Thermal Transitions in Dense Two-Colour QCD

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Motivation

- Sign problem means makes it impossible probe dense (baryon density $\mu > 0$) QCD at temperatures below deconfinement threshold
- QCD-like theories such as two-colour QCD (QC_2D) don't suffer from this sign problem
- Previous studies on smaller lattice volumes have investigated deconfinement transition and colour superfluid to normal matter transition
- Larger lattice volume of this study hopes to disentangle finite volume and finite temperature effects
- Fitting to larger number of diquark sources to allow for extrapolation to zero diquark source

Methodology

- Start off with the path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\Phi] \mathcal{O}[\Phi] e^{-S[\Phi]} \quad (1)$$

- Space-time is discretised using unimproved Wilson Fermions
- Gauge configurations produced with probability weight

$$e^{-S[U]} = \det M[U] e^{-S_G[U]} \quad (2)$$

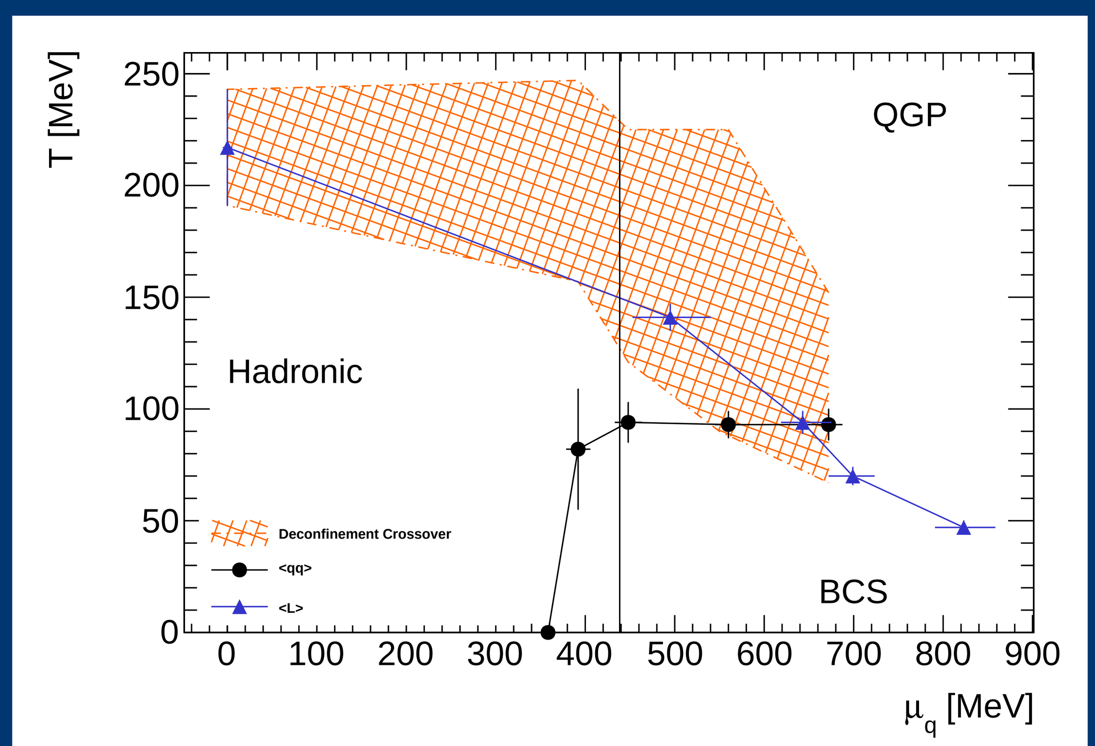
  using Hybrid Monte Carlo code from[1].
- At non-zero baryon density, fermion matrix acquires non-zero density of very small eigenvalues, slowing down the computation
- Diquark source j lifts these eigenvalues, with "physical" results recovered by extrapolation of j to zero.
- Temperature is given by

$$T = \frac{1}{a_\tau N_\tau} \quad (3)$$

- Varying the number of sites along the time direction allows us to complete a temperature scan

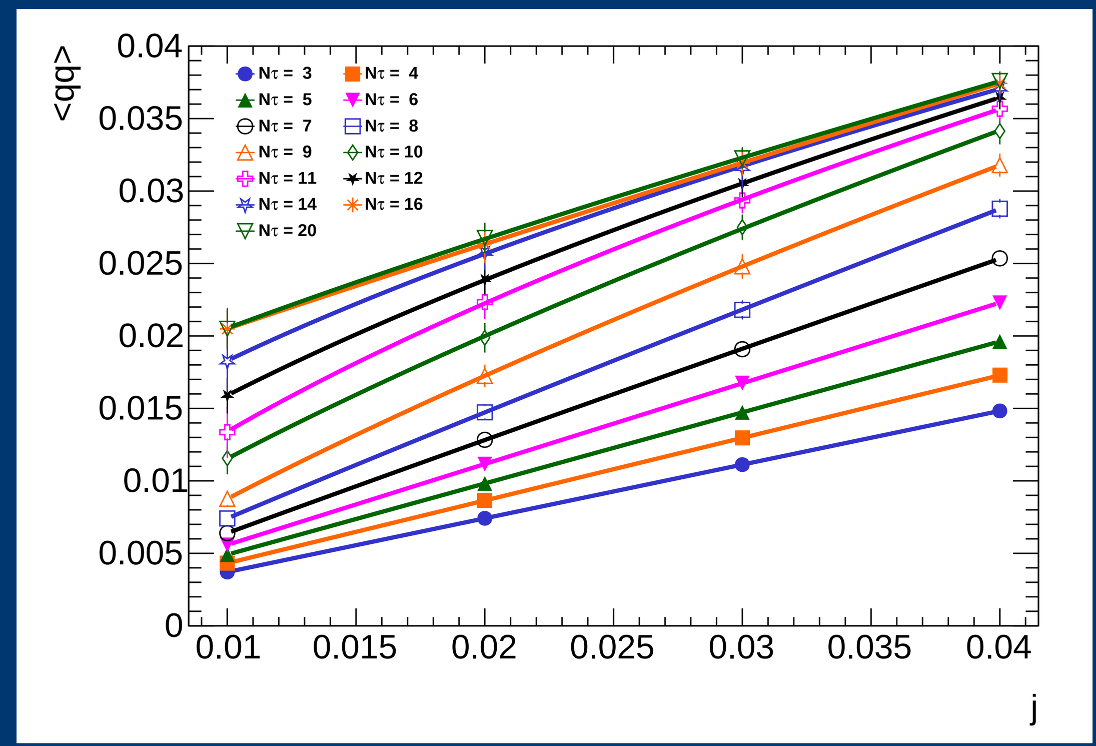
Lattice Setup

- Configurations were generated using the parameters for a coarse lattice, $\beta = 1.9$, $\kappa = 0.1680$ corresponding to a $m_\pi a = 0.68(1)$ and $\frac{m_\pi}{m_\rho} = 0.80(1)$.[2].
- All runs were conducted with a spatial extent $N_s = 24$ and a fixed chemical potential $a\mu = 0.400$.
- Temperatures were scanned at $N_\tau = 3$–20, giving temperatures in the range 55 MeV–365 MeV .



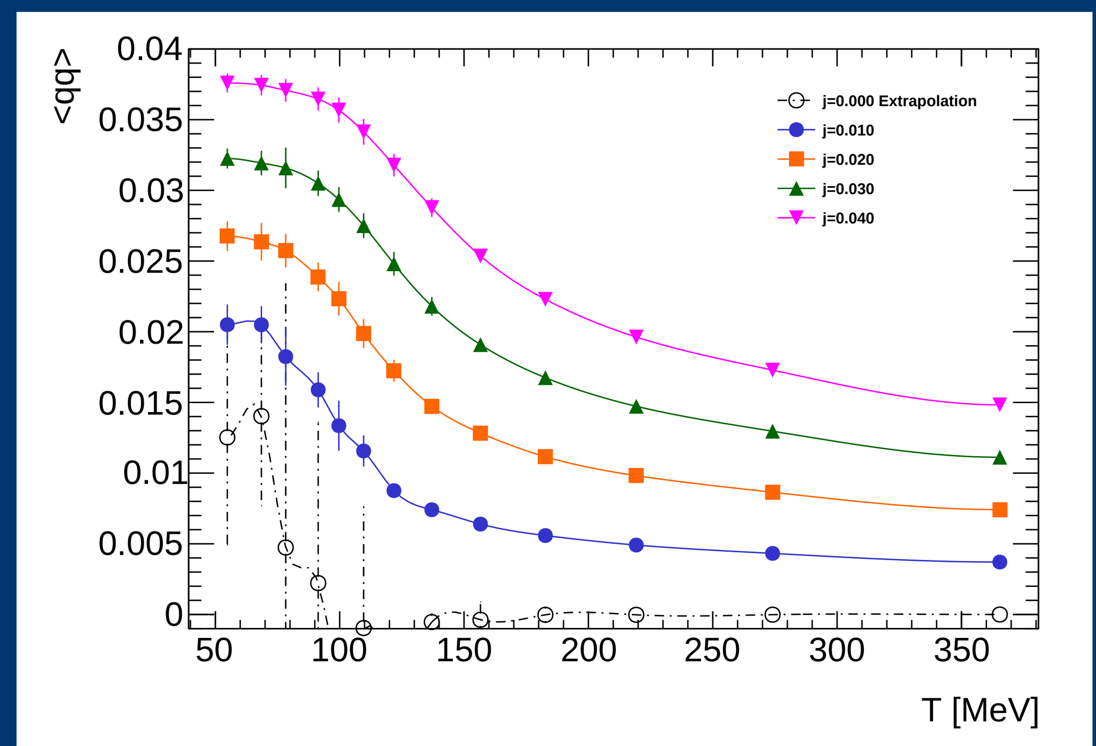
Phase diagram of QC_2D for $\frac{m_\pi}{m_\rho} = 0.80(1)$.[3] Orange hatched area is the deconfinement crossover, black circles are the diquark condensate and blue triangles the Polyakov line. Our temperature scan took place along the black vertical line corresponding to $a\mu = 0.400$.

Diquark Condensate



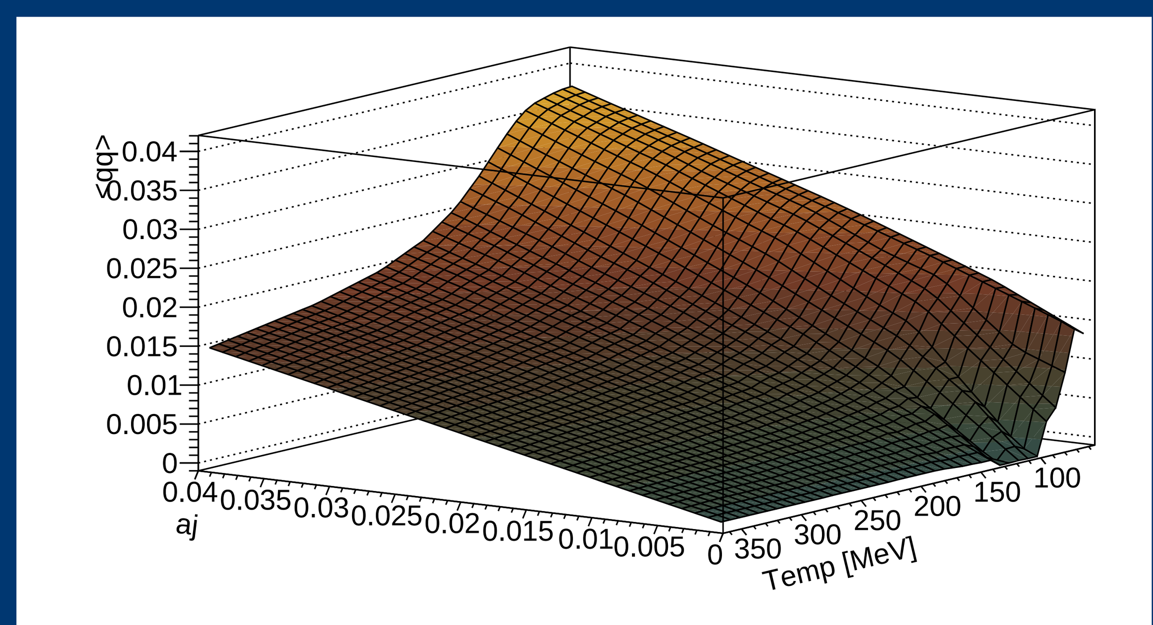
Diquark condensate vs Diquark Source

- First time we have done a run with this many diquark sources.
- The fit $\langle qq \rangle = a + bj^c$ was used.
- For low and high temperatures $\langle qq \rangle$ was found to be linear in j.
- Around the superfluid phase transition however the fit is non-linear, with the exponent reaching its minimum around the transition.
- An improved action and more data are required to nail down the transition temperature more precisely.



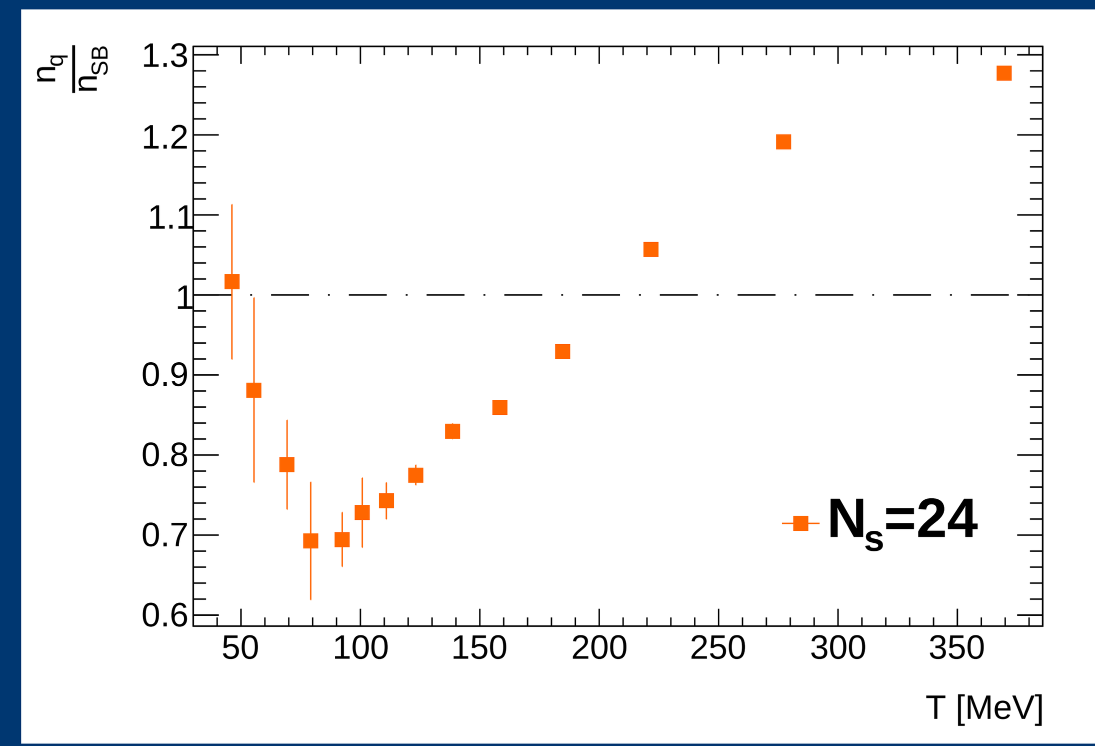
Diquark condensate vs Temperature

- Zero diquark values obtained by extrapolating the above fit to zero diquark source.
- Interpolated using a cubic spline.
- The superfluid phase transition occurs around $T \sim 100$ MeV.
- This indicates that the superfluid phase transition is indeed distinct from the deconfinement crossover.



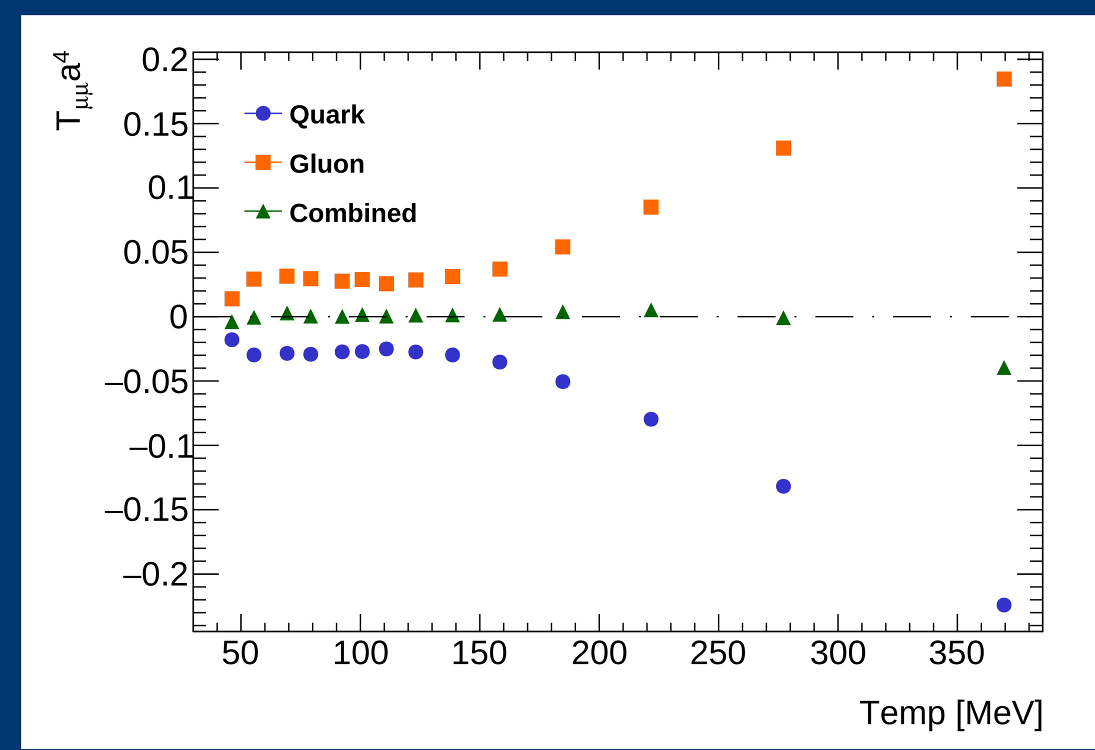
Surface plot of the diquark condensate as a function of Temperature and Diquark Source using a linear interpolation. We thought this looked rather nice.

Thermodynamics

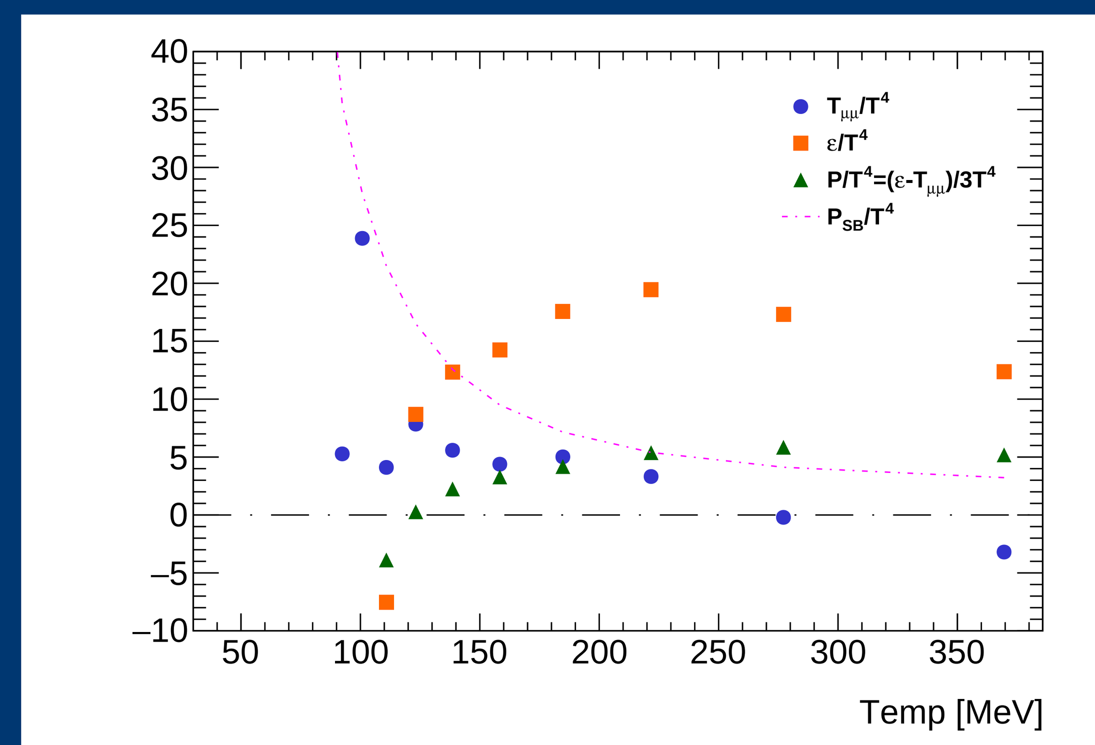


Quark number density. These result are compatible with previous results on a smaller volume in [3].

- Notice how the density dips in the same region as the superfluid phase transition.
- This is particular striking as $\frac{n_q}{n_{SB}}$ is not an order parameter for the phase transition.



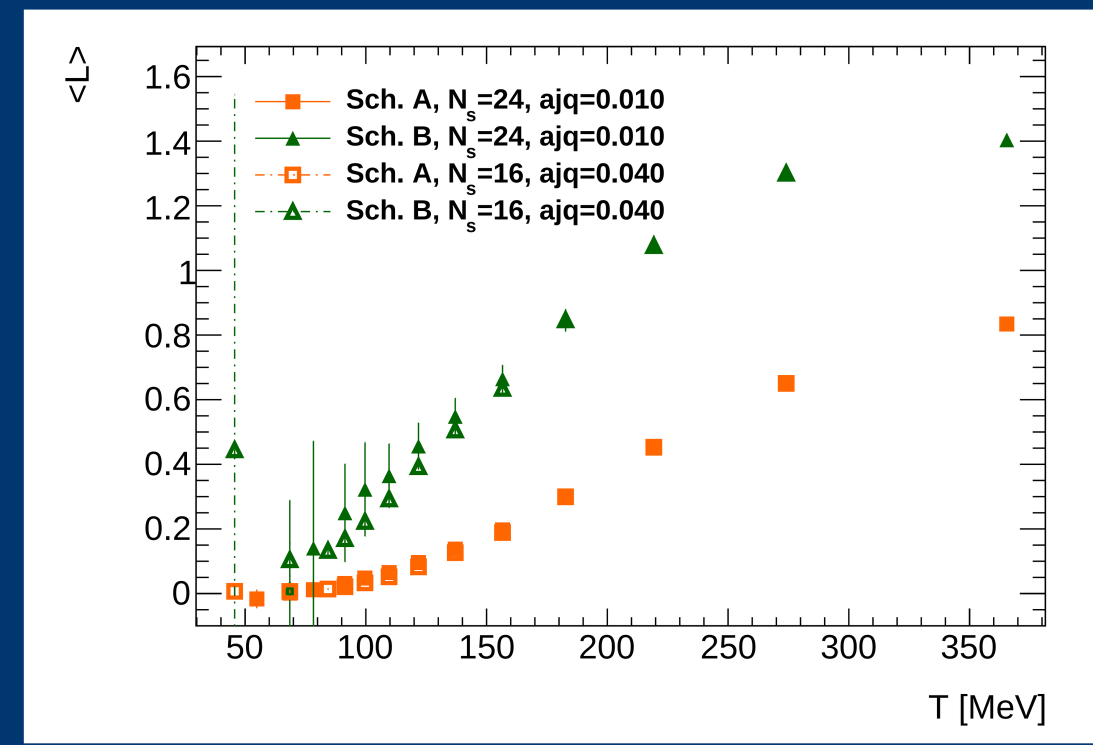
Trace Anomaly



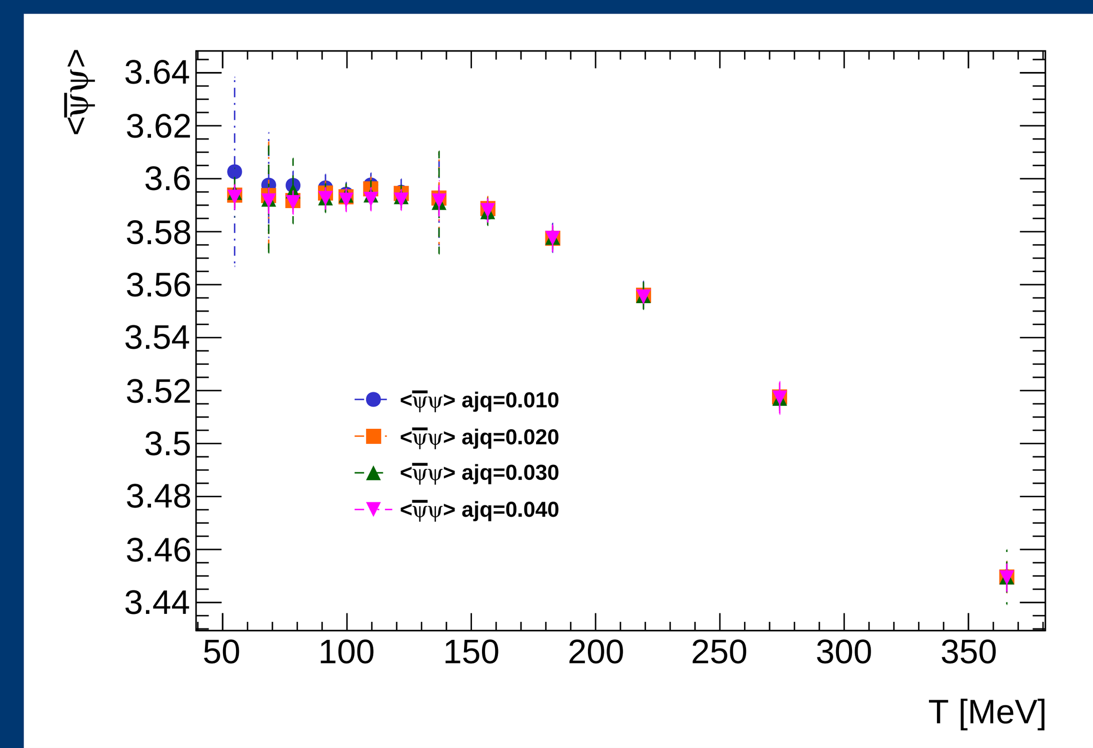
Dimensionless Thermodynamic Observables

- These results are qualitatively consistent with recent results from [4, 5].
- Pressure evaluated using derivitive method, with Karsch Coefficents evaluated in [6]
- Energy density behaviour is similar.
- However the conformal anomaly itself is smaller than expected.
- Also interesting is how the quark and gluon contributions nearly cancel each other out at most temperatures. This may be due to the choice of regularisation.
- Trace anomaly is constant up until $T \sim 150$ MeV. Sign of a second transition?

Deconfinement Transition



Renormalised Polyakov loop with two renormalisation schemes. These results are also compatible with earlier results on smaller volumes.
A: $L_R(N_\tau = 4, \mu = 0) = 0.5$
B: $L_R(N_\tau = 4, \mu = 0) = 1.0$



Unrenormalised and unsubtracted chiral condensate

The change in behaviour from constant to decreasing at $T \sim 150$ MeV suggests that the crossover coincides with the deconfinement crossover, not the superfluid transition.

Conclusions and Outlook

- The superfluid phase transition and deconfinement crossover are distinct
- Further investigation needed into unexpectedly small trace anomaly
- Work underway to implement a Symanzik improved fermion action
- Tuning has commenced on a finer lattice with lighter quarks

Acknowledgements

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References

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Ollscoil Mhá Nuad
Ollscoil na hÉireann Má Nuad