



# STATIC QUARK ANTI-QUARK INTERACTIONS AT NON-ZERO TEMPERATURE FROM LATTICE QCD

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+ HotQCD COLLABORATION

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The XVth Quark confinement and the Hadron spectrum conference 2022.

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# Introduction

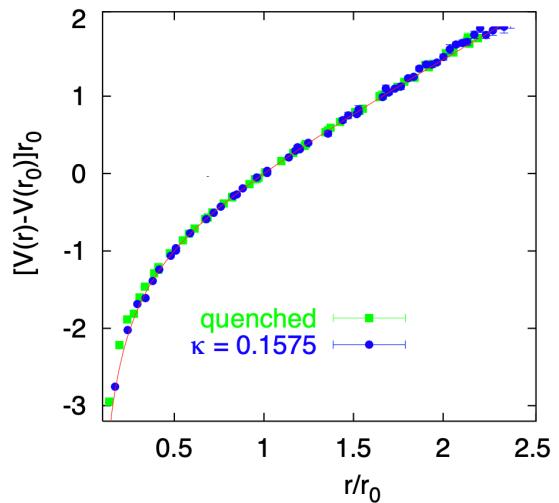
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Bound states of static quark and anti-quark pair: Probe for existence of Quark Gluon plasma in Heavy ion collisions (Alexander Rothkopf, Heavy quarkonium in extreme conditions, Physics Reports, Volume 858, 2020.).

- Time evolution in Real-Time suffers from sign problem. (QFT suffers from sign problem; see Rasmus and Daniel's talk)
- If separation of scales is present: Use EFTs (NRQCD and pNRQCD): describe physics in form of potential?

At T=0 Schrodinger like potential picture has been observed (G. Bali Phys.Rept. 343 (2001) 1-136).

$$i \partial_t W_{\square}(t, r) = \Phi(t, r) W_{\square}(t, r)$$
$$V(r) = \lim_{t \rightarrow \infty} \Phi(t, r)$$



# Introduction

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- It remains to be seen if the potential picture even holds for T>0, if it does how is it modified?
- Spectral function is a link between real and imaginary time: (A Rothkopf ,T Hatsuda, S Sasaki Phys.Rev.Lett. 108 (2012) 162001).

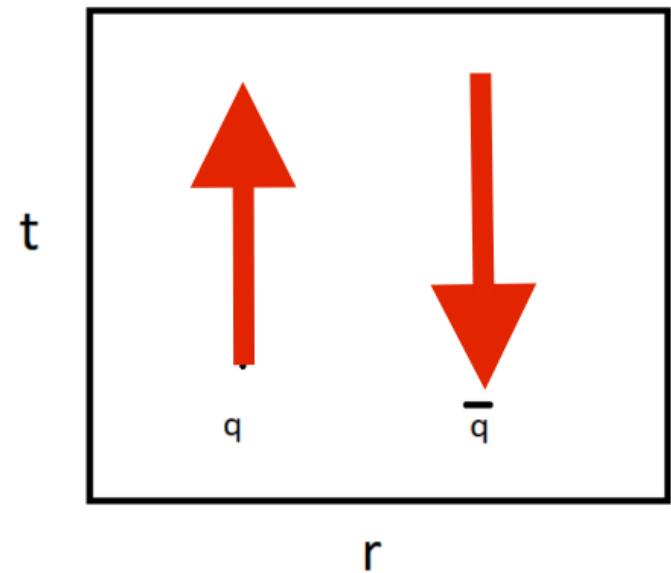
$$W_{\square}(r, t) = \int d\omega e^{-i\omega t} \rho_{\square}(r, \omega) \quad \longleftrightarrow \quad W_{\square}(r, \tau) = \int d\omega e^{-\omega\tau} \rho_{\square}(r, \omega)$$

- Computation of Spectral Function : Ill-posed Inverse problem
- Potential linked to the dominant peak position and width of Spectral Function (Yannis Burnier and Alexander Rothkopf 1208.1899).
- In HTL regime there exists a complex potential with screened real part (M.Laine et. al JHEP 03 (2007), 054).

# Lattice setup

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- (2+1)-flavour QCD - configurations generated by HotQCD and TUMQCD collaborations.
  - Using highly improved staggered quark (HISQ) action.
- $N_\sigma^3 \times N_\tau$  lattices.  $N_\tau = 10, 12, 16$  and  $N_\sigma/N_\tau = 4$
- Calculate Wilson Line correlator in Coulomb Gauge.
  - Fix box approach; temp range - 140MeV to 2GeV.
  - Pion mass 160MeV, Kaon mass physical.

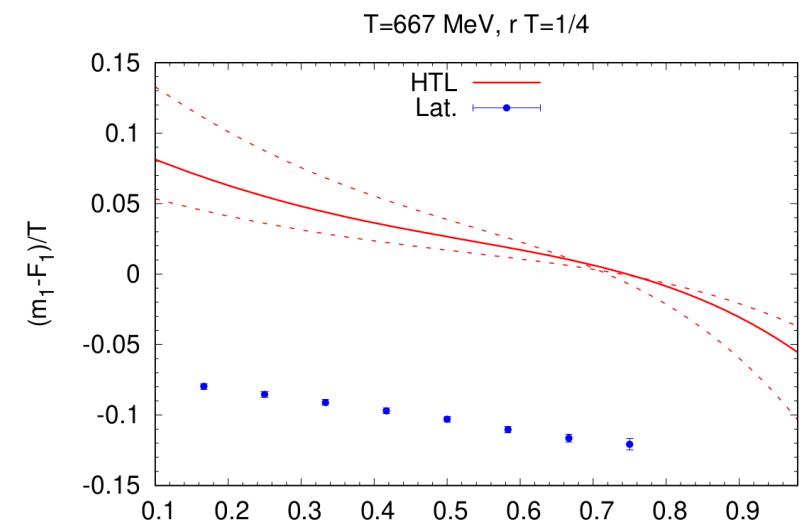
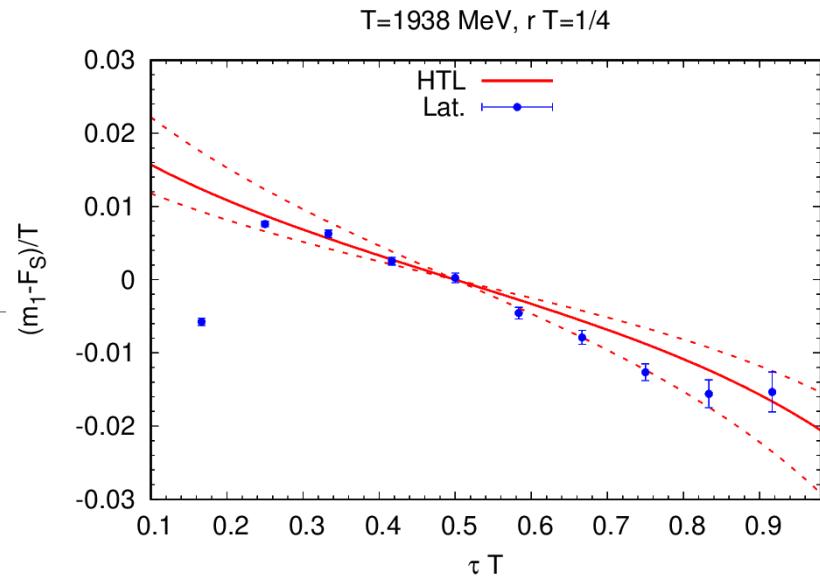


# Cumulants and HTL comparison

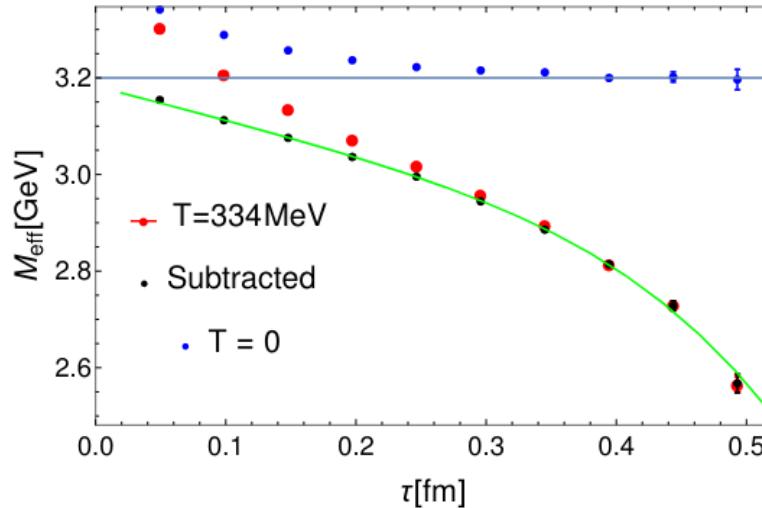
Define effective mass of the correlation function:

$$m_{eff}(r, \tau, T) = -\partial_\tau \ln W(r, \tau, T)$$

- Subtracting UV part using  $T=0$  correlator results in linear behavior at small tau.
- Plot shows effective mass subtracted from Free Energies.
- HTL does not quantitatively fit the data except for some specific T and separation distance.



# Spectral Function Model Fits



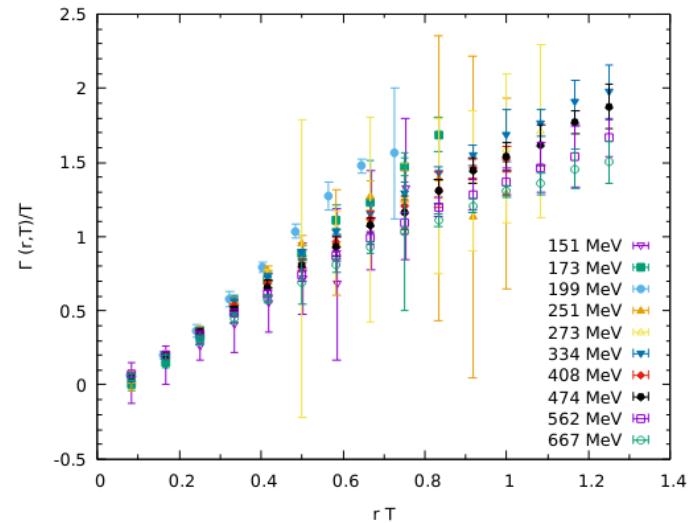
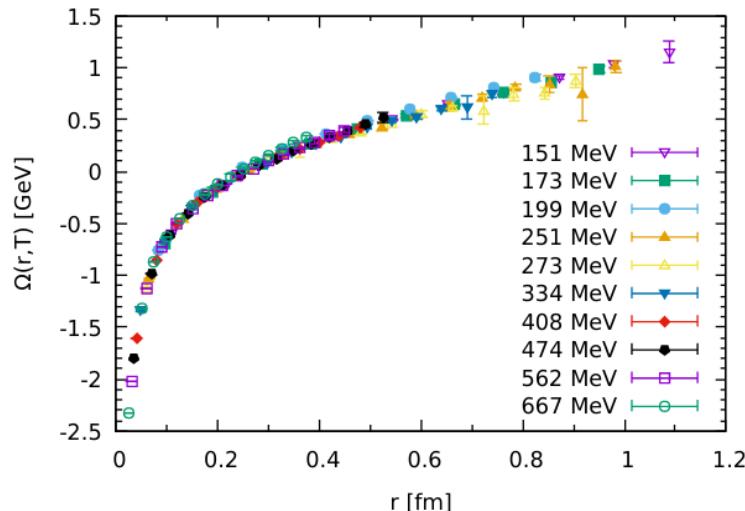
- Lattice data sensitive only to peak position ( $\Omega$ ) and effective width ( $\Gamma$ ).
- Parametrize Correlator as:  $C_{\text{sub}}(\tau, T) \sim \exp(-\Omega\tau + \frac{1}{2}\Gamma^2\tau^2 + O(\tau^3))$

$$\rho_r(\omega, T) = A(T) \exp\left(-\frac{[\omega - \Omega(T)]^2}{2\Gamma^2(T)}\right) + A^{\text{cut}}(T) \delta(\omega - \omega^{\text{cut}}(T))$$

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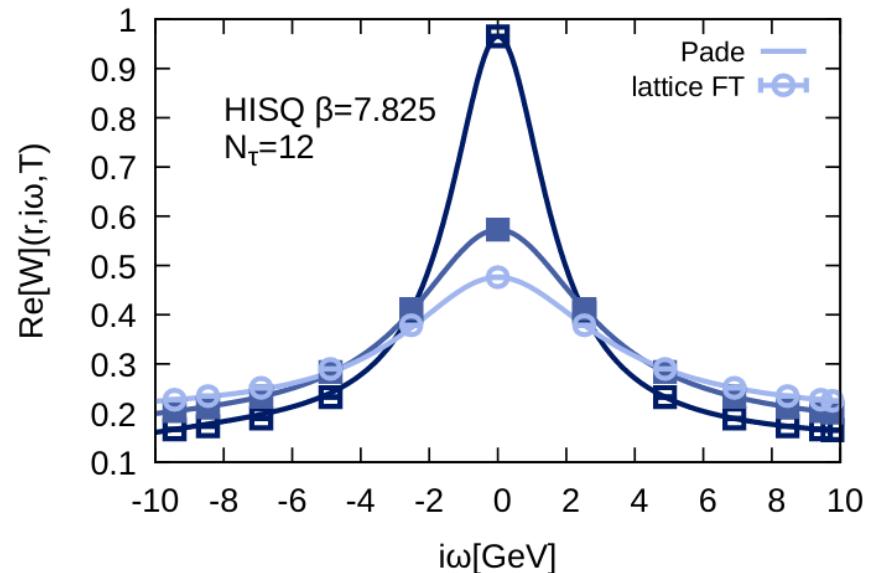
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# Pade' Interpolation

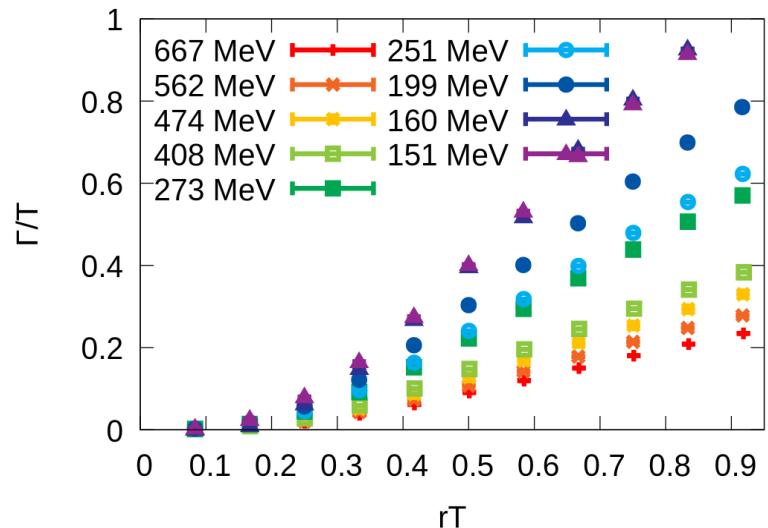
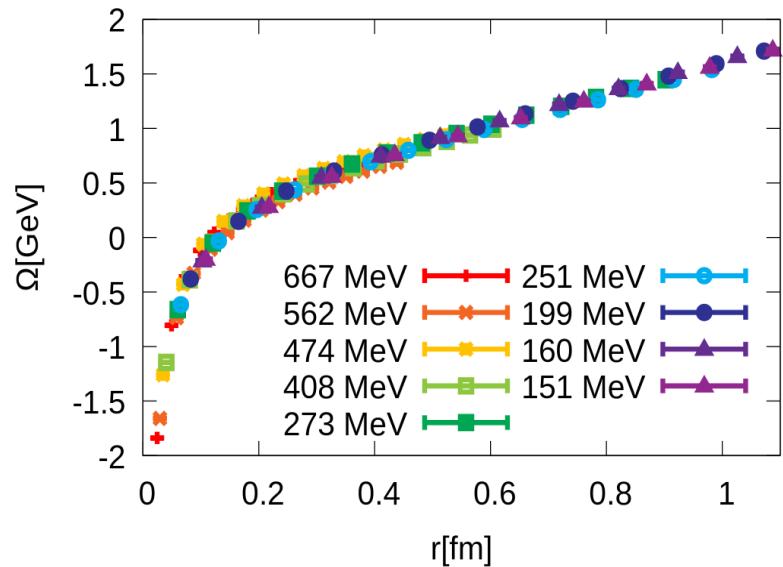
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- Transform the Euclidean correlator into Matsubara frequency space.
- Implement Pade approximation in the form of continued fraction according to Schlessinger prescription (L. Schlessinger, Phys. Rev. 167, 1411 (1968)).
- This is interpolation of data and not fitting. Does not require minimization.
- Obtain pole structure from rational function: Directly related to the peak position ( $\Omega$ ) and width ( $\Gamma$ ).



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# Spectral Function Extraction using Bayesian Method

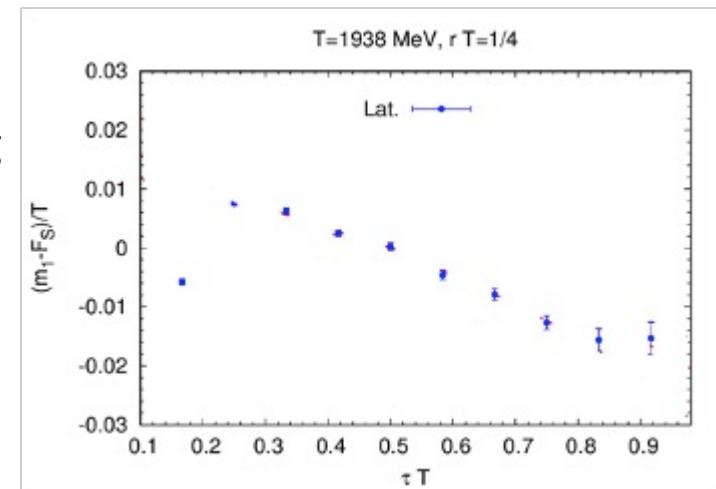
$$P[\rho|D, I] \propto P[D|\rho, I]P[\rho|I] = \exp[-L + \alpha S_{\text{BR}}]$$

$L$  is the usual quadratic distance used in chi-square fitting.

The prior probability  $P(\rho|I) = \exp(\alpha S_{BR})$  acts as a regulator

$$S_{\text{BR}} = \int d\omega \left( 1 - \frac{\rho(\omega)}{m(\omega)} + \log \left[ \frac{\rho(\omega)}{m(\omega)} \right] \right).$$

- Look for the most probable spectrum by locating the extremum of the posterior.
- Effective masses at high  $T$  show non-monotonicity at small  $\tau$ ; non-positive spectral function.--- cannot use Bayesian Methods.



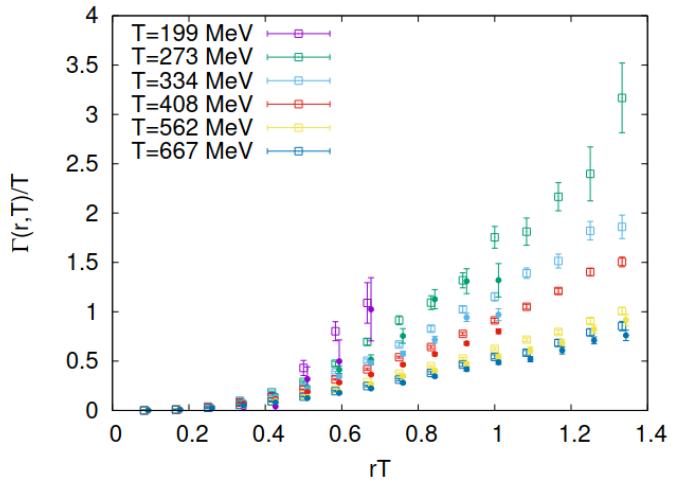
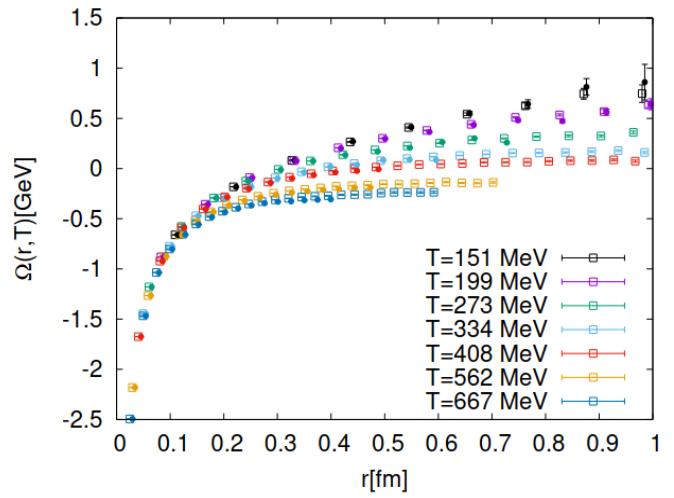
# HTL inspired fits

Peak position ( $\Omega$ ) and width ( $\Gamma$ ) interpreted as the real and imaginary part of thermal static energy Es (D. Bala and S. Datta, Phys. Rev. D 101, 034507(2020)).

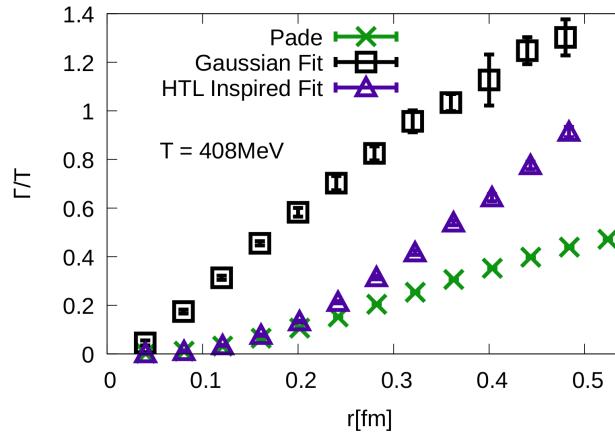
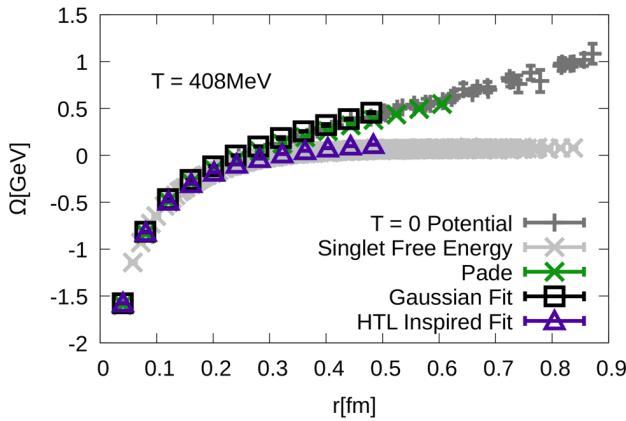
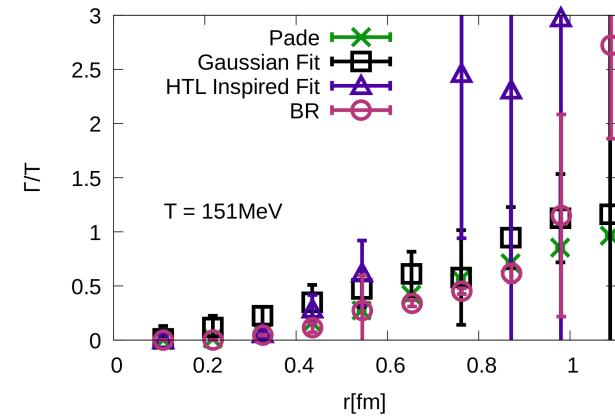
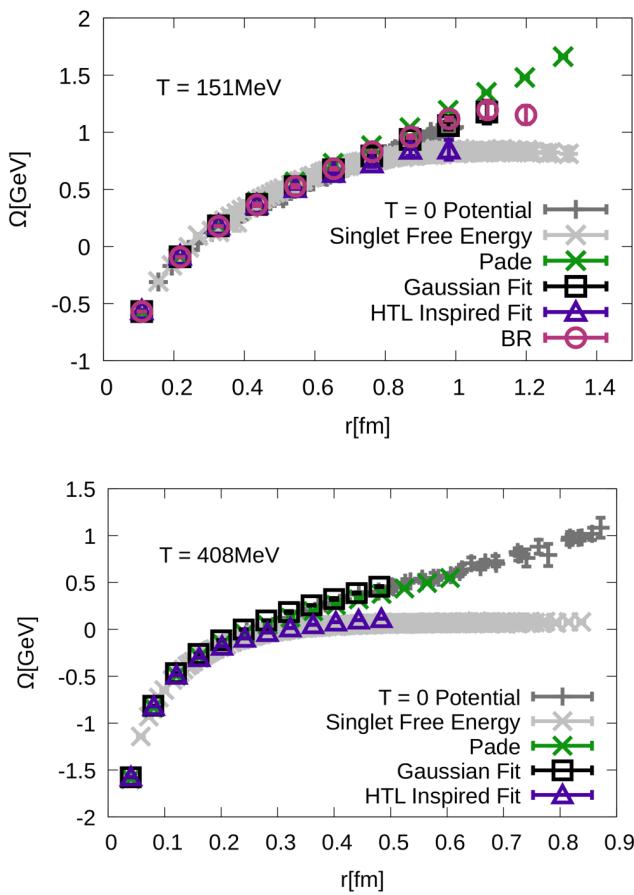
$$E_s(r, T) = \lim_{t \rightarrow \infty} i \frac{\partial \log W(r, t, T)}{\partial t} = \Omega(r, T) - i\Gamma(r, T).$$

$W(r, t, T)$  is the Fourier transform of the spectral function  $\rho_r(r, \omega)$

$$\begin{aligned} m_{eff}(r, n_\tau = \tau/a) a &= \log \left( \frac{W(r, n_\tau, N_\tau)}{W(r, n_\tau + 1, N_\tau)} \right) \\ &= \Omega(r, T) a - \frac{\Gamma(r, T) a N_\tau}{\pi} \log \left[ \frac{\sin(\pi n_\tau / N_\tau)}{\sin(\pi(n_\tau + 1) / N_\tau)} \right] \end{aligned}$$



# Comparison of Results



# Quenched QCD

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- We see that from the Gaussian fits and Pade' peak position ( $\Omega$ ) for HISQ is temperature independent; Quite puzzling.
- Results obtained are very different from previous studies of Quenched Lattices (1607.04049); Different methods used.
- Need further investigation:: Check new methods with Bayesian reconstruction.
- Check robustness of methods with new Quenched QCD lattices.

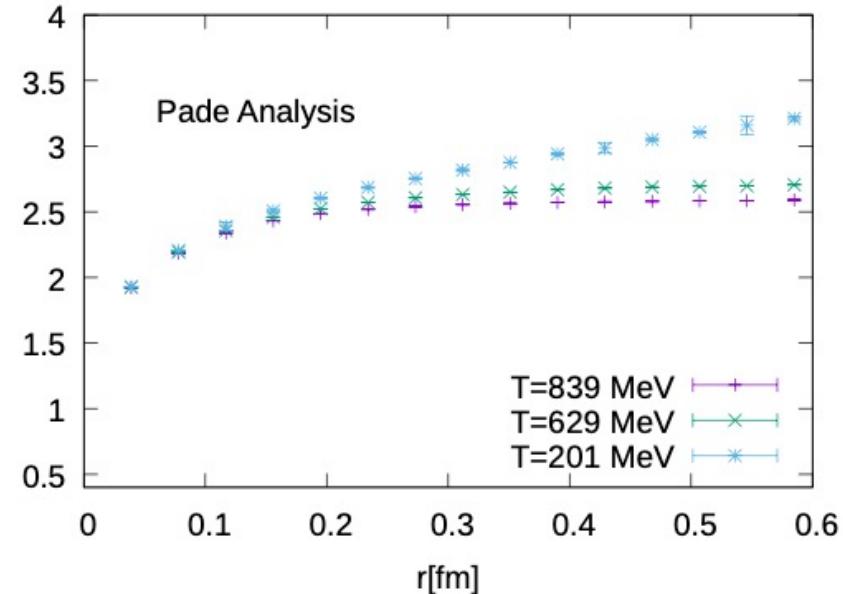
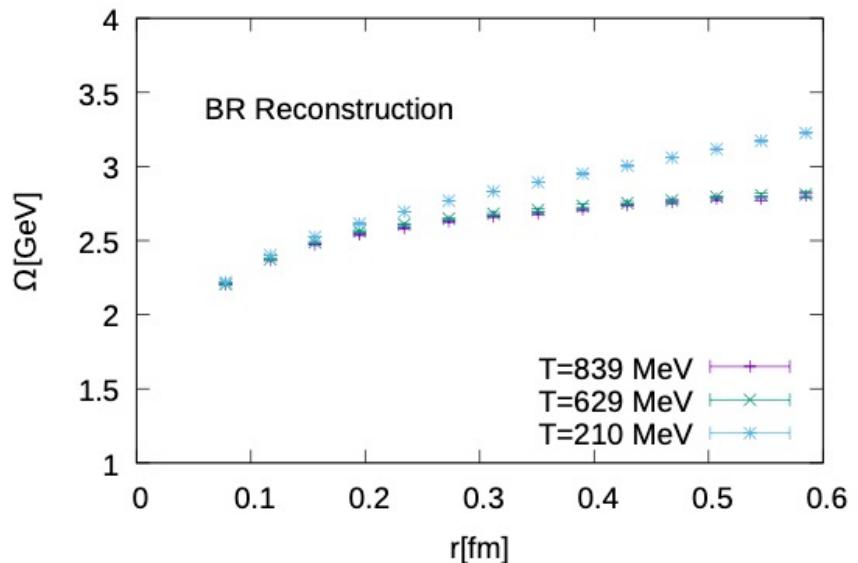
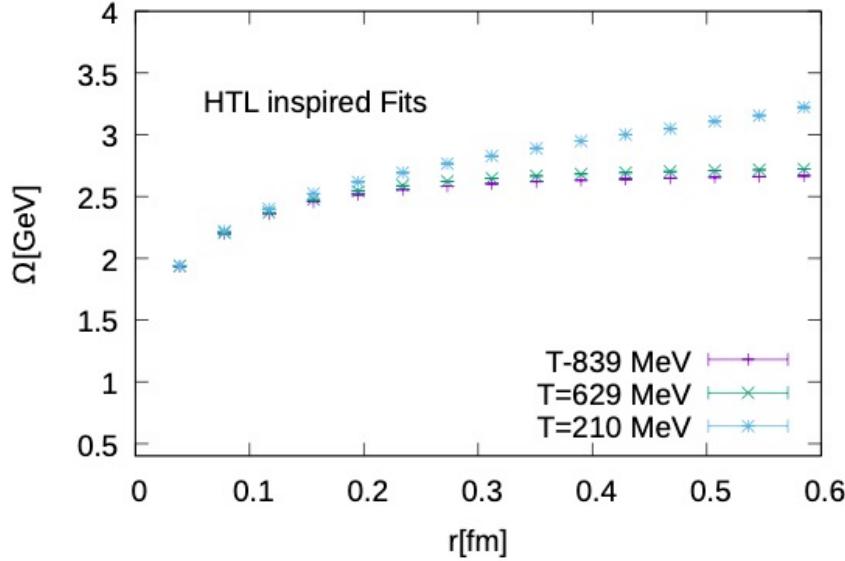
# Lattice setup

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- Using Wilson Action configurations generated using OpenQCD code from FASTSUM collaboration.
- $N_s^3 \times N_\tau$  lattices.  $N_\tau = 24, 32, 48, 96, 192$  and  $N_s = 64$ .
- $N_s=32$  used in previous studies showed artificial screening effects.
- Use Anisotropic Lattices:  $a_t / a_s = 4$  ( $\chi=3.5$ )
- Calculate Wilson Line correlator in Column Gauge (simulatedQCD code).
- Fix box approach; temp range 105-839 MeV .

# Preliminary Results

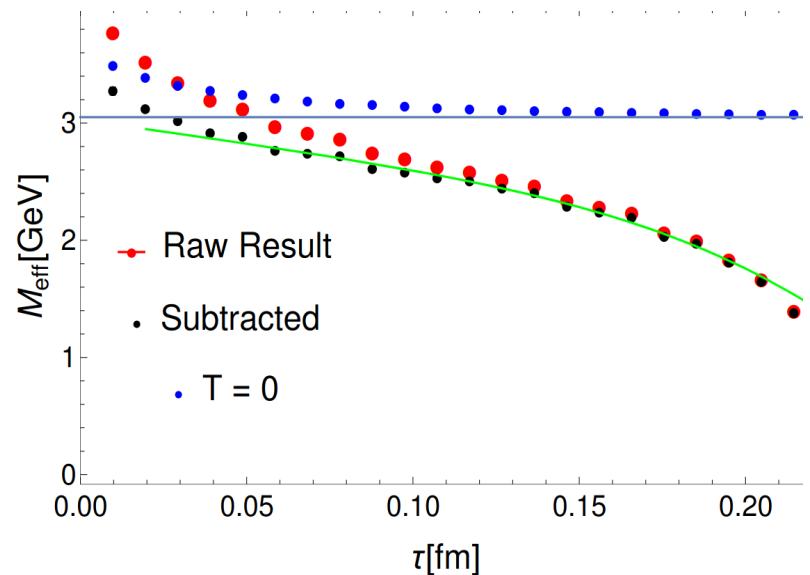
$T_c = 270 \text{ MeV}$



# Effective masses and correlator

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- $M_{\text{eff}}$  at  $r/a = 12$  for  $N_\tau = 96$  (blue),  $N_\tau = 24$  (T=839MeV) .
- Continuum extracted from  $N_\tau = 96$  and removed from  $N_\tau = 24$  (black).
- No positivity violation.
- Lines shows exponential and gaussian plus delta function fits.
- Small  $\tau$  behaviour shows T dependence.
- Cannot do Gaussian fits.



# Summary

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- Spectral functions of Wilson line correlators encode the real and imaginary part of the complex potential between static quark-antiquark pairs.
- We show analysis of spectral structure with four different methods.
- We see that from the Gaussian fits and Pade' peak position ( $\Omega$ ) is temperature independent in HISQ lattices.
- Results obtained are very different from previous quenched QCD studies.
- Peak position of new quenched study shows screening above  $T_c$
- Quenched study is still work in progress.
- Need to understand origin of differences between quenched and dynamical QCD.