

# A description of $J/\psi$ baryon decays within the factorisation framework

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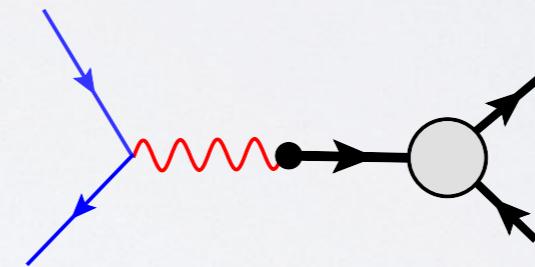
$J/\psi \rightarrow B\bar{B}$     **decays**

$$\Gamma[J/\psi \rightarrow B\bar{B}] = \frac{M_\psi \beta}{12\pi} \left( |\mathcal{G}_M|^2 + \frac{2m_B^2}{M_\psi^2} |\mathcal{G}_E|^2 \right) = \Gamma_{\perp} + \Gamma_{\parallel}$$

$$\Gamma_{\perp} \sim \mathcal{G}_M : J/\psi_{\perp} \rightarrow B(+) \bar{B}(-) \quad \Gamma_{\parallel} \sim \mathcal{G}_E : J/\psi_{\parallel} \rightarrow B(+) \bar{B}(+)$$

$$\mathcal{G}_E / \mathcal{G}_M \sim const \quad m_Q \rightarrow \infty$$

$$\frac{dN_B}{d\cos\theta} = \mathcal{N}(1 + \alpha_B \cos^2\theta)$$



$$\alpha_B = \frac{1 - 2\Gamma_{\parallel}/\Gamma_{\perp}}{1 + 2\Gamma_{\parallel}/\Gamma_{\perp}}$$

$$\frac{\Gamma_{\parallel}}{\Gamma_{\perp}} = \frac{2m_B^2}{M_\psi^2} \frac{|\mathcal{G}_E|^2}{|\mathcal{G}_M|^2}$$

$$m_Q \rightarrow \infty \quad \alpha_B = 1$$

Brodsky, Lepage 1981

$$\frac{4m_N^2}{M_\psi^2} \simeq 0.37$$

$$\frac{4m_N^2}{M_\gamma^2} \simeq 0.04$$

# $J/\psi$ baryonic decays

Data: BESIII/2008/2012/2016/2017/2019/2020/ & PDG

$$\frac{\Gamma_{\parallel}}{\Gamma_{\perp}} = \frac{2m_B^2}{M_{\psi}^2} \frac{|\mathcal{G}_E|^2}{|\mathcal{G}_M|^2}$$

$B(m_B, \text{MeV})$	$\text{Br}[J/\psi \rightarrow B\bar{B}] \times 10^3$	$\alpha_B$	$ \mathcal{G}_E / \mathcal{G}_M $	$\Gamma_{\parallel}/\Gamma_{\perp}$
$p(938)$	2.12(3)	0.59(1)	0.83(2)	0.13
$n(940)$	2.1(2)	0.50(4)(0.21)	0.95(6)	0.17
$\Lambda(1116)$	1.89(9)	0.47(3)	0.83(4)	0.18
$\Sigma^0(1193)$	1.17(3)	-0.45(2)	2.11(5)	1.31
$\Sigma^+(1189)$	1.5(3)	-0.51(2)	2.27(5)	1.53
$\Xi^+(1322)$	0.97(8)	0.58(4)	0.61(5)	0.13
$\Xi^0(1315)$	1.17(3)	0.66(3)	0.53(4)	0.10

Neutron: large syst. error      Sigma: large  $\mathcal{G}_E$  !

Claudson, Glashow, Wise 1982

$$\mathcal{G}_E/\mathcal{G}_M \simeq 1 \quad \alpha_N \approx \frac{1 - 4m_B^2/M_{\psi}^2}{1 + 4m_B^2/M_{\psi}^2} \approx 0.46$$

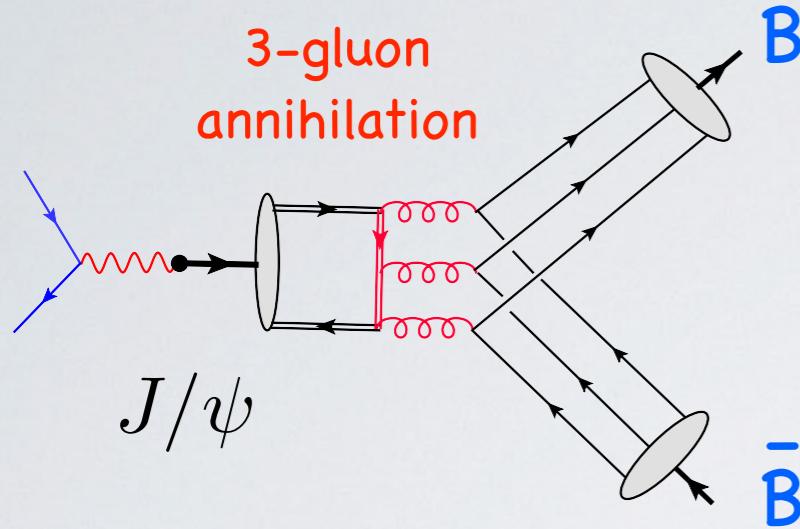
Carimalo, 1987       $\alpha_N = 0.70$       constituent quarks, non-relativistic nucleon WF

Murgia, Melis, 1995

$\alpha_N = 0.561 - 0.963$       constituent quark mass  $m_i = x_i m_N$   
in the factorisation formula for  $G_M$

# $J/\psi$ baryonic decays: EFT framework

QCD factorisation: NRQCD + collinear QCD copies



$$A[J/\psi \rightarrow B\bar{B}] = R_{10}(0) \textcolor{red}{T}[c\bar{c} \rightarrow B\bar{B}] + \text{corrections}$$

$$\textcolor{red}{T}[c\bar{c} \rightarrow B\bar{B}] = \int dx_i dy_i \textcolor{red}{H}(x_i, y_i) \phi_B(x_i) \phi_{\bar{B}}(y_i)$$

hard kernel  
nucleon distribution  
amplitudes

$$A[J/\psi \rightarrow B\bar{B}] = R_{10}(0) \phi_B * \textcolor{red}{H} * \phi_B$$

LO contribution: Brodsky, Lepage 1981/Chernyak et al, 1984, 1987

corrections: + rad. corrections (loops  $\mathcal{O}(\alpha_s)$ ) logarithmic corr's

+ relativistic corrections  $\mathcal{O}(v^2)$  higher order NRQCD operators

+ higher twist corr's  $\mathcal{O}(\Lambda/m_c)$  valence (3q) & higher Fock states

# General structure of the collinear expansion

$$\Gamma[J/\psi \rightarrow B\bar{B}] = \frac{M_\psi \beta}{12\pi} \left( |\mathcal{G}_M|^2 + \frac{2m_B^2}{M_\psi^2} |\mathcal{G}_E|^2 \right) = \Gamma_{\perp} + \Gamma_{\parallel}$$

$$\mathcal{G}_M = \mathcal{G}_M^{\text{lo}} + \mathcal{G}_M^{\text{nlo}} \quad \mathcal{G}_M^{\text{lo}} = R_{10} \phi_3 * H_{33} * \phi_3 \quad \text{twist-3 DAs}$$

$$\mathcal{G}_M^{\text{nlo}} / \mathcal{G}_M^{\text{lo}} \sim \Lambda^2 / m_Q^2 \quad \mathcal{G}_M^{\text{nlo}} = R_{10} \{ \phi_3 * H_{35} * \phi_5 + \phi_4 * H_{44} * \phi_4 \} \quad \text{twist-3,4,5 DAs}$$

$$\mathcal{G}_E = R_{10} \phi_3 * H_{34} * \phi_4 \quad \text{twist-3,4 DAs}$$

The hard kernels  $H_{34,35,44}$  are calculated NK, 2020/2022

and the collinear factorisation is verified

# Nucleon distribution amplitudes

## Kinematical twists & Fock expansion

twist-3 DAs       $\phi_3 \sim |qqq(^2S_{1/2})\rangle$       3 quark (valence)

$\mathcal{G}_M^{\text{lo}}$

twist-4 DAs       $\phi_4 \sim |qqq(^2P_{1/2})\rangle$       valence       $|L|=1$

$\mathcal{G}_E$      $\mathcal{G}_M^{\text{nlo}}$

+  $|qqq_8(^2S_{1/2}) + g_\perp\rangle$       4-particle Fock state      (neglected)

twist-5 DAs       $\phi_5 \sim |qq(D_\perp^2 q)(^2S_{1/2})\rangle_{kin} + |qqq(^4D_{1/2})\rangle$       valence, WW-parts only

$\mathcal{G}_M^{\text{nlo}}$

+  $|qqq_8(^2P_{1/2}) + g_\perp\rangle + |qqq(^2S_{1/2}) + 2g_\perp\rangle$       multi-particle Fock states  
(neglected)

various properties are studied in

Braun et al, 2000, 2008, 2013 Definitions/ Conformal expansions/ LL Evolution

Manashov, Anikin 2013, 2015 / Lorentz symm. relations (WW decompositions)

Shäfer, Wein 2015 / octet DAs, SU(3) relations & corrections

Kivel, 2022 / additional relations for tw.5 DAs, SU(3) breaking corrections

# Results

# Nucleon DAs

Other Baryon DAs      QCD SR Chernyak et al, 1989  
Lattice: Bali et al [RQCD Collaboration] 2019  
few unknown tw.4 moments were estimated using  $SU(3)_f$  sym.

charmonium WF       $|R_{10}(0)|^2 \simeq 0.81 \text{ GeV}^3$        $m_c = 1.48 \text{ GeV}$       Eichten Quigg, 1995

	$Br[J/\psi \rightarrow B\bar{B}] \times 10^3$	$Br[\text{exp}]$	$ \mathcal{G}_E / \mathcal{G}_M ^*$	$ \mathcal{G}_E / \mathcal{G}_M [\text{exp}]$
$p$	2.10	2.12(3)	$0.85^{+0.10}_{-0.12}$	0.83(2)
$n$	1.94	2.09(2)	$0.85^{+0.10}_{-0.10}$	0.95(6)
$\Lambda$	1.94	1.89(9)	$0.85^{+0.15}_{-0.07}$	0.83(4)
$\Sigma^0$	1.10	1.17(3)	$1.85^{+0.10}_{-0.24}$	2.11(5)
$\Sigma^+$		1.50(3)		2.27(5)
$\Xi^+$	1.38	0.97(8)	$0.65^{+0.10}_{-0.03}$	0.61(5)
$\Xi^0$		1.16(44)		0.53(4)

Branching's can only be described at relatively low scale

$$\mu^2 = 1.5 \text{ GeV}^2$$

$$\alpha_s = 0.35$$

\* the error bars indicate the uncertainty due to the mixing with e.m. amplitude

## Relativistic corrections

Motivation There are also data for excited  $\psi(2S)$  BESIII 2018

$$\psi(2S) : \alpha_p = 1.03 \pm 0.06 \pm 0.03 \Rightarrow |\mathcal{G}_E/\mathcal{G}_M| \approx 0 \quad ?$$

$$\psi(1S) : \alpha_p = 0.595 \pm 0.012 \pm 0.015 \quad |\mathcal{G}_E/\mathcal{G}_M| \approx 0.83(2)$$

13% rule:  $Q_B = \frac{Br[\psi(2S) \rightarrow B\bar{B}]}{Br[\psi(1S) \rightarrow B\bar{B}]} \approx \frac{Br[\psi(2S) \rightarrow e^+e^-]}{Br[\psi(1S) \rightarrow e^+e^-]} \simeq 0.133$

	$p\bar{p}$	$n\bar{n}$	$\Lambda\bar{\Lambda}$	$\Sigma^0\bar{\Sigma}^0$	$\Sigma^+\bar{\Sigma}^+$	$\Xi^+\bar{\Xi}^+$	Data BESIII
$Q_B$	0.139	0.146	0.204	0.210	0.072	0.276	

Can the relativistic corrections explain these observations ?

# Relativistic corrections

$$\Gamma[H({}^3S_1) \rightarrow e^+e^-] = \frac{8\pi e_c^2 \alpha^2}{3M_H^2} \left[ 1 - \frac{1}{6} \langle \mathbf{v}^2 \rangle_H + \dots \right]^2 \langle O_1 \rangle_H$$

Bodwin, Petrelli 2002

$$\langle \mathbf{v}^2 \rangle_H = \frac{\langle 0 | \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | H({}^3S_1) \rangle}{m_c^2 \langle 0 | \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* \psi | H({}^3S_1) \rangle}$$

$$\langle \mathbf{v}^2 \rangle_{J/\psi} \approx 0.225$$

Bodwin, et al 2008

$$\langle \mathbf{v}^2 \rangle_{\Upsilon(1S)} \approx -0.01$$

Bodwin, et al 2007

The relative correction of order  $\mathbf{v}^2$  and partial resummation to all orders to baryon decay amplitude  $\mathcal{G}_M$  can be done within the same framework.

The important difference is that baryon has structure. This provides strong numerical enhancement of the RC:

$$\Gamma[J/\psi \rightarrow p\bar{p}] = \text{const} \times \langle O_1 \rangle_H \underbrace{[2.54 + \delta_{\text{rad}} - 13.71 \langle \mathbf{v}^2 \rangle + 2.18 \langle \mathbf{v}^2 \rangle^2]}_{-2.44}^2$$

# Relativistic corrections

The structure of the  $v^2$  coefficient

asymptotic DA for simplicity  $\phi_3(x_i) \simeq f_N 120 x_1 x_2 x_3$

$$\Gamma[J/\psi \rightarrow p\bar{p}] = \text{const} \times \langle O_1 \rangle_H [J_0 + \delta_{\text{rad}} + J_2 \langle \mathbf{v}^2 \rangle]^2$$

Convolution integrals:

$$Dx_i = \delta(1 - x_1 - x_2 - x_3) dx_1 dx_2 dx_3 \quad 0 < x_i, y_i < 1 \quad D_i = x_i(1 - y_i) + y_i(1 - x_i) > 0$$

$$J_0 = \int Dx_i \int Dy_i \frac{12 x_1 y_3}{D_1 D_3} = 1.68 \quad J_2 = \int Dx_i \int Dy_i \frac{-4}{D_1 D_3} = -8.12$$

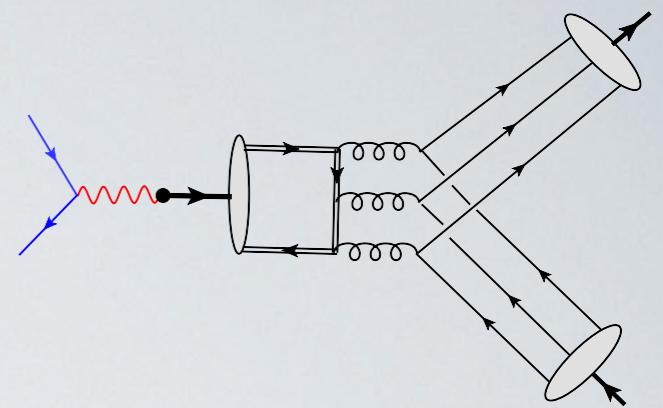
$$J_2 \langle \mathbf{v}^2 \rangle = -1.62$$

Possible scenario:  $\delta_{\text{rad}} + J_2 \langle \mathbf{v}^2 \rangle \ll J_0$

Rel. corr's & rad. corr's are much larger than collinear power corr's but, probably, cancel each other. This must be verified.

# Conclusions

All decay amplitudes associated with the 3g annihilation,  
are computed within the QCD EFT framework.



The considered models for baryon DAs provide reliable description of Br's for the relatively low norm. scale  $\mu^2 = 1.5 \text{ GeV}^2$  only

The interference of hadronic amplitudes with e.m. baryon FF's is numerically important.

The SU(3) breaking effects are important for description of  $\sum$  baryon

The NLO power corrections to  $\mathcal{G}_M$  are of relative order 3%-25%

The ratios of hadronic amplitudes are in agreement with the exp. data within 10-20% accuracy

However the relativistic corrections are of the same order as the leading-order term! This enhancement is closely related with the baryon structure.

There are indication that large relativistic corrections can cancel by the radiative corrections.

Thanks!