Heavy Quark Dynamics in a Strongly Magnetized Medium

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In collaboration with : Jinfeng Liao and Hongxi Xing

Outline

- **Motivation**
- **Formalism**
- **Results**

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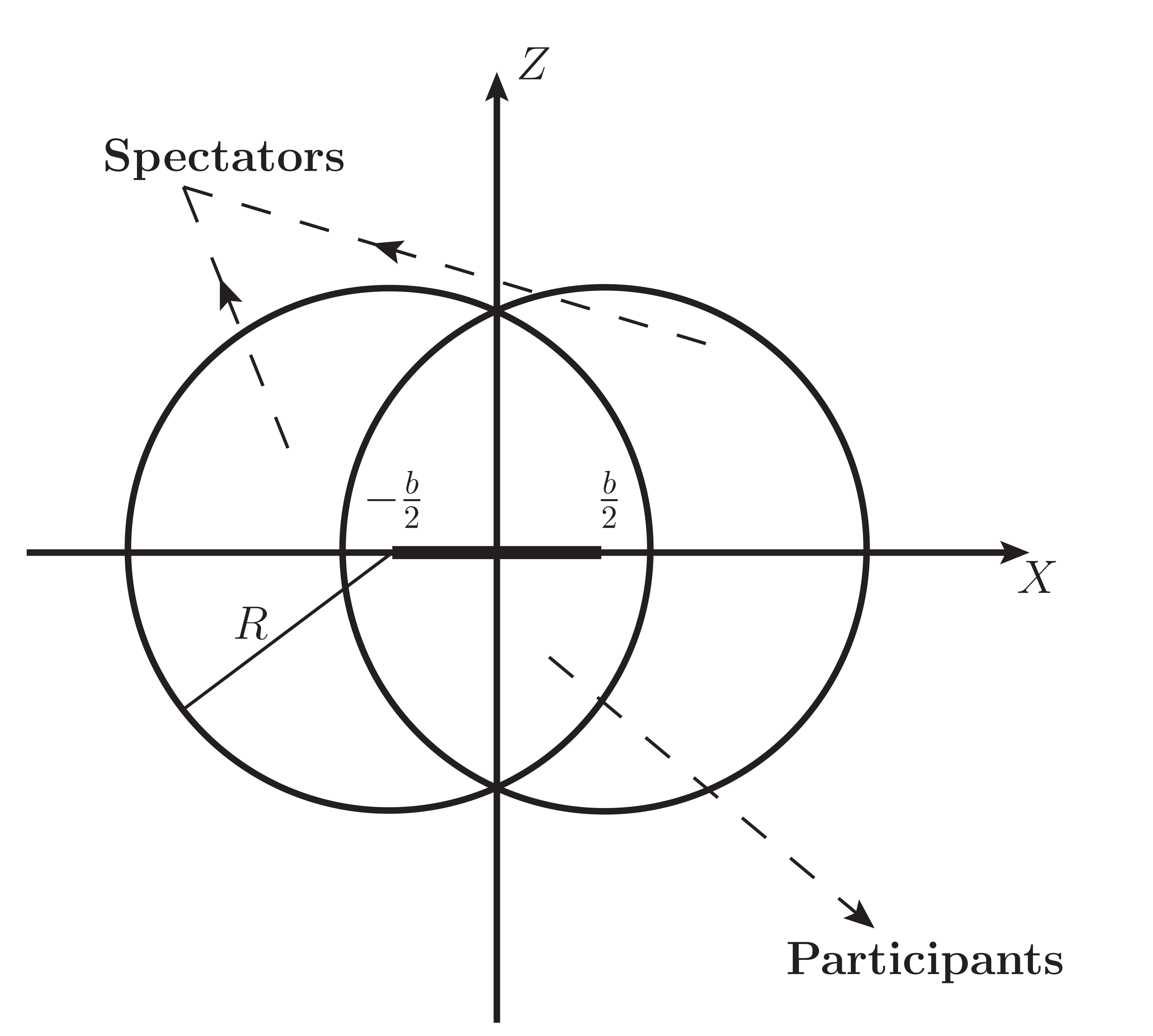
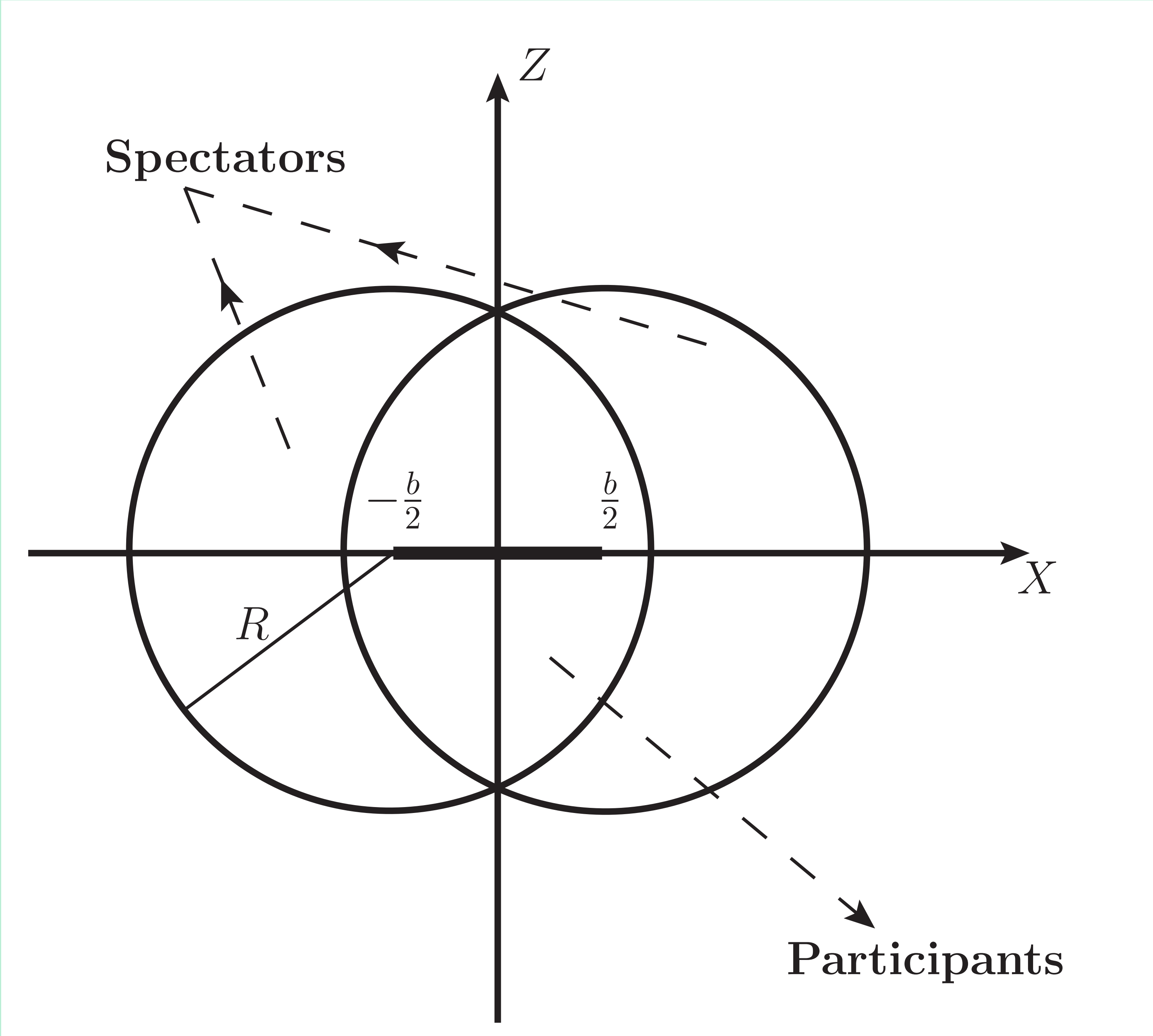
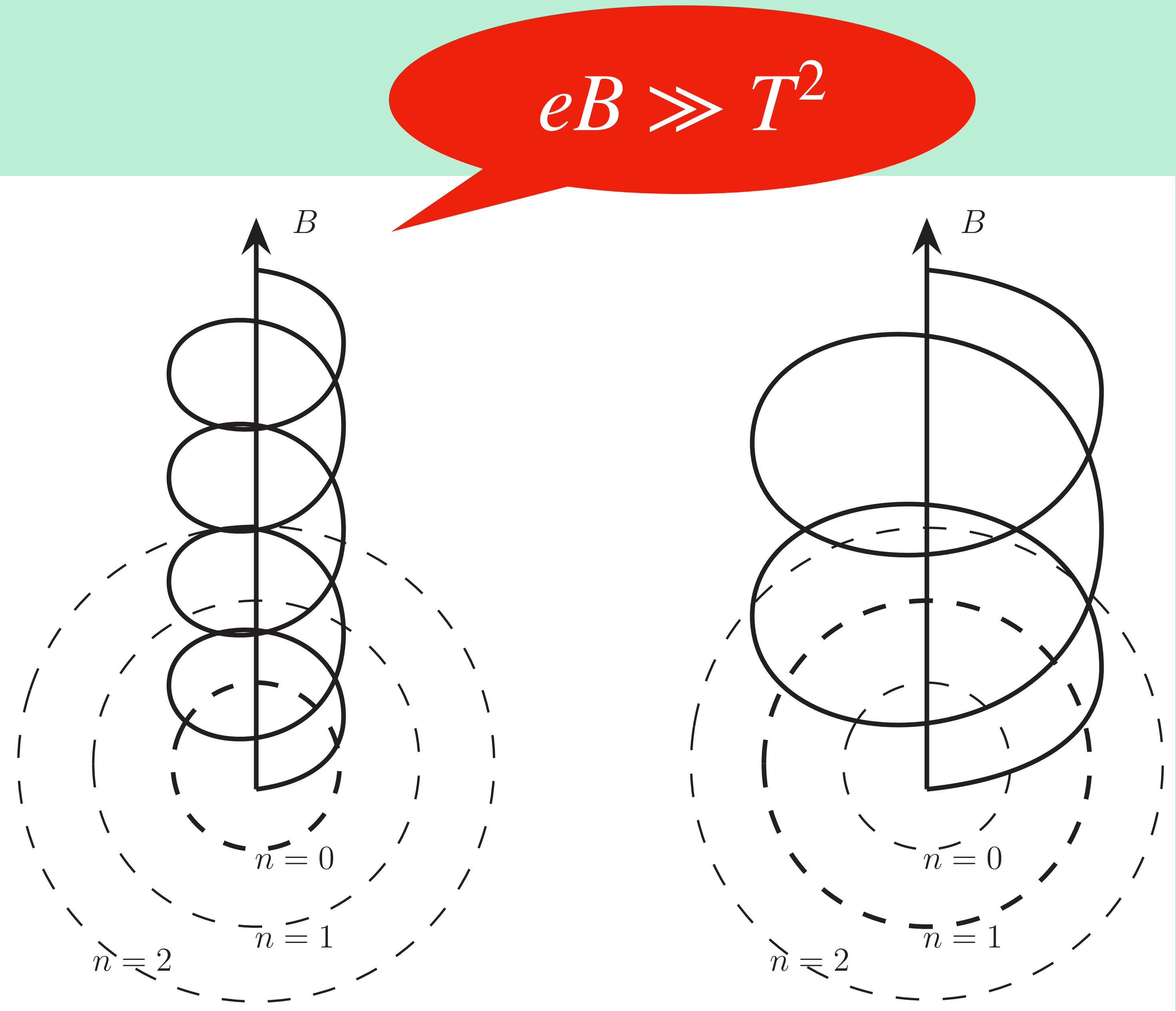
Magnetized medium



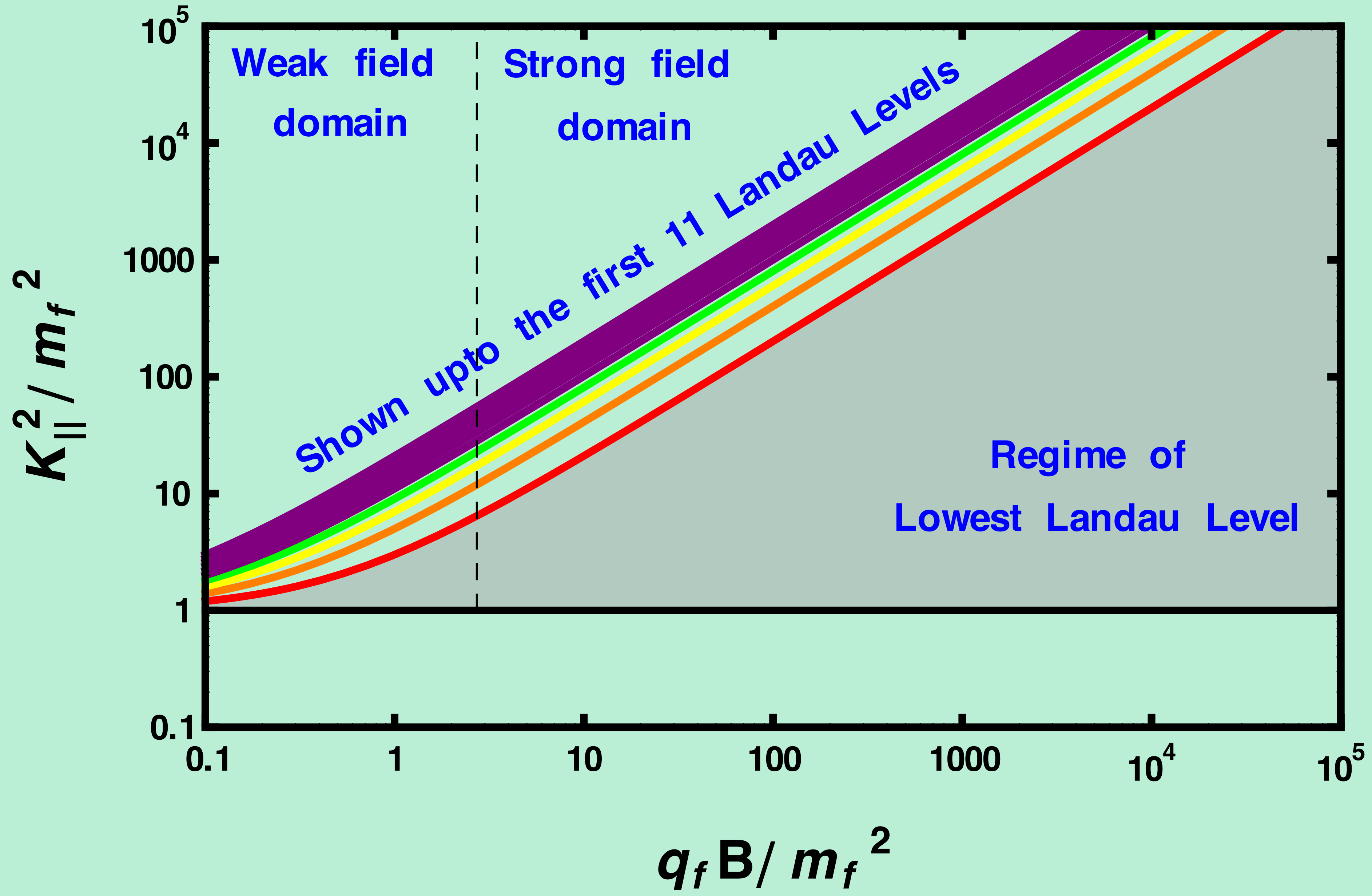
- **Strong magnetic fields are present in some stellar objects and non-central heavy ion collisions (e.g. $eB \sim \hat{O}(10)m_\pi^2$ at LHC).**

Magnetized medium





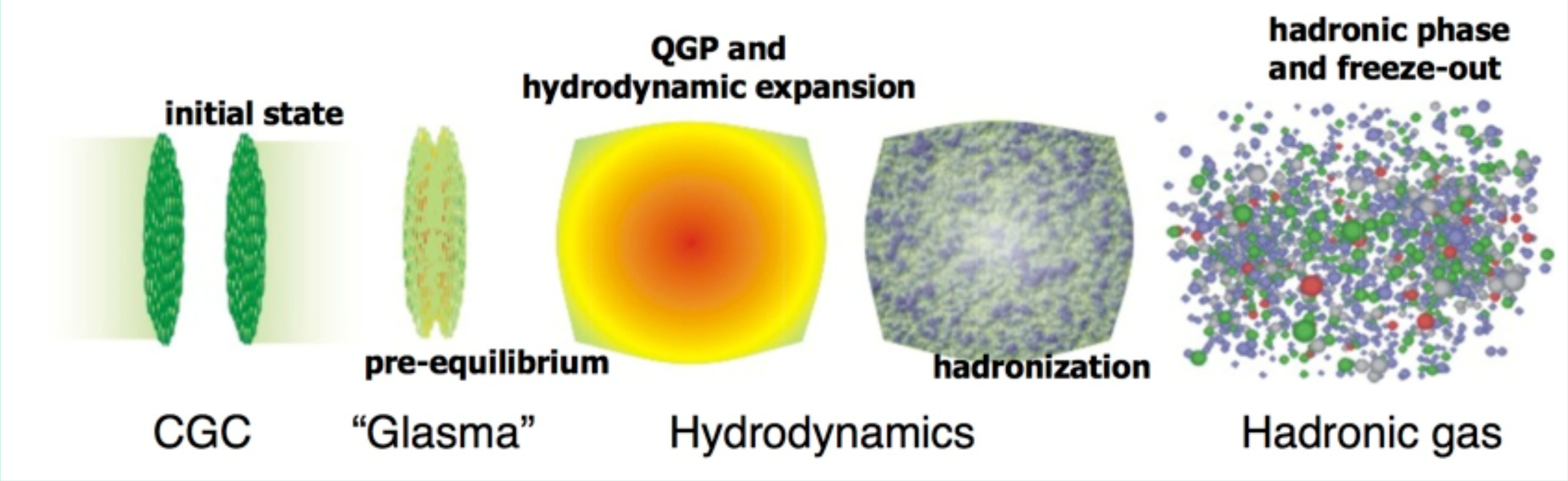
AB, Islam, Mustafa : 1602.06769



- Strong magnetic fields are present in some stellar objects and non-central heavy ion collisions (e.g. $eB \sim \hat{O}(10)m_\pi^2$ at LHC).

- Introduces extra scale eB in the medium in addition to T, $\mu \rightarrow$ triggers significant interest in theoretical understanding of the properties of a magnetized medium.

Heavy quark as a QGP signature



QGP Signatures

Static properties

Dynamic properties

- **Large mass compared to $T \rightarrow$ External to the bulk medium.**
- **Generated at the early stage $\rightarrow$ Less contaminated information of the QGP state.**

Heavy quark as a QGP signature

- HQs experience drag forces as well as random kicks from the bulk medium.
- A widely adopted approach is to use the Langevin equations for describing HQ in-medium evolution.
- Essential theoretical inputs : HQ momentum diffusion coefficients → influence phenomenological modelings of predictions for experimental observables. (R_{AA} and v_2)

QGP Signatures

Static properties	Dynamic properties
(e.g. Heavy quark diffusion coefficients)	(e.g. Heavy quark spectra)

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Static and Dynamic limits of Heavy Quark

$B = 0$

- **Static limit :** $M \gg T$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\ \delta_{ij}\delta(t-t')$$

Single diffusion coefficient κ

- **Dynamic limit :** $\gamma v \lesssim 1 \to p \lesssim M,\ M \gtrsim p \gg T$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa_{ij}(\mathbf{p})\ \delta(t-t')$$

where $$\kappa_{ij}(\mathbf{p}) = \kappa_L(p)\ \hat{p}_i\hat{p}_j + \kappa_T(p)\left(\delta_{ij} - \hat{p}_i\hat{p}_j\right)$$

Longitudinal (κ_L) and Transverse (κ_T) diffusion coefficients.

Static and Dynamic limits of Heavy Quark

$B \neq 0$

Two anisotropic directions - $\mathbf{v}$ and $\mathbf{B}$

$$M \gtrsim p > \sqrt{eB} \gg T$$

- **Case 1 :** $\mathbf{v} \parallel \mathbf{B}$

  Diffusion coefficients $\rightarrow \kappa_L,\ \kappa_T$

- **Case 2 :** $\mathbf{v} \perp \mathbf{B}$

  Diffusion coefficients $\rightarrow \kappa_1,\ \kappa_2,\ \kappa_3$

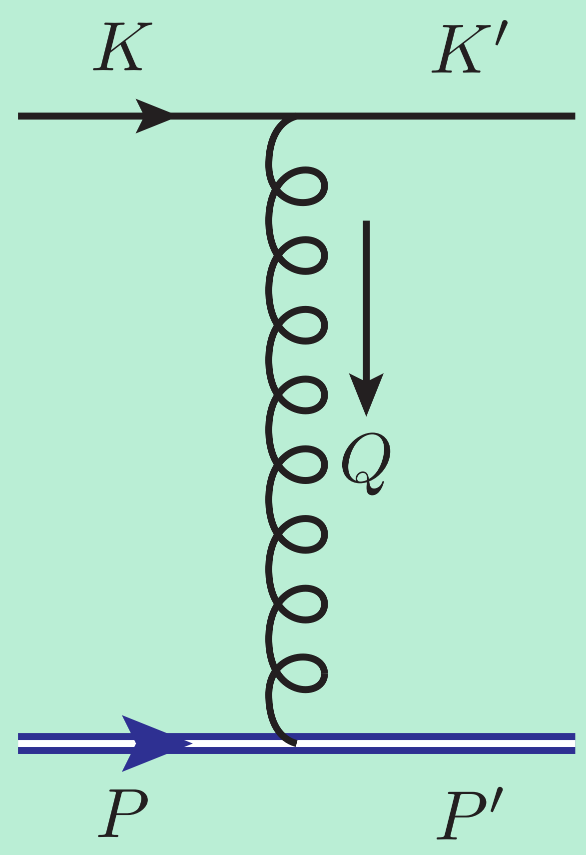
Scattering / Interaction rate

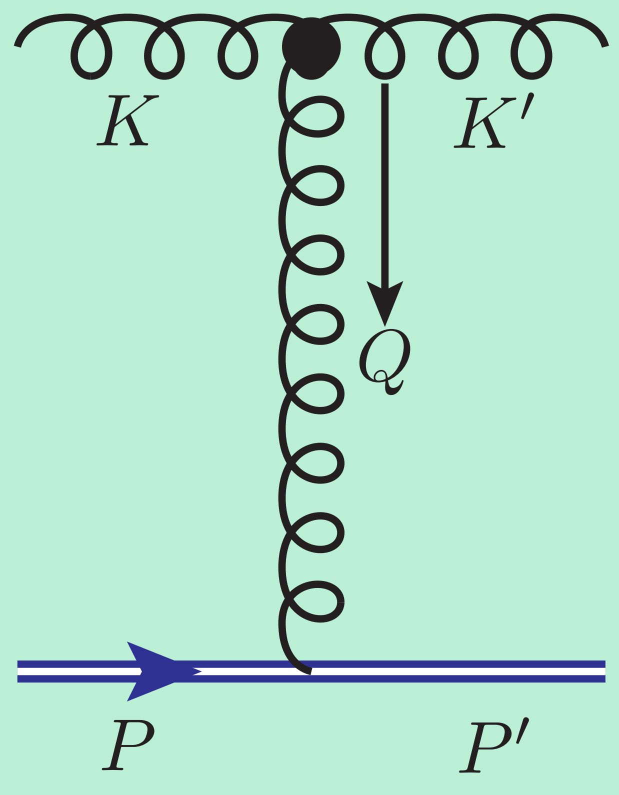
$$\kappa_i(p) = \int d^3q \frac{d\ \Gamma(v)}{d^3q}\ q_i^2$$

- $2 \leftrightarrow 2$ **scattering processes in a finite temperature medium**

  $qH \rightarrow qH$ **and** $gH \rightarrow gH$ **(**$q \rightarrow$ **quark,** $g \rightarrow$ **gluon and** $H \rightarrow$ **HQ).**

- **At leading order in strong coupling, these scatterings are mediated by one-gluon exchange.**

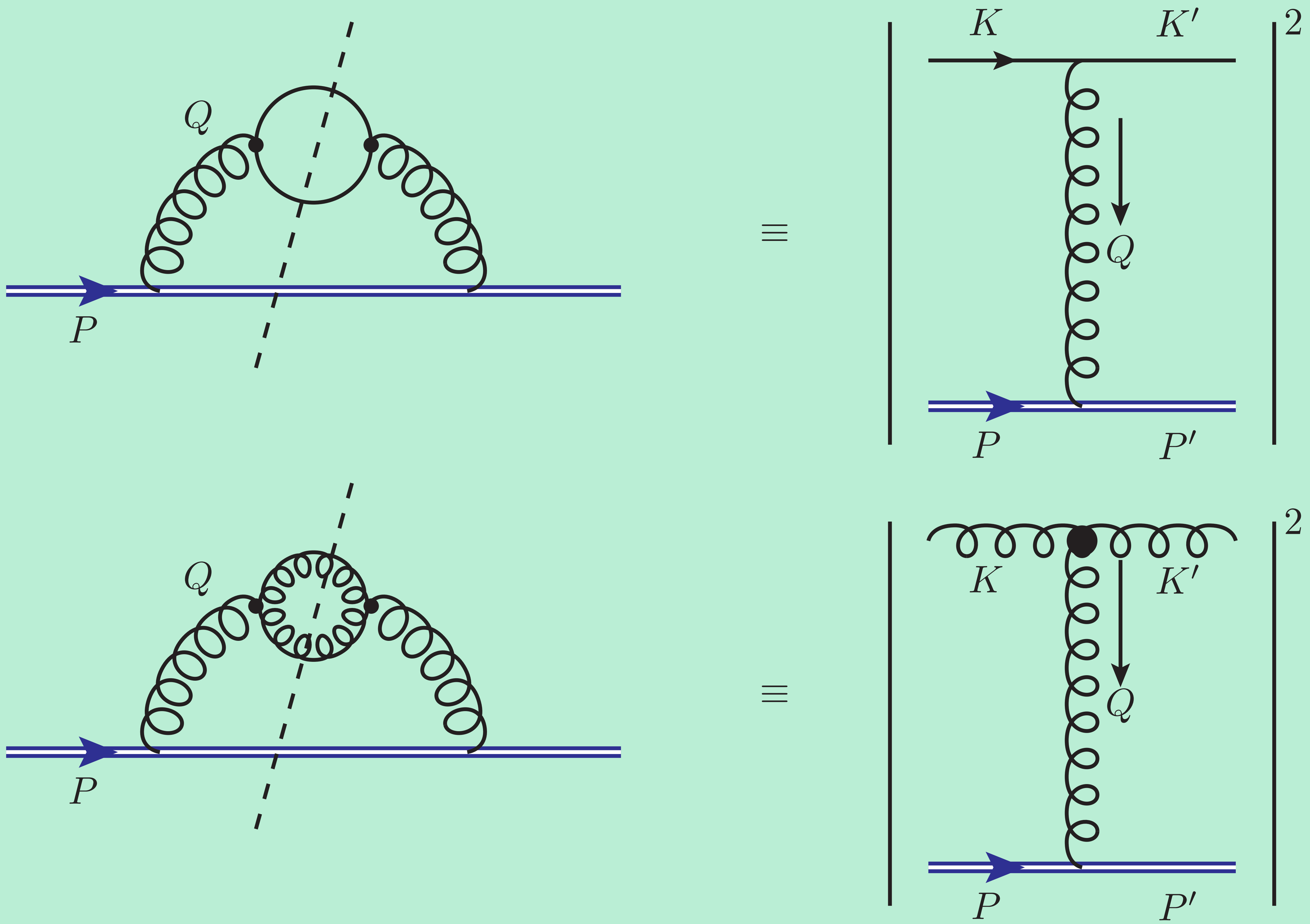




Scattering / Interaction rate

An effective way of expressing Γ is in terms of the cut/imaginary part of the HQ self energy $\Sigma(P)$

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\,\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[\left(\gamma_\mu P^\mu + M\right)\,\mathrm{Im}\,\Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$



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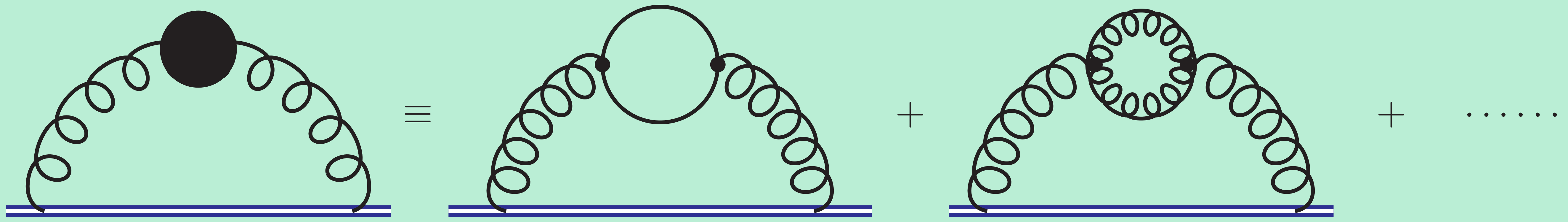

$$\Sigma(P) = ig^2 \int \frac{d^4Q}{(2\pi)^4}\mathscr{D}^{\mu\nu}(Q)\ \gamma_\mu\ S_m^s(P-Q)\ \gamma_\nu$$

Scattering / Interaction rate

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LLL

$$S_m^s(K) = e^{-k_\perp^2/|q_fB|}\ \frac{\gamma_\mu K_\parallel^\mu + M}{K_\parallel^2 - M^2}(1 - i\gamma_1\gamma_2)$$

Karmakar, AB, Haque, Mustafa : 1804.11336

$$\mathscr{D}^{\mu\nu}(Q) = \frac{\xi Q^\mu Q^\nu}{Q^4} + \frac{(Q^2-d_3)\Delta_1^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{\Delta_2^{\mu\nu}}{Q^2-d_2} + \frac{(Q^2-d_1)\Delta_3^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{d_4\Delta_4^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2}$$

where $$\Delta_1^{\mu\nu} = \frac{1}{\bar{u}^2}\bar{u}^\mu\bar{u}^\nu,\ \Delta_2^{\mu\nu} = g_\perp - \frac{Q_\perp^\mu Q_\perp^\nu}{Q_\perp^2},\ \Delta_3^{\mu\nu} = \frac{\bar{n}^\mu\bar{n}^\nu}{\bar{n}^2},\ \Delta_4^{\mu\nu} = \frac{\bar{u}^\mu\bar{n}^\nu + \bar{u}^\nu\bar{n}^\mu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}},$$

with $$d_1(Q) = \Delta_1^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_2(Q) = \Delta_2^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_3(Q) = \Delta_3^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_4(Q) = \frac{1}{2}\Delta_4^{\mu\nu}\Pi_{\mu\nu}(Q)$$

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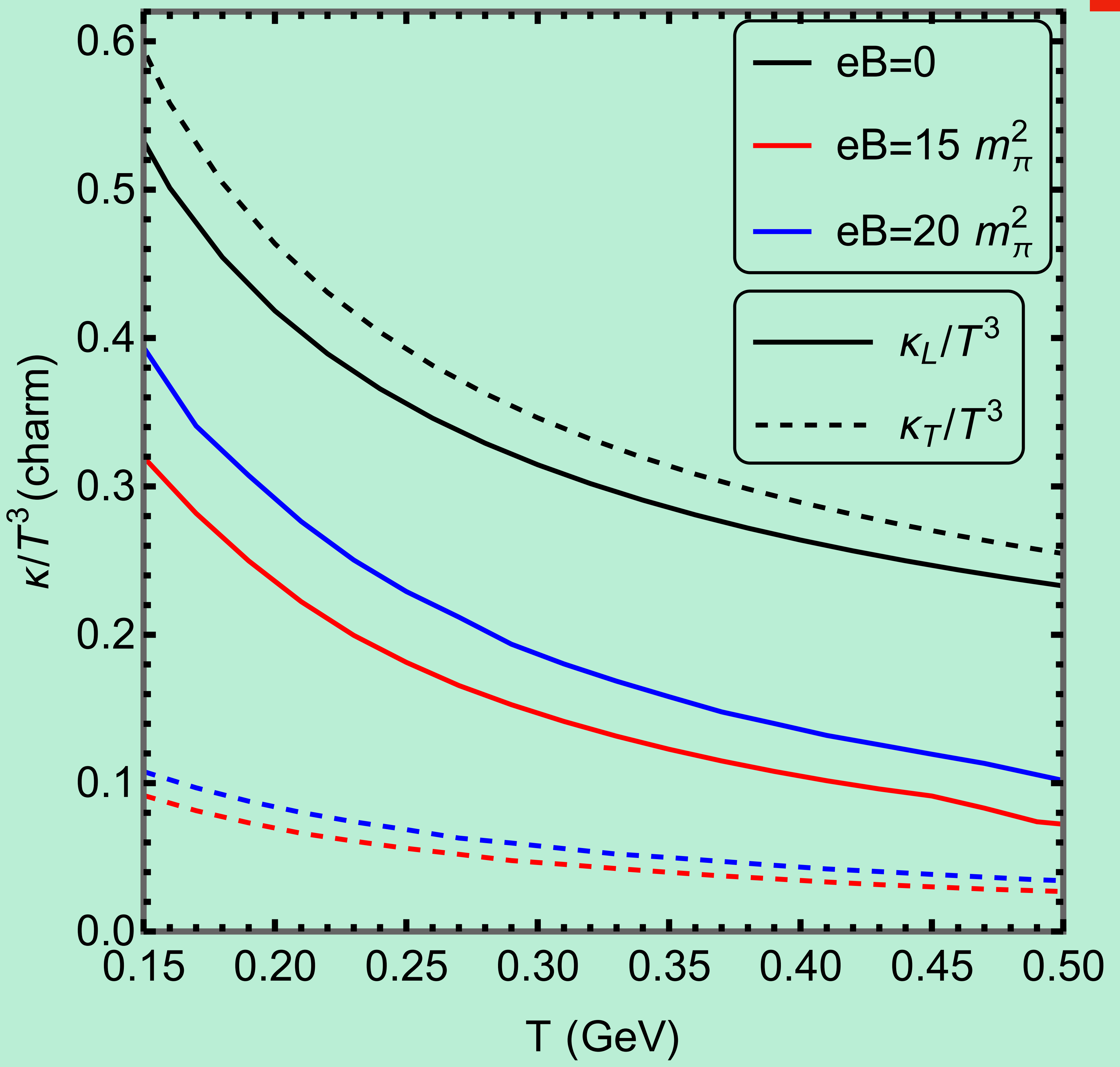
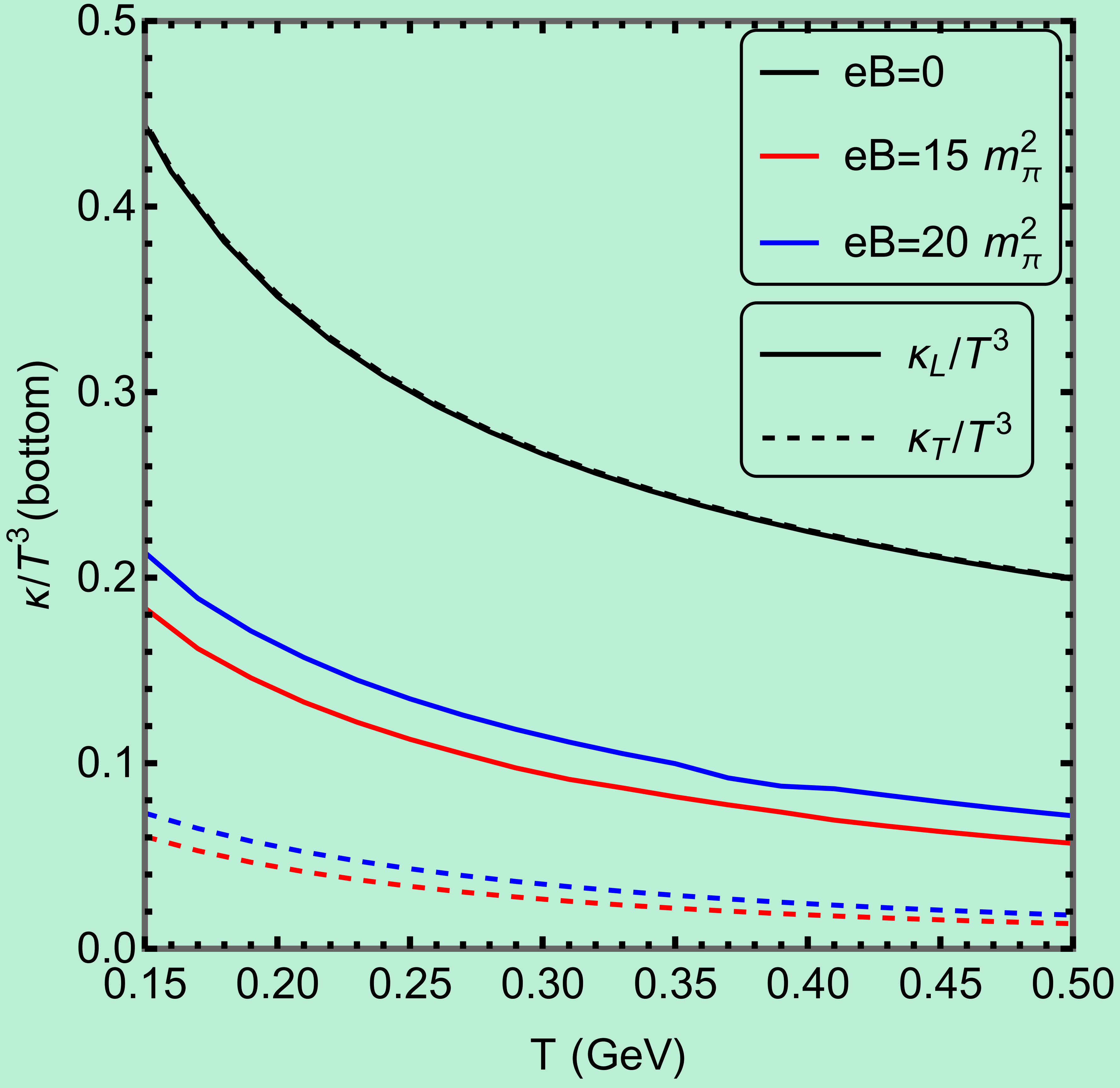
Results : $v \parallel B$

AB, Liao, Xing : 2105.02167





Variation of the scaled HQ longitudinal (solid lines) and transverse (dashed lines) momentum diffusion coefficients (for $\mathbf{v} \parallel \mathbf{B}$) with T for three different values of eB, i.e. $eB = 0,\ 15m_\pi^2,\ 20m_\pi^2$ and for both charm (left panel) and bottom (right panel) quarks. HQ momentum is taken to be 1 GeV.

Results : $v \parallel B$

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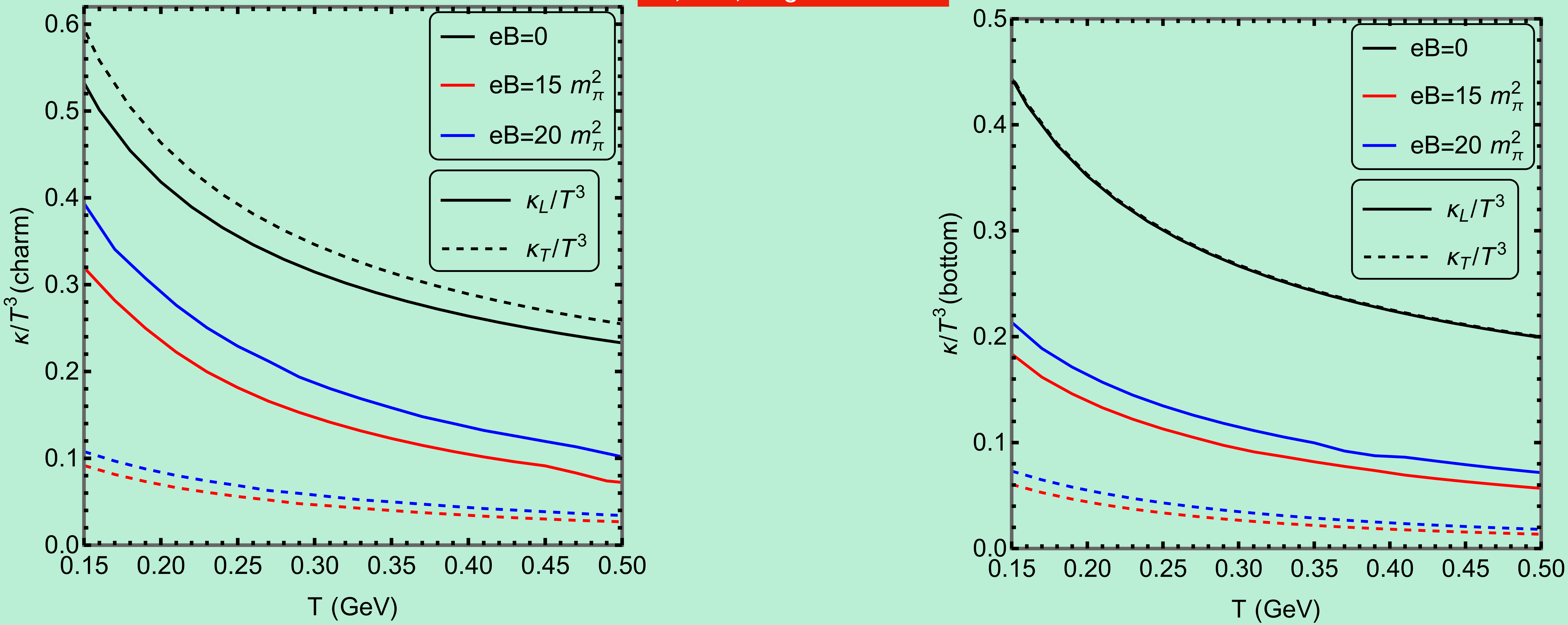
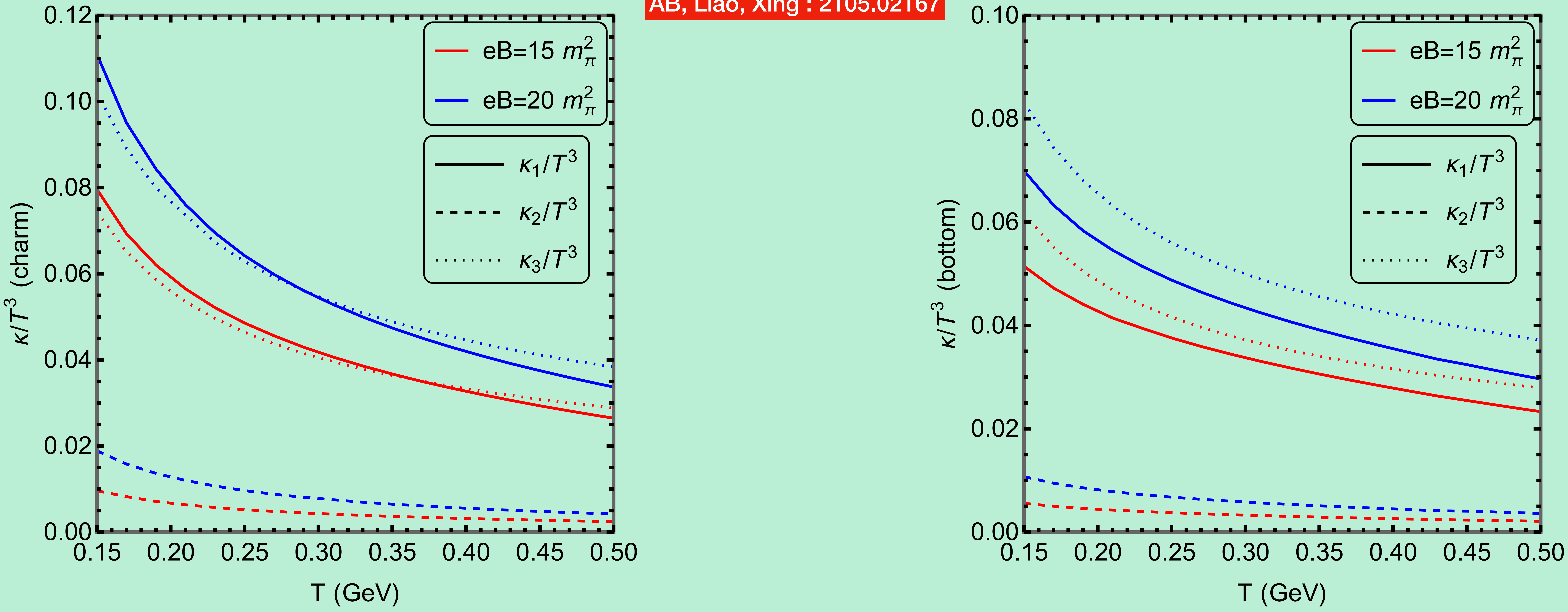


- Within strong field limit : $eB(\uparrow) \rightarrow \kappa_L/\kappa_T\ (\uparrow)$
- W.r.t the $eB = 0$ case, the values for κ_T appear to be significantly reduced by finite magnetic fields.

Results : $v \perp B$

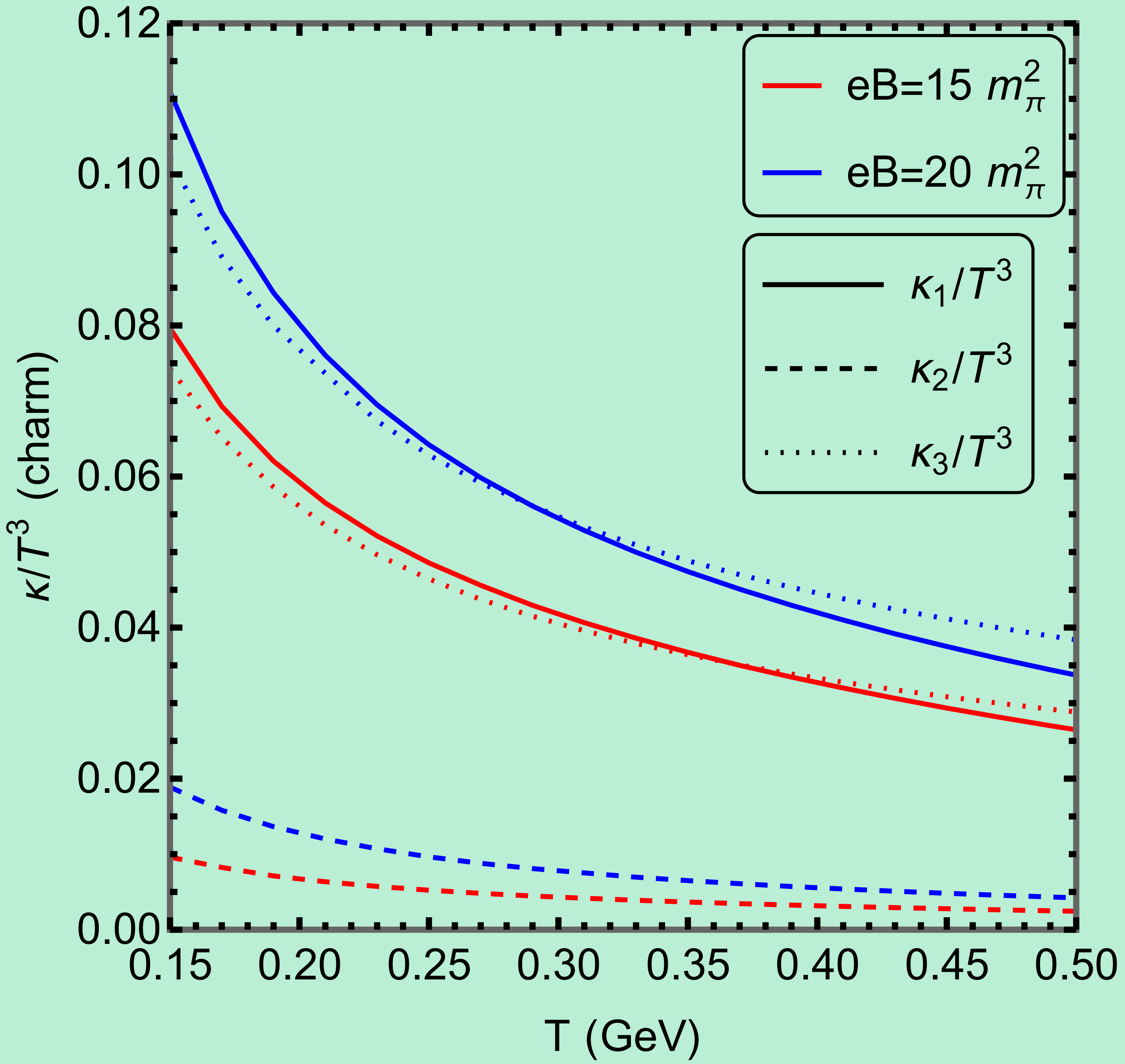
AB, Liao, Xing : 2105.02167

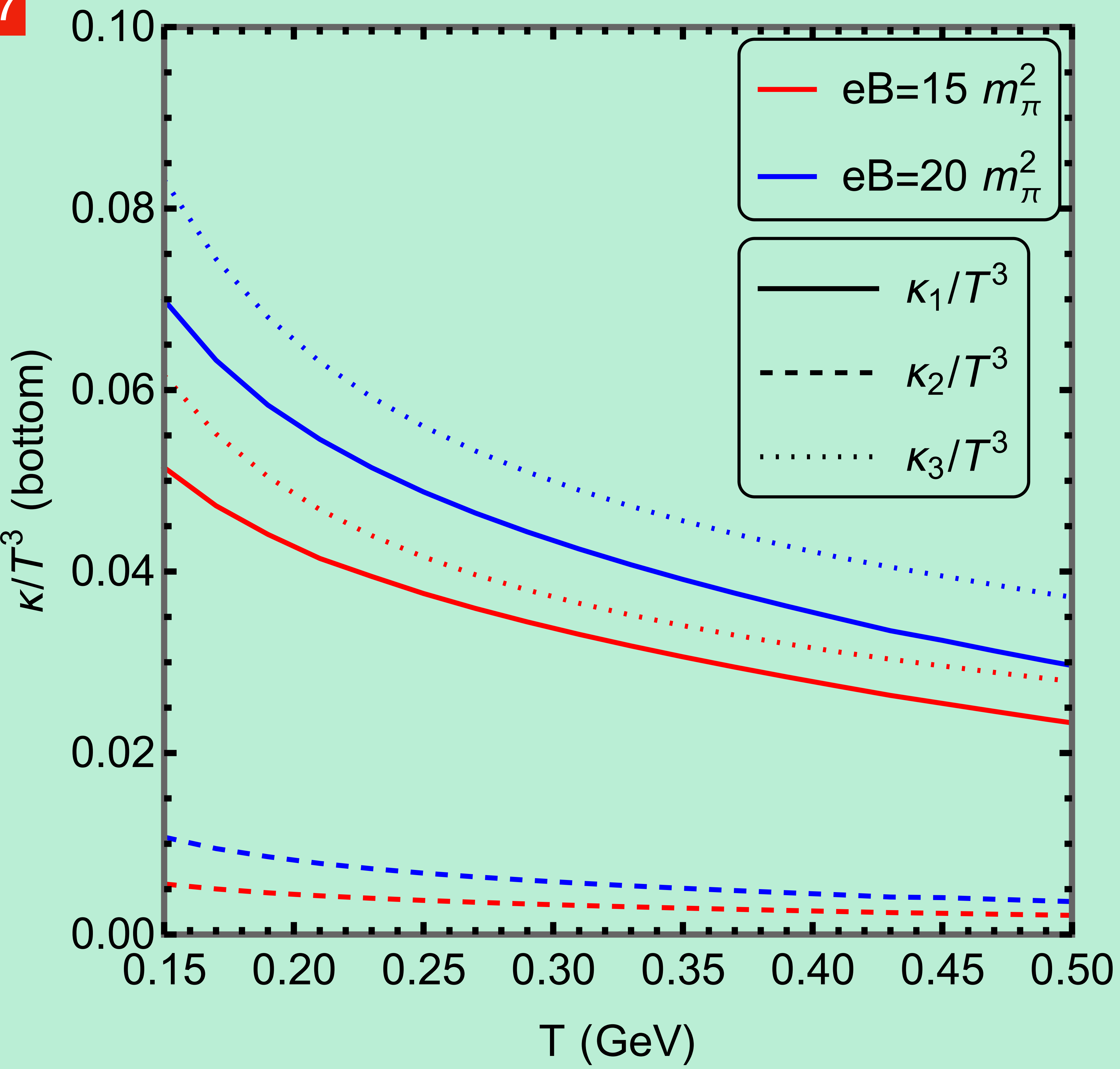


Variation of the scaled charm (left panel) and bottom (right panel) quark momentum diffusion coefficients (for $\mathbf{v} \perp \mathbf{B}$) with T for two different values of eB, i.e. $eB = 15m_\pi^2$, $20m_\pi^2$ for scaled κ_1 (solid lines), κ_2 (dashed lines) and κ_3 (dotted lines). HQ momentum is taken to be 1 GeV.

Results : $v \perp B$

AB, Liao, Xing : 2105.02167





- Within strong field limit : $eB(\uparrow) \rightarrow \kappa_1/\kappa_2/\kappa_3\ (\uparrow)$
- For bottom quarks, $\kappa_3 > \kappa_1$; For charm quarks κ_1 dominates at lower T and κ_3 at higher T (crossover).
- κ_2 is almost an order of magnitude lower than κ_1/κ_3 .

Conclusion

- To incorporate the soft gluonic momenta in our evaluation, we have worked with the recently obtained effective HTL gluon propagator in a hot and magnetized medium.

- Applying the static limit ($\mathbf{v} \rightarrow 0$) in our general result we have explicitly checked that our result matches with the previous studies within static limit.

Fukushima, Hattori, Yee, Yin
1512.03689

- Present beyond the static limit calculation can also be adapted to numerically evaluate the fully anisotropic drag/diffusion coefficients for the HQ velocity in arbitrary direction.

Conclusion

- Strong eB has considerable influence on diffusion coefficients for eB values accessible in high energy HIC.

- Compared to $eB = 0$, the values of the coefficients appear to be significantly reduced by finite eB. Such a behaviour may be related to the LLL approximation which reduces the available scattering states of the medium quarks.

- Phenomenologically, this may suggest a suppression of the heavy quark diffusion at the very early stage of the QGP evolution when the magnetic field is very strong.

- With future simulations of heavy quark transport with eB-dependent diffusion coefficients, one could hope for putting constraints on the lifetime of magnetic field in HIC.

- Outlook : Arbitrary LL, Phenomenological implications of our theoretical results.

THANK YOU FOR YOUR ATTENTION.