

QCD confronts heavy flavor and exotic hadrons

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Quark Confinement and Hadron Spectrum Conference

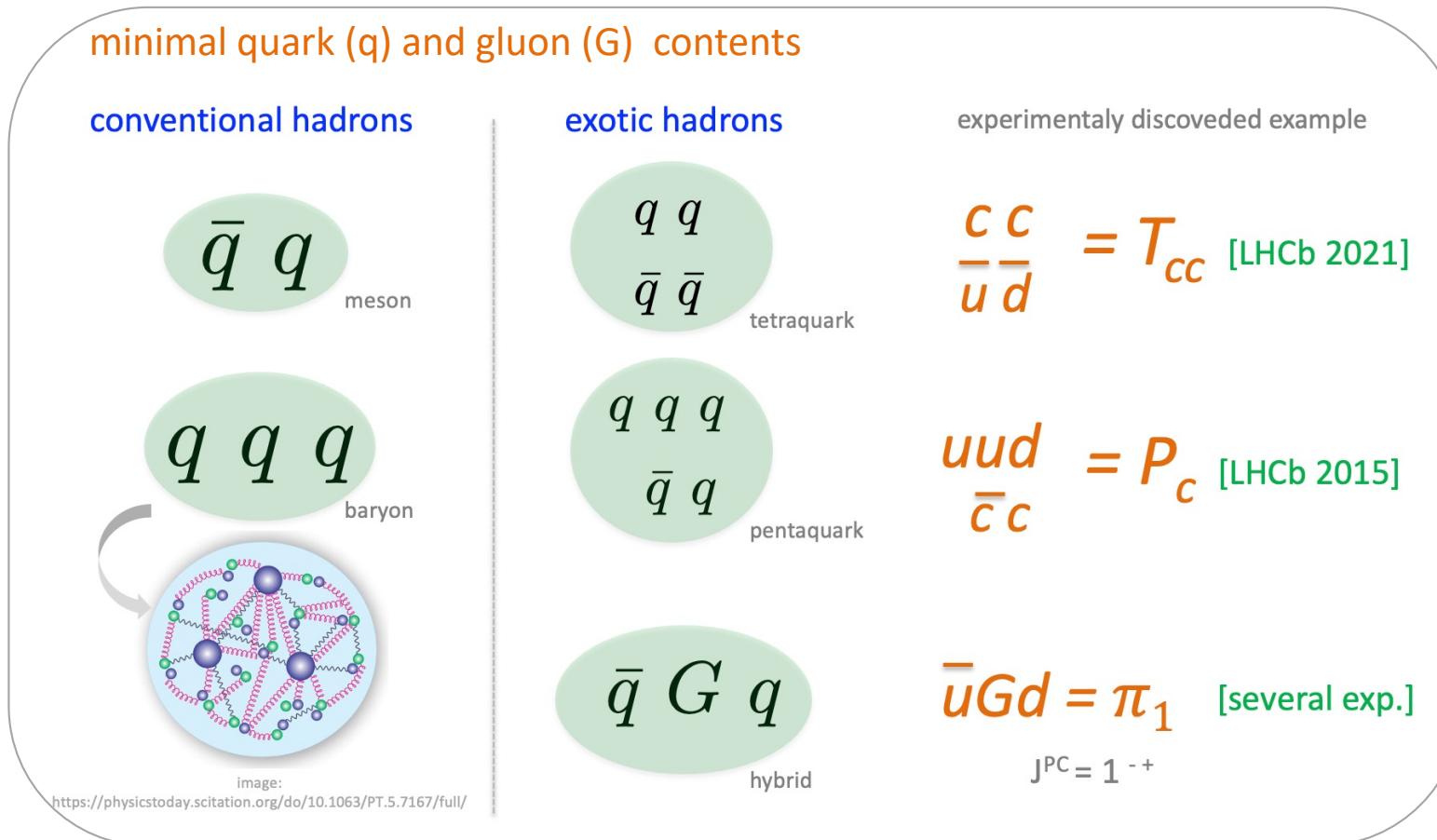
1-6 Aug, 2022, University of Stavanger, Stavanger, Norway

Hadrons

$G=\text{gluon}$, $q=\text{quark}=u,d,s,c,b$

Today we know (from exp and theory) that hadrons with the following minimal quark and gluon contents.

There may be more categories, but these are not reliably confirmed yet.



LHCb discoveries:
talk by Mikhasenko

This talk: how well QCD confronts (heavy flavor and exotic) hadrons

- intro on theory approaches with few examples
- journey to various hadron sectors
- T=0 throught

QCD: $\mathcal{L}_{QCD} = \frac{1}{4}G_a^{\mu\nu}G_a^{\mu\nu} + \bar{q}i\gamma_\mu(\partial^\mu + ig_sG_a^\mu T^a)q - m_q\bar{q}q$ $g_s \ll 1$ at hadronic energy scale

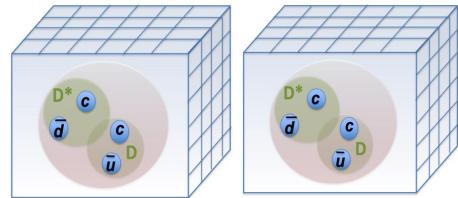
Non-perturbative theory approaches to QCD

Lattice QCD

Main quantity extracted: eigen-energy E_n

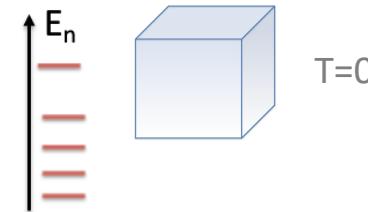
$$\sum_n |n\rangle\langle n| \xrightarrow{\text{Euclidian time}} e^{-iE_n t_M} C_{ij}(t) = \langle 0 | \mathcal{Q}_i(t) \mathcal{Q}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{Q}_i | n \rangle e^{-E_n t} \langle n | \mathcal{Q}_j^+ | 0 \rangle$$

t, J^P, I $t=0, J^P, I$



often “non-precision” studies: single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

E_n allow to extract stable hadrons and decaying resonances (as I'll explain later)



This talk: mostly
lattice, lattice + EFT

recent reviews:

N. Brambilla et al. 1907.07583, Phys. Rept.

M. Mai, U. Meissner, C. Urbach, 2206.01477

N. Brambilla, 2111.10788

Effective Field Theory (EFT)

- Lagrangian for effective d.o.f. based on symmetries of QCD
- Taylor expansion in a small quantity ($1/m_Q, p, E, \dots$)
- drawback: dependence on (unknown) parameters

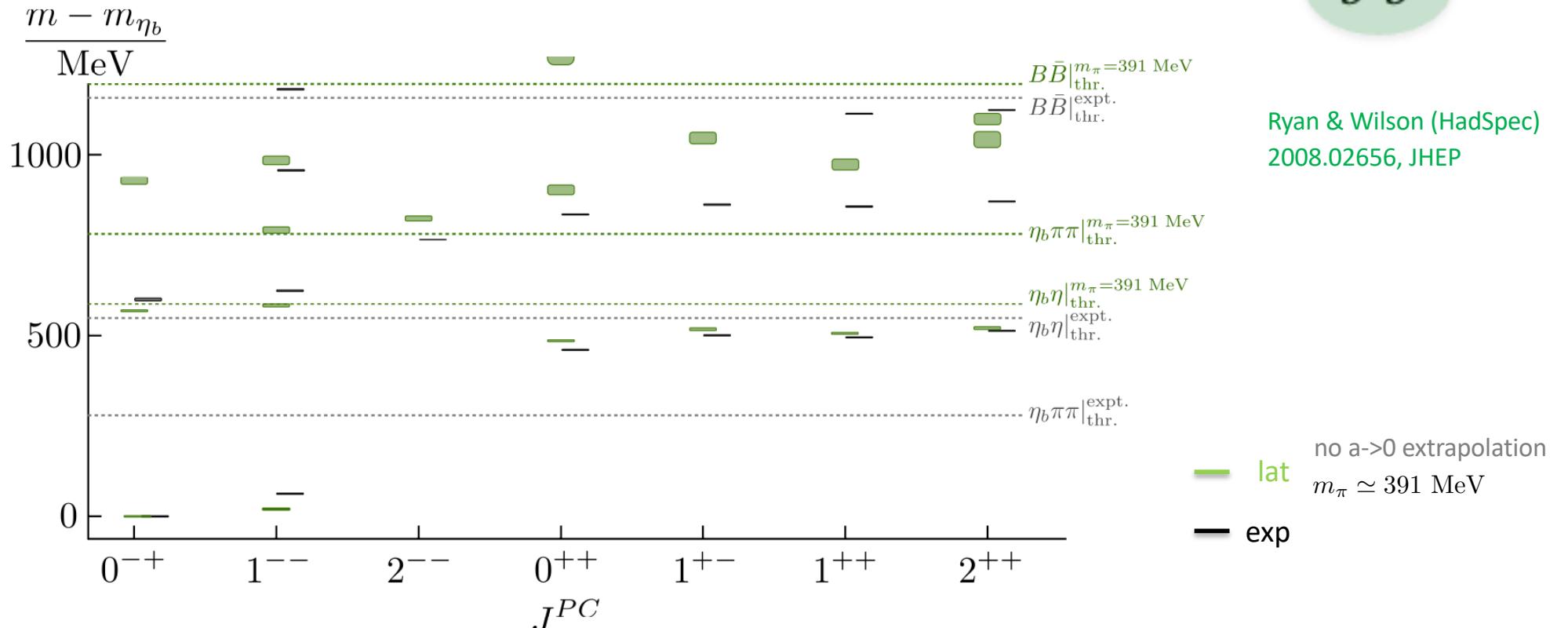
Functional methods (Dyson-Schwinger & Bethe-Salpeter equations): parallel talk by C. Fischer

...

Strongly stable hadrons (well below threshold)

$$m = E \ (\vec{p}=0)$$

Example: bottomonia : reacheest spectrum of (almost) strongly stable hadrons



lowest lying $\underline{b}\underline{b}$ and $\underline{c}\underline{c}$ with m_π^{phy} and $a>0$, QCD+QED

HPQCD, 2101.08103, JHEP, 2204.02137: also other observables

	lat	exp
$m(\Upsilon) - m(\eta_b)$ [MeV]	57.5 (2.3) (1.0)	62.3 (3.2)

systematic error from omitted $\underline{b}\underline{b}$ annihilation

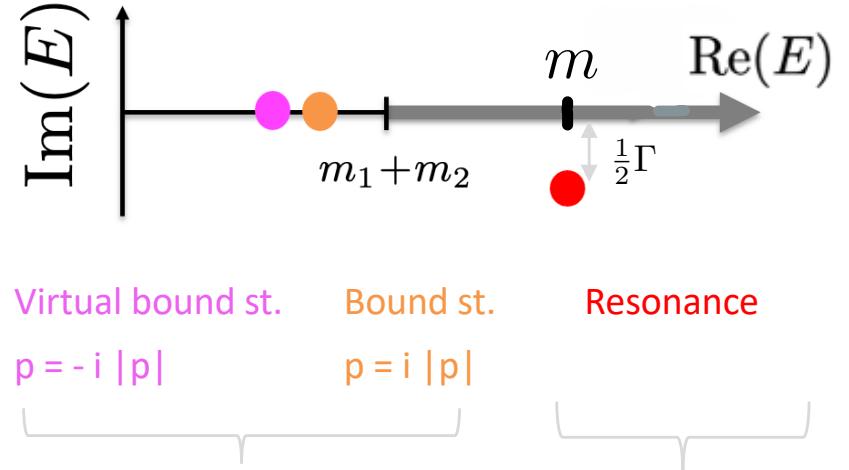
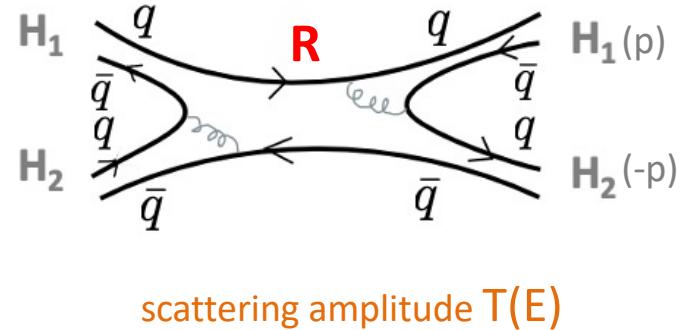
Ryan & Wilson (HadSpec)
2008.02656, JHEP

General conclusion on strongly stable hadrons:

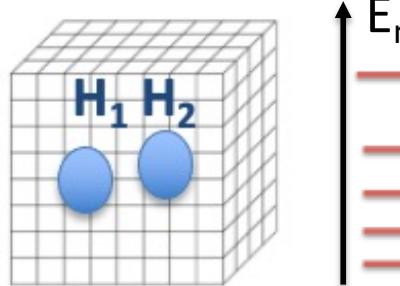
- many precision lattice studies
- QCD confronts exp data well !

Resonances $R \rightarrow H_1 H_2$, bound states near threshold

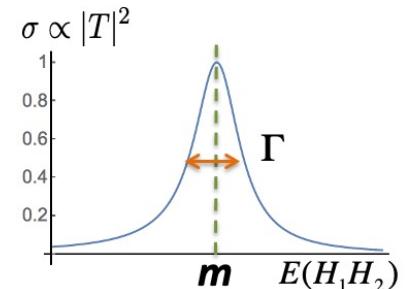
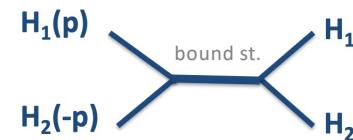
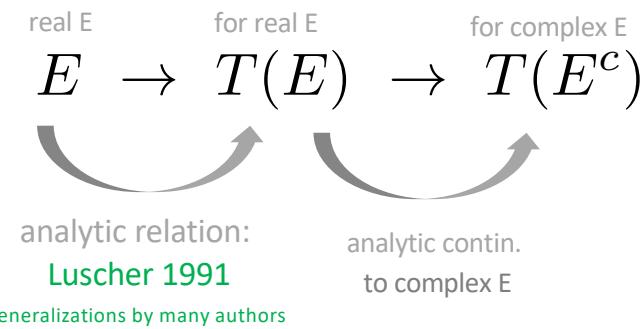
see also: plenary by Pelaez



Scattering amplitude $T(E)$ from lattice QCD

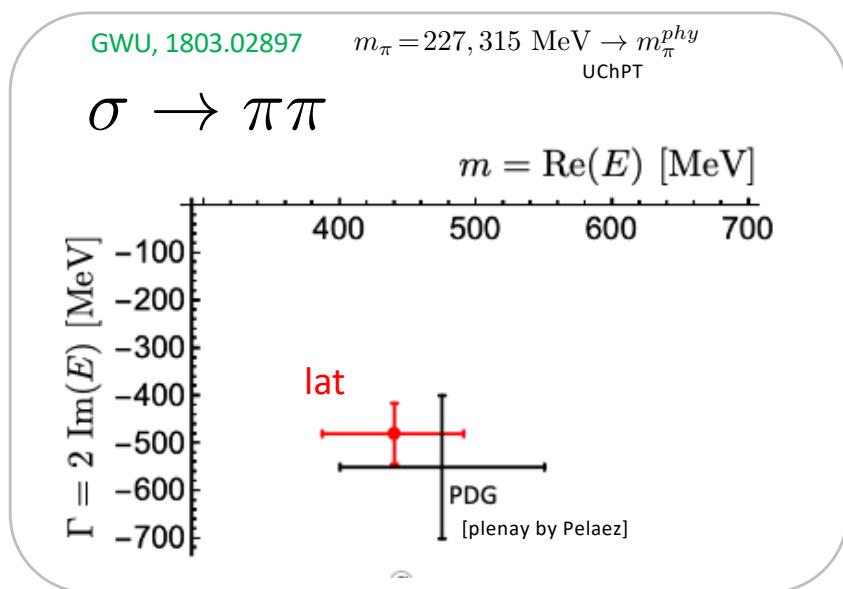


many resonances and bound states extracted in this way by now
(apologies for not covering all)



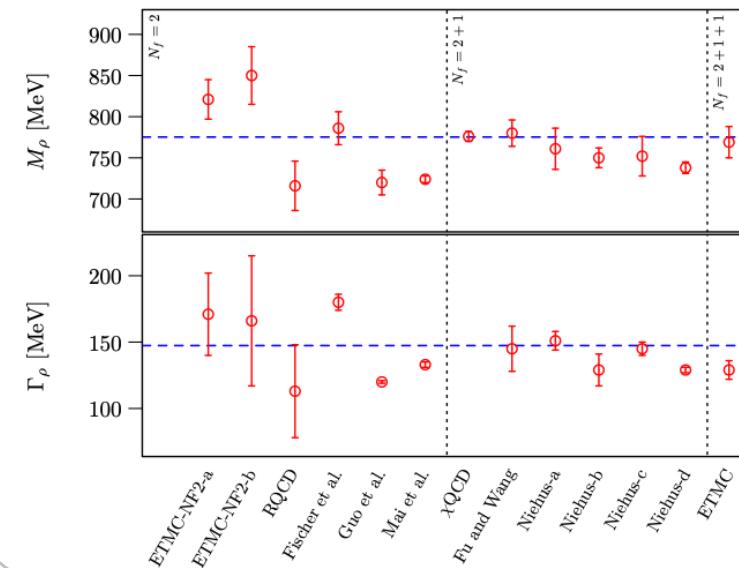
Resonances $R \rightarrow H_1 H_2$

examples

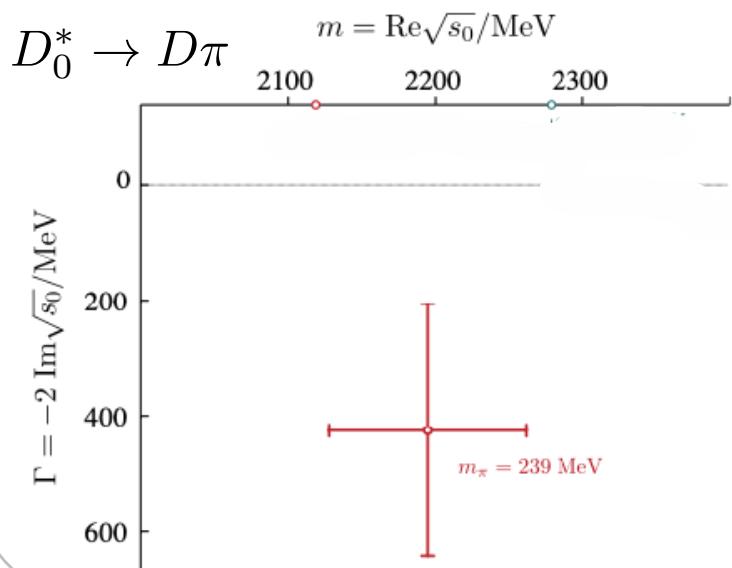


$$\rho \rightarrow \pi\pi$$

from review
Mai, Meissner, Urbach 2206.01477



HadSpec, 2102.04973 JHEP



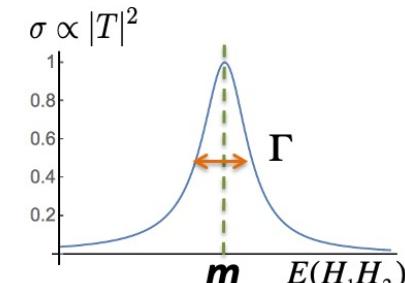
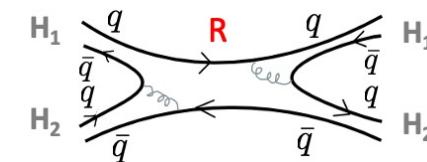
$$J^P = 0^+$$

PDG: puzzle

$-m[D_0^*] \simeq m[D_{s0}^*(2317)]$

lat: OK

$m[D_0^*] < m[D_{s0}^*(2317)]$



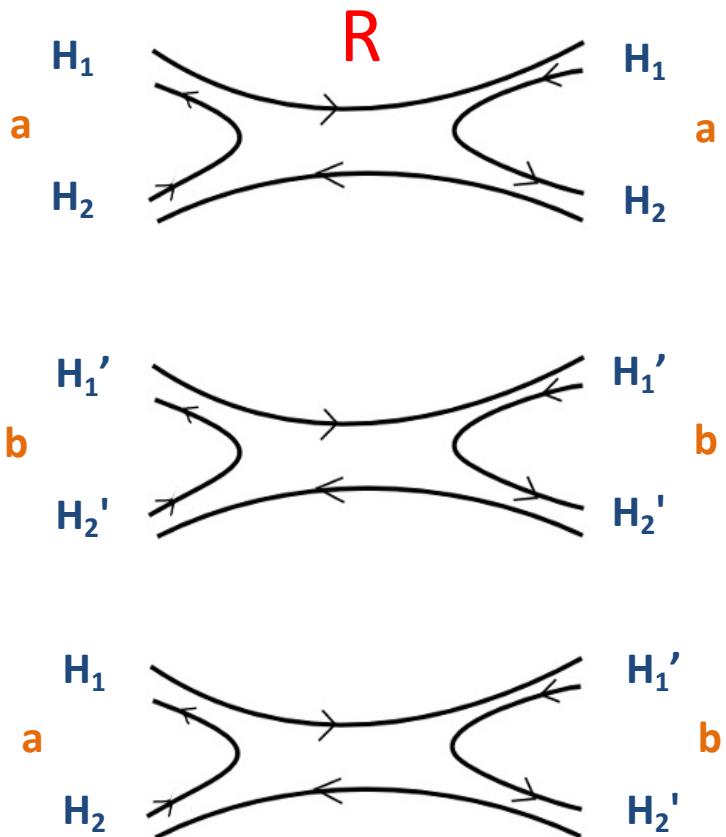
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Hadron spectroscopy from lattice

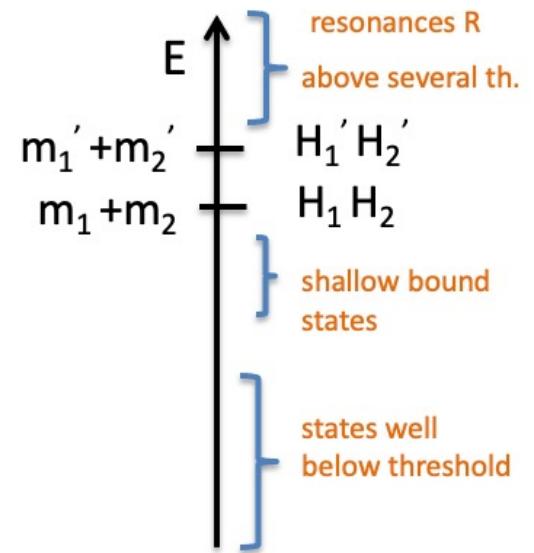
see also: plenary by Pelaez

Resonances from coupled-channel scattering

$$R \rightarrow H_1 H_2, H'_1 H'_2, \dots$$

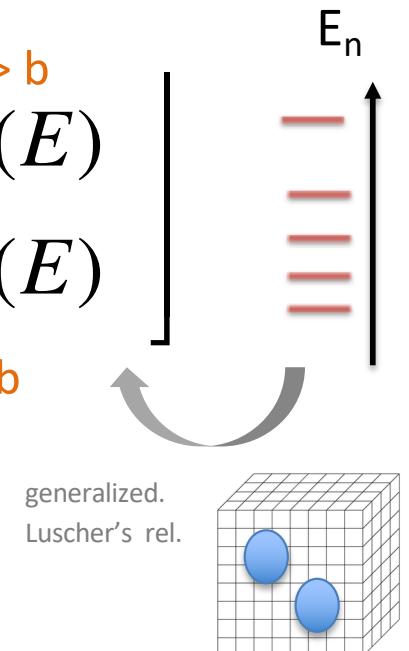


channel a : $H_1 H_2$
 channel b : $H'_1 H'_2$



$$T(E) = \begin{bmatrix} a \rightarrow a & a \rightarrow b \\ T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \\ b \rightarrow a & b \rightarrow b \end{bmatrix}$$

- lattice QCD studies extracted $T(E)$ for several resonances

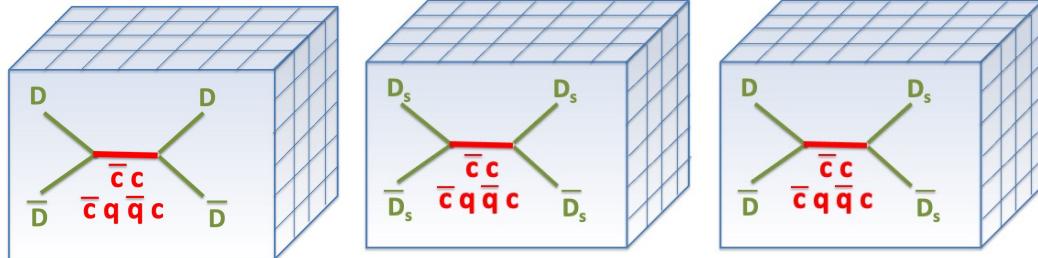


Resonances from coupled-channel scattering

- most results by HadSpec. coll.: mostly light meson sector

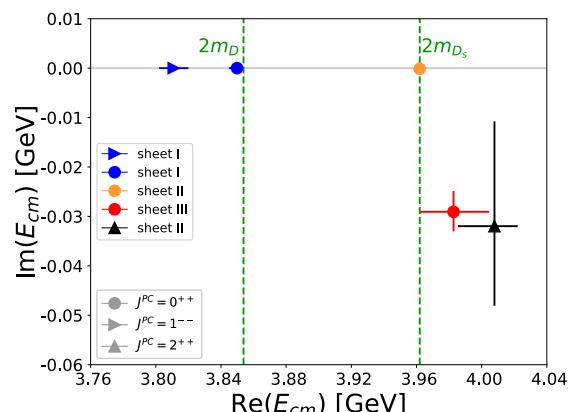
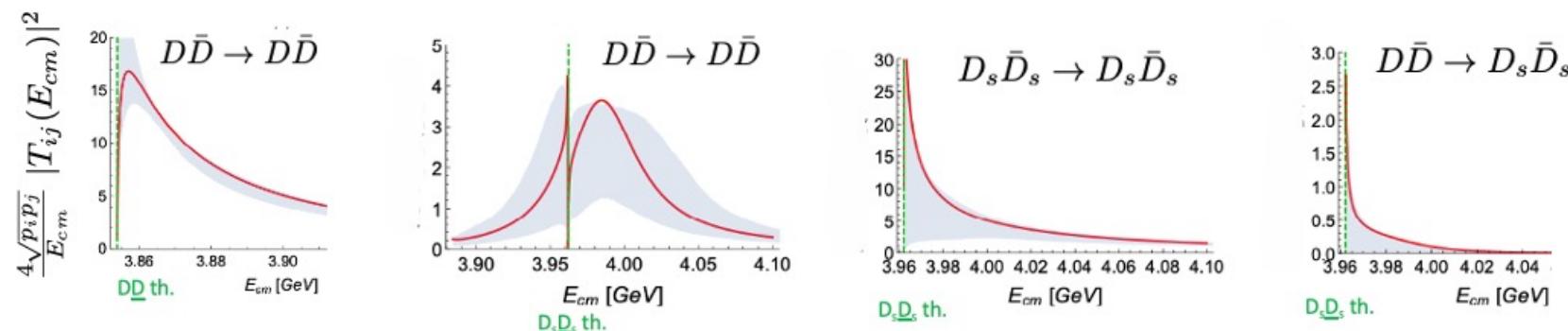
- example in heavy-quark sector

$\bar{c}c$, $\bar{c}q\bar{q}c$ $q=u,d,s$ $I=0$



two coupled channels

$$D\bar{D} - D_s\bar{D}_s$$



S.P. , Collins, Padmanath, Mohler, Piemonte

2011.02542 JHEP

physics interpretation: two slides later

journey to various hadron sectors

(with heavy quarks)

exciting experimental discoveries by:

LHCb, Belle, BesIII, Babar, ...

most of discovered exotic hadrons contain heavy quarks:

(these are more likely to be quasi-bound due to small kinetic energy of Q)

$Q=c,b$ $q=u,d,s$

$\bar{c}c$, $\bar{c}c \bar{q}q$
 $\bar{b}b$, $\bar{b}b \bar{q}q$

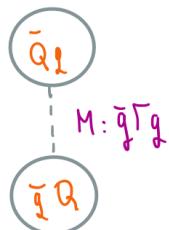
$QQ \bar{q}\bar{q}$

$\bar{Q}q$, $\bar{Q}q \bar{q}q$

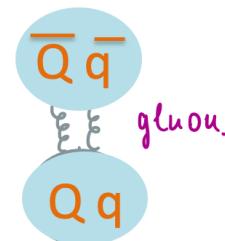
dibaryons

possible
binding
mechanisms

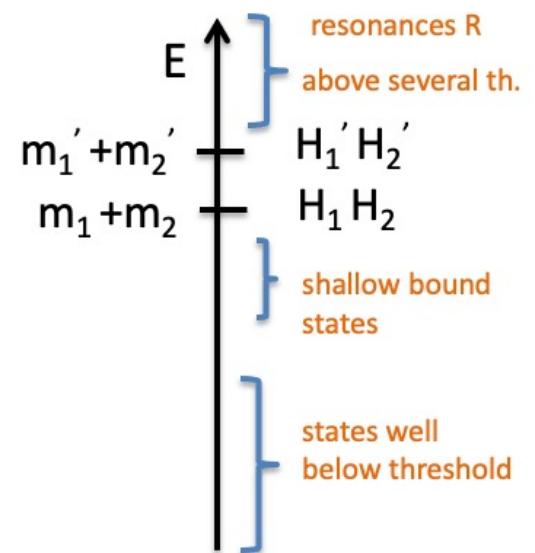
"molecule"



analogous



...



How challenging is a given state for ab-initio study? General rule:

more strong decay channels -> more challenging

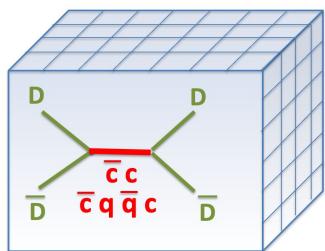
charmonium(like) sector

$\bar{c}c$, $\bar{c}q\bar{q}c$

Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$

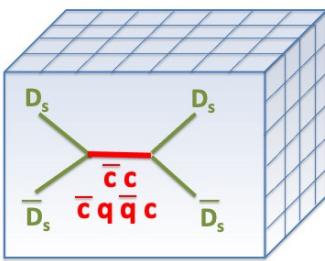
$q=u,d,s$ $I=0$



$\bar{D}_s D_s$ $J^P=0^+$ state

likely related to $X(3915)$ / $\chi_{c0}(3930)$ / $X(3960)$
[BaBar, LHCb 2009.00026, LHCb 2022 indico..../1176505]

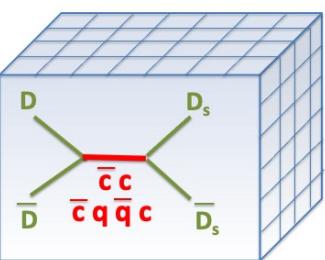
explaining why it has narrow width to $\bar{D}D$.
Supported by some pheno studies:
Lebed, Polosa 1602.08421, Oset et al . 2207.08490,
Guo et al, 2101.01021,



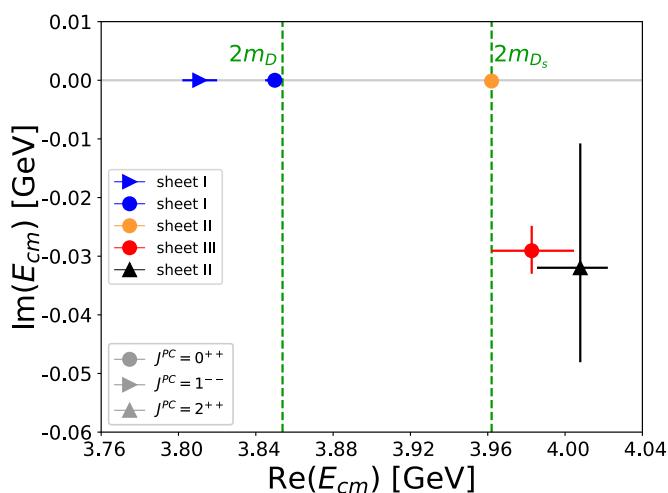
$\bar{D}D$ $J^P=0^+$ state

predicted in models [Oset et al,
0612179 PRD, Hildago Duque et al
1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive re-analysis of exp.
[Deineka, Danilkin et al 2111.15033, poster]



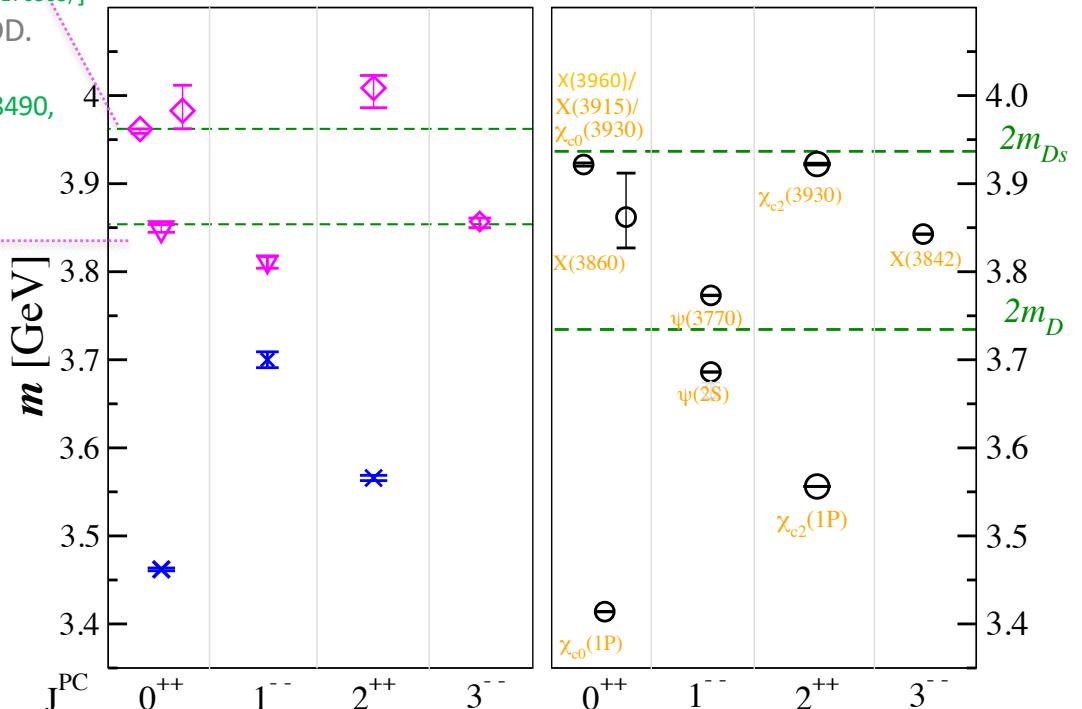
+ expected conventional charmonia



$m_\pi \simeq 280$ MeV

Lat

Exp



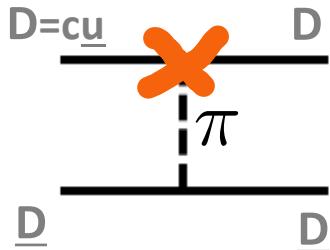
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02542 JHEP, 1905.03506 PRD, 2111.02934

Likely interpretation of some near-threshold states: “molecules” attracted by V exchange

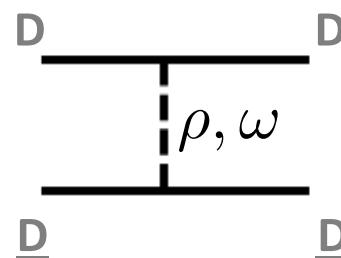
a number of pheno studies
 Oset et al, 0612179 PRD,
 Wu, Molina, Oset, Zou, 1007.0573, PRL
 Guo et al, 2101.01021,...

$\bar{c}q\bar{q}c$

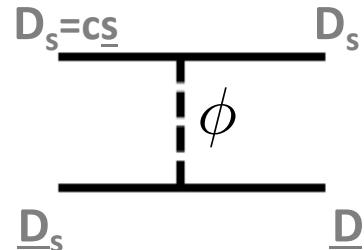
$I=0$
 $J^P=0^+$



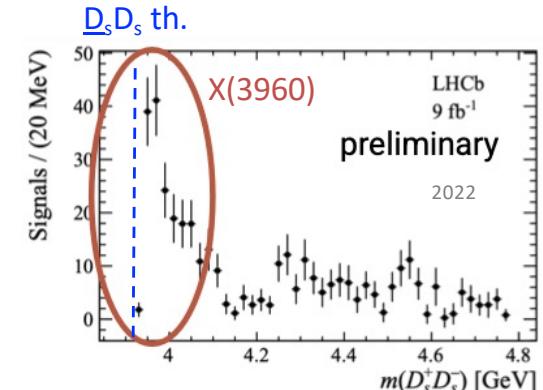
$\bar{D}D$



$\bar{D}_s D_s$



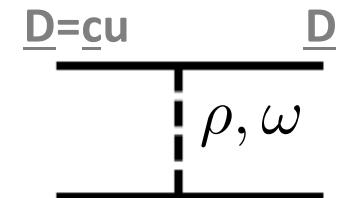
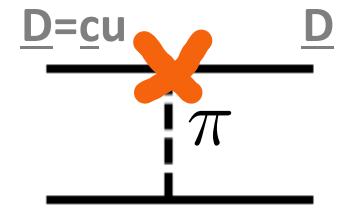
indico.cern.ch/event/1176505/, july 2022



LHCb discoveries:
 talk by Mikhaseenko

$c\bar{c}uud$

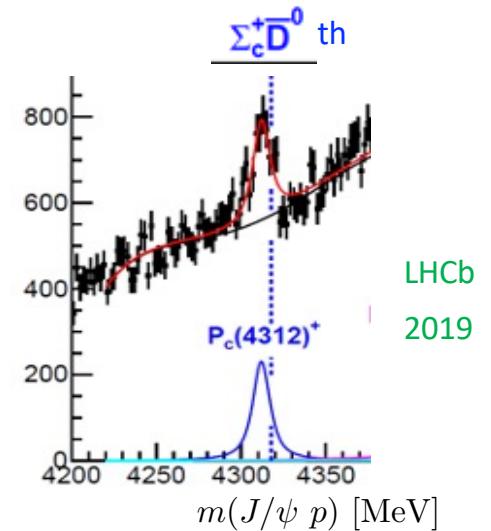
$\bar{D}\Sigma_c$



$\Sigma_c = udc$

Σ_c

currently to challenging for lattice



Charmonium(like) resonances

$\bar{c}cd\bar{u}$

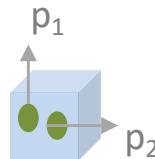
$Z_c(3900)$

$$\bar{c}cd\bar{u} \rightarrow D\bar{D}^*, J/\psi\pi$$

status from lattice QCD: still challenging, giving perhaps puzzling results

- E_n close to the non-interacting E_n^{ni}

$$E^{n.i.}(H_1 H_2) = \sqrt{\vec{p}_1^2 + m_1^2} + \sqrt{\vec{p}_2^2 + m_2^2} \quad \vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$

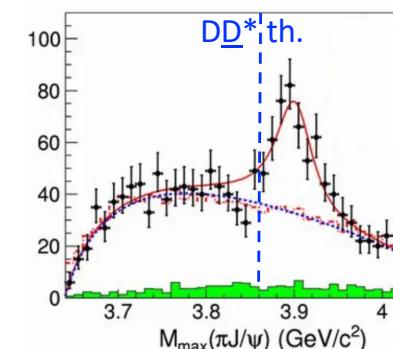


- very small interaction between mesons observed
- this should already constrain some models and interpretations
- perhaps consistent with virtual bound st. found by some approaches?
[talk by Rodriguez Entem]

consistent conclusions from studies via Luscher's method
[Leskovec Mohler Lang SP: 1308.2097, 1405.7623
HadSpec 1709.01417
Liuming Liu et al. 1907.03371, 1911.08560,
M. Sadl, SP, Padmanath, Collins @ lattice 2022]

- indication of large coupling between both channels
that could be responsible for $Z_c(3900)$ peak in exp

[BessIII & Belle 2013]



HALQCD method

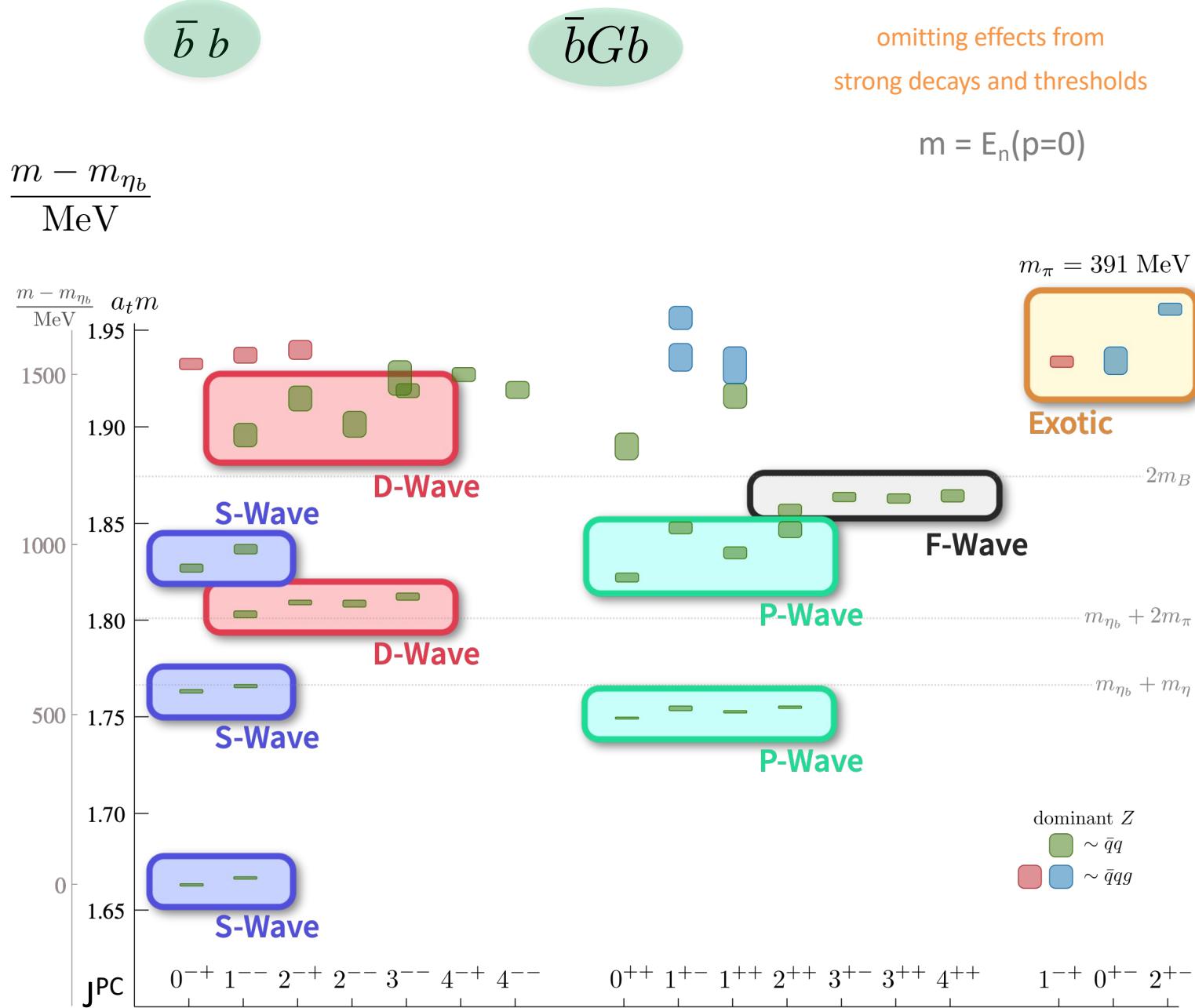
Ikeda et al., HALQCD, 1602.03465, PRL

bottomonium(like) sector

$\bar{b}b$

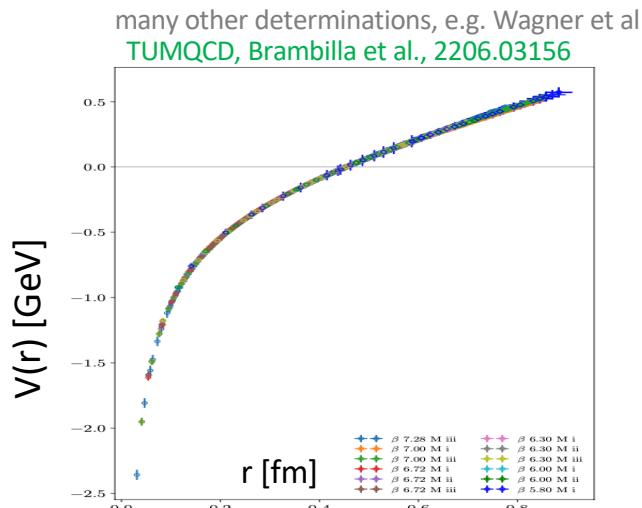
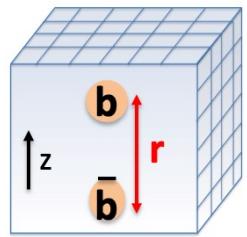
$\bar{b}b\bar{q}q$

Bottomonia and bottomonium hybrids, I=0



Lattice QCD
relativistic b quarks
Ryan & Wilson (HadSpec)
2008.02656, JHEP
(same result as shown earlier)

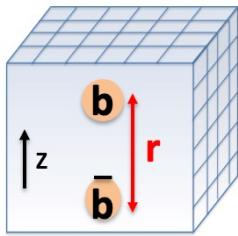
Bottomonia (I=0) from static potentials & Born Oppenheimer approximation & EFTs



masses of low-lying
confinement
determination of α_s

$\bar{b} b$

e.g. works of TUMQCD,
Leino et al. [2020-2022]



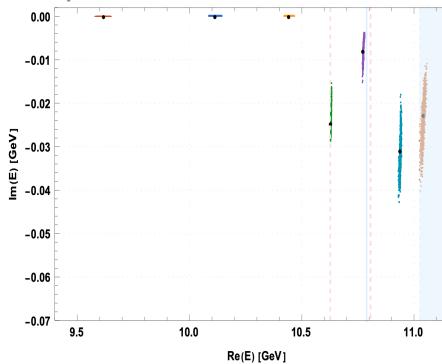
string breaking
SESAM, Bali et al, 2005, 0505012



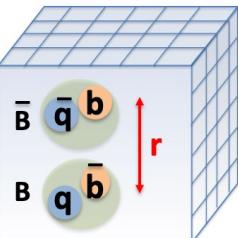
Bicudo, Cardoso, Mueller, Wagner, 2205.11475,
 $\sim J=0$: here; $\sim J=1,2,3$ in the paper;
 J =total ang. mom. without heavy quark spins

$\bar{b} b$ $\bar{b} b \bar{q} q$

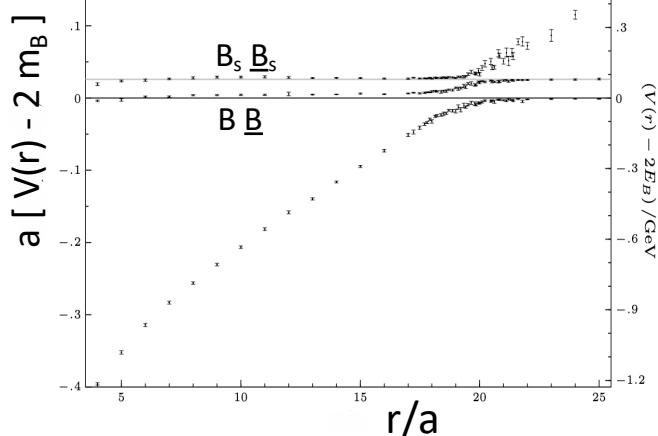
poles



see also: Castella, 2207.09365

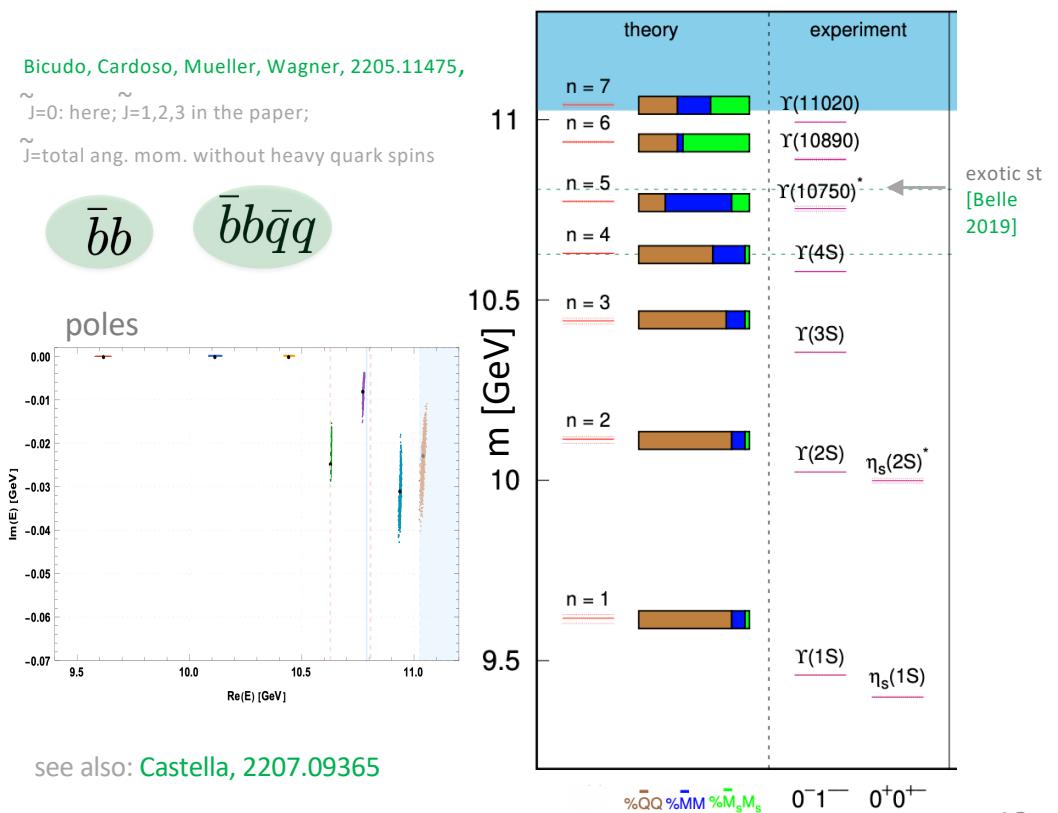


$q=u,d,s$



Sasa Prelovsek

QCD confronts heavy flavor and exotic hadrons



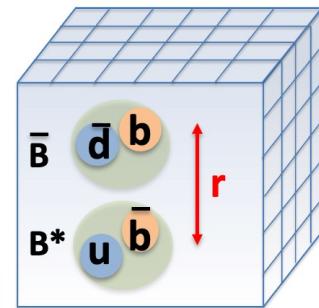
Bottomonium-like states from static potentials & Born Oppenheimer approximation & EFTs

$I=1, J^P=1^+$

$\bar{b}b\bar{d}u$

$\bar{b}b\bar{d}u \rightarrow B\bar{B}^*$

$\Upsilon\pi$



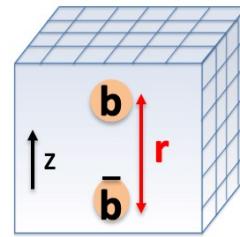
$I=0$, various J^P

$\bar{b}b$

$\bar{b}Gb$

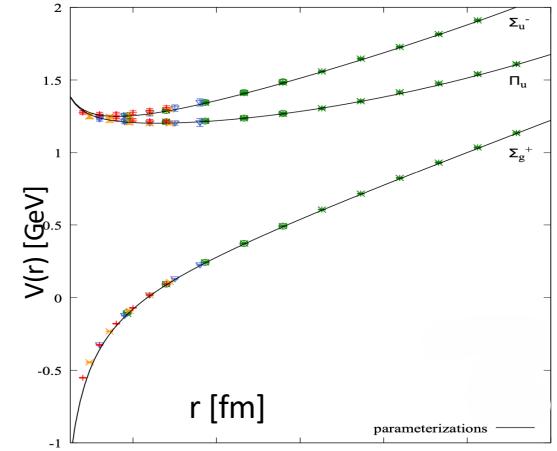
omit strong decays

quenched



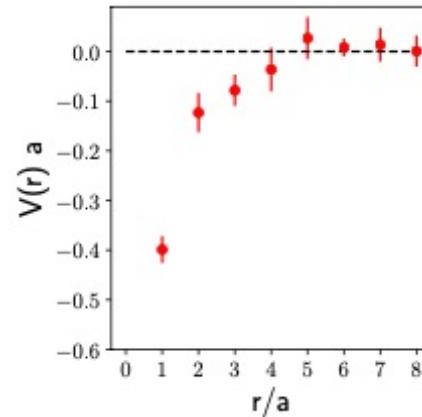
Juge, Kuti, Morningstar, 1997, 1998

Schlosser & Wagner, 2111.00741



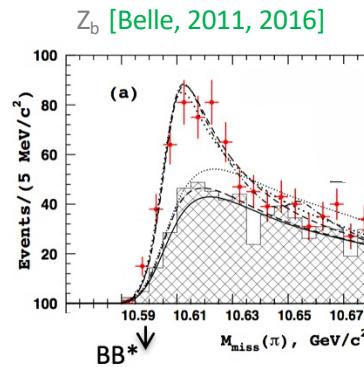
QCD confronts heavy flavor and exotic hadrons

Peters, Wagner, Bicudo
SP, Bahtiyar, Petkovic, Sadl 2019, 2020



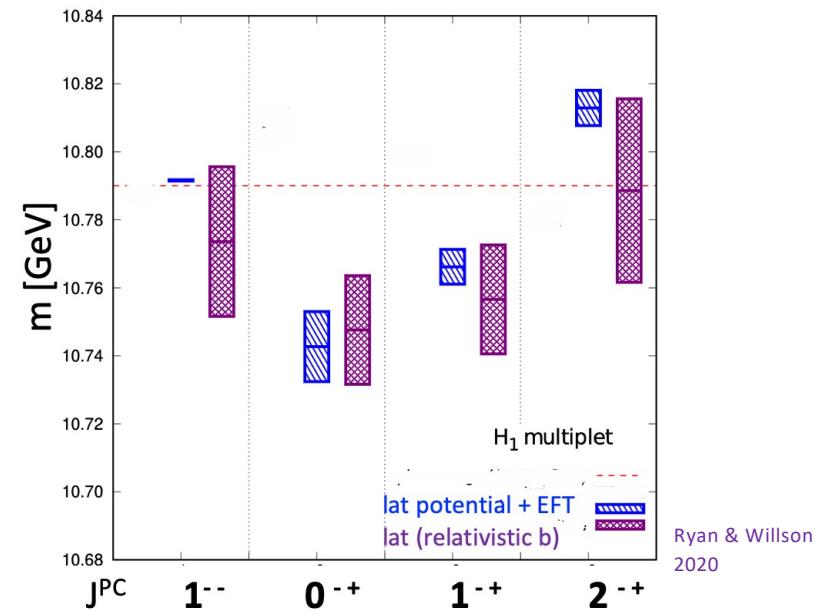
Brambilla, 2111.10788;
Soto & Castella, 2005.00552

bound state just below BB^* th.
likely related to

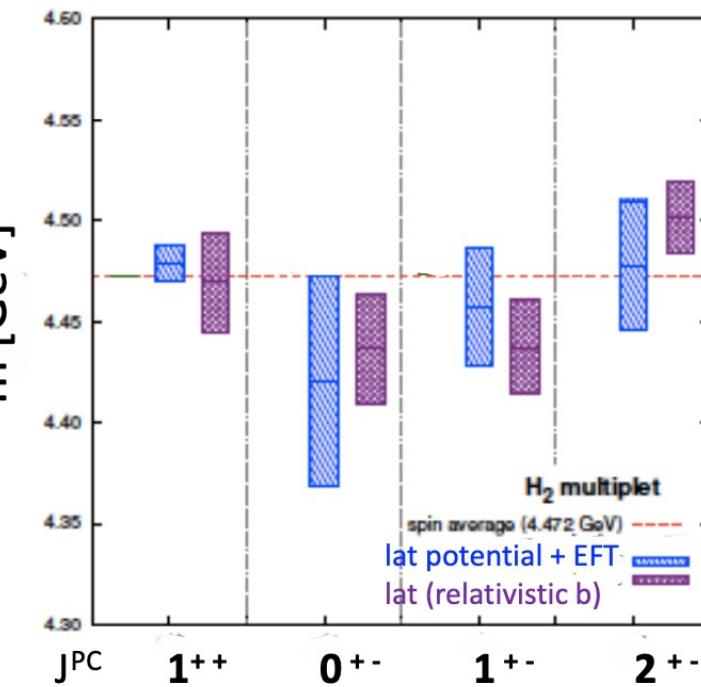
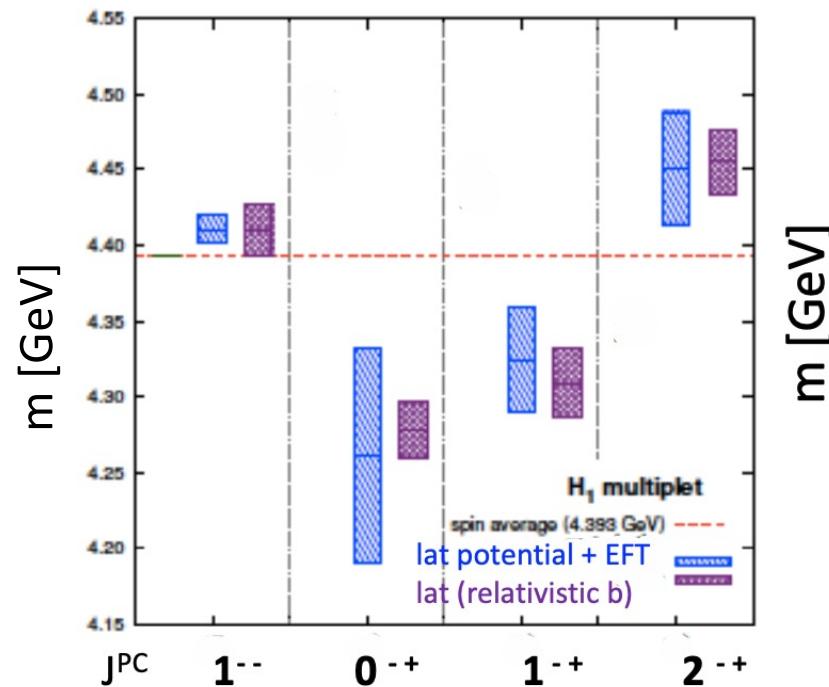


$\bar{b}G\bar{b}$

Segovia, Tarrus; Brambilla MITP 2022
(unpublished, with 1/mb spin effects)



see also: Brambilla et al, 1805.07713, 1908.11699, PRD



TUMQCD, Brambilla et al.
Cheung et al, HadSpec, 2016

Doubly heavy tetraquarks

$QQ'\bar{q}\bar{q}'$

$Q=c,b \quad q=u,d,s$

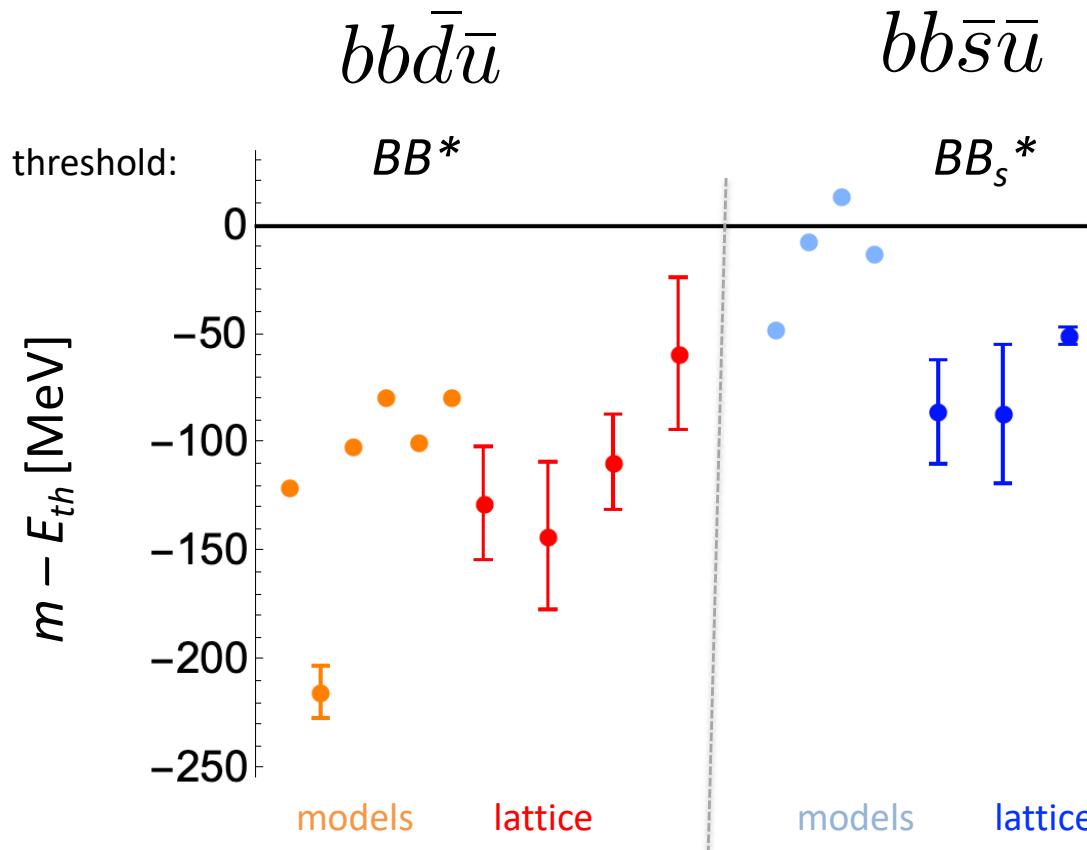
Doubly bottom tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$

$bb\bar{s}\bar{u}$

$I=0, J^P=1^+$



references from left to right

models (many more references):

Eichten and Quigg (2017) 1707.09575 PRL

Karliner and Rosner (2017) 1707.07666 PRL

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results

[Leskovec, Meinel, Pflaumer, Wagner \(2019\) 1904.04197](#)

Junnarkar, Mathur, Padmananth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)

Eichten and Quigg (2017) 1707.09575 PRL

Parket al. (2018) 1809.05257

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

lattice: most updated results

[Meinel, Pflaumer, Wagner \(2022\) 2205.13982](#)

Junnarkar, Mathur, Padmananth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

lattice	$m_{u/d}$	a [fm]
Leskovec, Pflaumer, Wagner, Meinel	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.08-0.11
Junnarkar et al.	$m_{u/d} \rightarrow m_{u/d}^{phy}$	$a \rightarrow 0$
Francis et al	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.09
Bicudo et al.	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.08

extracted also
 $BB_{(s)}^*$ scattering amplitudes

earlier results of Frances et al in 1810.10550, 1607.05214 PRL

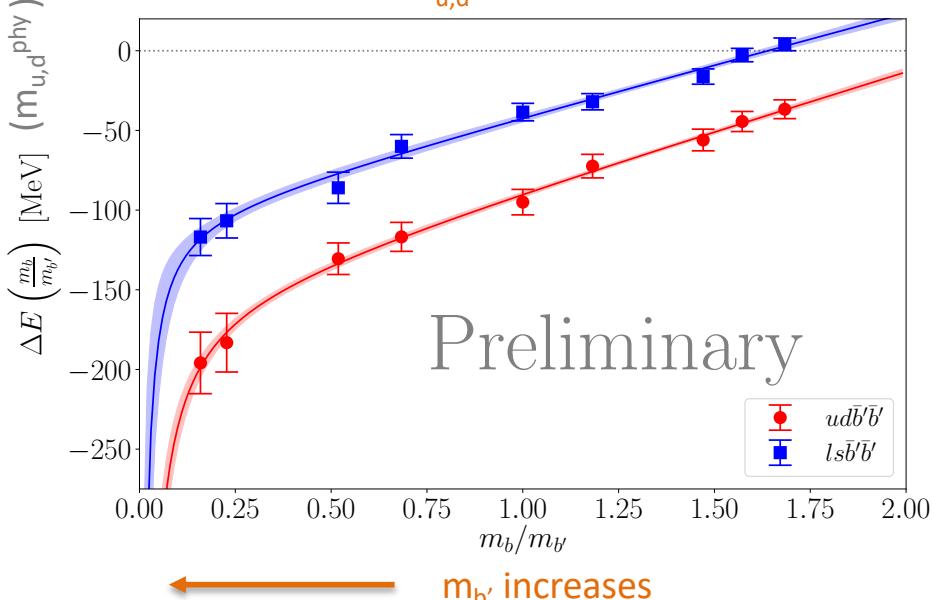
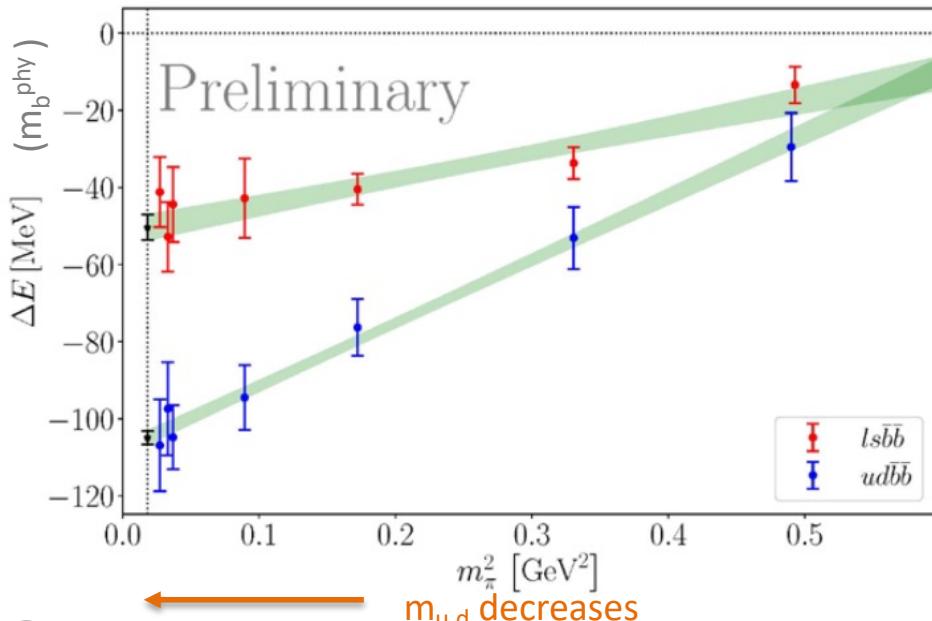
Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$

$bb\bar{s}\bar{u}$

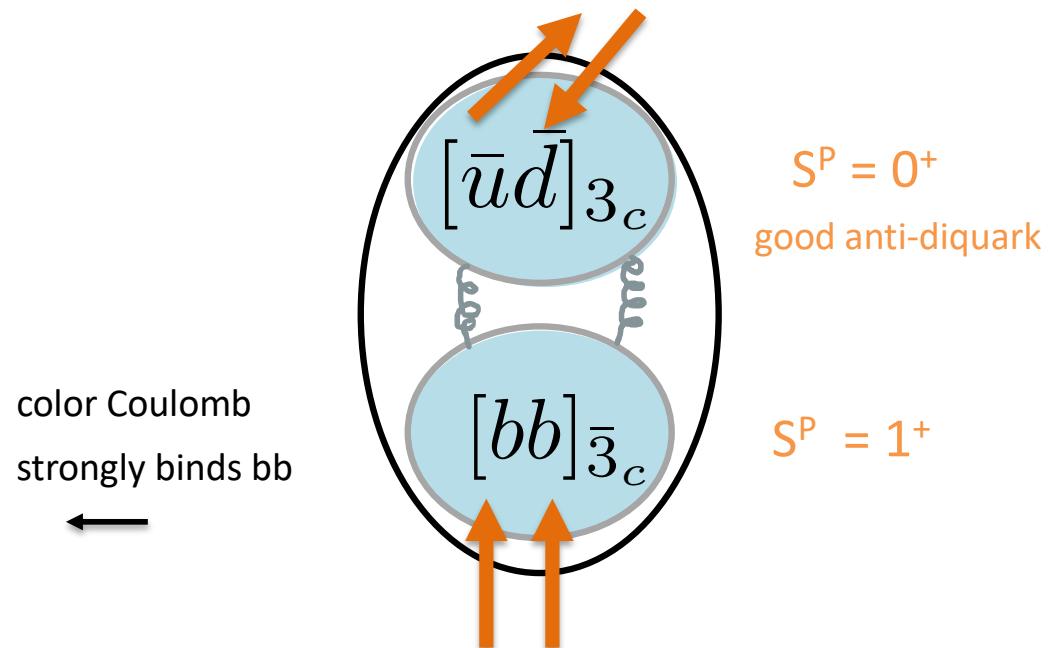
$I=0, J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550 , PoS LATTICE2021 (2022) 144
supports internal structure below

supported also by almost all model studies
Karliner and Rosner (2017), Janc and Rosina (2004), ...



these are the only tetraquarks where lattice finds support for significant $[qq][\bar{q}\bar{q}]$

good and bad diquark properties:

Francis et al, 2201.03332

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bcd\bar{u}$ $ccd\bar{u}$

Theoretically expected near or above threshold

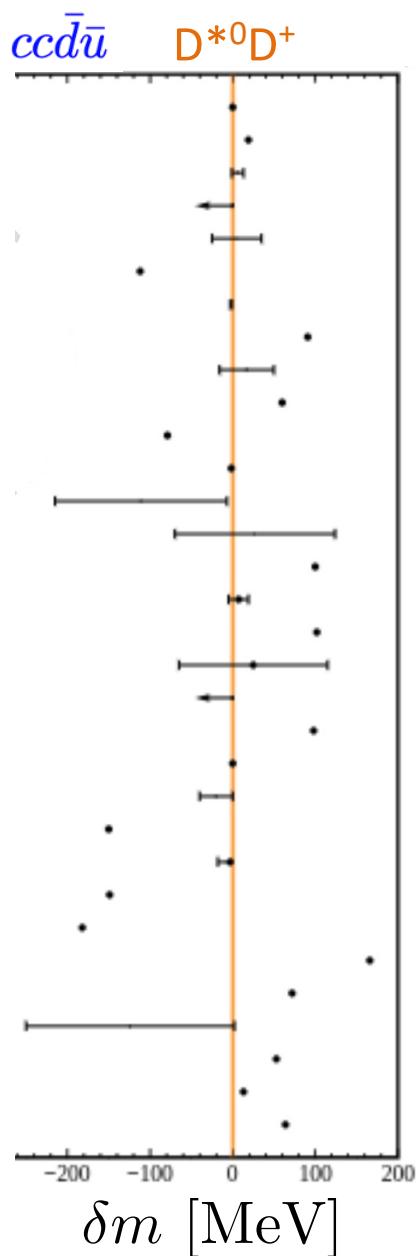
States near or above threshold have to be identified as poles in scattering $T(E)$

Other $QQ'\bar{q}\bar{q}'$ and J^P

$b\bar{c}\bar{d}\bar{u}$ $c\bar{c}\bar{d}\bar{u}$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering $T(E)$



J. Carlson <i>et al.</i>	1987
B. Silvestre-Brac and C. Semay	1993
C. Semay and B. Silvestre-Brac	1994
S. Pepin <i>et al.</i>	1996
B. A. Gelman and S. Nussinov	2003
J. Vijande <i>et al.</i>	2003
D. Janc and M. Rosina	2004
F. Navarra <i>et al.</i>	2007
J. Vijande <i>et al.</i>	2007
D. Ebert <i>et al.</i>	2007
S. H. Lee and S. Yasui	2009
Y. Yang <i>et al.</i>	2009
G.-Q. Feng <i>et al.</i>	2013
Y. Ikeda <i>et al.</i>	2013
S.-Q. Luo <i>et al.</i>	2017
M. Karliner and J. Rosner	2017
E. J. Eichten and C. Quigg	2017
Z. G. Wang	2017
G. K. C. Cheung <i>et al.</i>	2017
W. Park <i>et al.</i>	2018
A. Francis <i>et al.</i>	2018
P. Junnarkar <i>et al.</i>	2018
C. Deng <i>et al.</i>	2018
M.-Z. Liu <i>et al.</i>	2019
G. Yang <i>et al.</i>	2019
Y. Tan <i>et al.</i>	2020
Q.-F. Lü <i>et al.</i>	2020
E. Braaten <i>et al.</i>	2020
D. Gao <i>et al.</i>	2020
J.-B. Cheng <i>et al.</i>	2020
S. Noh <i>et al.</i>	2021
R. N. Faustov <i>et al.</i>	2021

Theoretical predictions for T_{cc} mass ($I=0, J^P=1^+$)

[compilation by Polyakov]

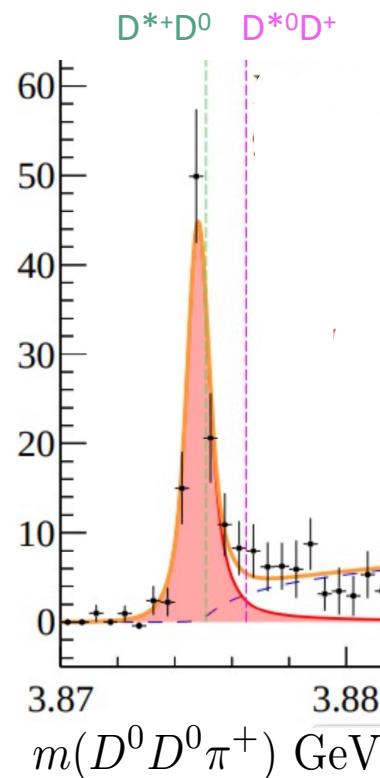
Other $QQ'\bar{q}\bar{q}'$ and J^P

$bcd\bar{u}$ $ccd\bar{u}$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering $T(E)$

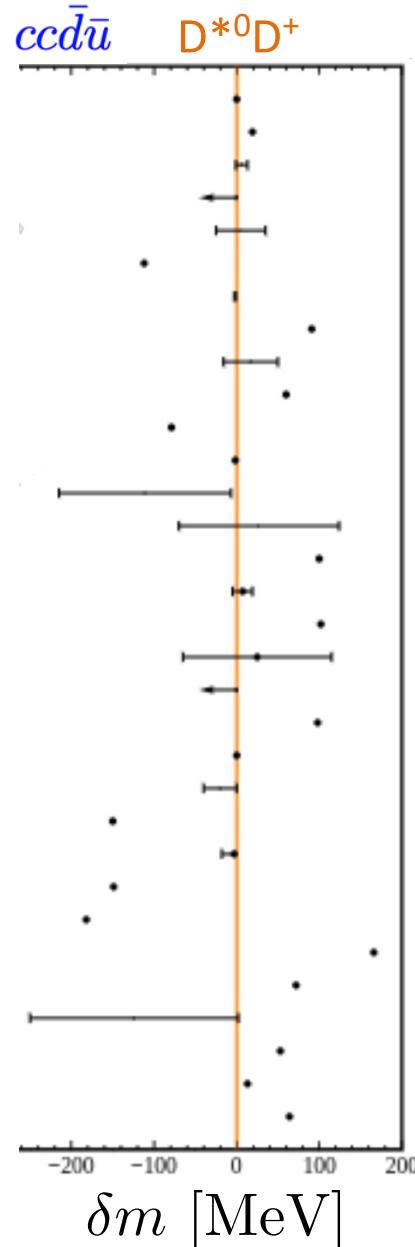
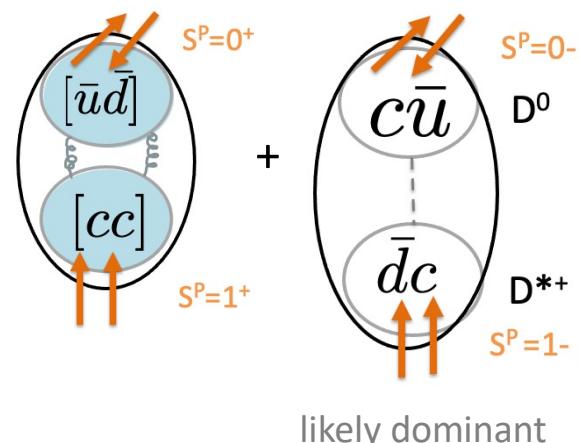
Doubly charm tetraquark T_{cc}



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics



Theoretical predictions for T_{cc} mass ($I=0, J^P=1^+$)

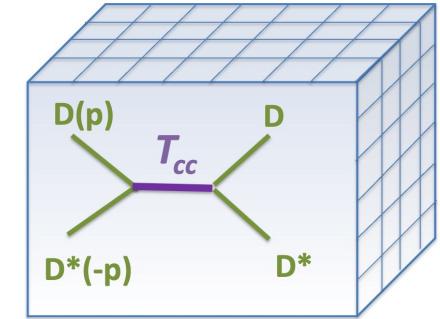
[compilation by Polyakov]

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G. K. C. Cheung <i>et al.</i>	2017
W. Park <i>et al.</i>	2018
A. Francis <i>et al.</i>	2018
P. Junnarkar <i>et al.</i>	2018
C. Deng <i>et al.</i>	2018
M.-Z. Liu <i>et al.</i>	2019
G. Yang <i>et al.</i>	2019
Y. Tan <i>et al.</i>	2020
Q.-F. Lü <i>et al.</i>	2020
E. Braaten <i>et al.</i>	2020
D. Gao <i>et al.</i>	2020
J.-B. Cheng <i>et al.</i>	2020
S. Noh <i>et al.</i>	2021
R. N. Faustov <i>et al.</i>	2021

Doubly charm tetraquark T_{cc} from lattice QCD

$cc\bar{d}\bar{u}$

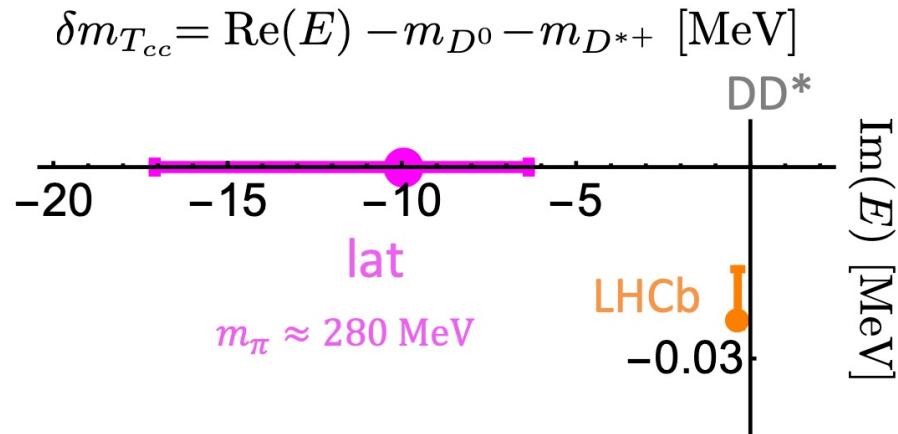
$J^P = 1^+, I=0$



What is virtual bound state? See supplemental of [2202.10110, PRL](#)

Pole of $T(E)$

● virtual bound state pole $p = -i|p|$

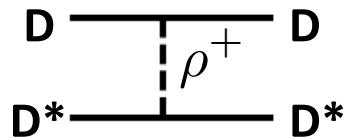


- ❖ The first lat. study that extracts $T(E)$:
- ← [\[Padmanath, SP: 2202.10110, PRL\]](#) $m_\pi \approx 280$ MeV
- ❖ Evidence for pole related to T_{cc}
- ❖ For $m_{u,d} > m_{u,d}^{\text{phy}}$ one expects decreased attraction
- T_{cc} : bound state becomes virtual bound state
this is indeed found on the lattice

another recent lattice study: [CLQCD, Chen et al. 2206.06185](#)

comparison of $I=0,1$: attraction in $I=0$ channel arises mainly from

$m_\pi \simeq 350$ MeV

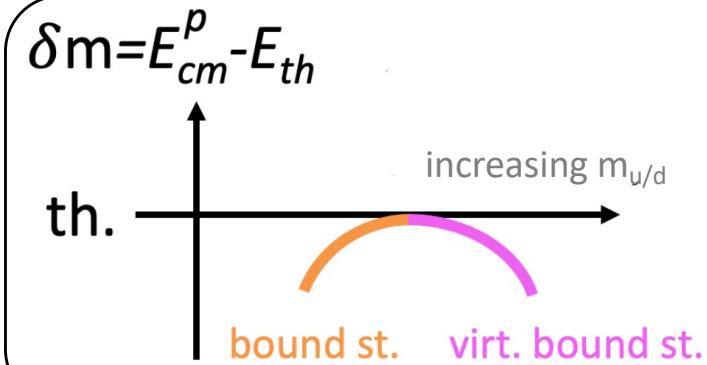


previous simulations extracted only eigen-energies:

[\[Junnarkar, Mathur, Padmanath, 1810.12285 PRD ; HadSpec 1709.01417 JHEP\]](#)

Dyson Schwinger: [Fischer et al. 2111.15310](#): $I=0$ state near threshold found

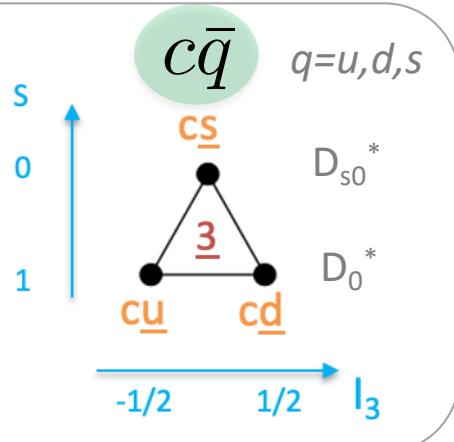
Sketch of expected binding energy



Scalar heavy-light mesons

$J^P = 0^+$

Conventional quark model



new paradigm supported by:

- lattice
- effective models ChPT+HQET
- reanalysis of exp data
- states circled by blue seem to feature in the spectrum

Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL

Lang et al, 1403.8103, PRD

RQCD, 1706.01247, PRD

HadSpec 2008.06432, JHEP

S=0 Mohler et al. 1208.4059, PRD

HadSpec, 1607.07093, JHEP

HadSpec 2102.04973, JHEP

HadSpec 2205.05026

S=-1 HadSpec, 2008.06432, JHEP

New paradigm

Du et al, 1712.07957, PRD

Albaladejo et al, 1610.06727, PLB

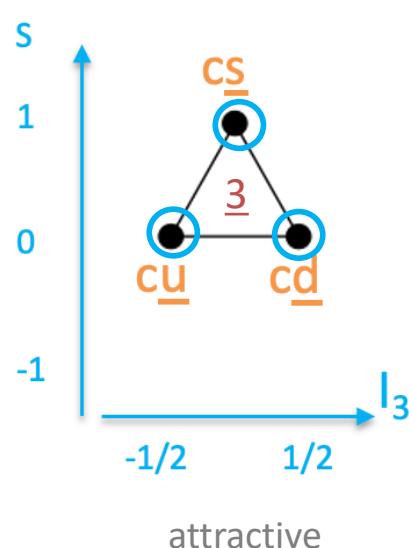
Lutz et al (2003), 0307133, PLB

$$c\bar{q} + c\bar{q} q\bar{q} \quad q=u,d,s \quad n=u,d$$

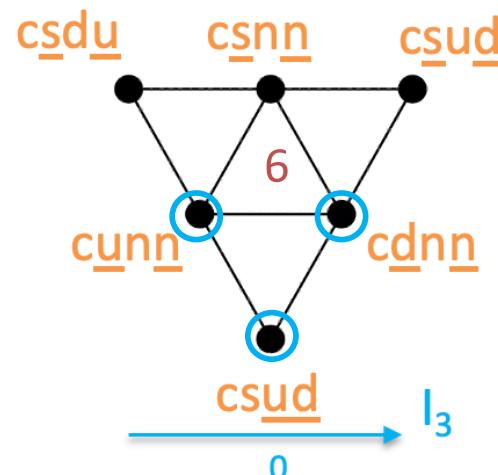
$$\underline{3} \otimes \underline{8} = \underline{3} \oplus \underline{6} \oplus \underline{15} \quad \text{SU}(3)_F$$

Beveren, Rupp; Dmitrasinovic

lat: 2.1-2.2 GeV (pole)
PDG: 2.3 GeV (BW)
(see backup)



mix
 $SU(3)_F$



mixes with repulsive 15

2.4-2.5 GeV
reanalysis of lat
1607.07093 by
Albaladejo 1610.06727

virtual bound state
HadSpec 2008.06432
partner of X(2900)
[LHCb 2009.00025] ?

Di-baryons

$$\text{binding energy} \quad \Delta E = m - m_{B1} - m_{B2}$$

H-dibaryon

many lattice studies:

NPLQCD, CallAT, Mainz, ...

mostly at $\mu_u = \mu_d = \mu_s$

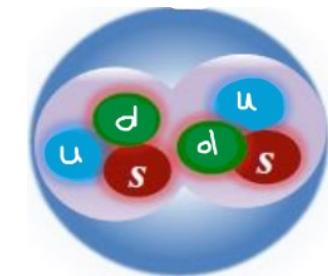
[Mainz group, 2103.01054 PRL]

$$m_\pi = m_K \simeq 420 \text{ MeV}$$

$$\Delta E = -4.56 \pm 1.13_{\text{stat}} \pm 0.63_{\text{syst}} \text{ MeV.}$$

significant discretization effects on ΔE found

physical m_q : ??
experimentally not confirmed (yet)



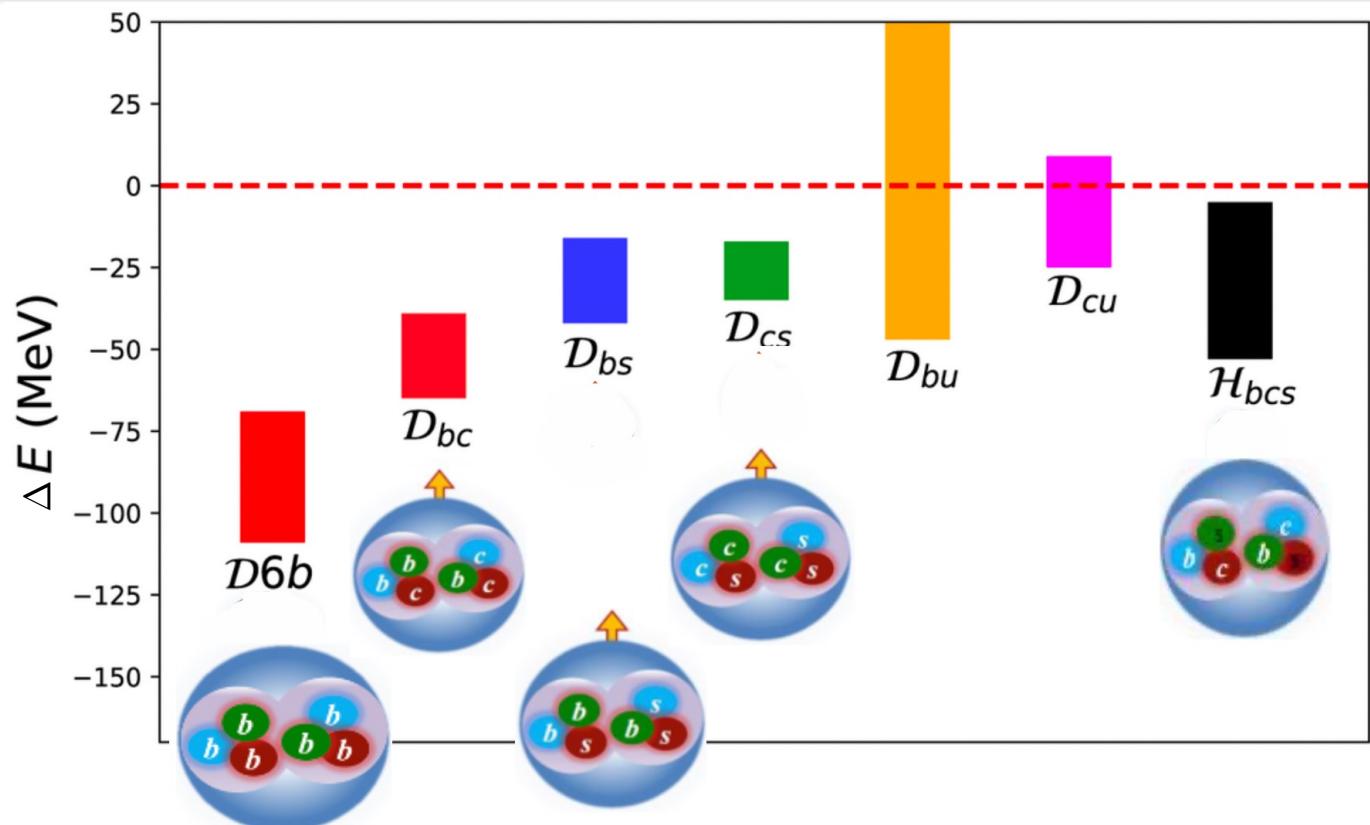
$\Lambda \Lambda$

dibaryons with heavy quarks

Junnarkar Mathur
1906.06054, PRL 2019

Mathur, Padmanath, Chakraborty
2205.02862

Junnarkar, Mathur, 2206.02942



Conclusions

Compliments to experiments for GREAT results !!

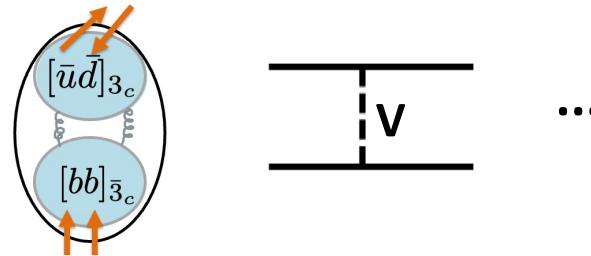
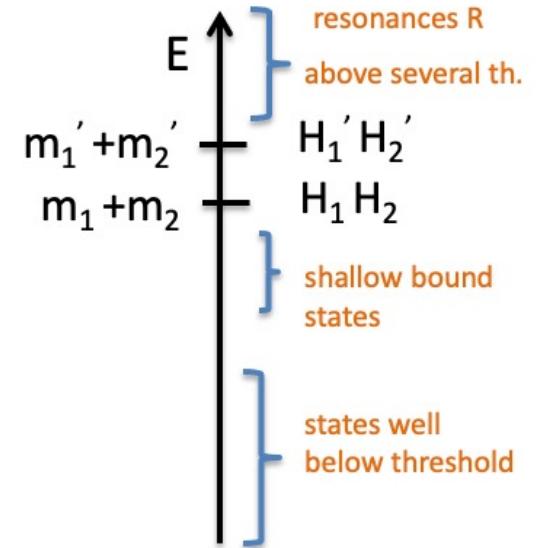
Status on hadrons from Lattice , Lattice + EFTs:

- hadrons that are not resolved (yet)
strongly decay via many decay channels: P_c , $Z_c(4430)$, $X(6900)$, ...
- available: valuable results on conventional and exotic hadrons
strongly stable ; strongly decaying to 1,2,3 channels

support for specific binding mechanisms

one picture can not explain all exotic hadrons

for each exotic hadron there is at least one viable picture



A direction that might lead to a valuable insight into dynamics of hadrons:

identify and inspect states that can be rigorously studied by theory and well explored by experiments
quark-mass dependence of these states in theory ... could lead to further clues about their nature

backup

Hadrons

$$\bar{Q}Q\bar{q}q,$$

$$\bar{Q}Qqqq$$

$Q=c,b$ $q=u,d,s$

$$Z_b = \bar{b}b\bar{d}u \text{ [Belle 2011]}$$

$$Z_c = \bar{c}c\bar{d}u \text{ [BessII, Belle, 2013]}$$

$$P_c = \bar{c}cuud \text{ [LHCb 2015]}$$

GRRR!

challenging for ab-initio study
due to many decay channels:

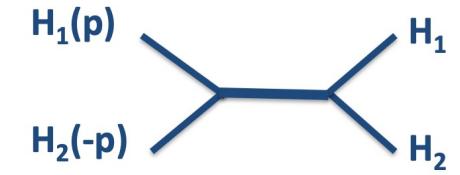
$$\bar{Q}Q\bar{q}q \rightarrow \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$$

$$\bar{Q}Qqqq \rightarrow \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$$

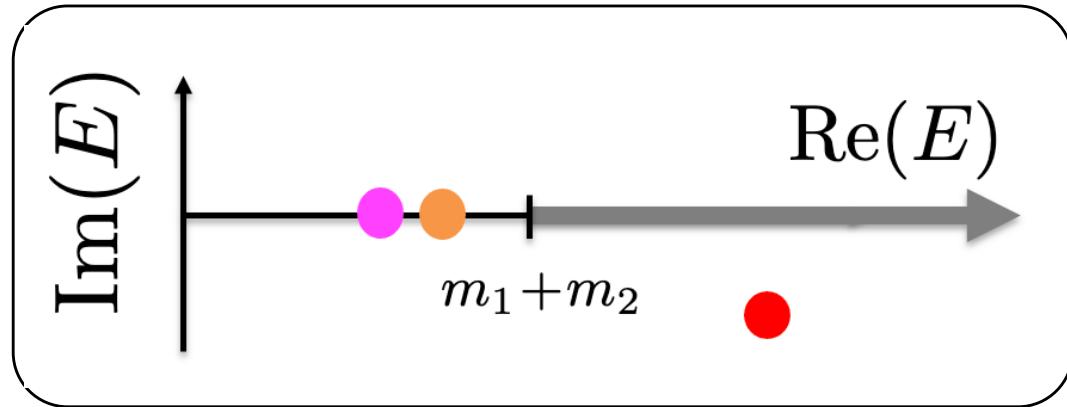
Only partial conclusions are available
from ab-initio approaches
[reviewed e.g. in S.P. 2001.01767]
I'll discuss other approaches
(also due to lack of time).

Definitions: bound state, virtual bound state & resonance

$$T(E) \propto \frac{1}{E^2 - m^2} \quad T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$



Poles of $T(E)$, $E=E_{cm}$



Virtual bound st.

$$p = -i|p|$$

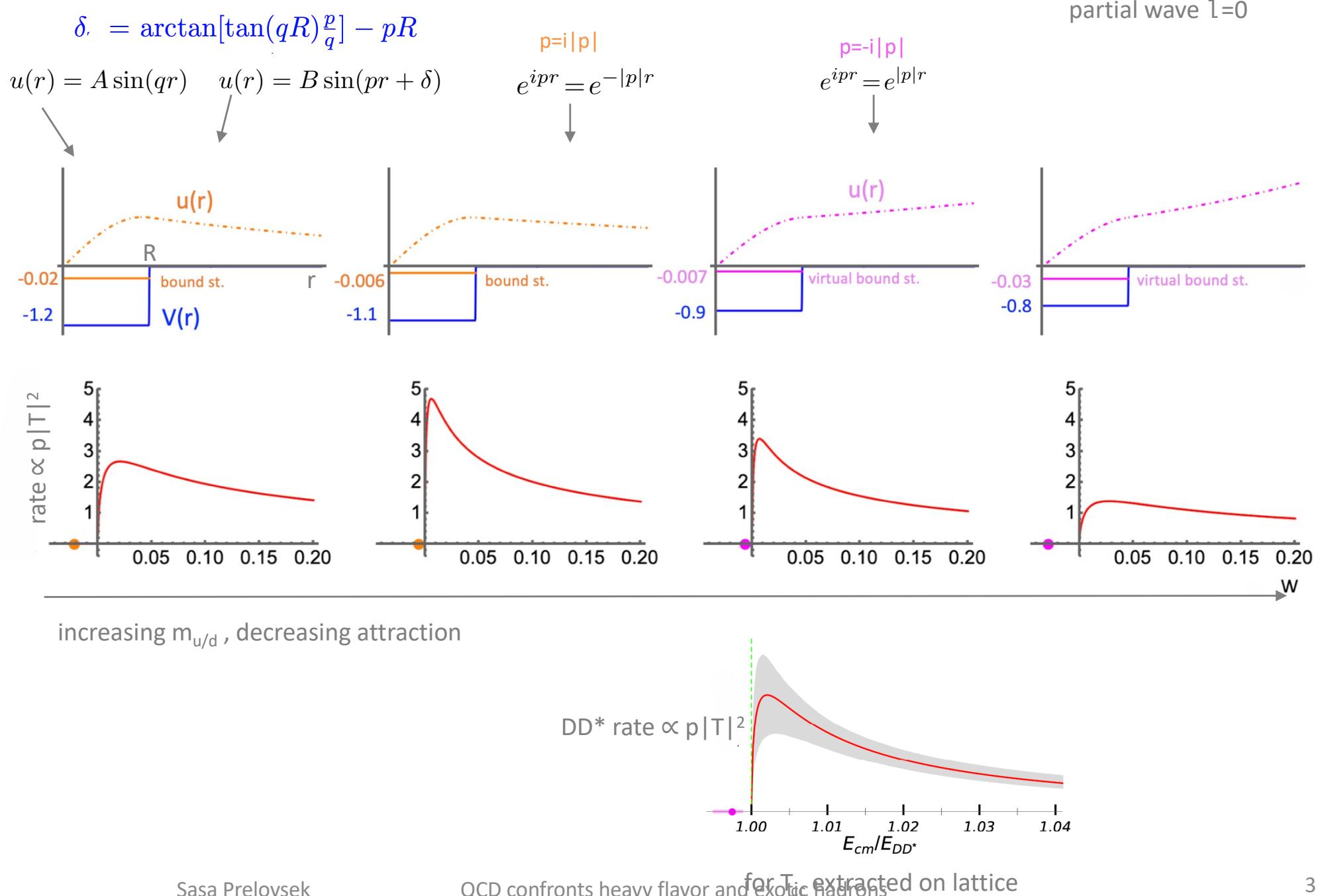
Bound st.

$$p = i|p|$$

Resonance

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} < m_1 + m_2$$

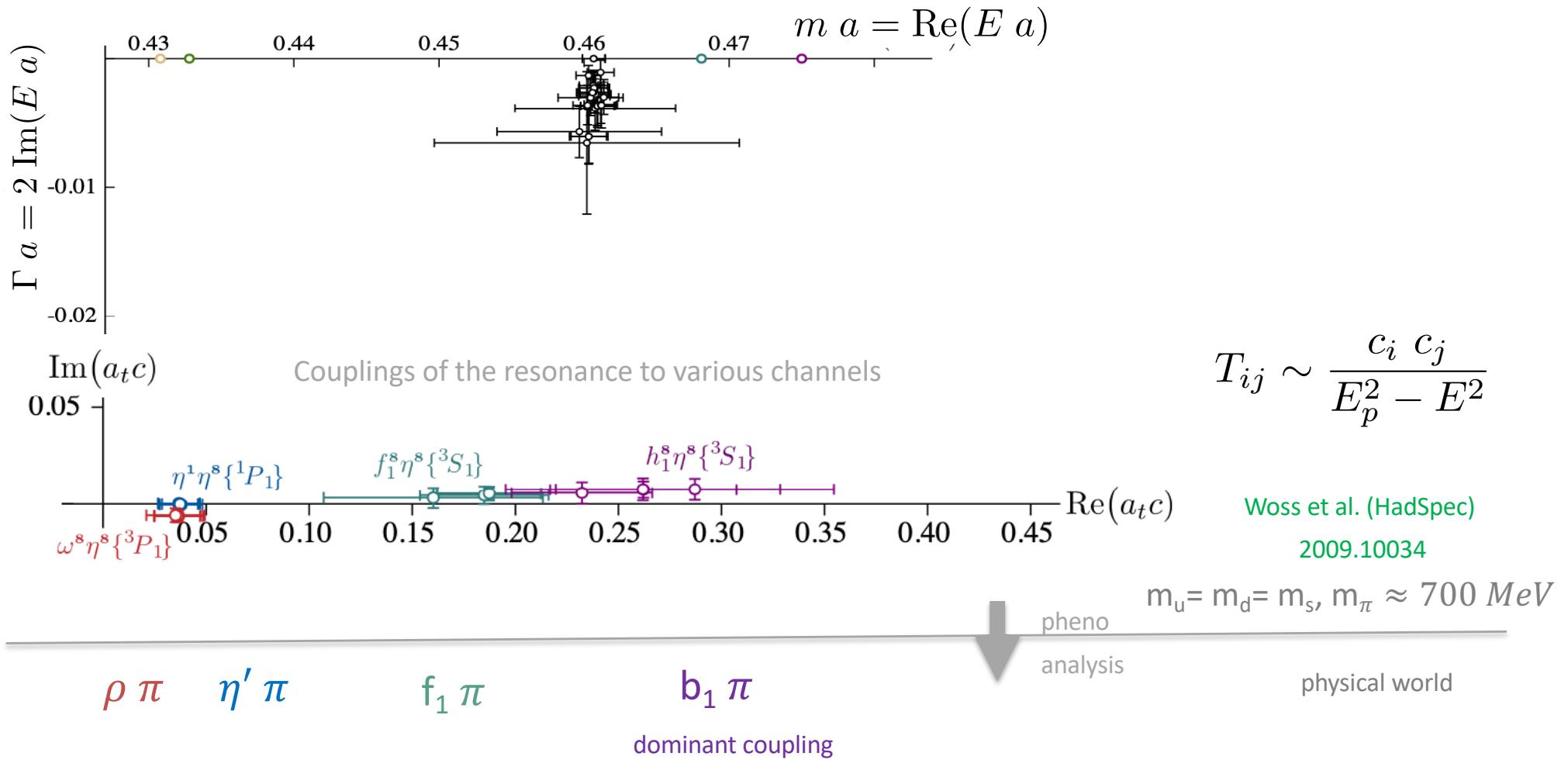
Bound and virtual bound state: simplest example scattering in square-well potential in QM



light hybrid meson π_1 from lattice

$\bar{d}Gu$

$J^{PC} = 1^{-+}$

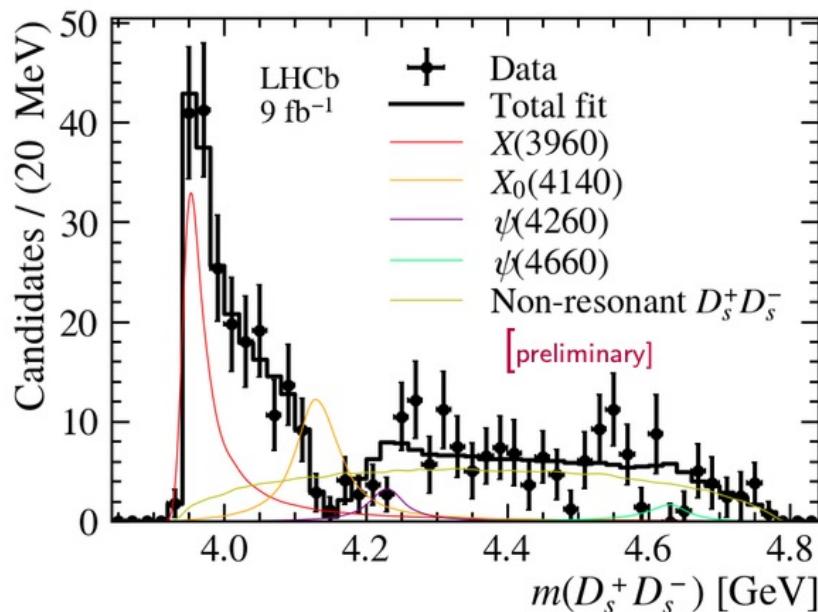


resemblance to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

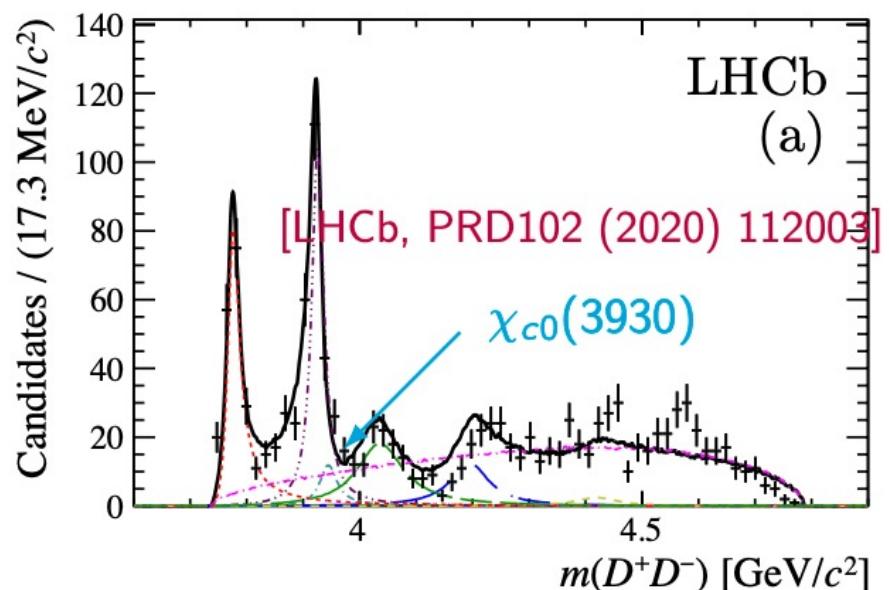
$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Is $X(3960)$ the same as $\chi_{c0}(3930)$ from D^+D^- ?

$B^+ \rightarrow (D_s^+ D_s^-) K^+$ by LHCb:



$B^+ \rightarrow (D^+ D^-) K^+$ by LHCb:



- Assuming to be the same, $\mathcal{B}(\chi_{c0} \rightarrow D^+ D^-)/\mathcal{B}(\chi_{c0} \rightarrow D_s^+ D_s^- P) \sim 0.3$
large molecular component, or large tetraquark component, $T_{\psi\phi}$
- [JHEP 06 (2021) 035] finds a state coupled to $D_s^+ D_s^-$ on the lattice