

Random quantum circuit sampling

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- ① Variational methods
- ② Estimation through extrapolation
- ③ The Ising model
- ④ Annealing
- ⑤ Conclusions

① Variational methods

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③ The Ising model

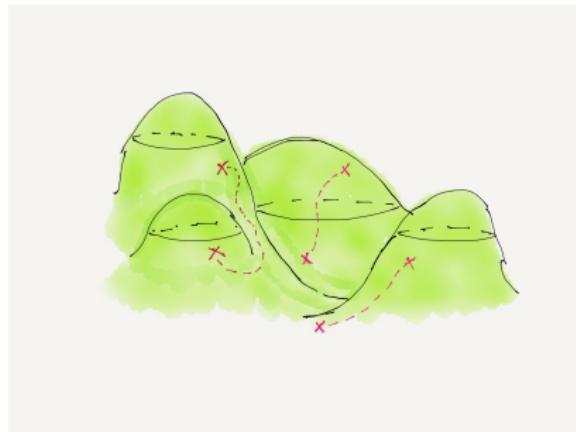
④ Annealing

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Variational methods

Foundation:

$$\begin{aligned} E_0 &\leq \langle \psi | H | \psi \rangle \\ &= \langle + | U^\dagger H U | + \rangle \end{aligned}$$



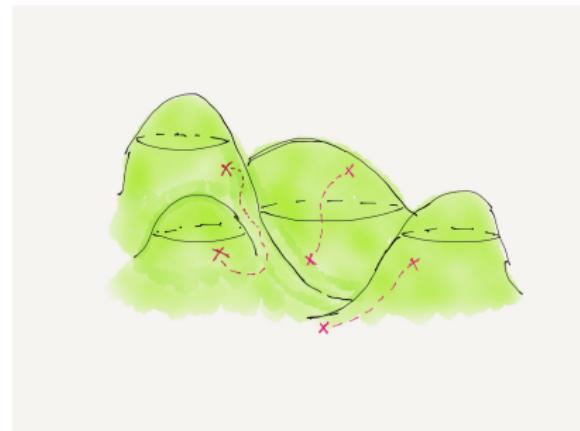
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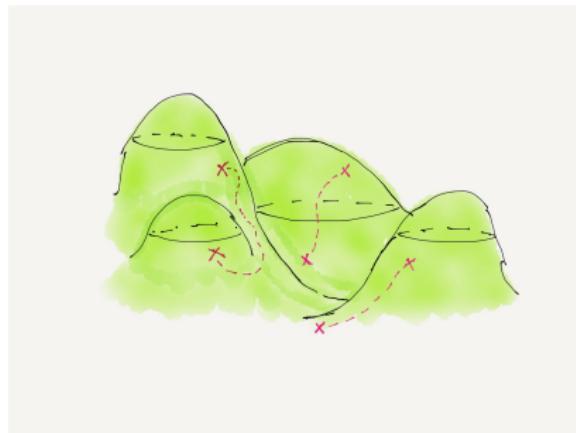
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Most common: Use gradients

[Farhi et al., 2014, Kandala et al., 2017, Hadfield et al., 2019,
Cerezo et al., 2021, Zhang et al., 2021]

Variational methods

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→ penalizes circuits



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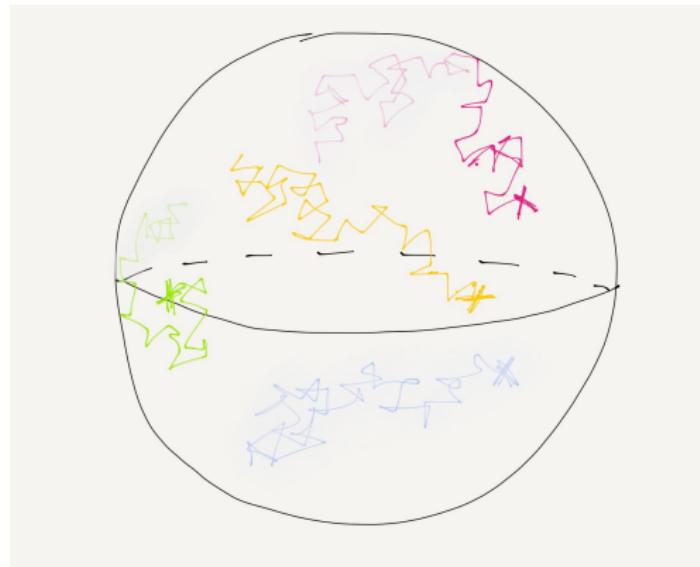
Variational methods

- There is some cost function
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 - Requires many measurements
- Adjust the parameters of U
- Can add a stochastic element



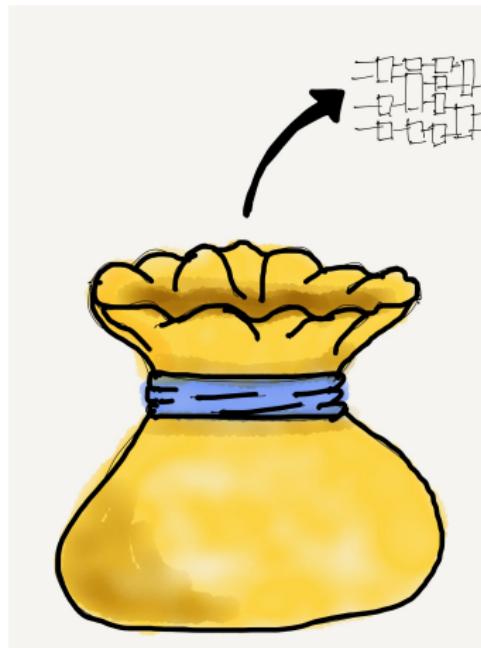
Variational methods

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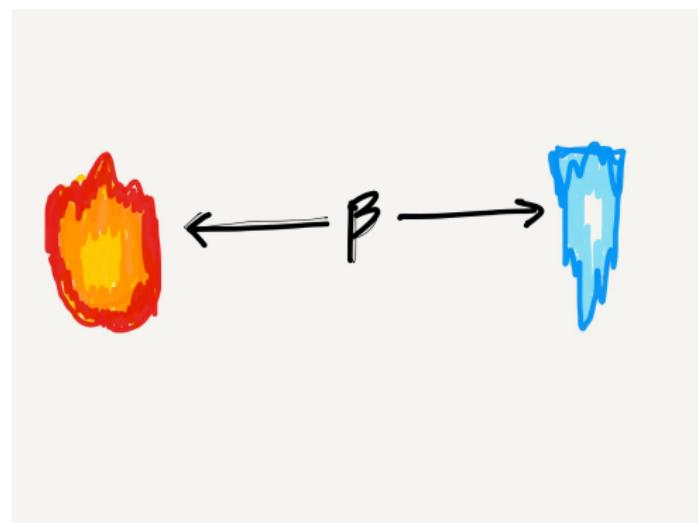
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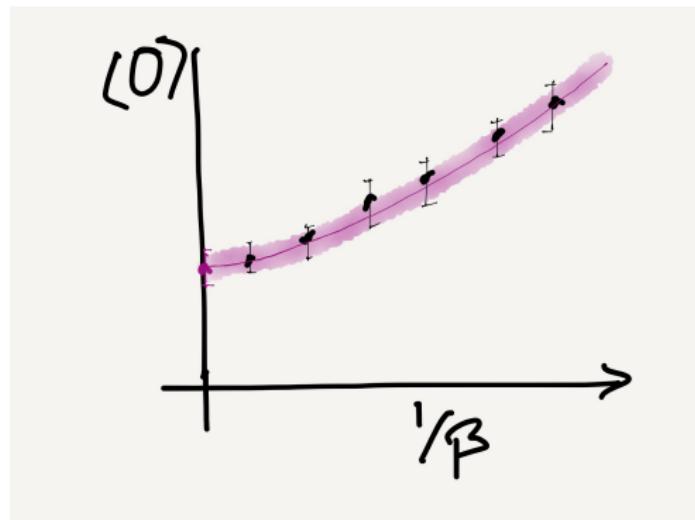
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Variational methods

- Here we consider a random walk
- Draw random samples of quantum circuits (U)
- Systematically control the energy, (β)
- *Extrapolate to lower energies.*



Estimation through extrapolation

Strategy:

- 1) Sample circuits at finite β

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- By-passes complexity of ground-state search
- No explicit circuit construction
- Controlled circuit depth

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Cons:

- Ansatz dependent
- New systematic errors
- New statistical errors

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Sampling

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The Hamiltonian $\mathcal{E} \equiv \langle \psi | H | \psi \rangle$ (average energy)

Sampling

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Sampling

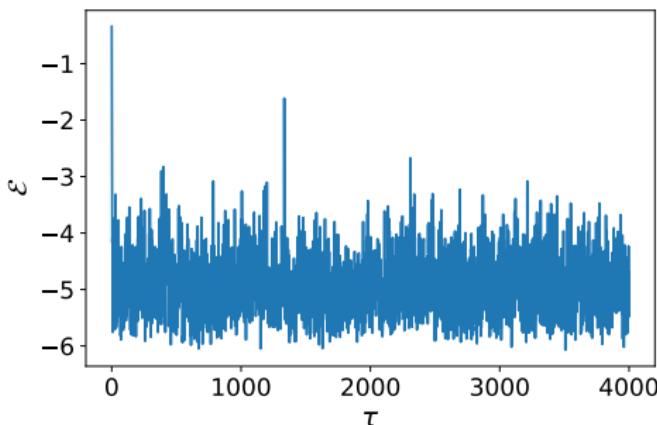
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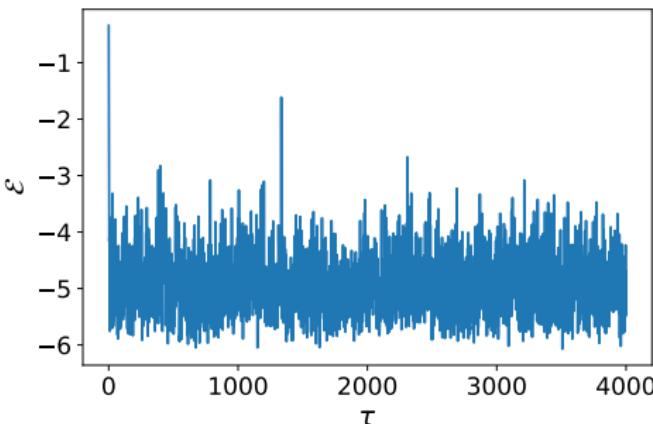
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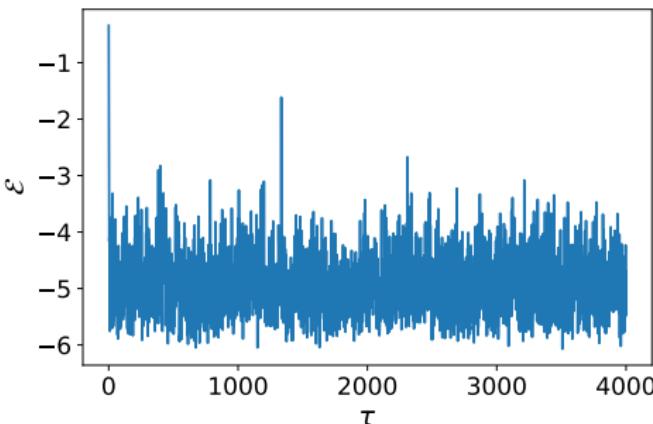
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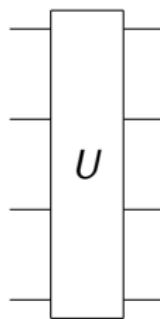
- Random walk with Metropolis
- For fixed β , particular U s are favored.
- The average energies fluctuate about some mean, $\bar{\mathcal{E}}$
- As β increases, $\bar{\mathcal{E}}$ decreases.



Sampling U

How to do it?

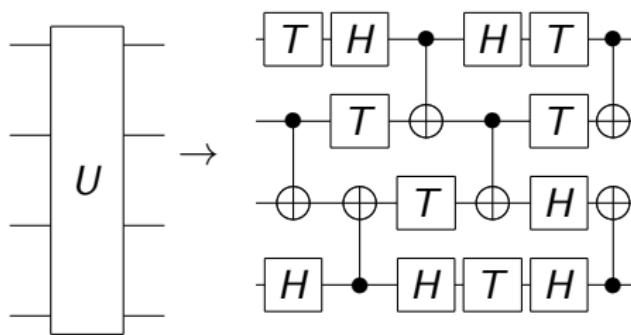
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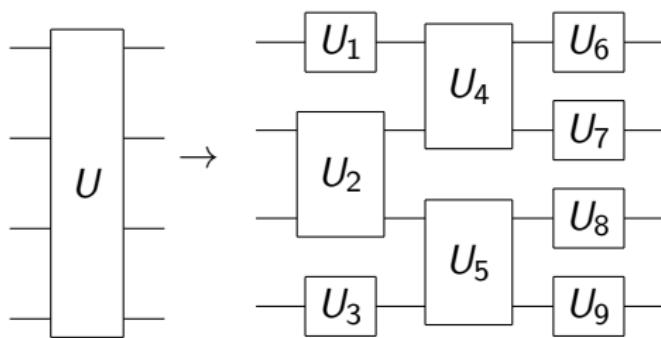
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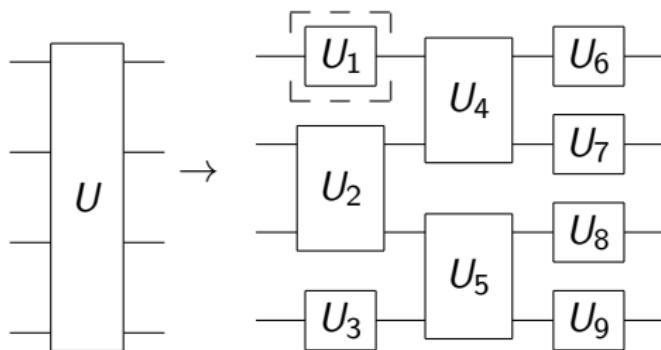
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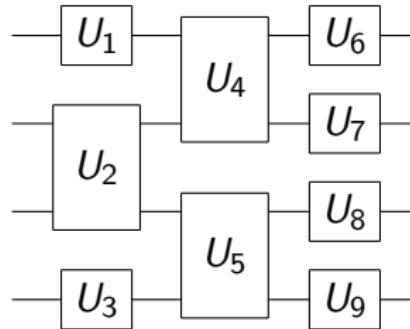
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- Gate d.o.f.



Metropolis on quantum gates

1) Calculate initial energy,

$$E_0 = \langle + | U^\dagger H U | + \rangle$$

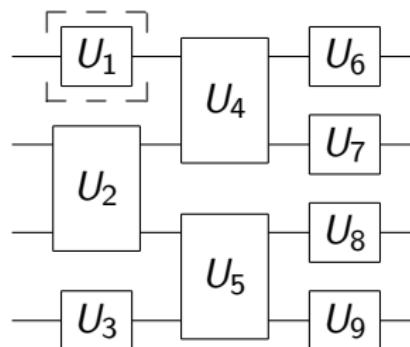


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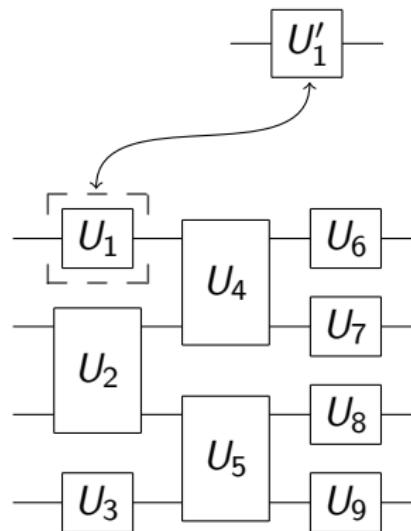
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- 2) Pick a gate to change: U_1



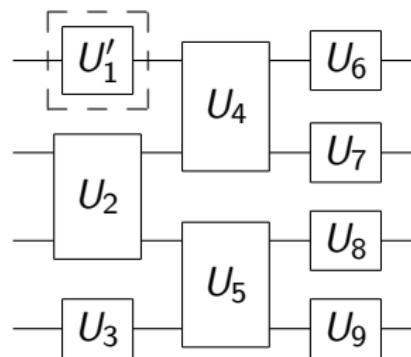
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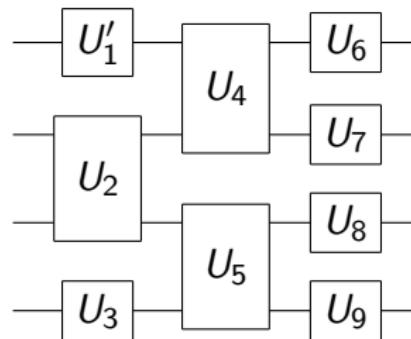
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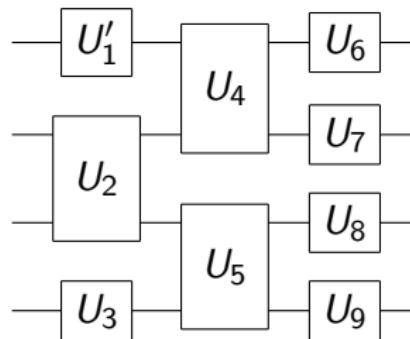
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- 5) Calculate
 $E_f = \langle + | U'^\dagger H U' | + \rangle$
- 6) Accept U' with probability
 $\min[1, e^{-\beta(E_f - E_0)}]$



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Ising model

The target Hamiltonian:

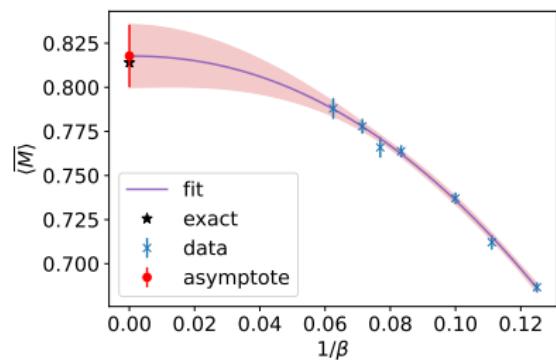
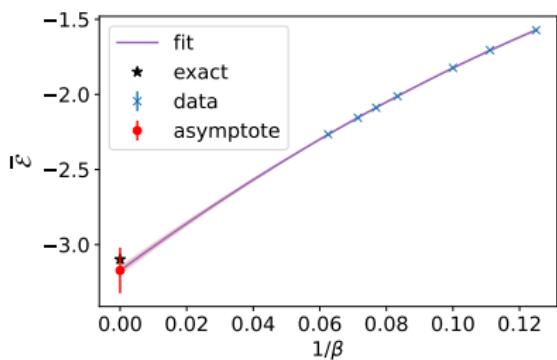
$$H = - \sum_i Z_i Z_{i+1} - h_x \sum_i X_i$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

We calculate the average energy $\mathcal{E} = \langle H \rangle$ and the magnetization $M = \langle \sum_i X_i \rangle$

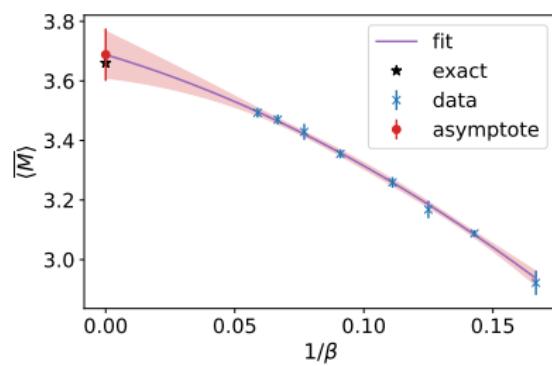
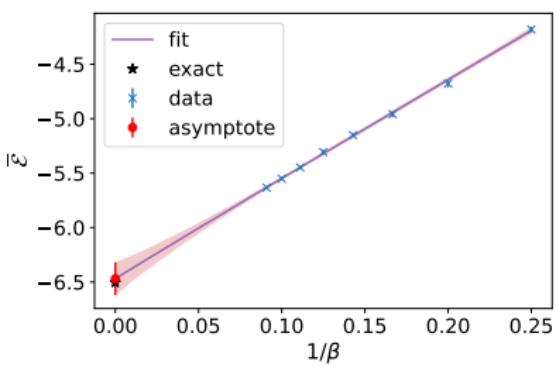
Results

$h_x = 0.25$



Results

$$h_x = 1.5$$



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Annealing

In contrast...

- Approximate the ground state

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Annealing

In contrast...

- Approximate the ground state
- Slowly lower “temperature”
- System “jiggles” into low energy
- Exponentially long wait...
- Force it to go faster

The $O(2)$ nonlinear sigma model

$$H = \sum_i (L_i^z)^2 - \mu \sum_i L_i^z - \frac{J}{2} \sum_i (U_i^+ U_{i+1}^- + U_i^- U_{i+1}^+)$$

- Possesses vortices

The $O(2)$ nonlinear sigma model

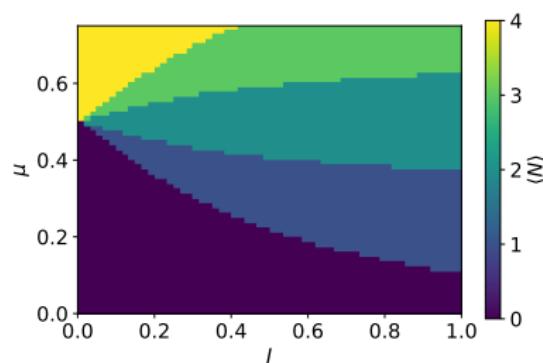
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- Possesses vortices
- Confining topological phase transition

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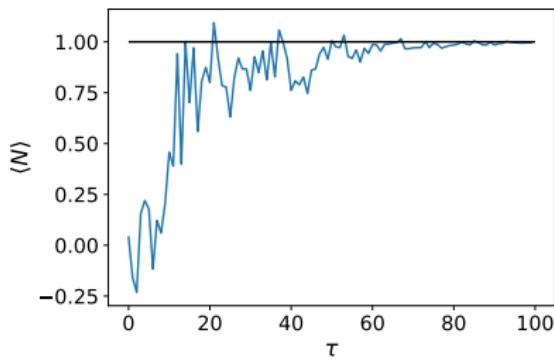
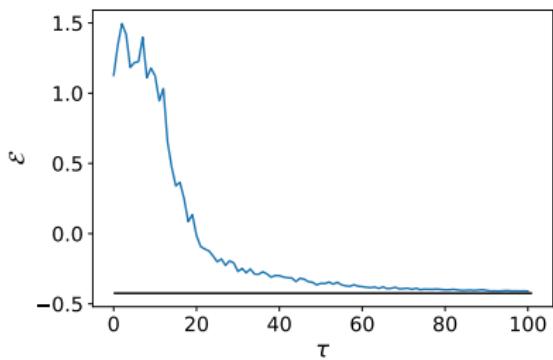
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- Possesses vortices
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- μ coupled to total charge
- $\sum_i L_i^z \equiv N$

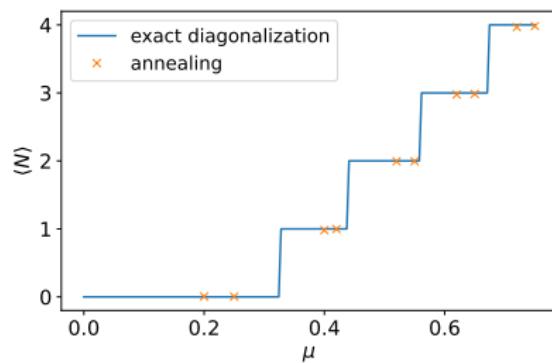
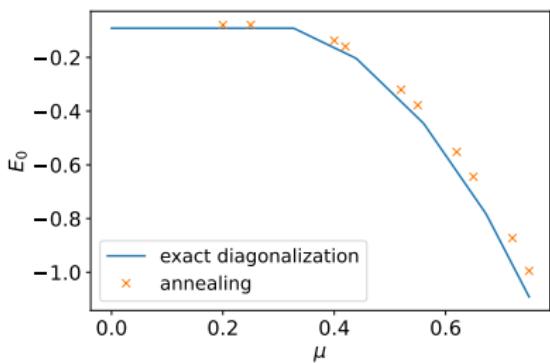


Results

Gradually increase β during sampling:



Results



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Conclusions

- Variational methods a possible avenue for near-term quantum computing
- Sampling random circuits provides an alternative to gradient-based methods
- Extrapolations for estimation
- Expand exploration into circuit depth
[Grimsley et al., 2019, Bilgis et al., 2021]
- Approximate ground state
- arXiv:2111.14676

Variational methods
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Estimation through extrapolation
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The Ising model
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ooooo

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Thank you!

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