Universal properties of neutron-rich nuclei near the neutron drip line

Dam Thanh Son (University of Chicago)

XVth Quark Confinment and the Hadron Spectrum
Stavanger, 4 August 2022

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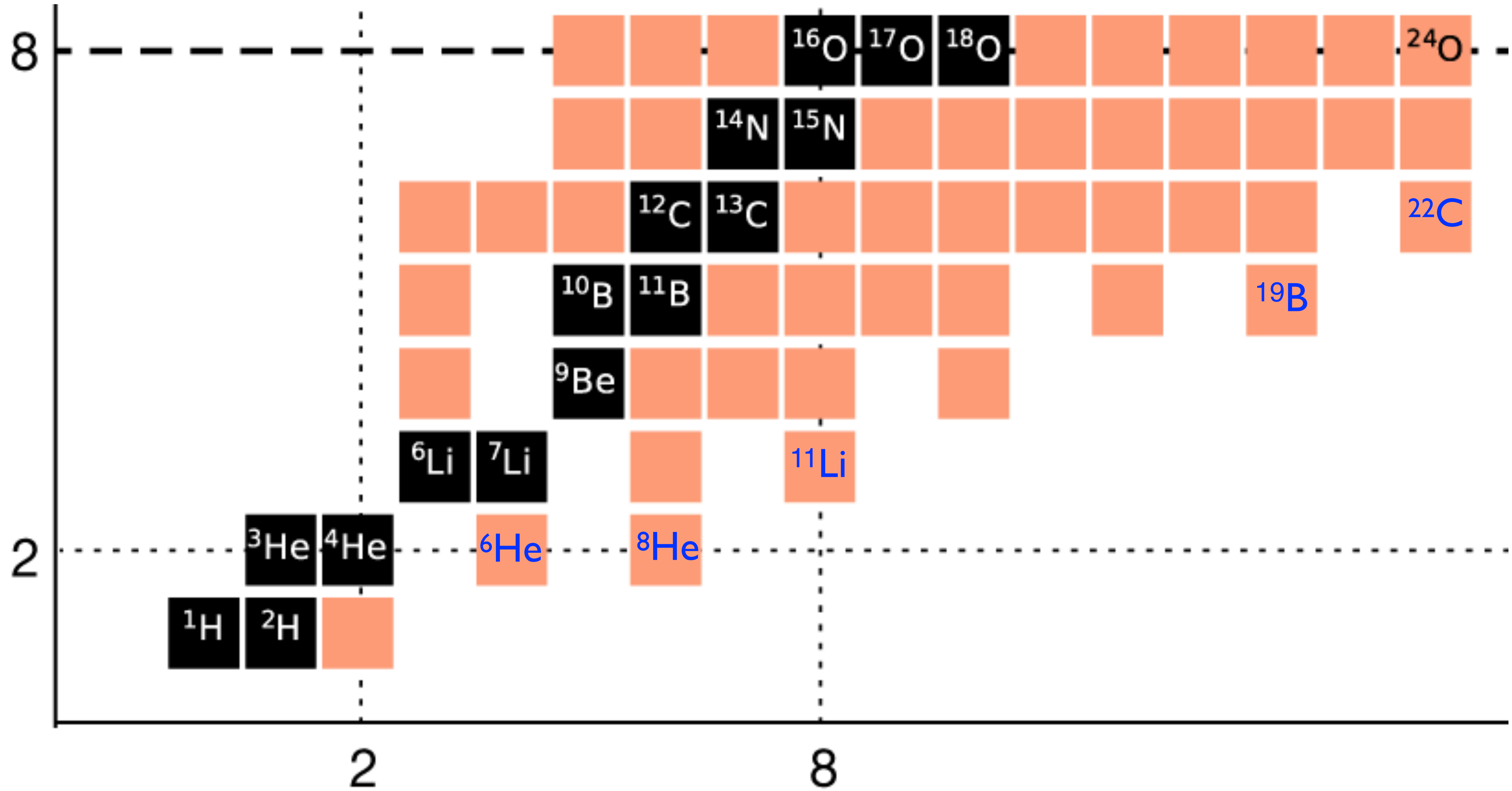
References

Masaru Hongo and DTS, PRL **128**, 212501 (2022) [arXiv:2201.09912]

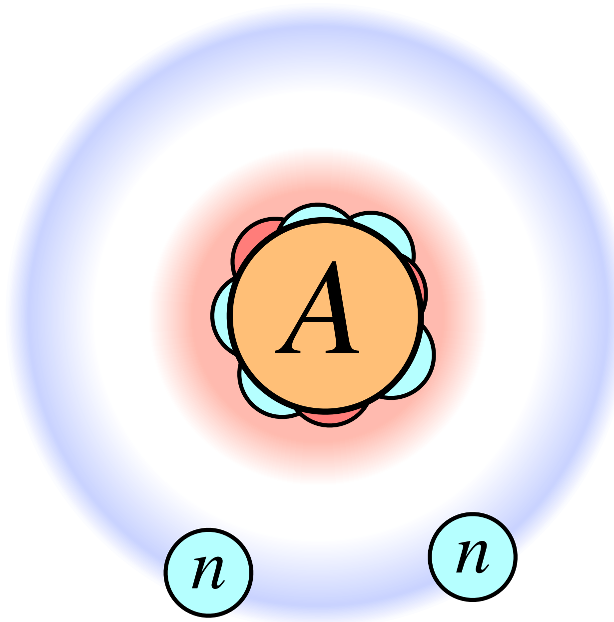
Plan

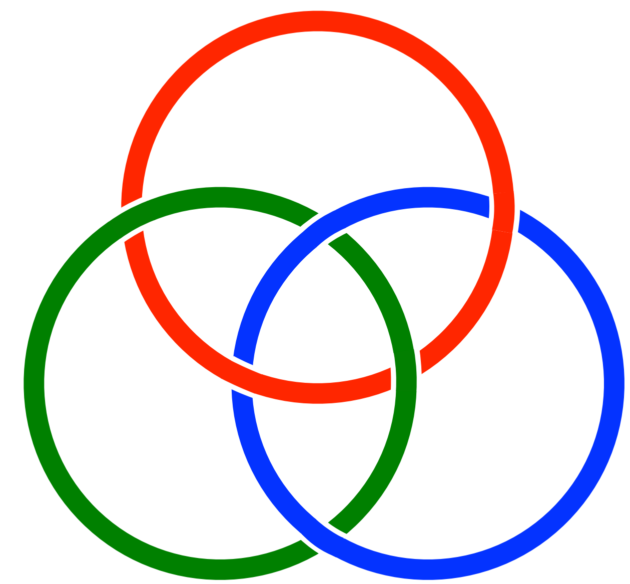
- Neutron-rich nuclei
- Two-neutron Borromean halo nuclei
- Neutrons as near-unitarity fermions: scaling dimensions of operators
- Coupling of neutron sector to the core nucleus: a renormalizable field theory



Tsunoda et al. Nature 587, 66 (2020)

Two-neutron halo nuclei





- Near the neutron drip line, sometimes one has "Borromean" nuclei
  - (Z, A) is bound (core)
  - (Z, A+1) is unbound
  - (Z, A+2) is bound
- Some examples: ^{6}He, ^{8}He, ^{11}Li, ^{22}C

Two small energies

- Interaction between neutrons fine-tuned (almost bound state):

$$a \approx -19 \text{ fm} \qquad \epsilon_n = \frac{\hbar^2}{m_n a^2} \approx 0.12 \text{ MeV}$$

- small 3-body binding energy (2n separation energy)

$$B(^6\text{He}) = 0.975 \text{ MeV}$$
$$B(^{11}\text{Li}) = 0.369 \text{ MeV}$$
$$B(^{22}\text{C}) < 0.18 \text{ MeV?}$$ Hammer Ji Phillips 2017

- Compare to the more typical energy scale

$$r_0 \approx 2.75 \text{ fm} \qquad \frac{\hbar^2}{m_n r_0^2} \approx 5.5 \text{ MeV}$$

Questions

- Is the 3-body system universal?

  Can any physical quantity can be written as

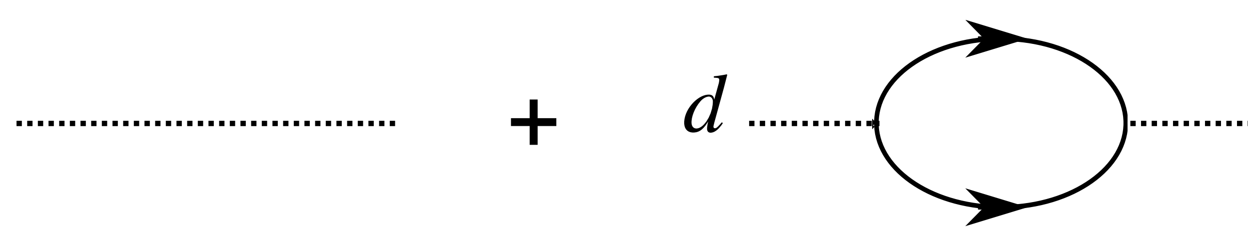
$$O = B^{\Delta_O} F_O\left(\frac{B}{\epsilon_n}\right), \quad O(\omega) = B^{\Delta_O} F_O\left(\frac{\omega}{B}, \frac{B}{\epsilon_n}\right)$$

- Answer: almost

Fine tuning in neutrons sector

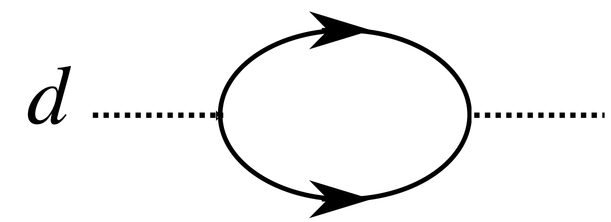
- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$
- Introducing auxiliary field d (“dimer”)
- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$
- Full dimer propagator:

$G_d^{-1}(p) =$



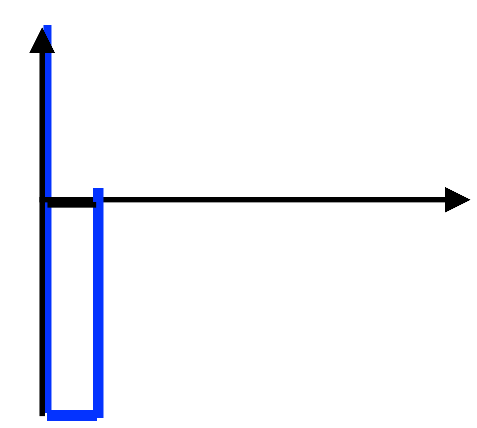
$$= \frac{1}{c_0} + \Sigma(p)$$

Renormalization



- $G_d^{-1}(\omega, \mathbf{p}) = c_0^{-1}$ + one-loop integral
- $= c_0^{-1} + \Lambda + \left(\frac{p^2}{4m} - \omega\right)^{1/2}$
- Fine-tuning: $c_0^{-1} + \Lambda = \frac{1}{a}$ (a = scattering length)
- When $1/a = 0$: bound state at threshold, "unitarity regime"

$$G_d(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$



Nonrelativistic power counting

- Set $m = 1$

$$S = \int dt\, d^3\mathbf{x}\, \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2} \right) \psi \qquad [x] = -1, \;\; [t] = -2$$

$$[\psi] = \frac{3}{2}$$

- From the propagator of d we find: $[d] = 2$
- OPE:

$$\psi(\mathbf{x})\psi(\mathbf{0}) = \frac{d(\mathbf{x})}{|\,\mathbf{x}\,|} + \cdots$$

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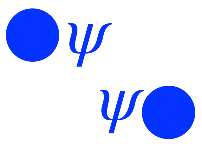

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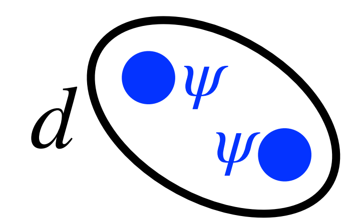

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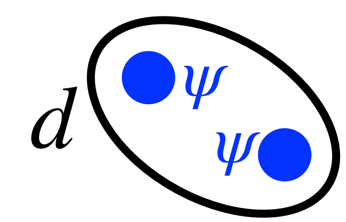

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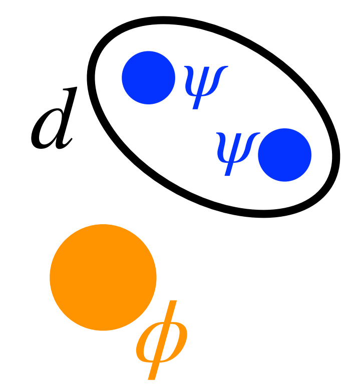

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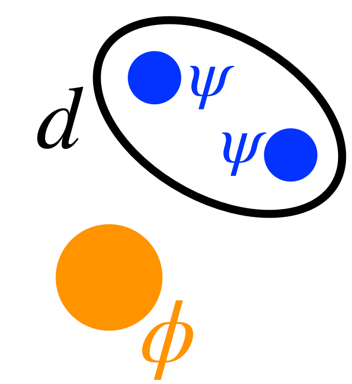

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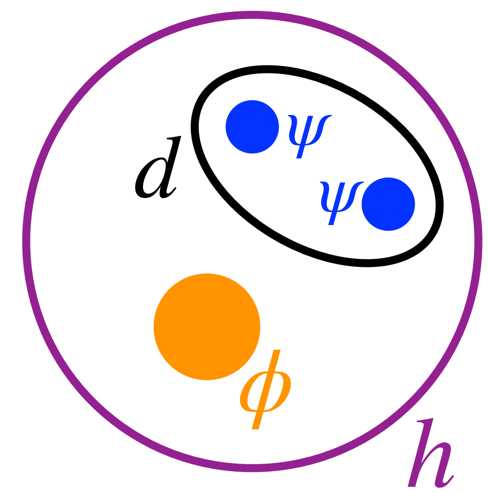

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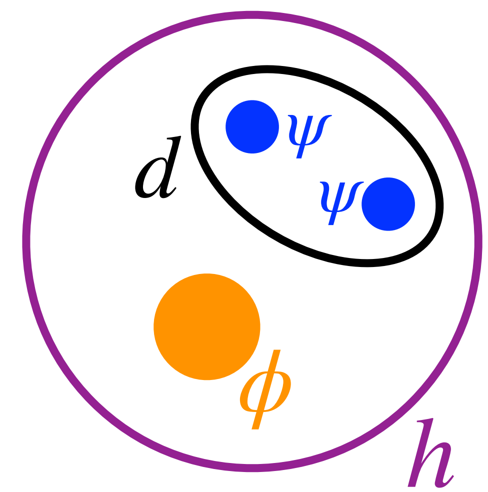

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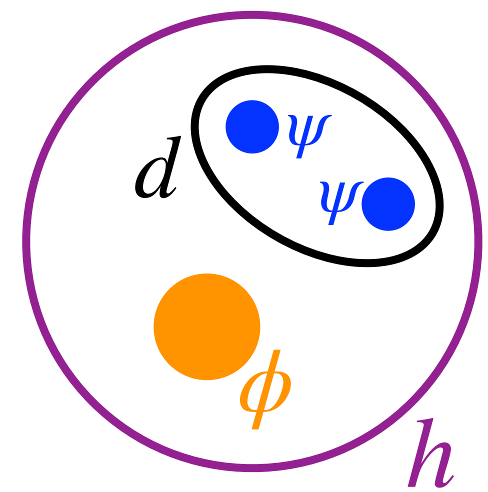

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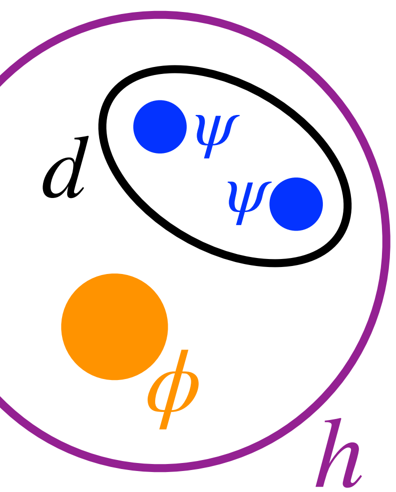
  - leading-order EFT renormalizable

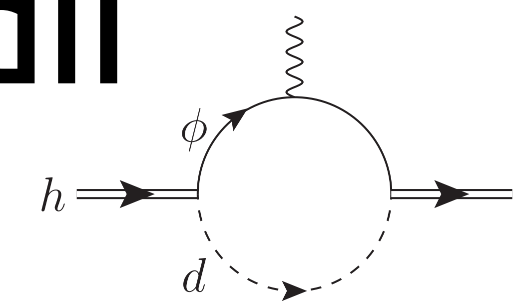


Effective Lagrangian

$$\mathscr{L} = h^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_h} + B\right)h + \phi^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_\phi}\right)\phi + g(h^\dagger\phi d + \phi^\dagger d^\dagger h)$$
$$+\psi^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$$

Logarithmic running of g ($g \to 0$ in the IR, Landau pole in UV)

Charge and matter radii



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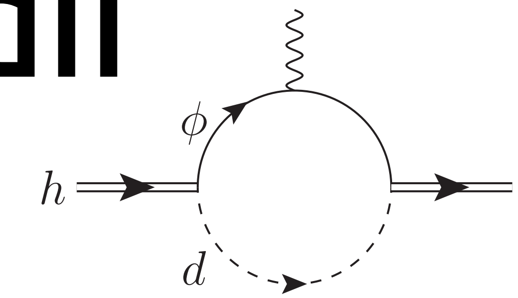

- Charge radius $\langle r_c^2 \rangle = \dfrac{4}{\pi} \dfrac{A^{1/2}}{(A+2)^{5/2}} \dfrac{g^2}{B} f_c(\beta),$

$$\beta = \sqrt{\frac{\epsilon_n}{B}}$$

$$f_c(\beta) = \frac{1}{1-\beta^2} - \frac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}$$

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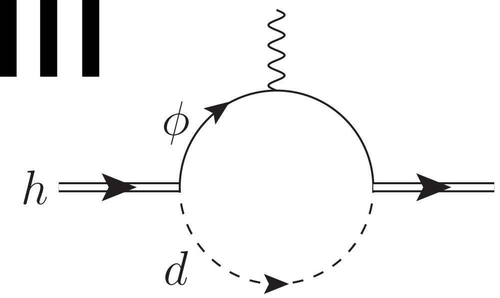

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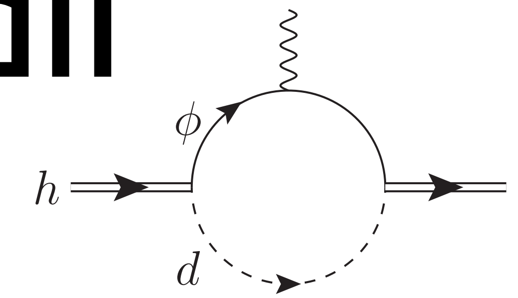
- Matter radius $\langle r_m^2 \rangle = \dfrac{2}{2\pi} \dfrac{A^{3/2}}{(A+2)^{5/2}} \dfrac{g^2}{B} [f_c(\beta) + f_n(\beta)]$

$$f_n(\beta) = \frac{1}{\beta^3} \left[\pi - 2\beta + (\beta^2 - 2) \frac{\arccos \beta}{\sqrt{1-\beta^2}} \right]$$

Charge and matter radii



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- Each are not universal (g^2), but universal ratio

$$\frac{\langle r_m^2 \rangle}{\langle r_c^2 \rangle} = \frac{A}{2} \left[1 + \frac{f_n(\beta)}{f_c(\beta)} \right] = \begin{cases} \frac{2}{3} A & B \gg \epsilon_n \\ A & B \ll \epsilon_n \end{cases}$$

E1 dipole strength function

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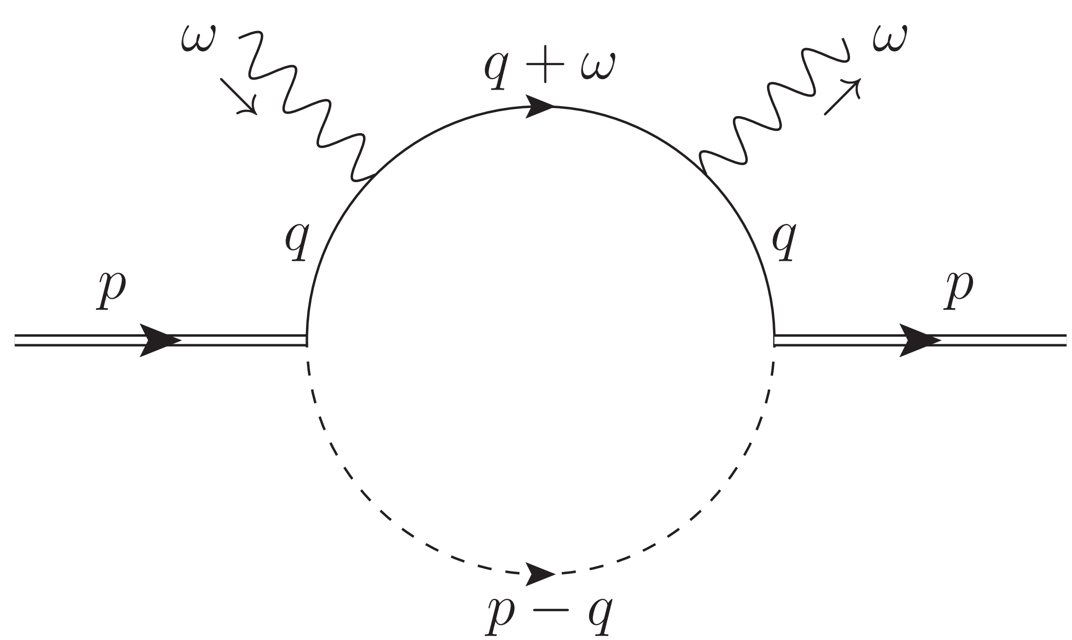


Dipole strength in unitarity limit

- When neutrons are in the unitarity limit $a = \infty$

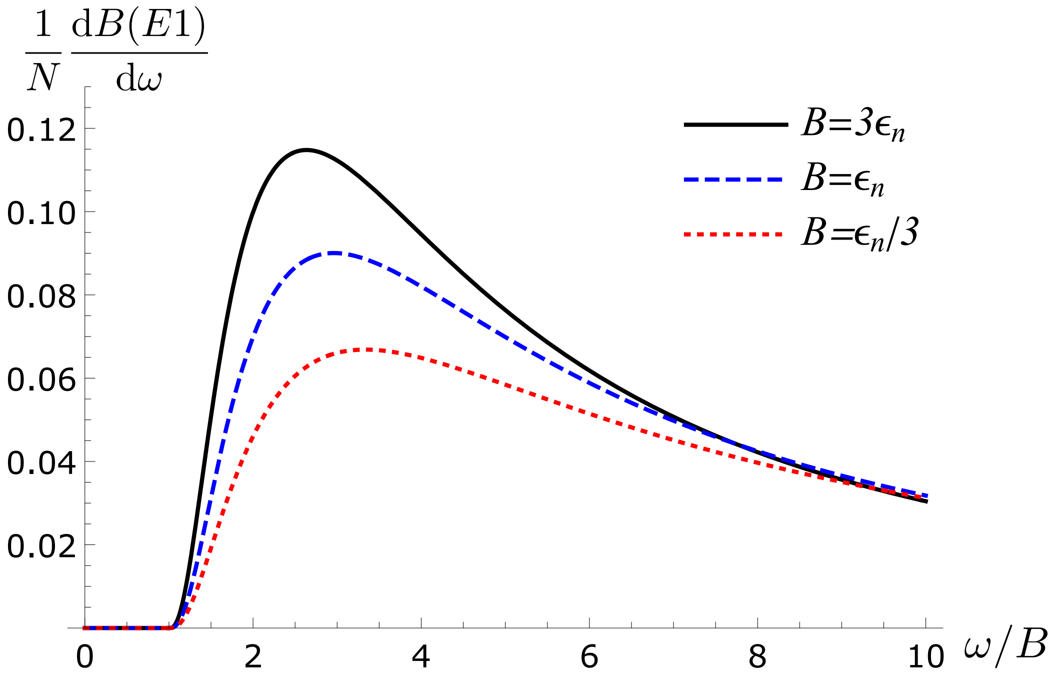
$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} \sim g^2 \frac{(\omega - B)^2}{\omega^4}$$

Result for dipole strength

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{12g^2}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{(\omega-B)^2}{\omega^4} \times f_{E1}\left(\frac{1}{-a\sqrt{\omega-B}}\right), \quad (29)$$

where

$$f_{E1}(x) = 1 - \frac{8}{3}x(1+x^2)^{3/2} + 4x^2\left(1+\frac{2}{3}x^2\right). \quad (30)$$



consistency check: sum rules

$$\int_0^\infty \mathrm{d}\omega \, \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \langle r_c^2 \rangle,$$

$$\int_0^\infty \mathrm{d}\omega \, \omega \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{3}{A(A+2)},$$

Corrections to EFT

- Corrections to EFT are irrelevant terms EFT
- Effective range in *n*-*n* scattering: $r_0 d^\dagger(\mathrm{i}\partial_t - \frac{1}{4}\nabla^2)d$
- *s*-wave core-neutron scattering $a_{cn}\phi^\dagger\psi^\dagger\psi\phi$
  - exp upper bound on *n*-^{20}C scattering length: correction < 25%
- *p*-wave core-neutron resonance (i.e., ^{5}He) can also be included

Conclusion

- Weakly bound two-neutron halo nuclei are simple enough to be described by EFT
- Logarithmic running of coupling
- Ratios of lengths and shape of E1 dipole function are universal
- Corrections: *n*-*n* effective range (relatively easy), core-neutron scattering length or *p*-wave resonance (3-loop graphs)

Extra slides

Efimov effect?

- When the core-neutron scattering length is also large: Efimov effect, Borromean bound state inevitable
- But 3-body bound state can exist without the Efimov effect

strength of core-n attraction

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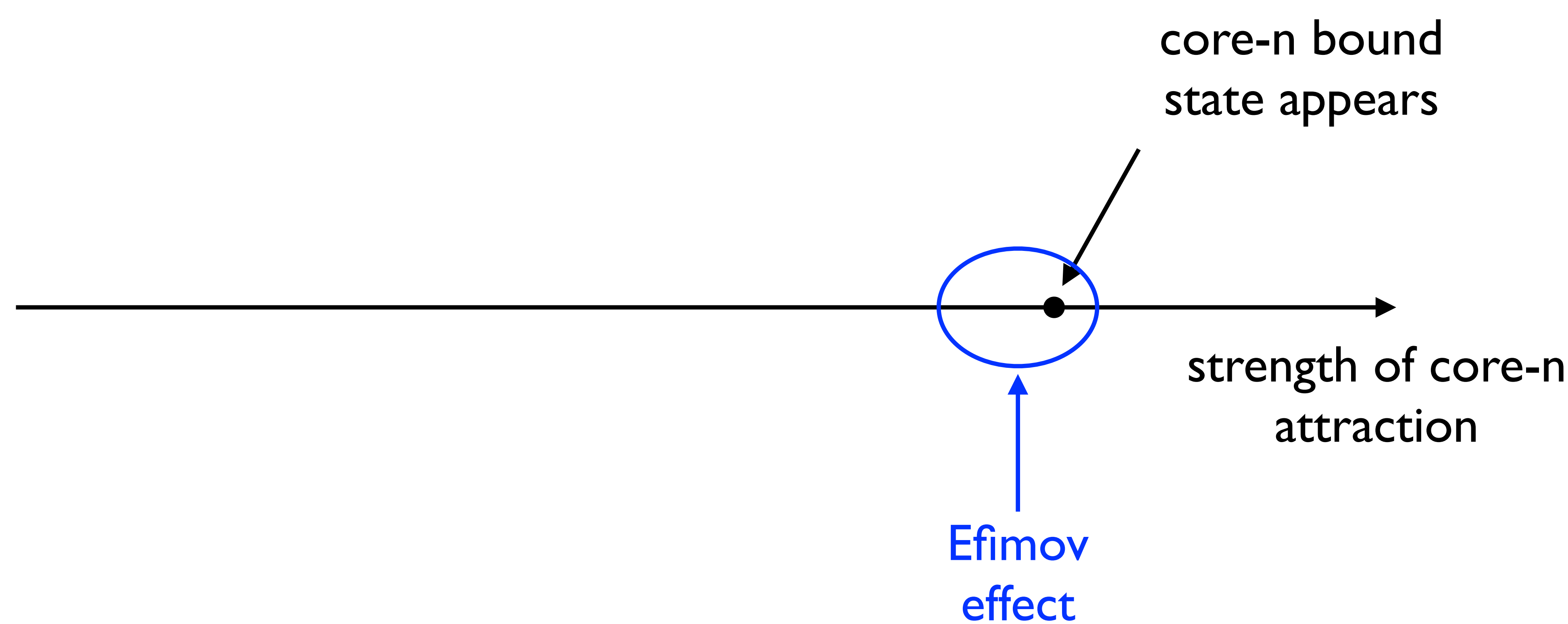

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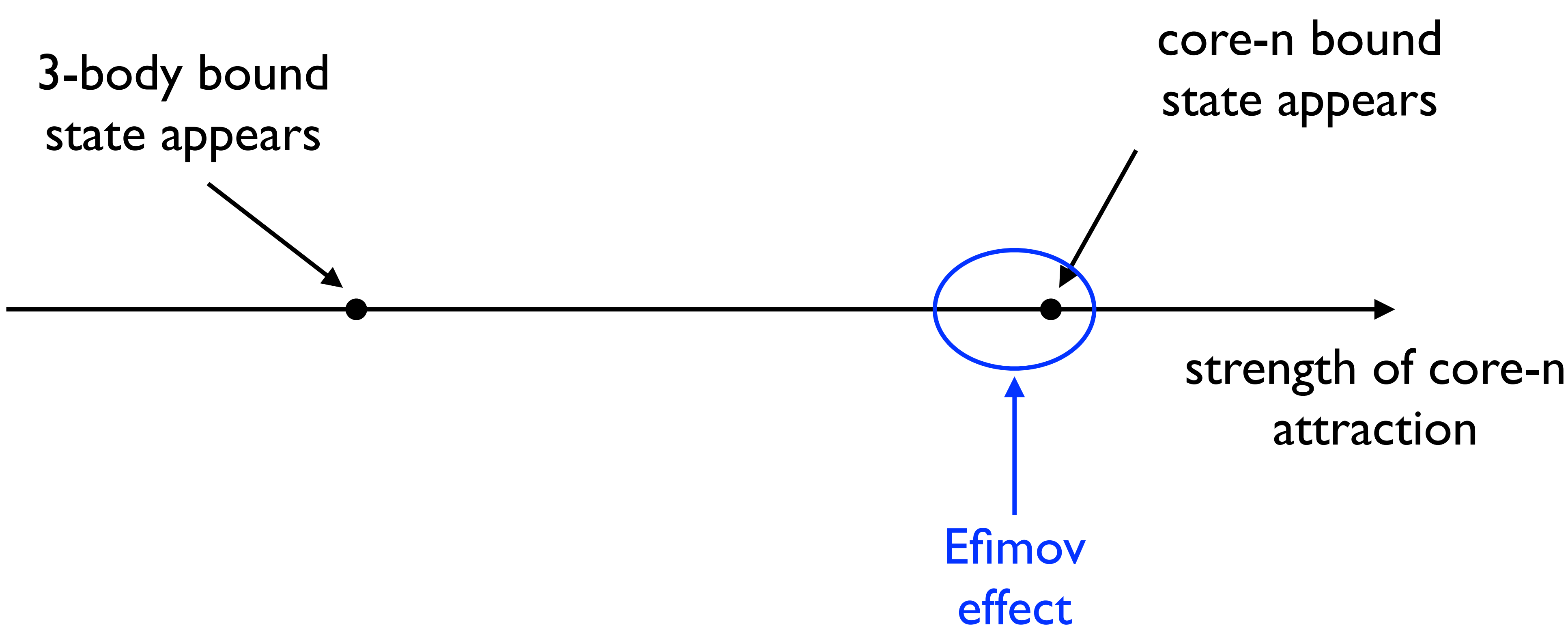

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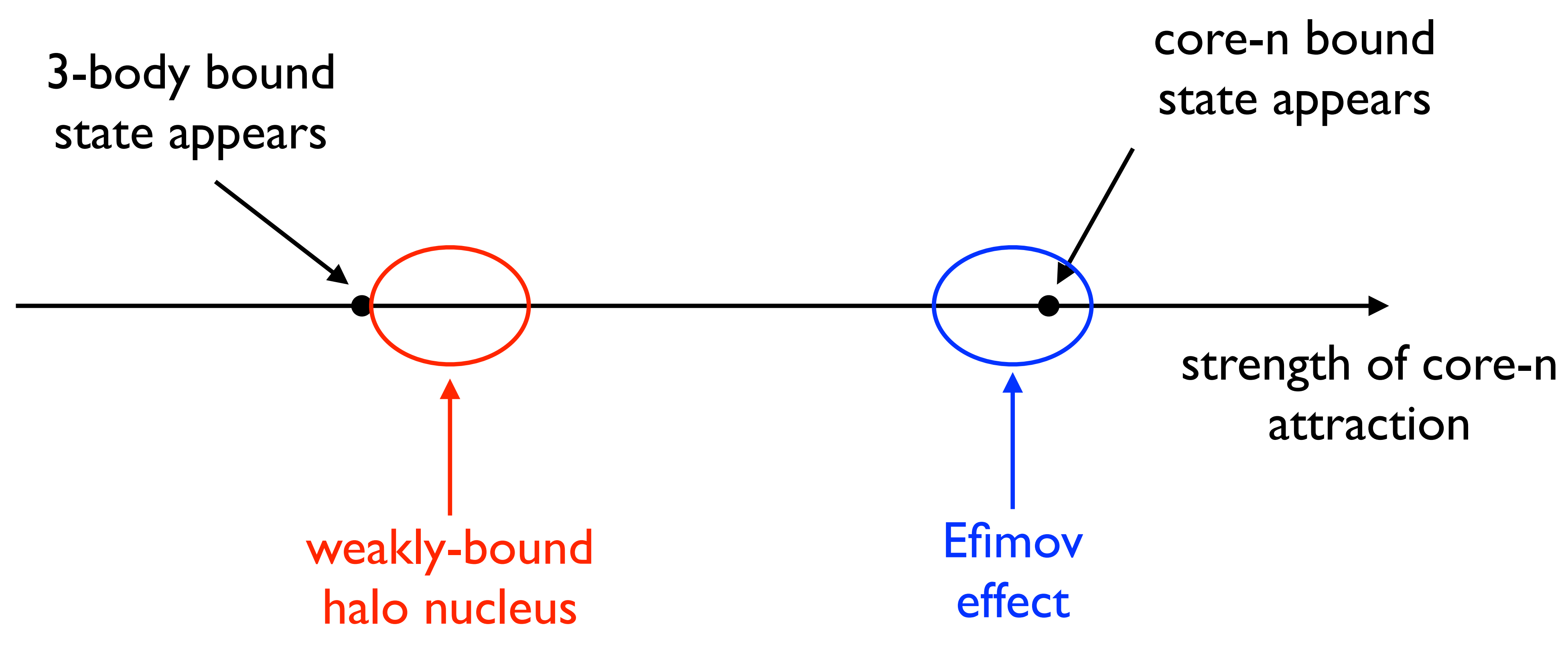

Example: Carbon-22

- $|a(n^{20}C)| < 2.8$ fm Mosby et al 2013: non-Efimovian
- large matter radius Togano et al 2016 $\rightarrow$ small binding energy
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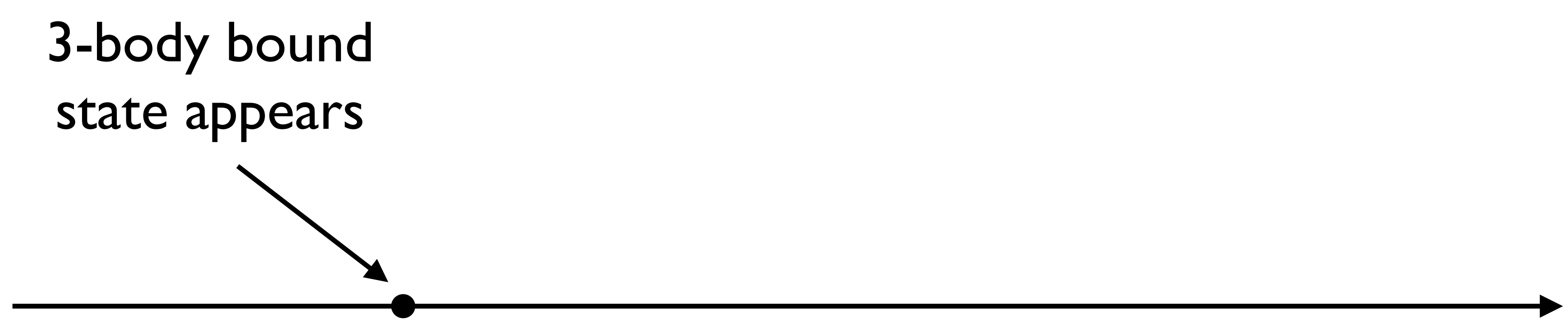

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