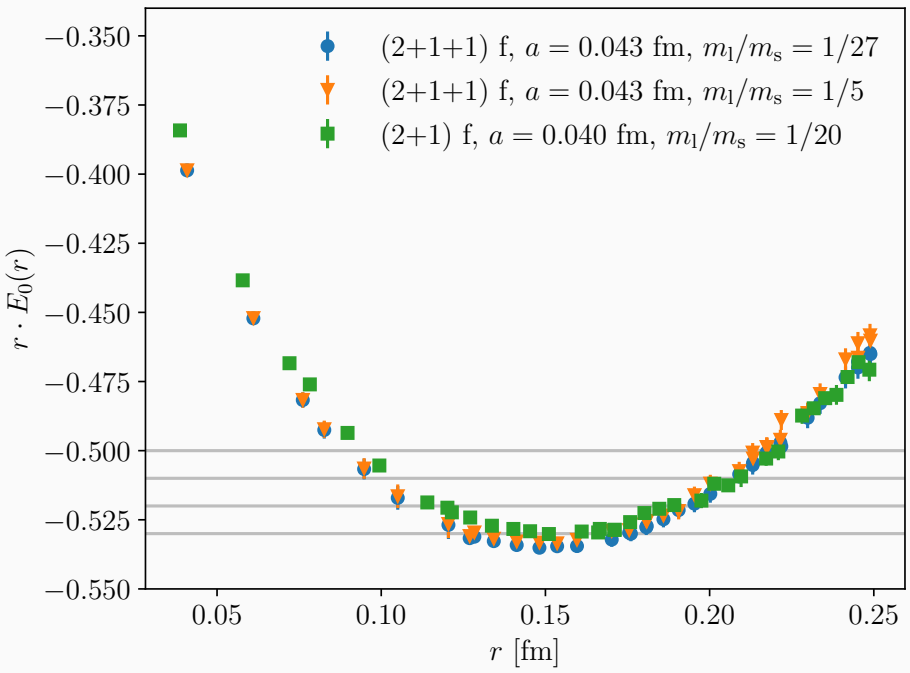
$$\tfrac{1}{3}|\mathrm{r}| \leq \tau_{\mathrm{min},3} \leq \tau_{\mathrm{min},2} - 2a \qquad \text{for } N_\mathrm{st} = 3, \quad \Rightarrow \text{ cross-check}$$

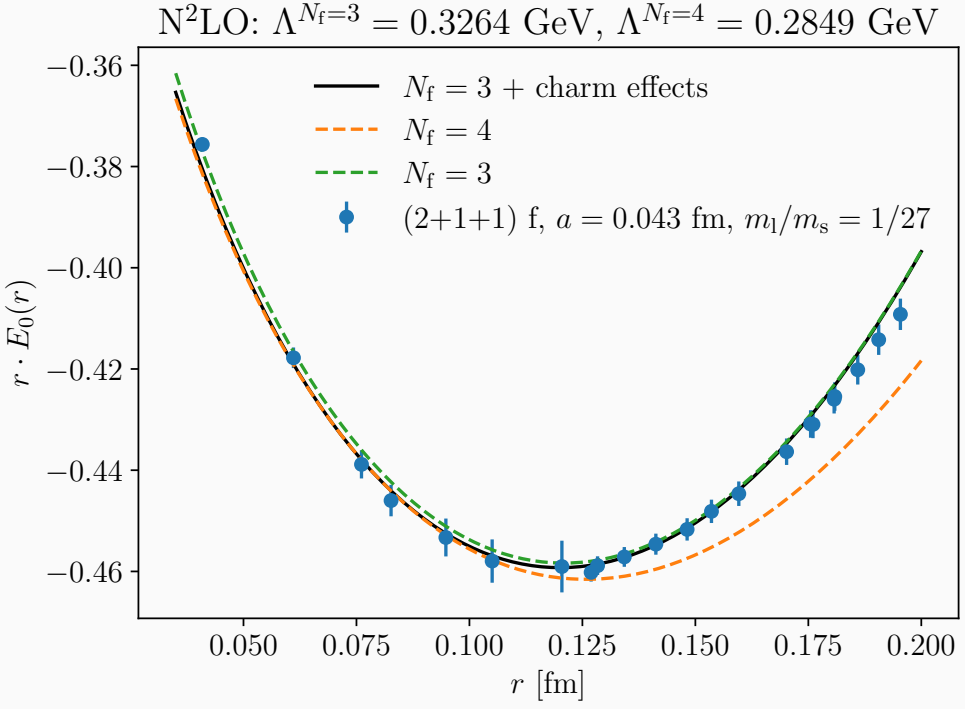
- Prior for $E_1 - E_0$ from[2]
- Measurements with and without one step of HYP-smearing[1]
- Use tree-level improvement for the distance r to correct for artifacts

Extension to 1-loop ongoing: G. v. Hippel, *et.al.*, in preparation: TUM-EFT 171/22

[1] A. Hasenfratz, *et.al.*, PRD64 (2001); [2] K. Juge, *et.al.*, PRL90 (2003)

Charm quark mass effects on the lattice





- Clearly visible difference between the behaviors of 2+1[1] and 2+1+1
- Curve with charm effects follows the data better than curves without
- Use $\Lambda_{\overline{\mathrm{MS}}}$ that fits the data well, $r0\Lambda_{\overline{\mathrm{MS}}}$ compatible with previous determinations

[1] A. Bazavov, *et.al.*, PRD100 (2019)

Lattice scales and the string tension

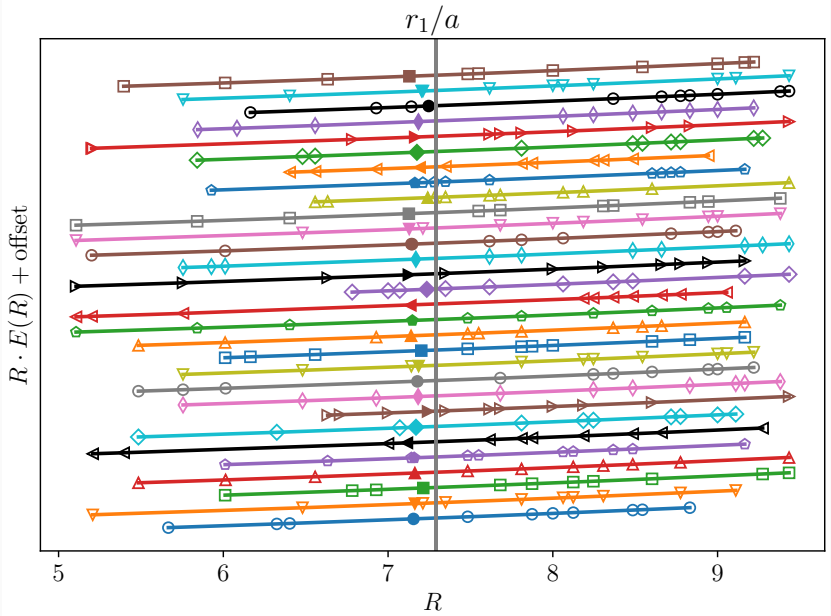
- Static energy allows determination of lattice scales r_i and the string tension σ

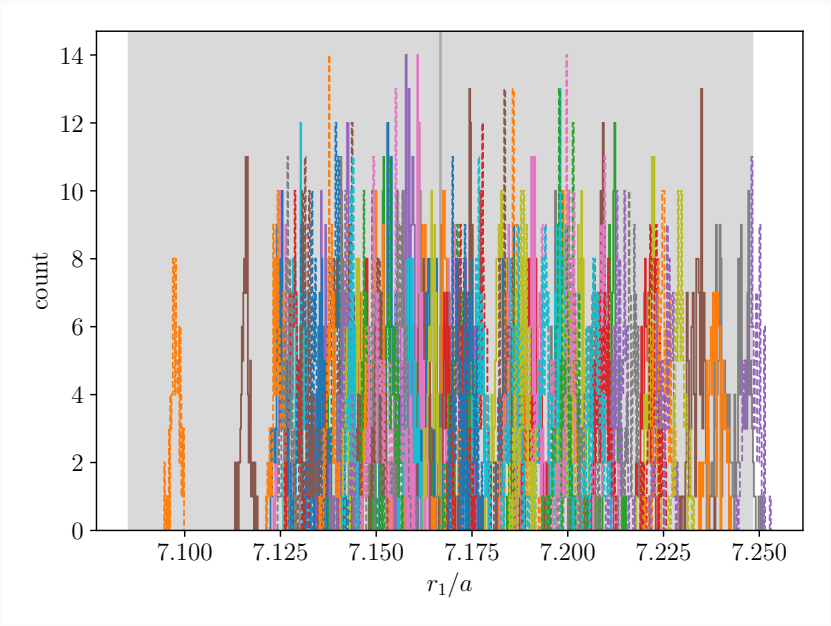
$$r_i^2 F(r_i) = \begin{cases} 1.65, & i = 0\,^{1} \\ 1.0\ , & i = 1\,^{2} \\ 0.5\ , & i = 2\,^{3} \end{cases} \quad , \quad \begin{matrix} r_0 \sim 0.475\,\mathrm{fm} \\ r_1 \sim 0.3106\,\mathrm{fm} \\ r_2 \sim 0.145\,\mathrm{fm} \end{matrix}$$

- $r_2 \sim 1/m_c$, scales r_i expected to be affected by charm differently
- Locally fit the data using Cornell ansatz

$$E(R, a) = -\frac{A}{R} + B + \sigma R$$

- Asymmetric random picking for systematics





[1] R. Sommer, NPB411 (1994); [2] C. Bernard, *et.al.*, PRD62 (2000); [3] A. Bazavov, *et.al.*, PRD97 (2018)

Continuum limit

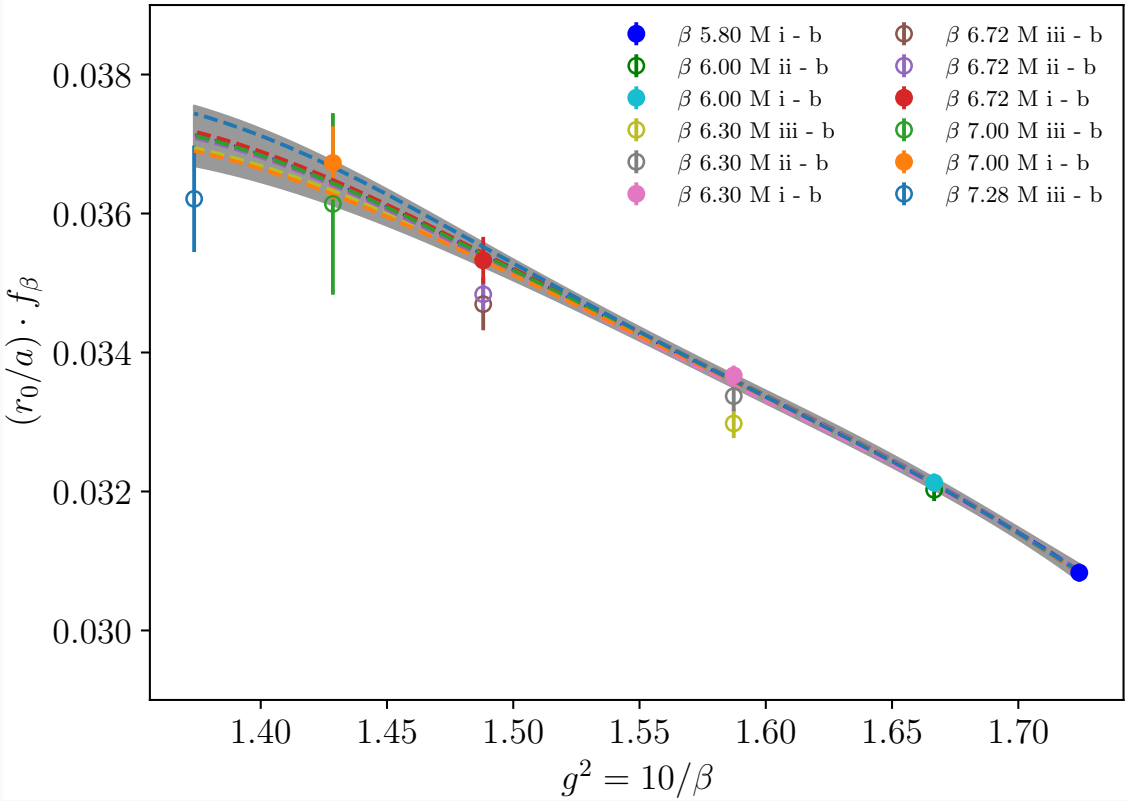
$$\frac{a}{r_i} = \frac{C_0 f_\beta + C_2 g_0^2 f_\beta^3 + C_4 g_0^4 f_\beta^3}{1 + D_2 g_0^2 f_\beta^2}$$

$$f_\beta = (b_0 g_0^2)^{-b_1/(2b_0^2)} e^{-1/(2b_0 g_0^2)},$$

$$C_0 = C_{00} + C_{01l}\frac{am_l}{f_\beta} + C_{01s}\frac{am_s}{f_\beta} + C_{01}\frac{am_{tot}}{f_\beta} + C_{02}\frac{(am_{tot})^2}{f_\beta},$$

$$C_2 = C_{20} + C_{21}\frac{am_{tot}}{f_\beta},$$

$$am_{tot} = 2am_l + am_s + am_c,$$

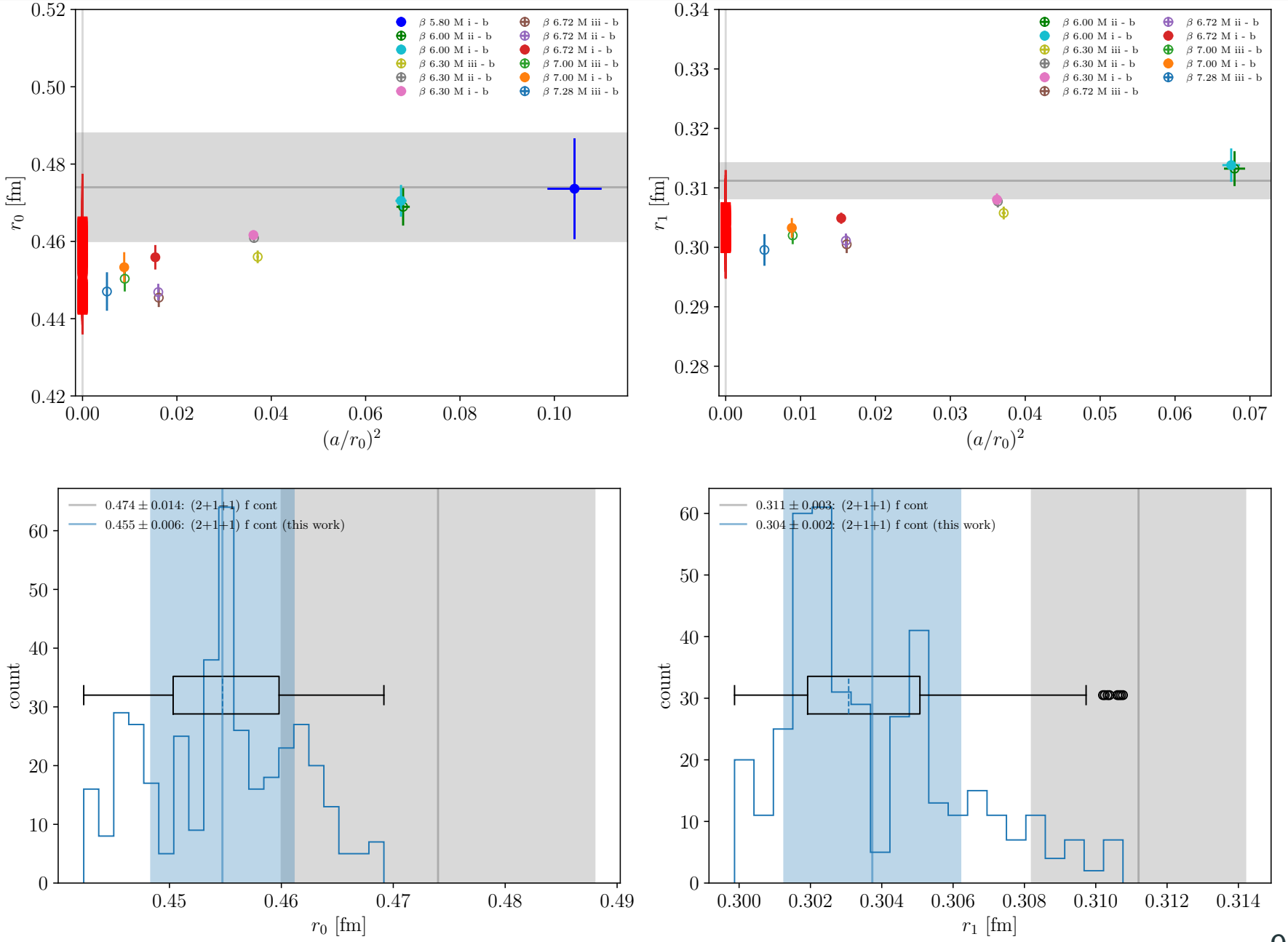


- Smooth the data with Allton ansatz
- Leading discretization effects $\alpha_s^2 a^2$ and a^4
- Lattice spacing dependence: $x = (a/r_{0,1})^2$
- Light quark mass dependence: $y = (am_l)/(am_s)$

$$\xi = \xi_0 + \alpha^2[\xi_1 x + \xi_2 x y^{(1,2)}] + \xi_3 x^2 + \xi_4 y\,, \quad \text{(l,q;l,qm;mc)}$$

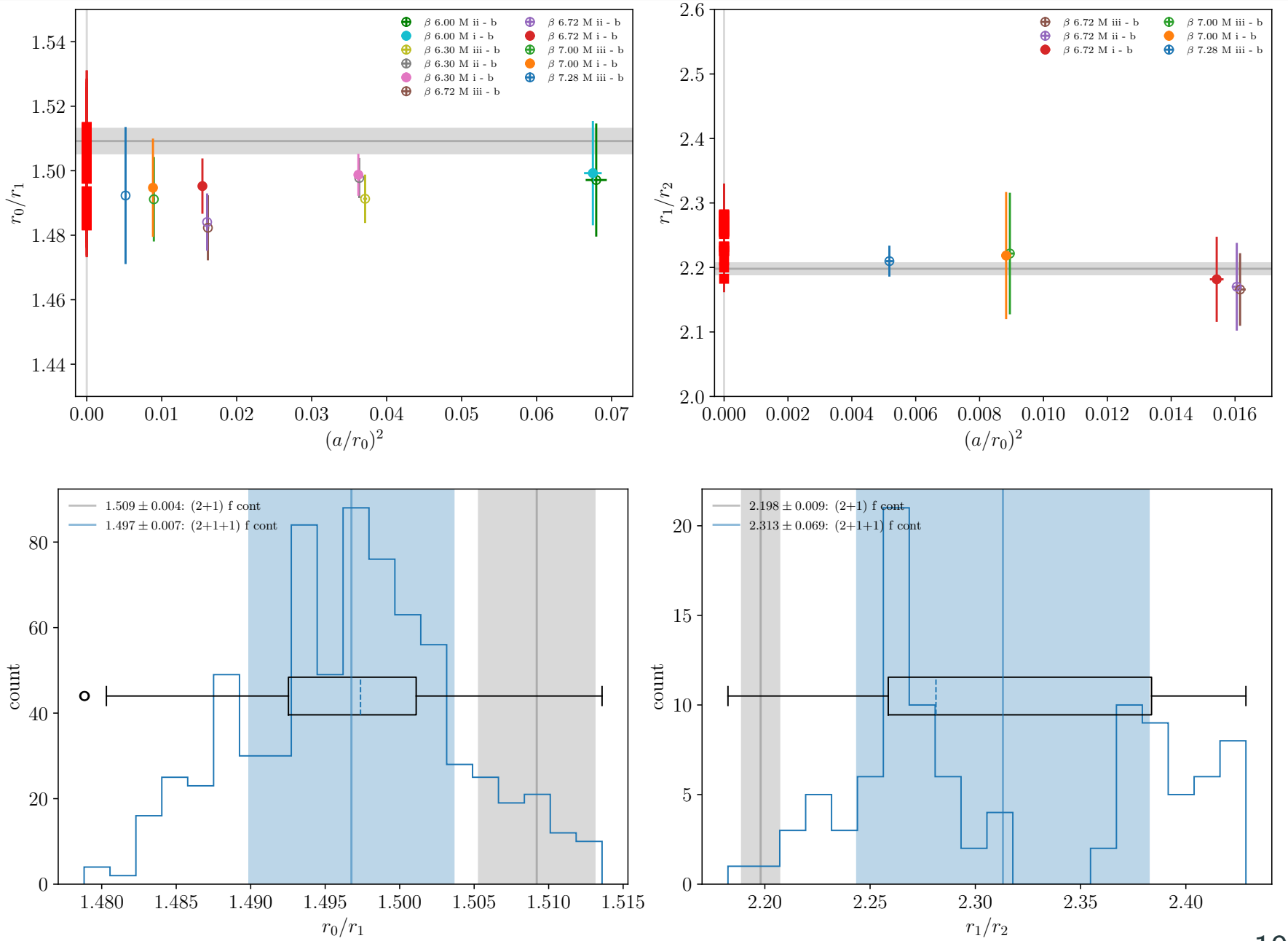
- Where $\alpha = 1$ or $\alpha = g_0^2/(4\pi u_0^2)$

Extrapolation of individual scales



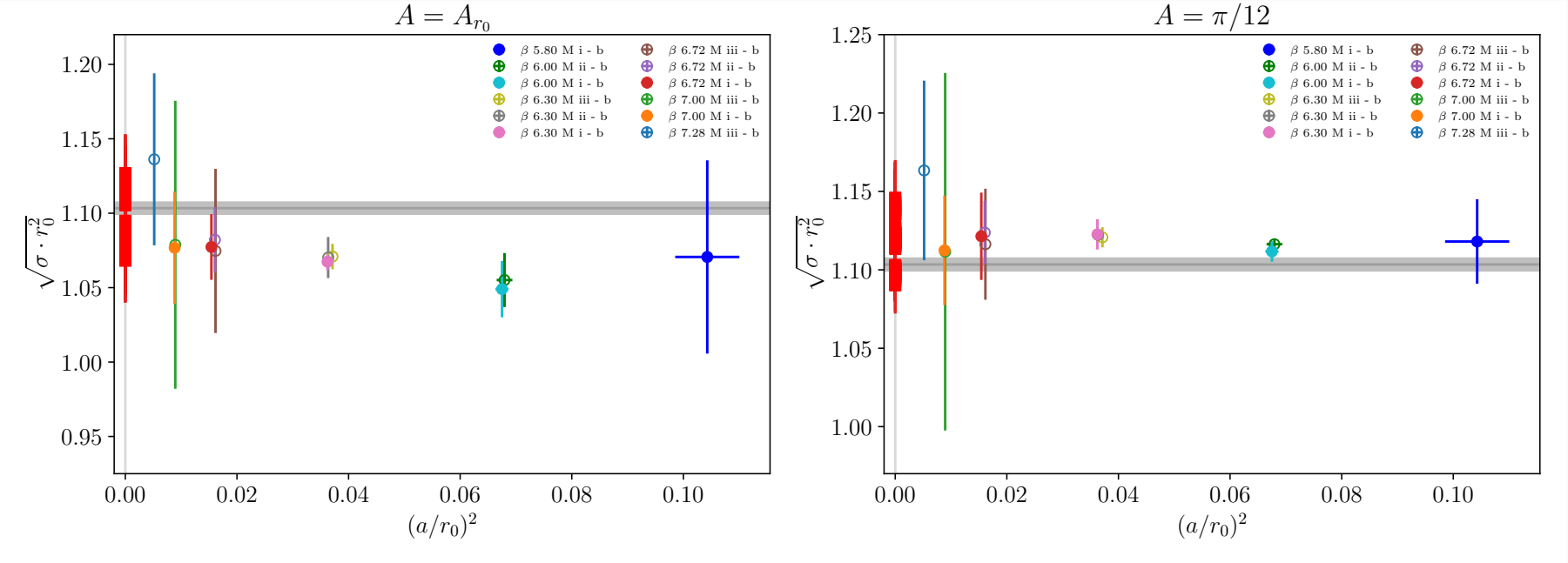
Grey bands from 2+1+1: N. Carrasco *et.al.*, Nucl.Phys. B887 (2014); R. J. Dowdall, *et.al.*, PRD88 (2013)

Extrapolation of ratios of scales



Grey bands from 2+1: A. Bazavov, *et.al.*, PRD97 (2018); A. Bazavov, *et.al.*, PRD90 (2014)

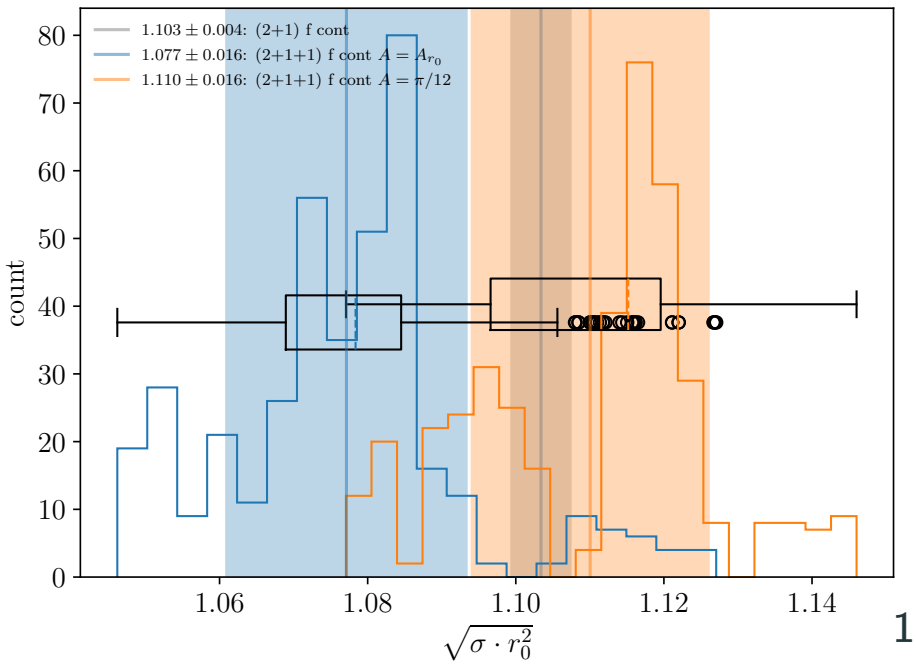
Final numbers and comparison to literature



- Extraction r lies between Coulomb and string regime

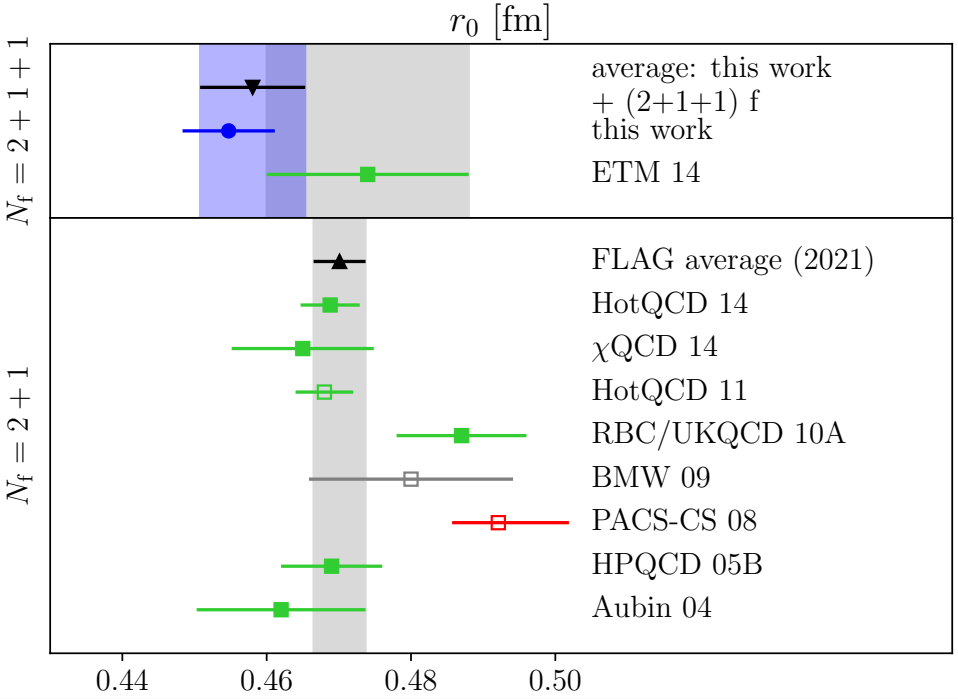
$\Rightarrow$ Two choices for fit parametr A

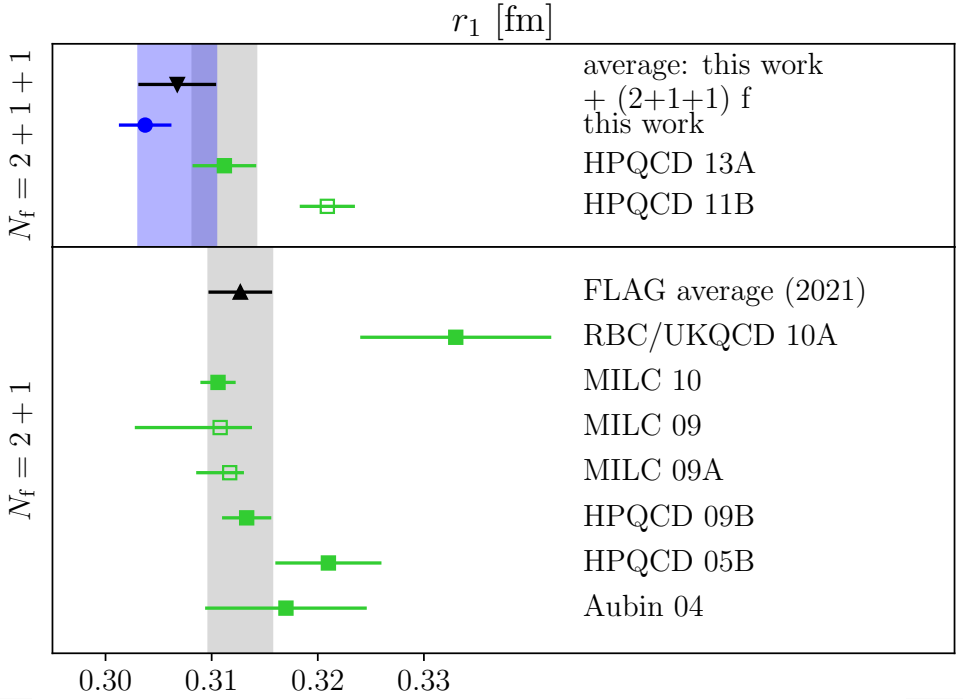
- Consistent with 2+1 flavor



Grey bands from 2+1: M. Cheng, *et.al.*, PRD77 (2008)

Final numbers and comparison to literature





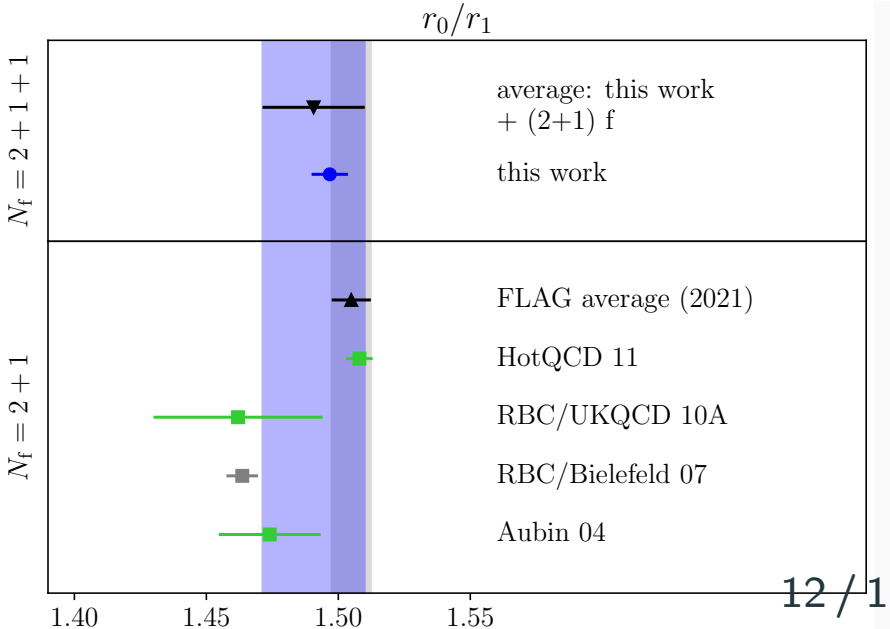
$$r_0 = 0.4547(64)\ \text{fm},$$

$$r_1 = 0.3037(25)\ \text{fm}, \quad r_0/r_1 = 1.4968(69),$$

$$r_2 = 0.1313(41)\ \text{fm}, \quad r_1/r_2 = 2.313(69).$$

$$\sqrt{\sigma r_0^2} = 1.077 \pm 0.016 \quad (A = A_{r_0}),$$

$$\sqrt{\sigma r_0^2} = 1.110 \pm 0.016 \quad (A = \pi/12).$$



Conclusions

- We have computed the static energy $E_0(r)$ with 2+1+1 flavors
- Determine scales r_0, r_1, and r_2, their ratios, and string tension σ
- Can determine all these scales simultaneously
- We can see charm decoupling well in the data
  - Perturbative charm effects give better description of the data
  - Scales closer to inverse charm mass differ more from their 2+1 flavor counterparts
- Future prospects: Measure $\Lambda_{\overline{\mathrm{MS}}}$

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Thank you for your attention!