Muon $g-2$ HVP contribution from the lattice

Round table discussion about standard model anomalies at ConfXV

Lukas Varnhorst

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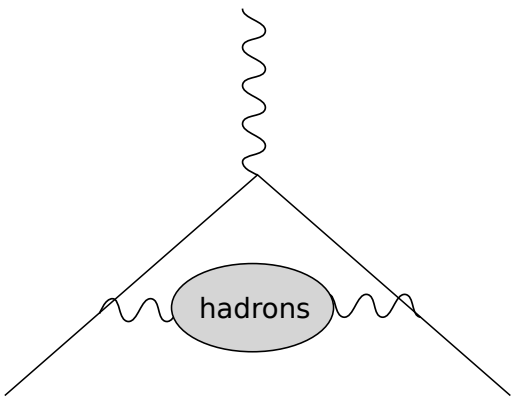
BMW collaboration

BERGISCHE UNIVERSITÄT WUPPERTAL

Muon g-2 from the theory side

Free Dirac fermions are have a gyromagnetic factor $g = 2$. Interactions change this values $\rightarrow$ Study the deviation $g - 2$ or $a = (g - 2)/2$.



For the muon, electroweak, Higgs and QCD contribution are relevant.

Uncertainty dominated by the Hadron vacuum polarization (HVP) contribution.

HVP contribution can not be determined by perturbation theory. $\rightarrow$ Data-driven approach or lattice calculation.

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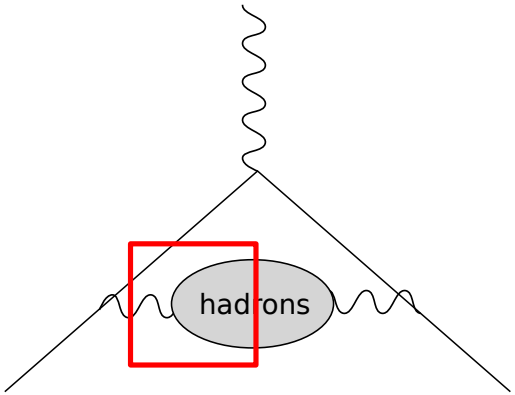

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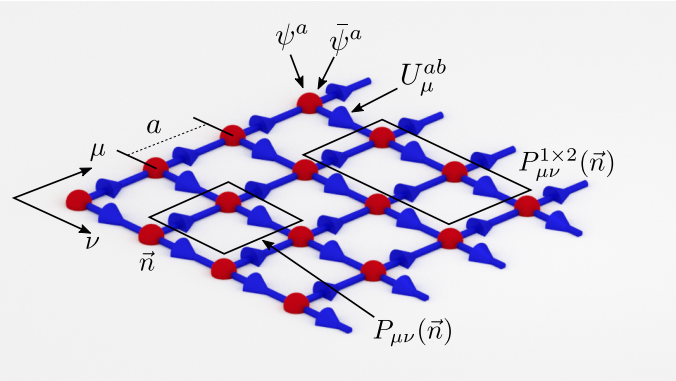
Data driven approaches rely on dispersion theory: Express the HVP contribution in terms of the hadronic R-ratio.

Lattice approach



In lattice QCD, euclidean space time is discretized and quark and gluon fields are put on the sites and links of a space-time lattice.

Use powerfull computers to solve the path integral numerically.

Leading HVP contribution can be written as

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty \mathrm{d}t K(t) G(t)$$

where $K(t)$ is a known kernel and

$$G(t) = \frac{1}{3e^2} \sum_{\vec{x},\mu=1,\ldots,3} \langle J_{\mu,t,\vec{x}} J_{\mu,0} \rangle$$

Divide HVP contribution into parts:

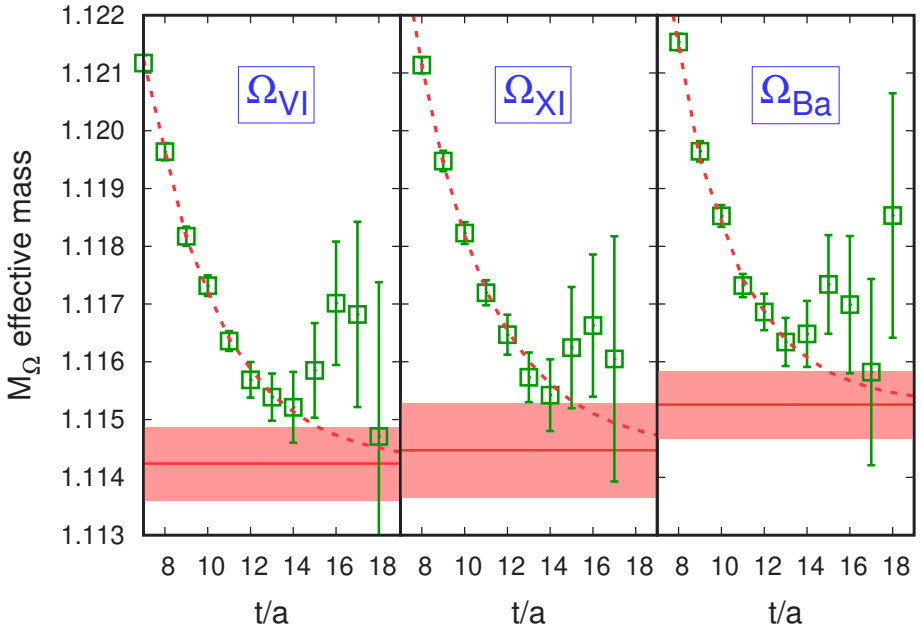
$$a_\mu = a_\mu^{\text{light}} + a_\mu^{\text{strange}} + a_\mu^{\text{charm}} + a_\mu^{\text{disc}} + a_\mu^{\text{pert}}$$

Ingredients of the calculation

Key challenge: a_μ^{HVP} has to be determined with sub percent precision.
$\rightarrow$ All systematics must be controlled to this precision

Important ingredients of the calculation:

- Setting the lattice scale



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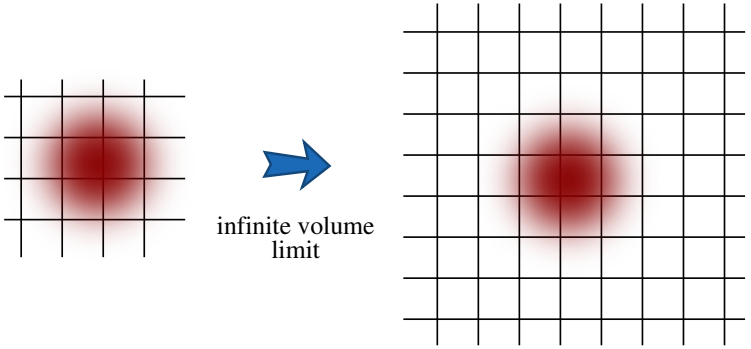

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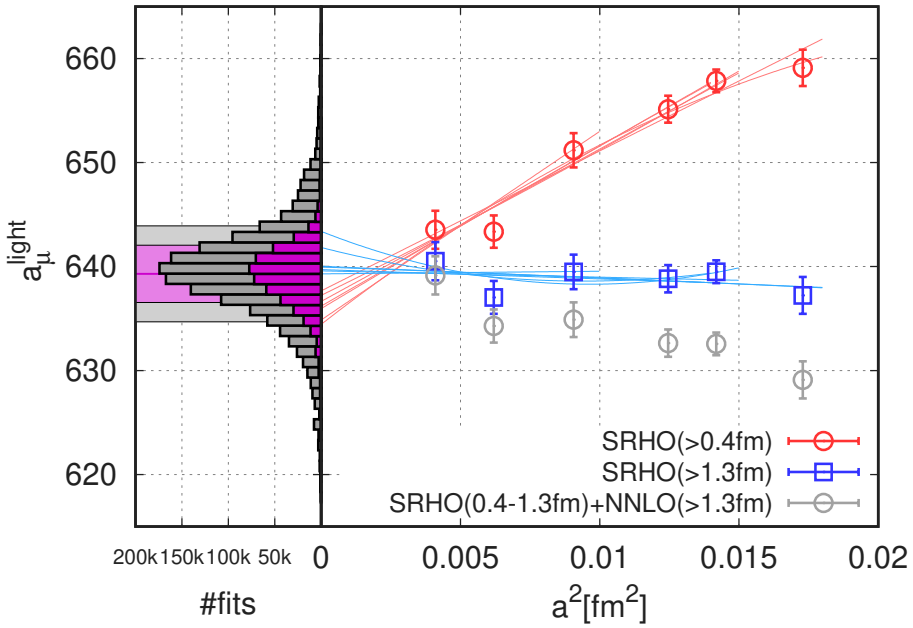

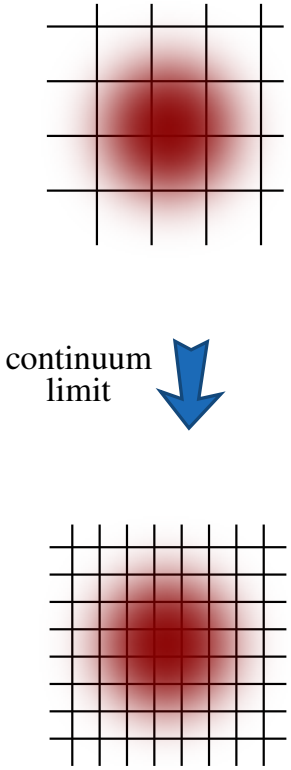

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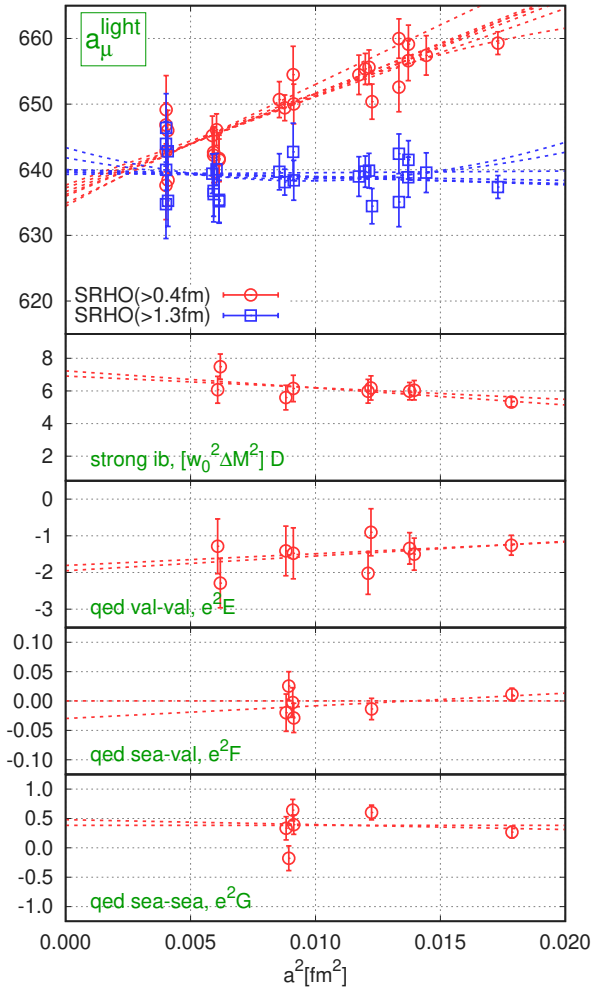
- Isospin breaking contributions



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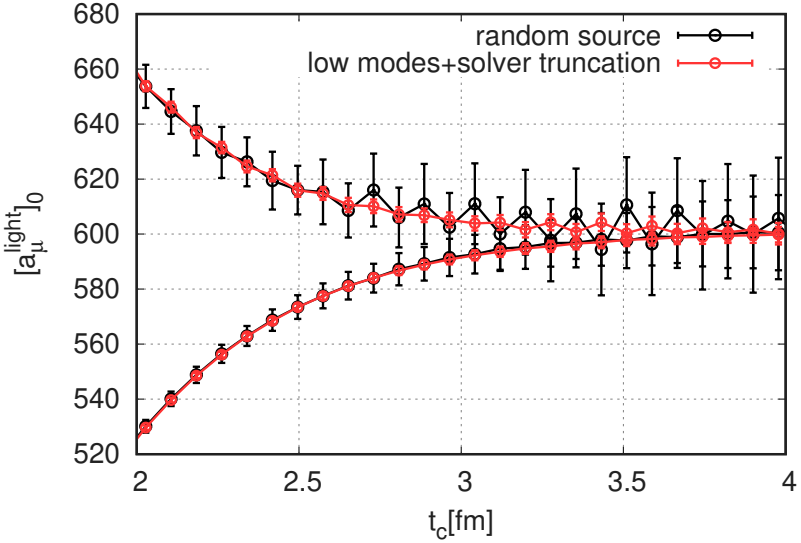
- Noise reduction and bounding method



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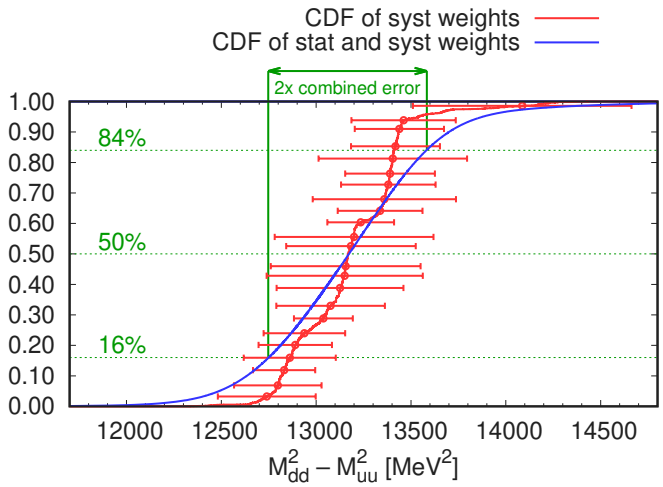
- Global fits and systematic error estimation



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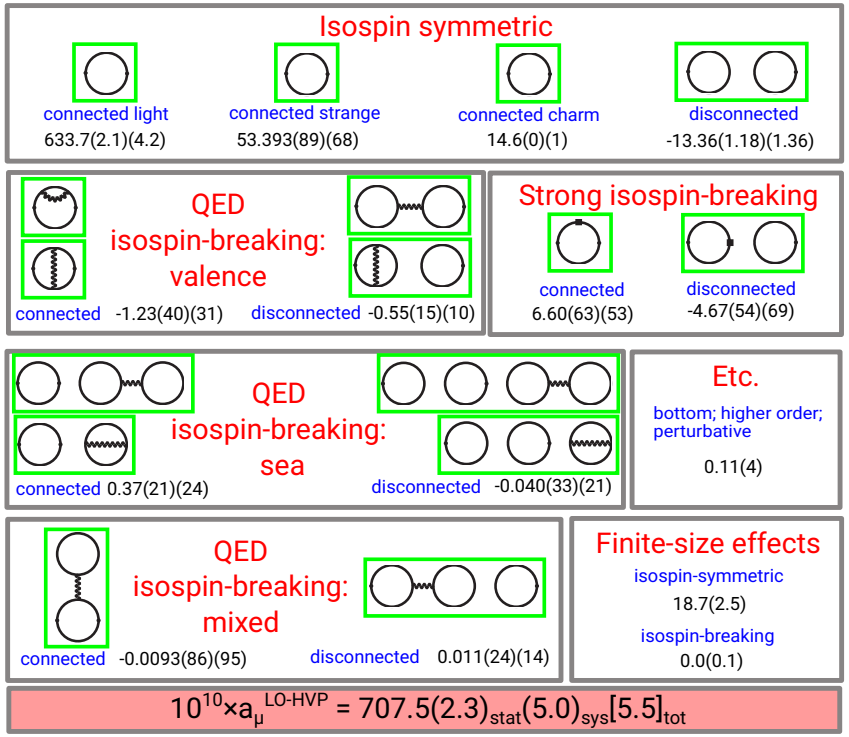
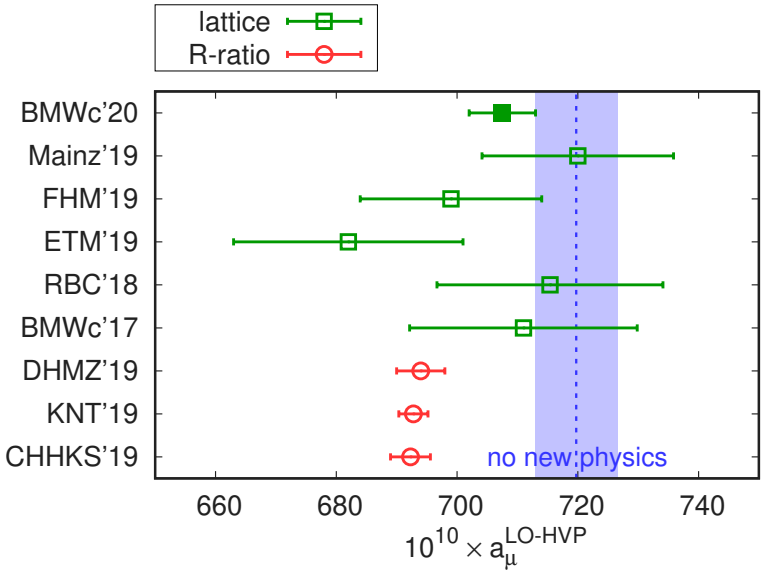
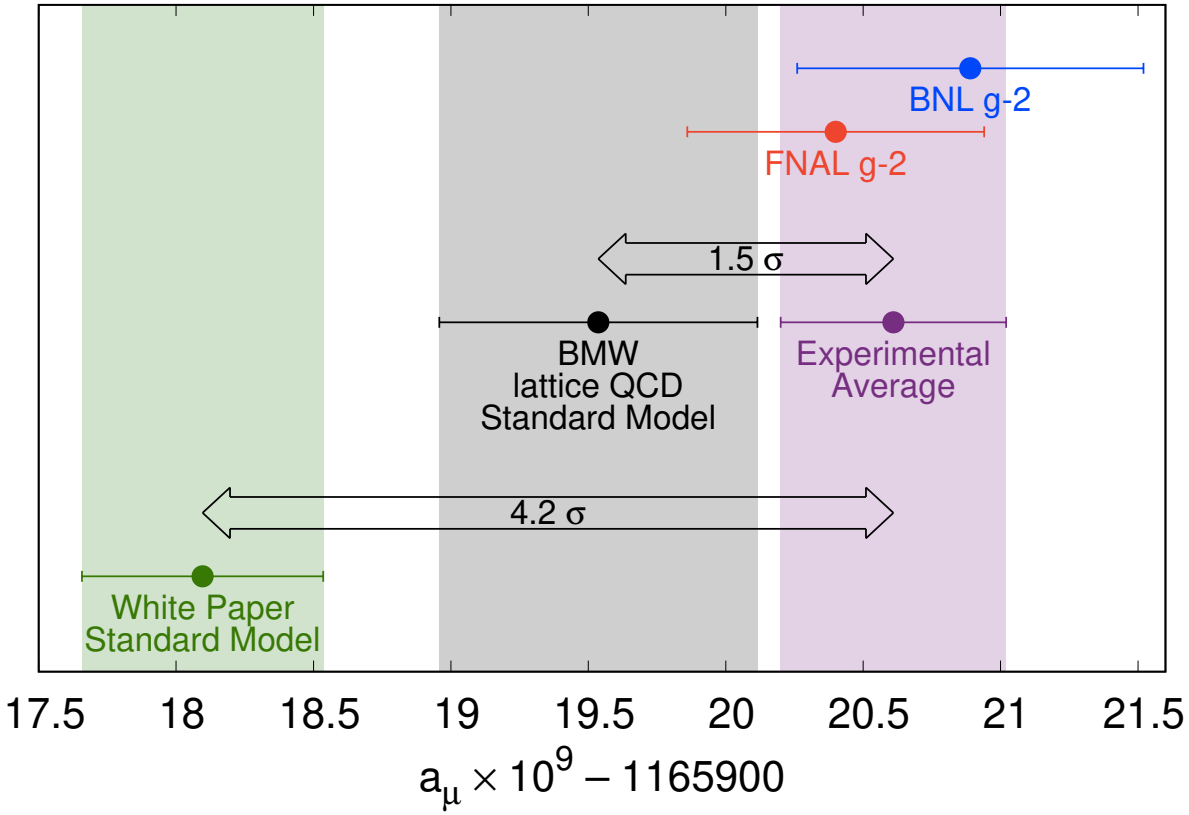
- ... many more ...

Lattice results for a_μ





Comparison of Results



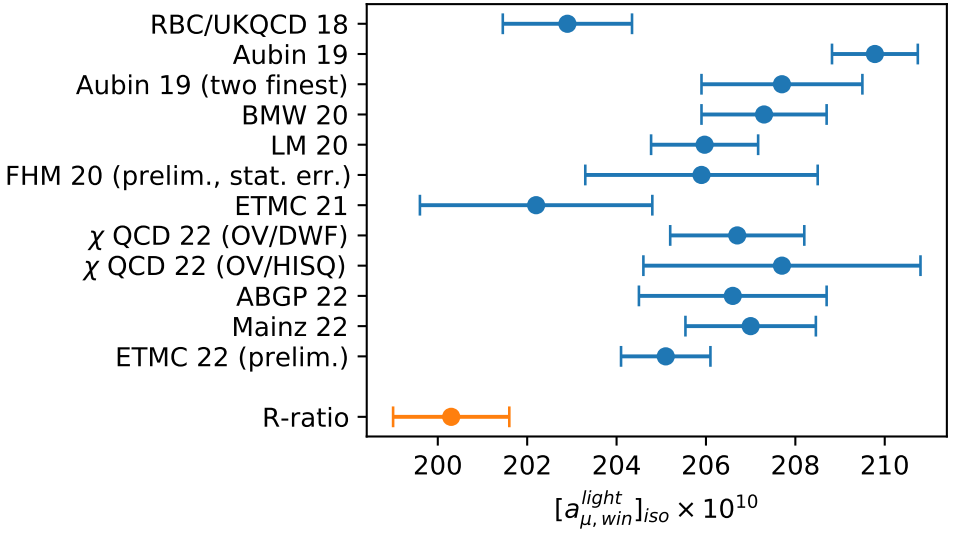
[BNL] G. W. Bennett *et al.* [Muon g-2], Phys. Rev. D **73** (2006), 072003 doi:10.1103/PhysRevD.73.072003 [arXiv:hep-ex/0602035 [hep-ex]].
[FNAL] B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801 doi:10.1103/PhysRevLett.126.141801 [arXiv:2104.03281 [hep-ex]].
[White paper] T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè and G. Colangelo, *et al.* Phys. Rept. **887** (2020), 1-166 doi:10.1016/j.physrep.2020.07.006 [arXiv:2006.04822 [hep-ph]].
[BMWc] S. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato and K. K. Szabo, *et al.* Nature **593** (2021) no.7857, 51-55 doi:10.1038/s41586-021-03418-1 [arXiv:2002.12347 [hep-lat]].

Window observable

One can define a window observable where the lattice correlation function is multiplied by a window function

$$W(t;t_1,t_2) = \Theta(t;t_1,\Delta) - \theta(t,t_2,\Delta) \quad \text{with} \quad \theta(t;t',\Delta) = \frac{1}{2} + \frac{1}{2}\tanh\left[\frac{t-t'}{\Delta}\right]$$

$$(t_1, t_2, \Delta) = (0.4, 1.0, 0.15)\,\mathrm{fm}$$



The light, isospin symmetric part of the window quantity is much easier to calculate on the lattice and is a good consistency check.

Many lattice collaborations have calulated that value.

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