Lattice QCD for Heavy-Ion Collisions: Status Update

Brookhaven National Laboratory

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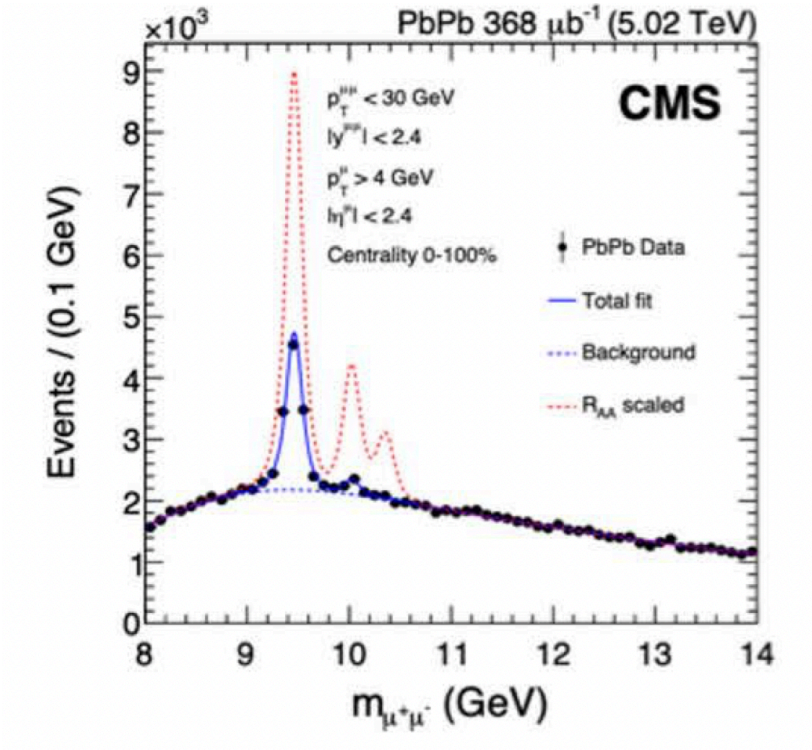
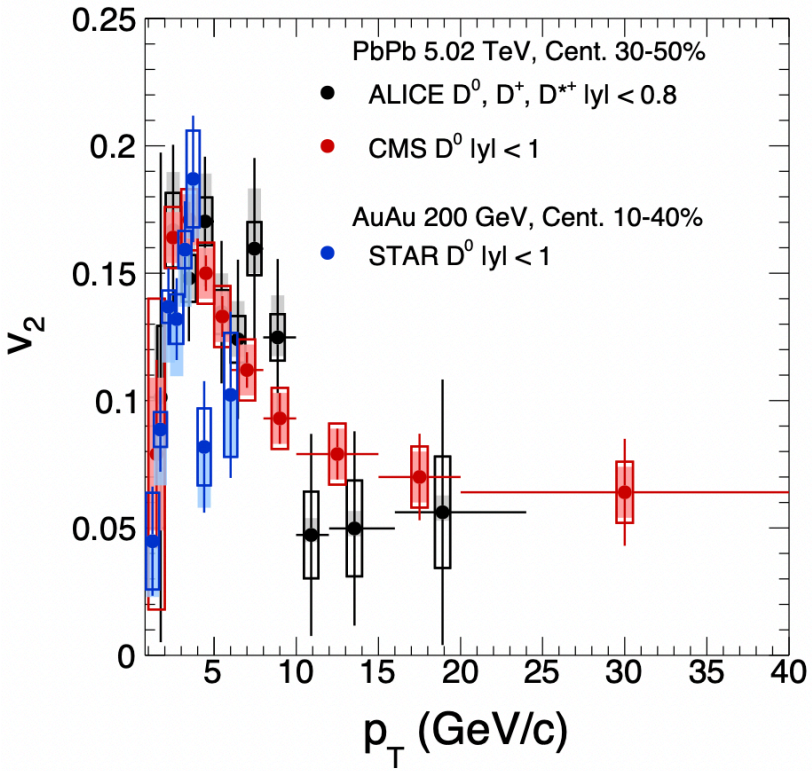
August 2022, Stavanger, Norway

lattice QCD: hot and heavy

- LHC
- sPHENIX
- STAR



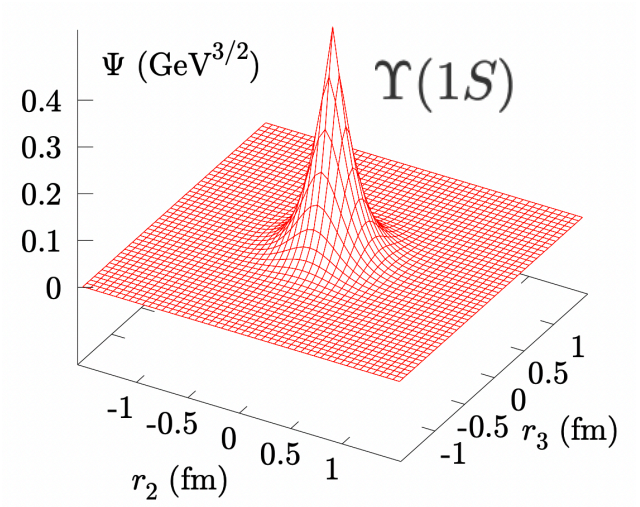


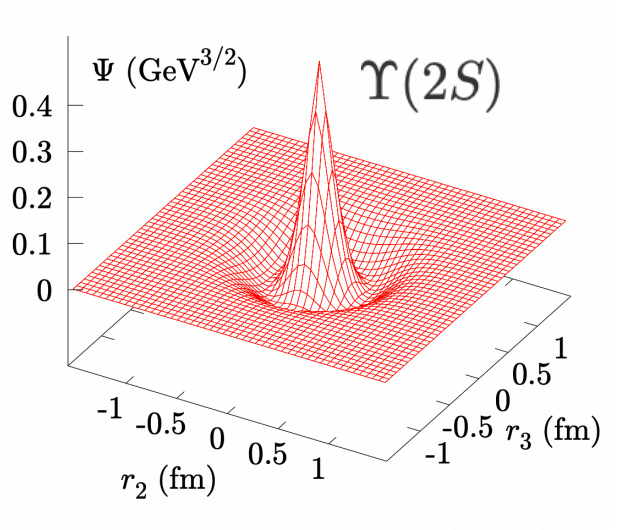
- bottomonia properties in QGP ?
- heavy-quark energy loss / diffusion constant ?

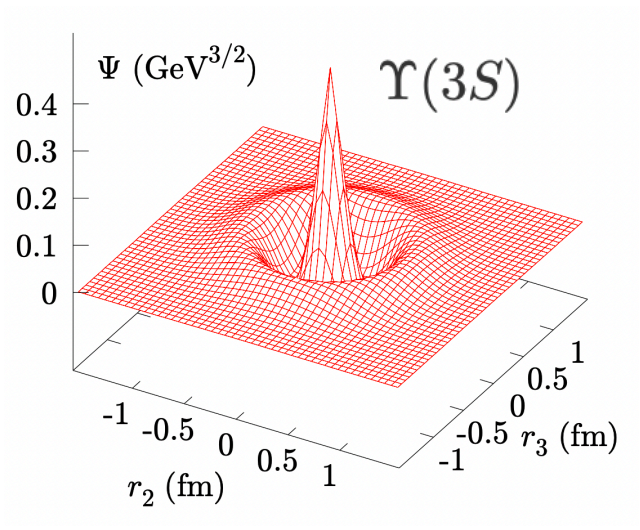
bottomonia in QGP

update: first results for up to 3S & 2P states

novel sources respecting symmetries of the state + variational analysis

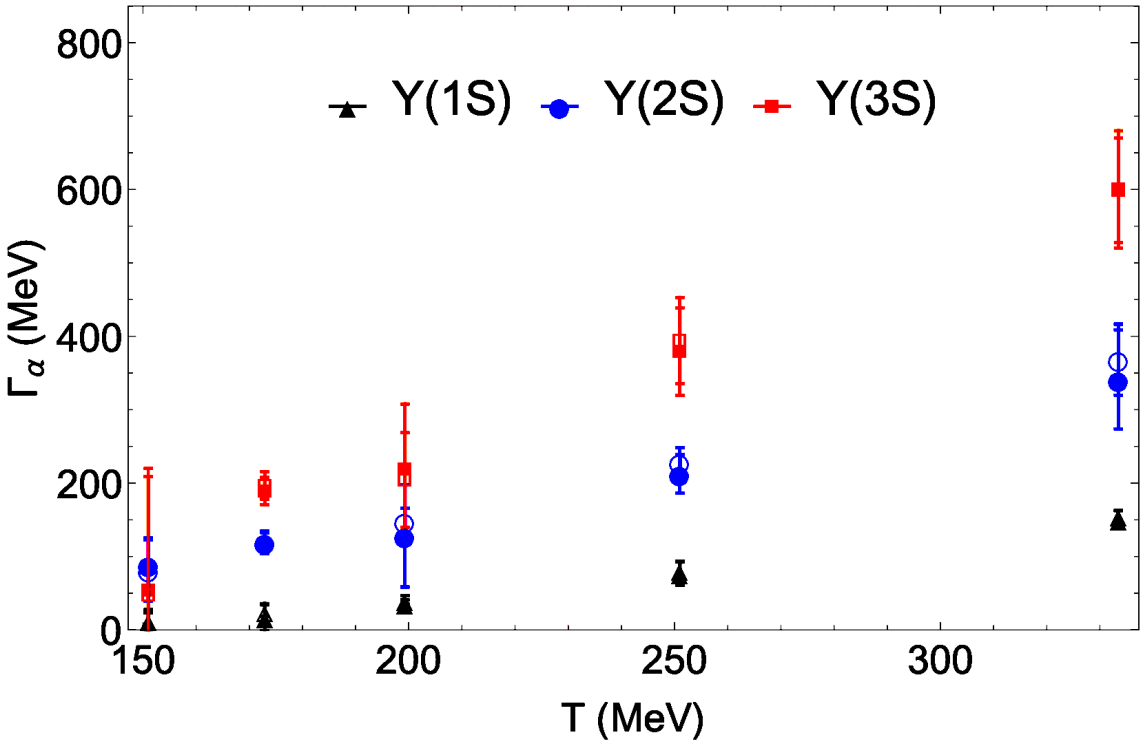


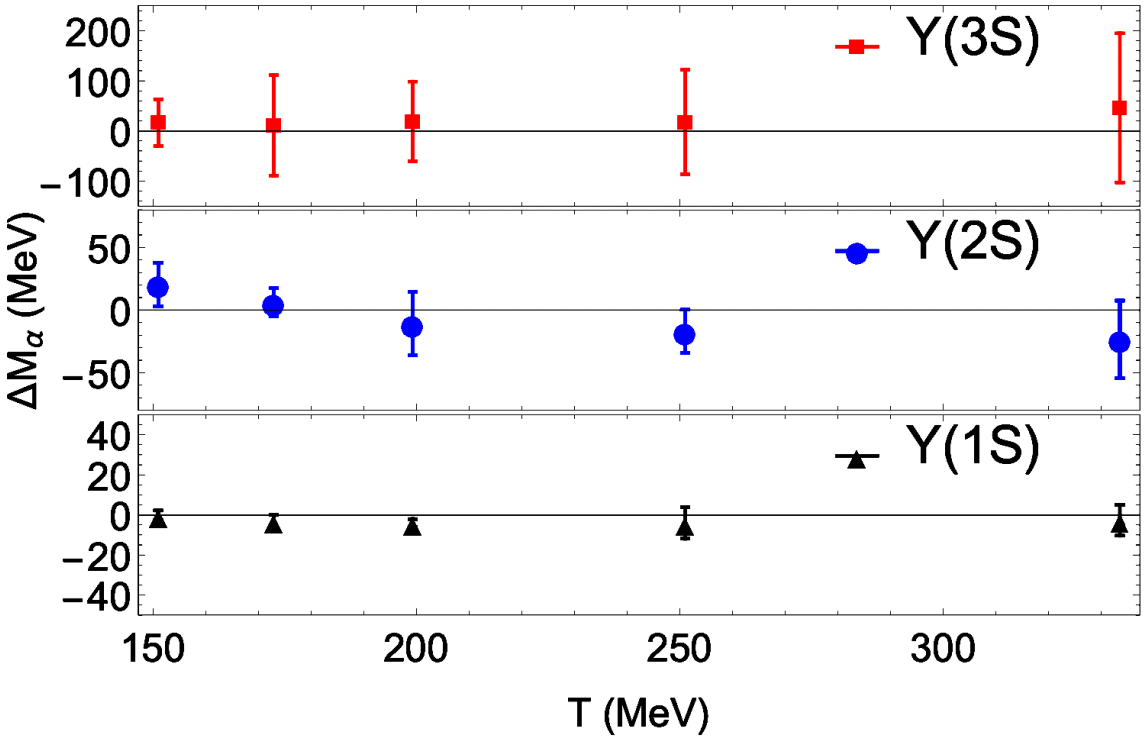




thermal widths: large

thermal mass shifts: small



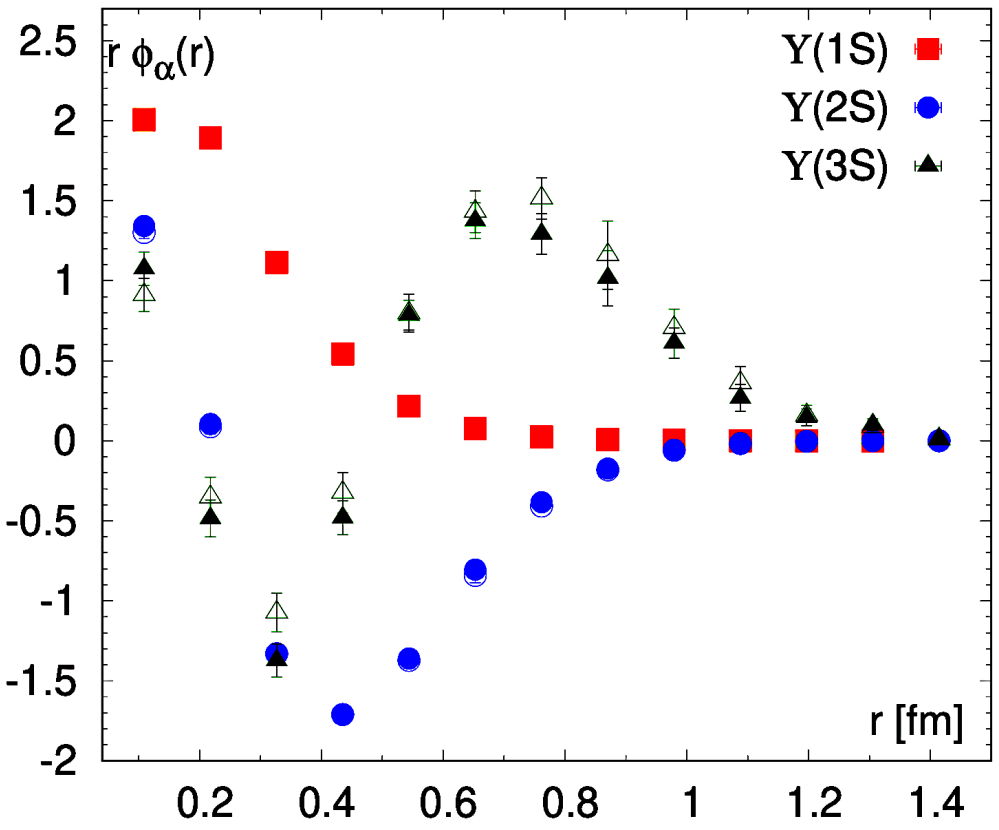


Rasmus Larsen et.al.: Phys. Lett. B800, 135119 (2020)

Bethe-Salpeter amplitudes

T=0: open symbols

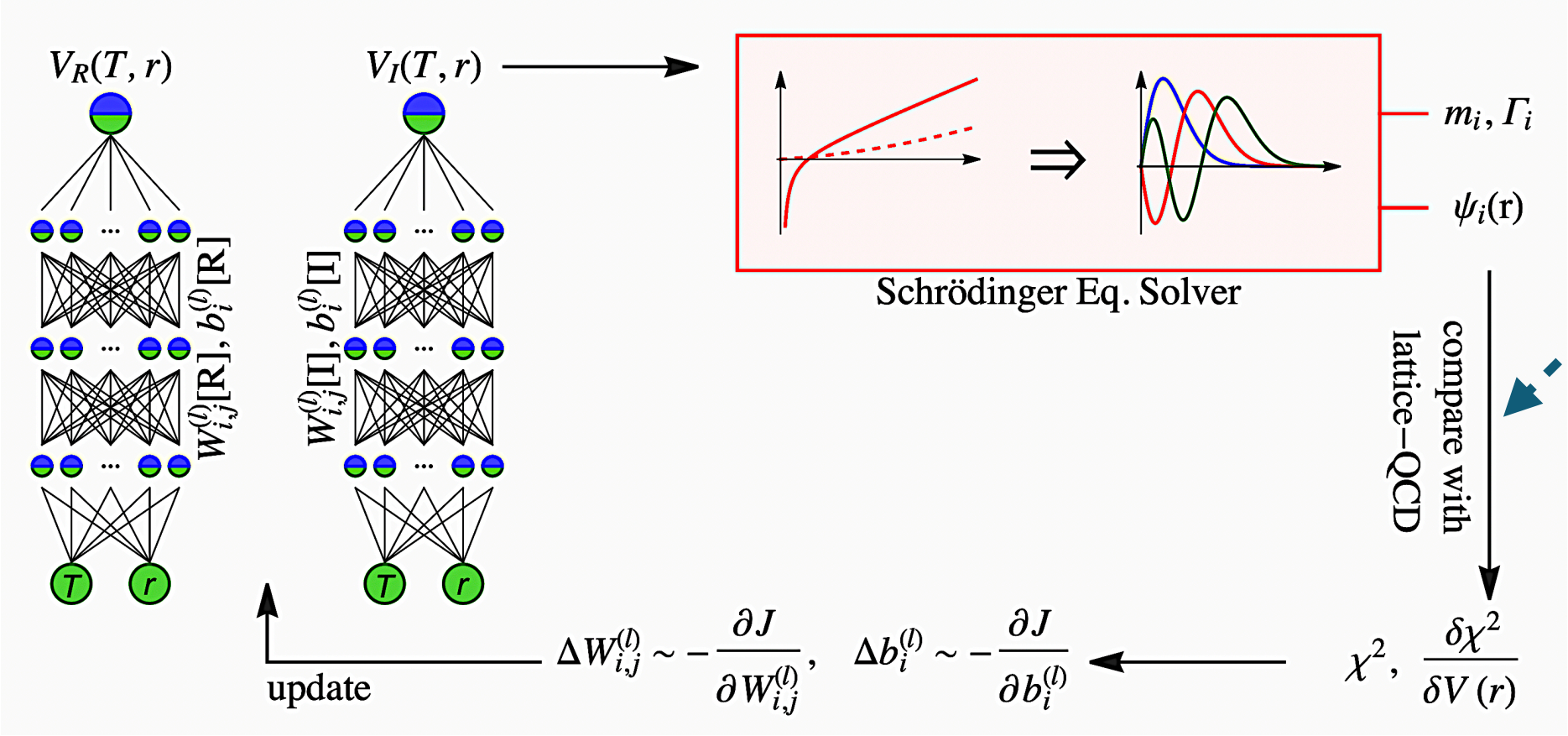
T=150 MeV: filled symbols

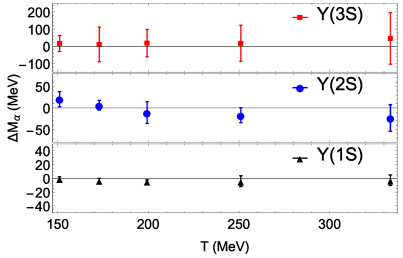


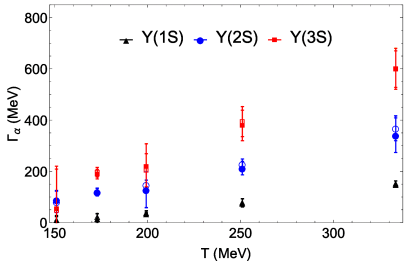
Rasmus Larsen et.al.: Phys. Rev. D102, 114508 (2020)

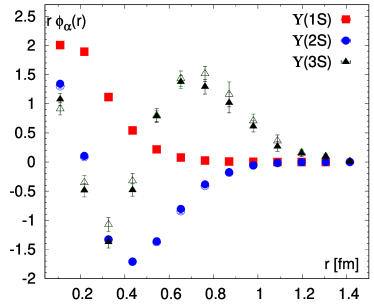
from bottomonia to heavy-quark potential in QGP

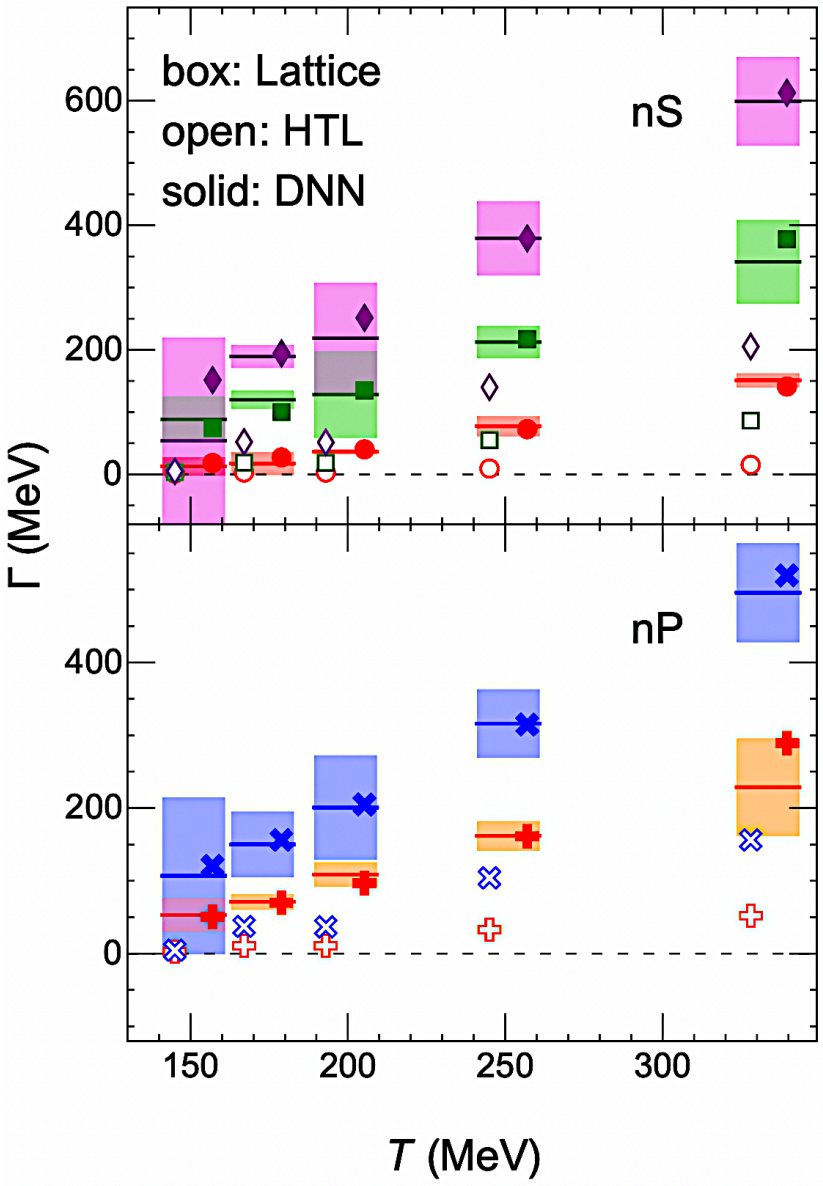
update: complex heavy-quark potential from LQCD results on $\Upsilon(nS)$, $\eta(nP)$

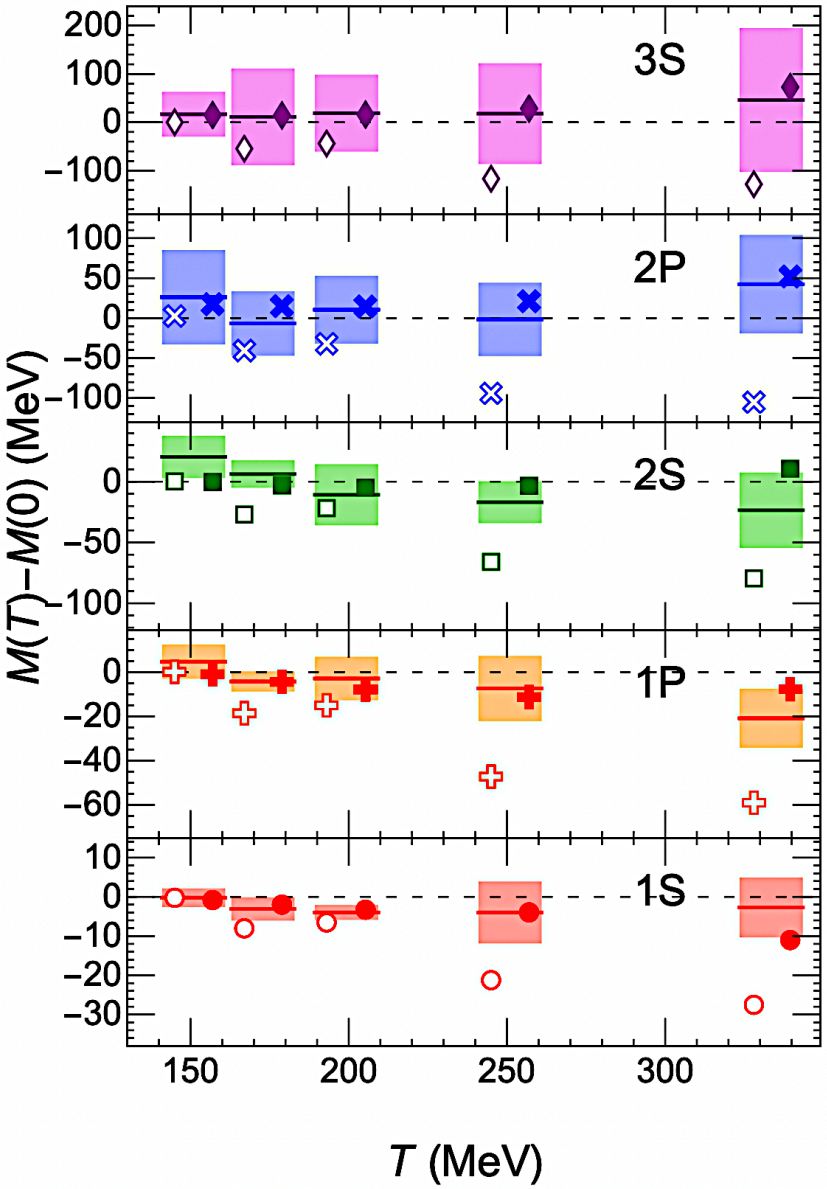










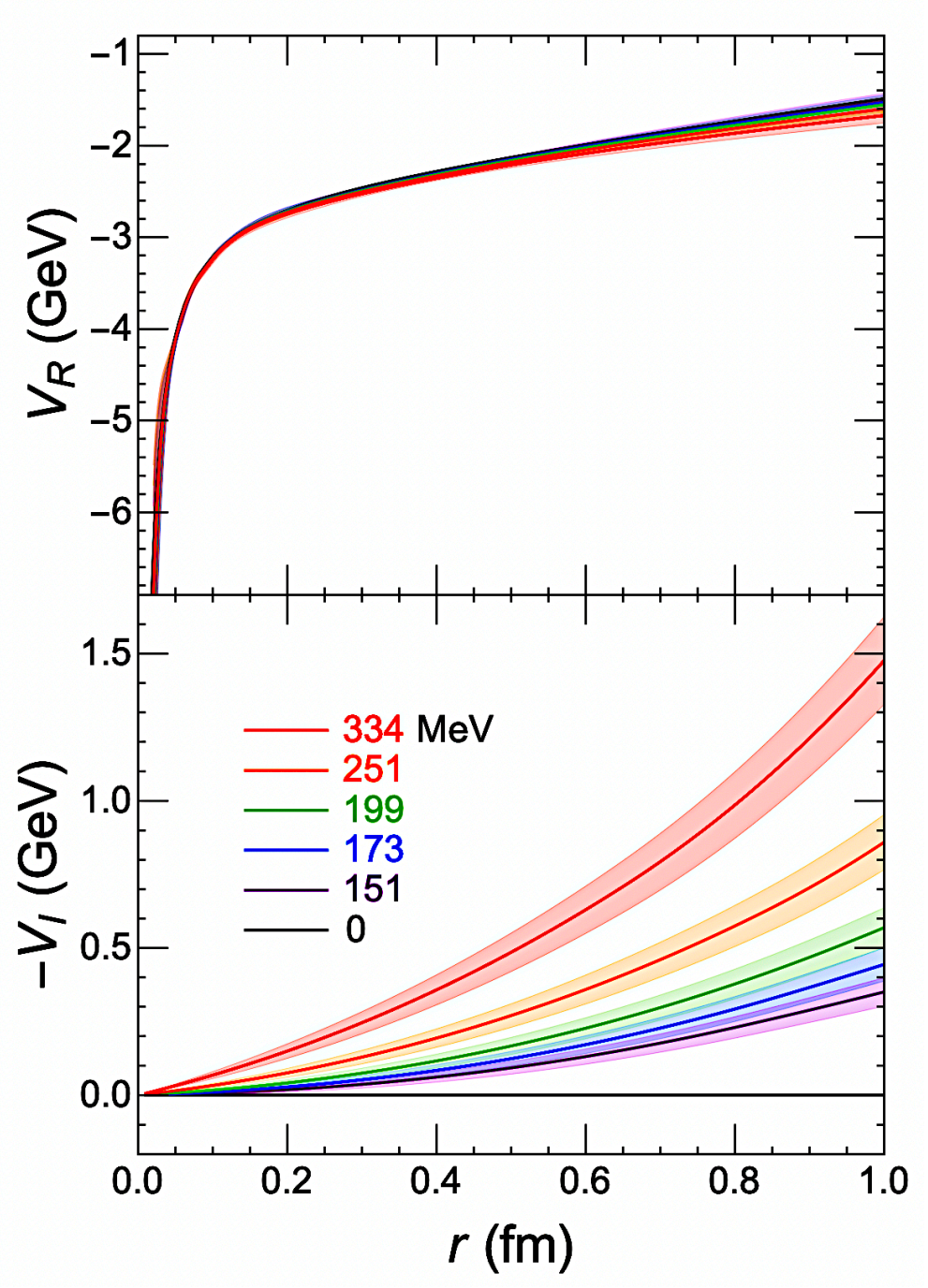


Shuzhe Shi, Kai Zhou et.al.: Phys. Rev. D105, 014017 (2022)

$\mathrm{Re}[V(r,T)]$: nearly T independent

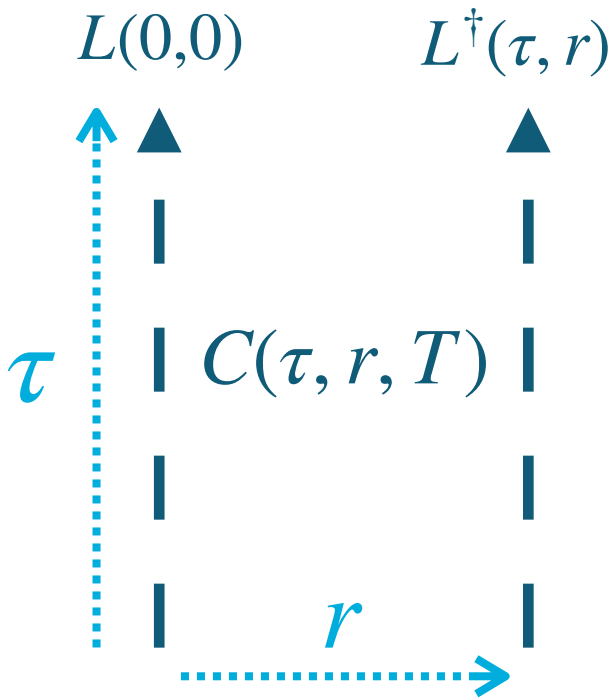
where is color screening in QGP ?

$\mathrm{Im}[V(r,T)]$: large, T dependent

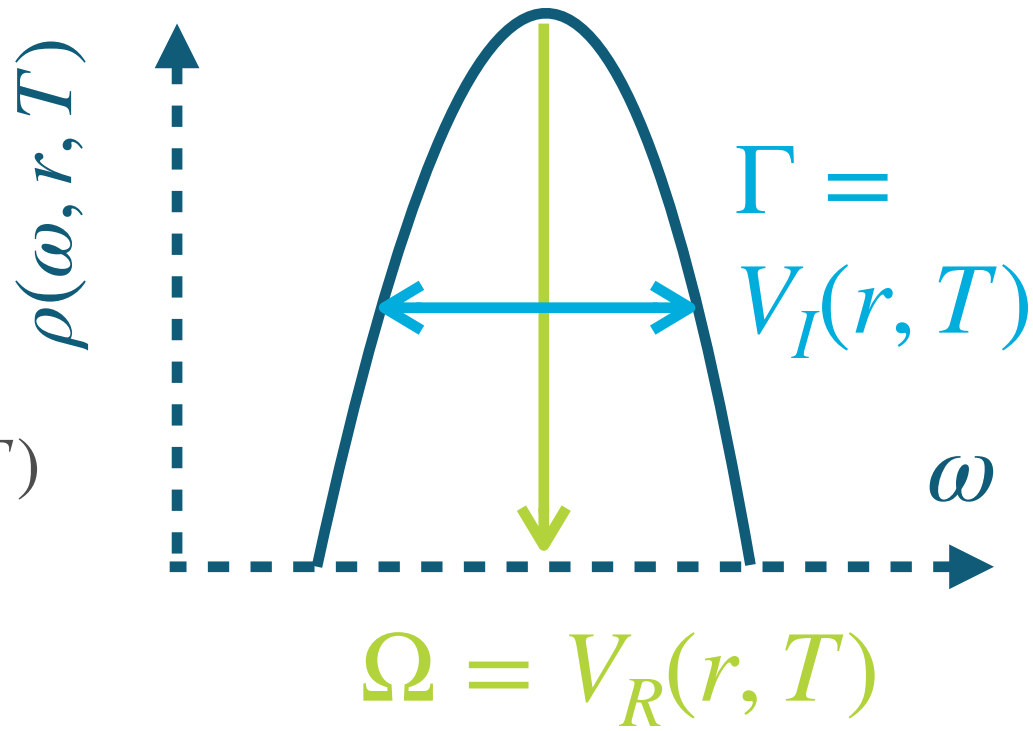


static-quark potential in QGP

update: first results for full QCD



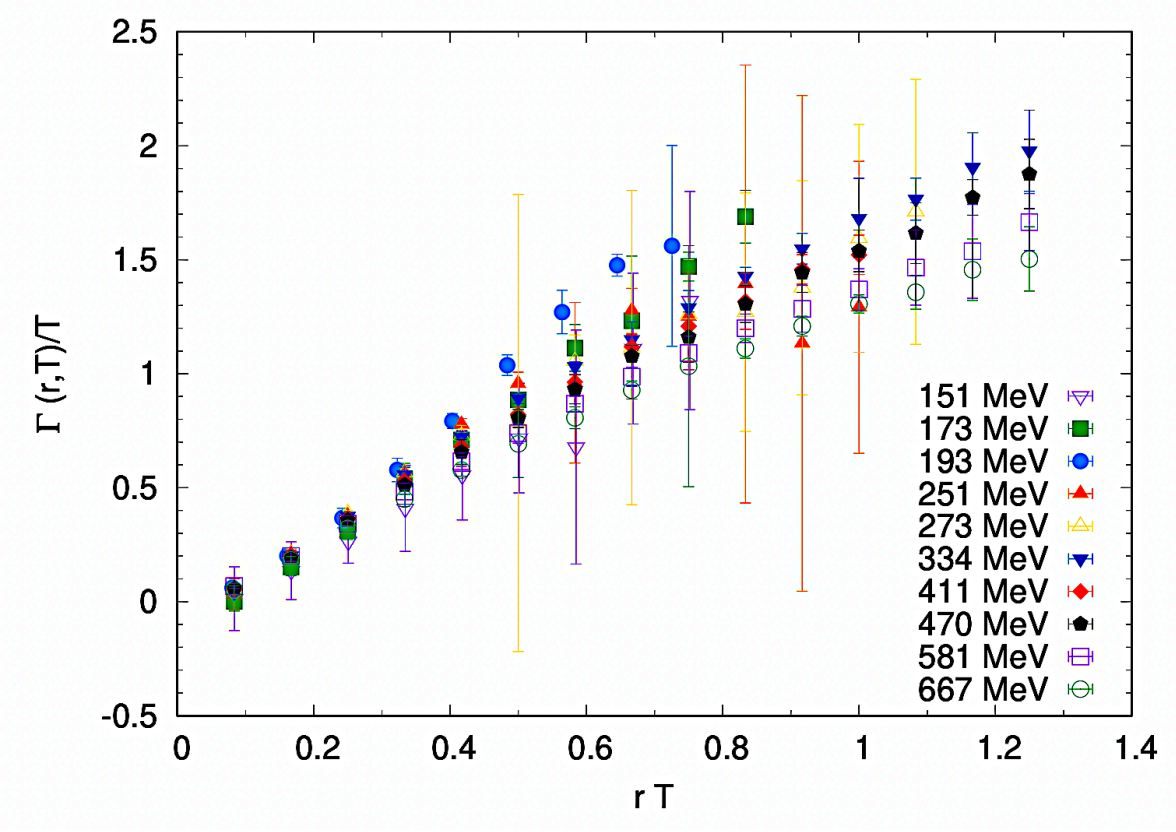
$$C(\tau, r, T) = \int_0^\infty d\omega \rho(\omega, T)$$

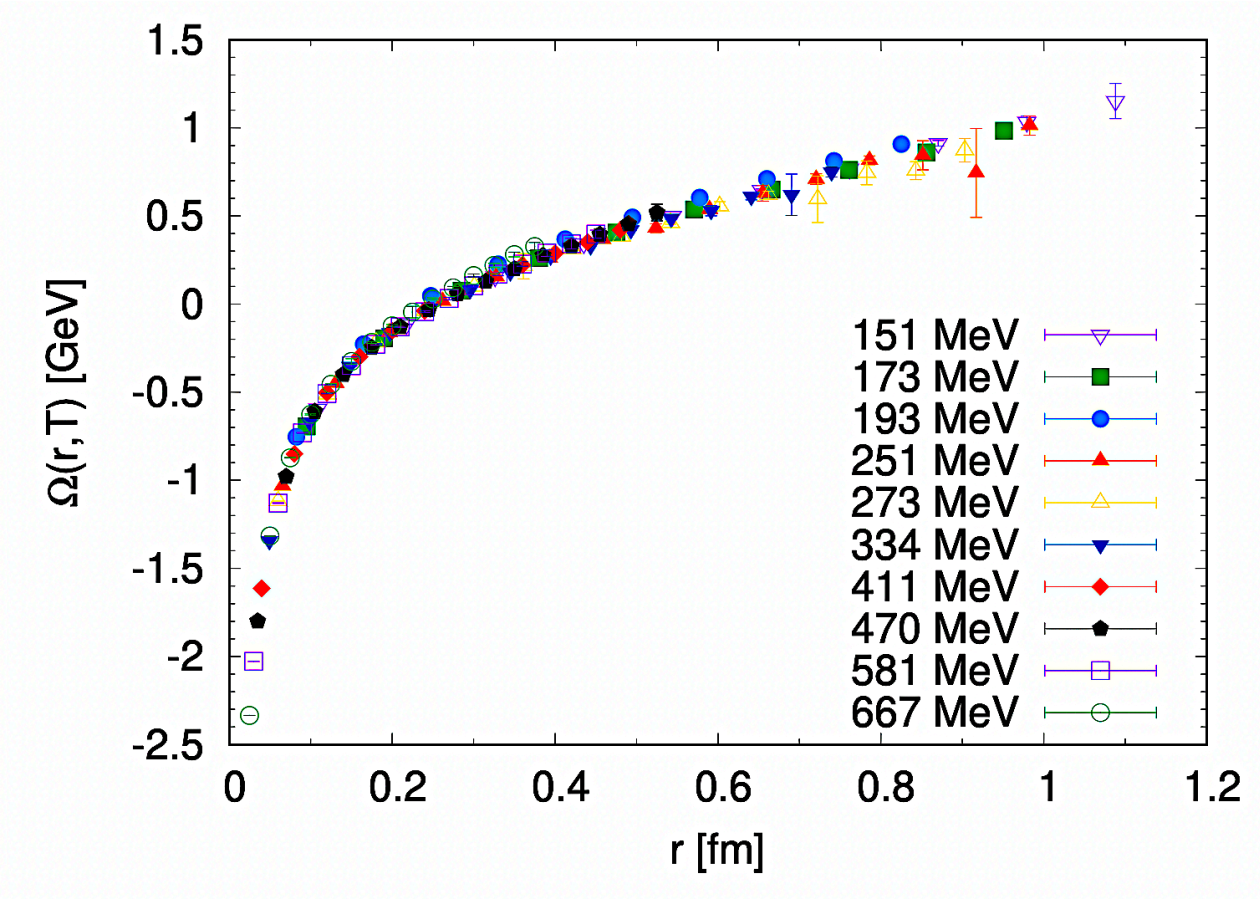


- subtract continuum using vacuum correlation
- characterize the dominant peak in $\rho(\omega, T)$

$\mathrm{Im}[V(r,T)]$: large, T dependent

$\mathrm{Re}[V(r,T)]$: nearly T independent



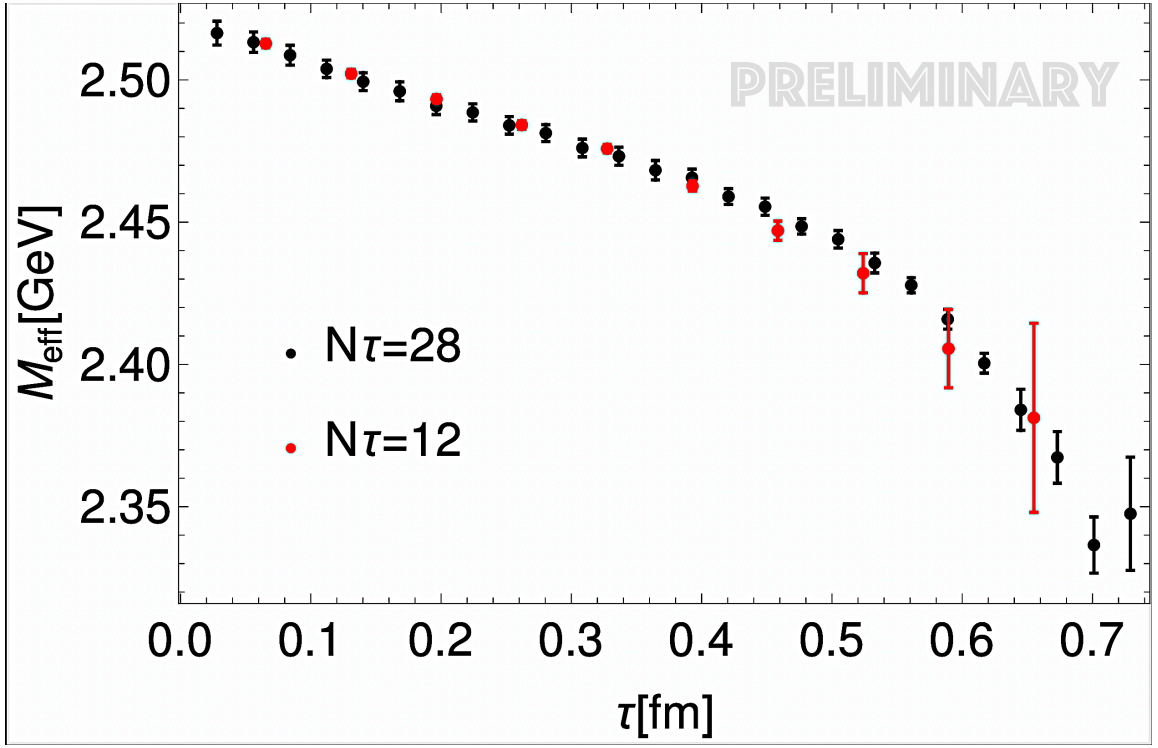


what happened to QGP color screening?

Rasmus Larsen et.al.: Phys. Rev. D105, 054513 (2022)

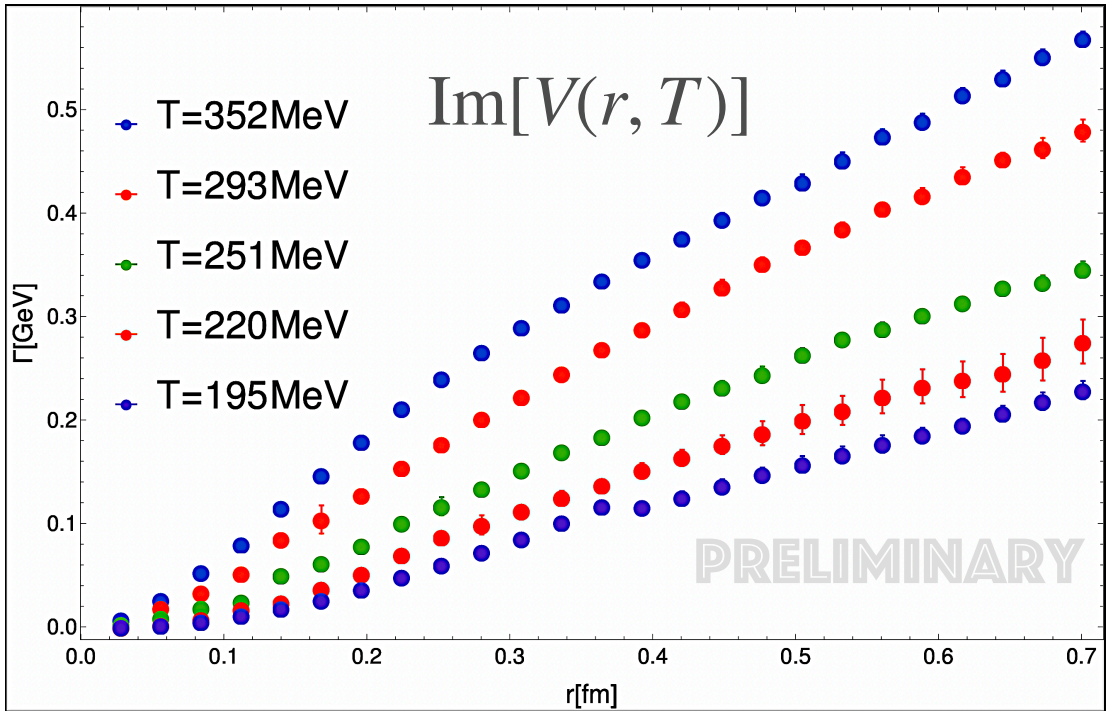
static-quark potential: finer resolution and more precise data

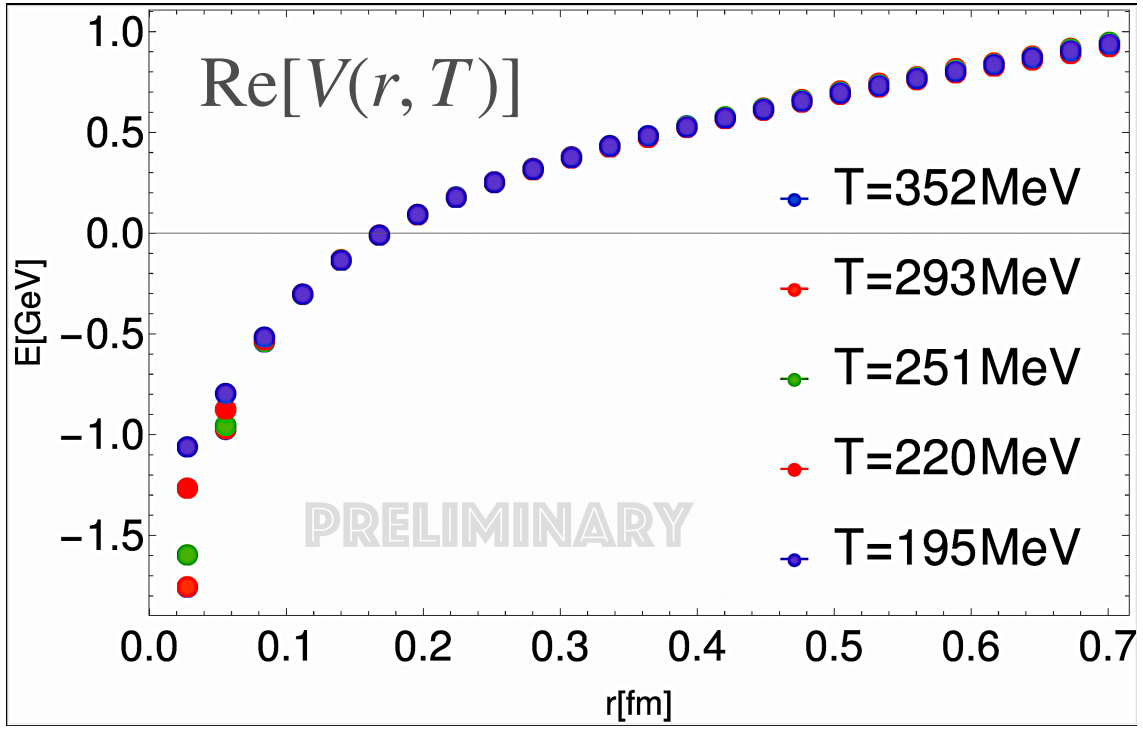
excellent agreement among large and small lattices



continuum-subtracted effective mass

reconfirm results





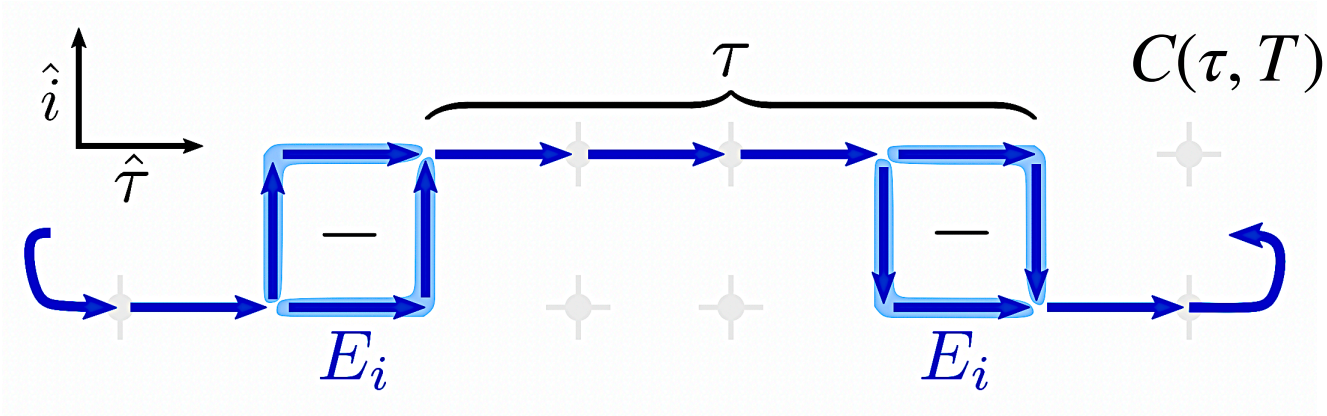
Rasmus Larsen et.al.: in preparation

due to dynamical quarks?

Gaurang Parkar's talk: Aug. 5, 15:40, Track C

heavy-quark diffusion constant

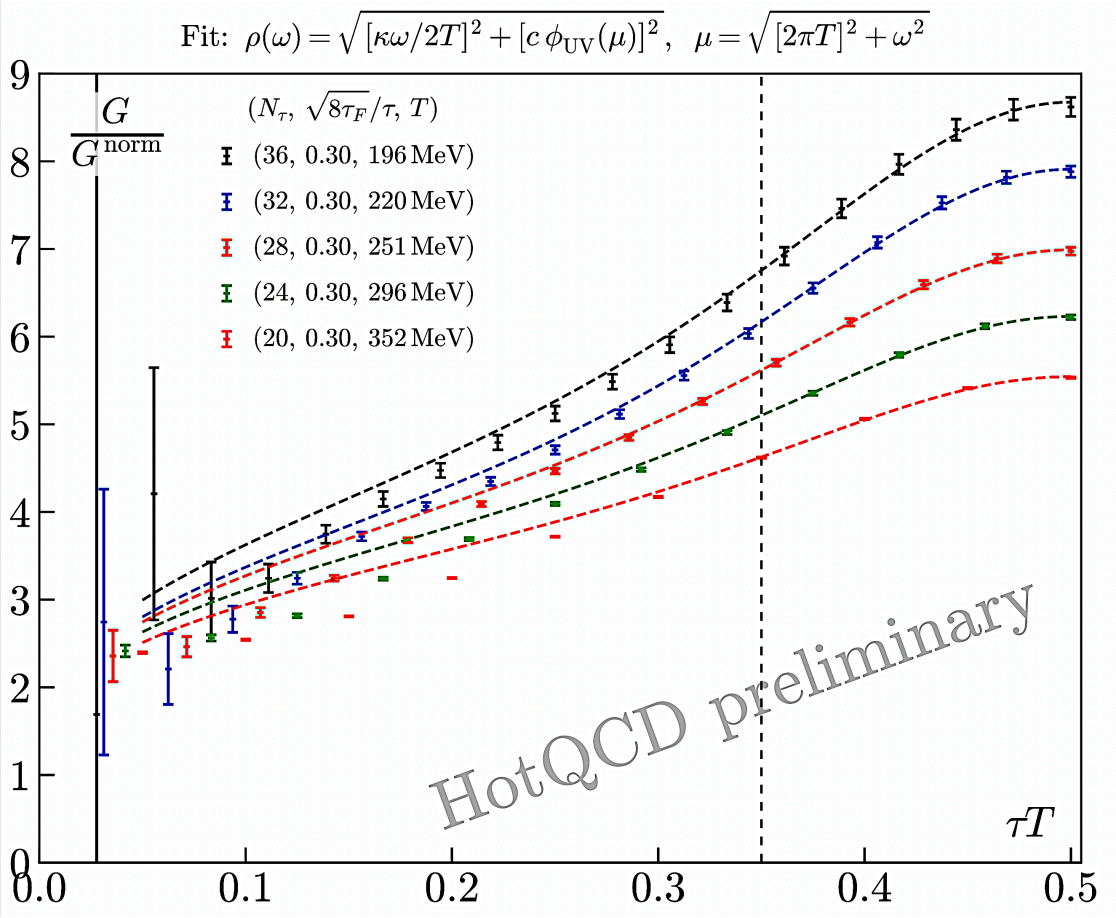
update: first results for full QCD



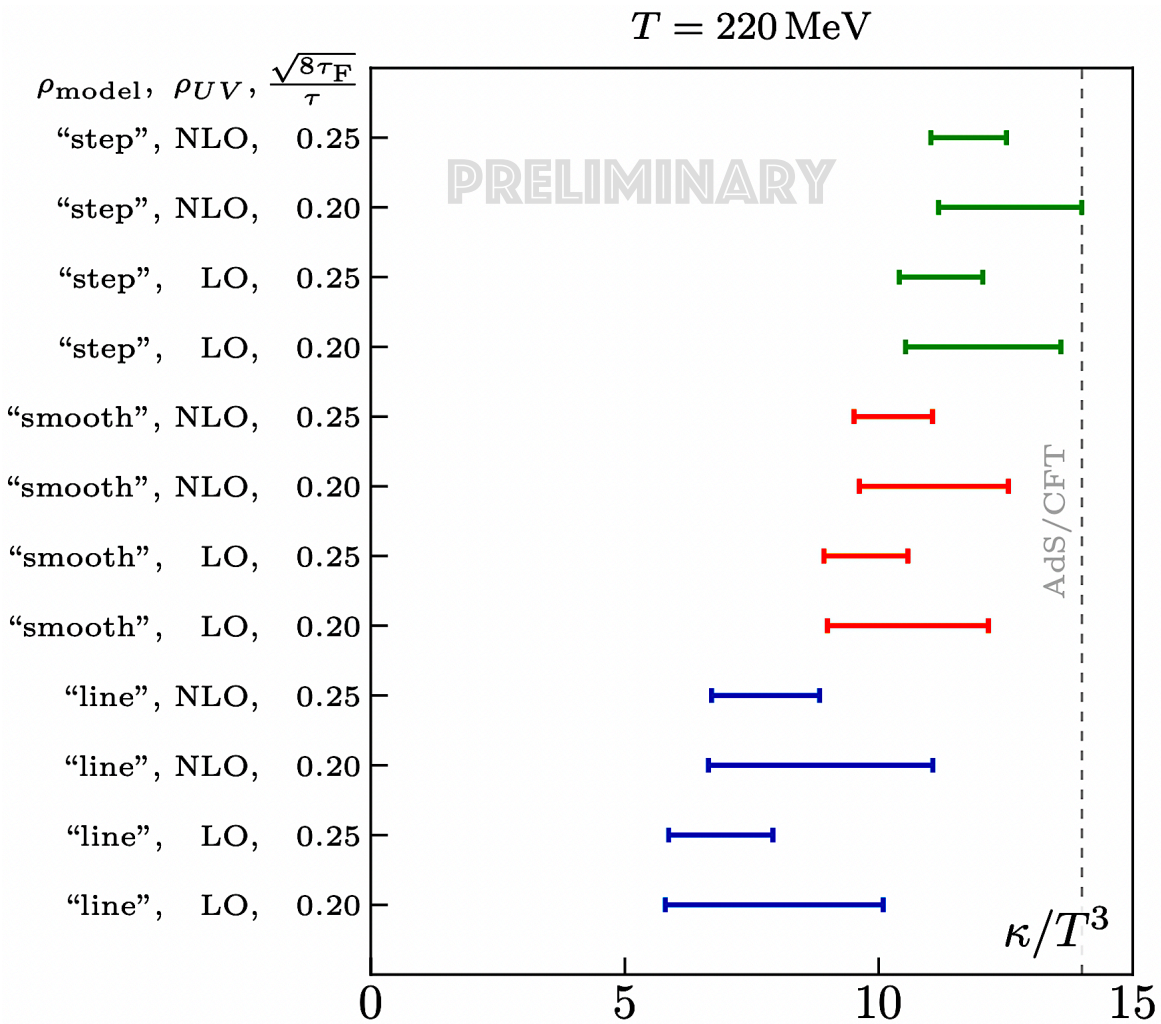
$$C(\tau, T) = \int_0^\infty d\omega \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh[\omega/(2T)]} \rho(\omega, T)$$

correlation of chromo-electric fields

$$D^{-1} = \frac{\kappa}{2T^2} = \frac{1}{T} \lim_{\omega \to 0} \frac{\rho(\omega, T)}{\omega}$$



fit model $\rho(\omega)$ to correlators

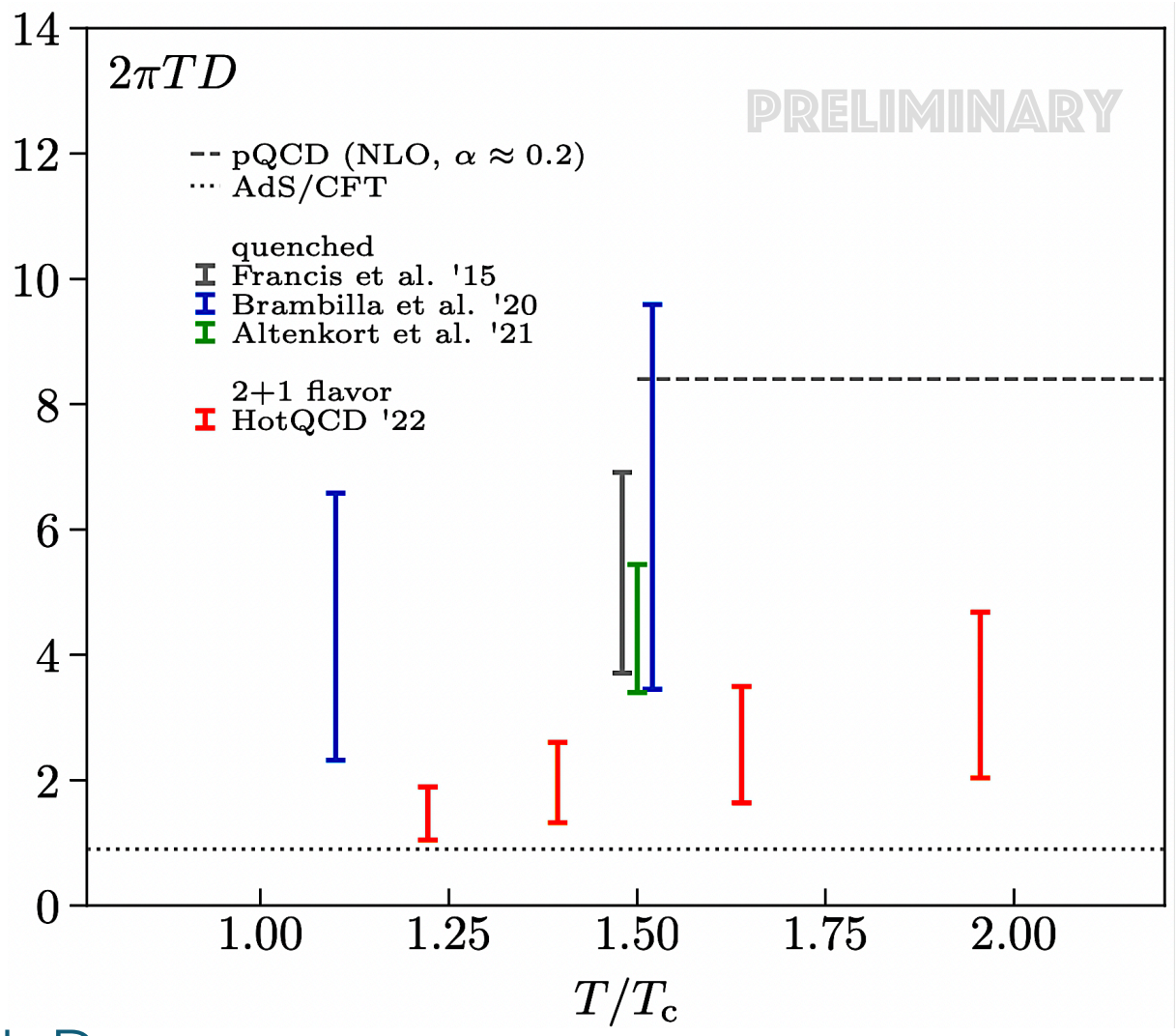


fit multiple models for $\rho(\omega)$

Luis Altenkort et.al.: in preparation

quenched lattice QCD results:

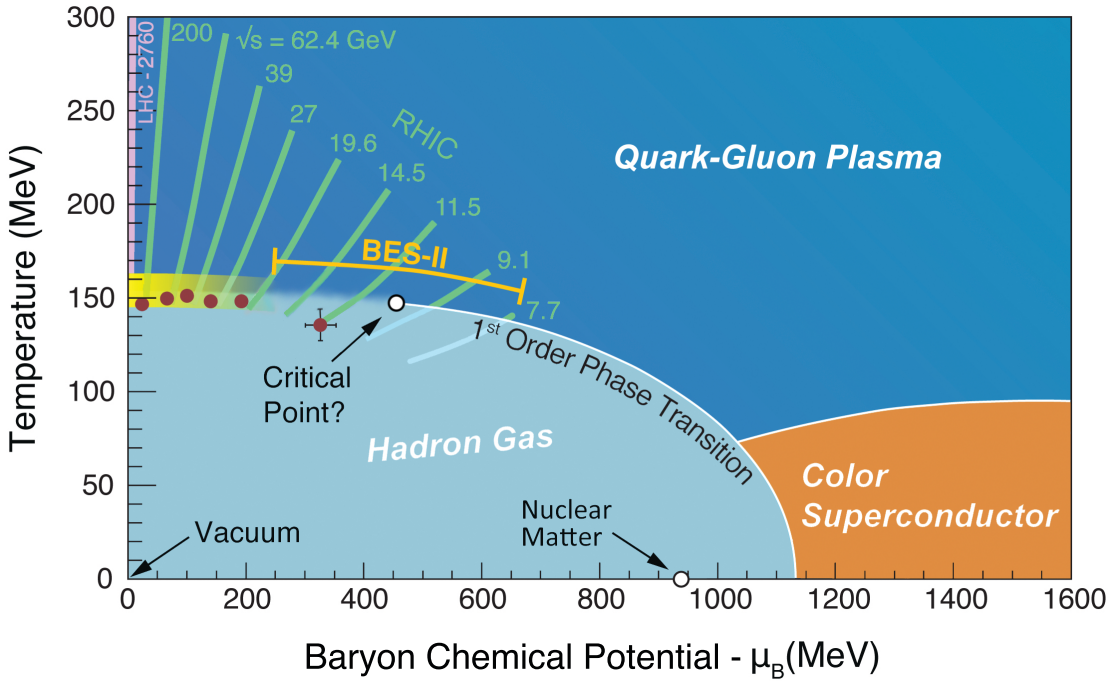
- Francis et.al., Phys. Rev. D92, 116003 (2015)
- Brambilla et.al., Phys. Rev. D102, 7, 074503 (2020)
- Altenkort et.al., Phys. Rev. D103,1, 014511, (2021)
- Brambilla et.al., 2206.02861



Hai-Tao Shu's talk: Aug. 4, 16:10, Track D

lattice QCD: hot and dense

- RHIC BES-II
- NA61/SHINE
- HADES
- CBM



- QCD equation of state ?
- location of the QCD critical point ?

Taylor expansion in μ_B

update: state-of-the-art Taylor expansion

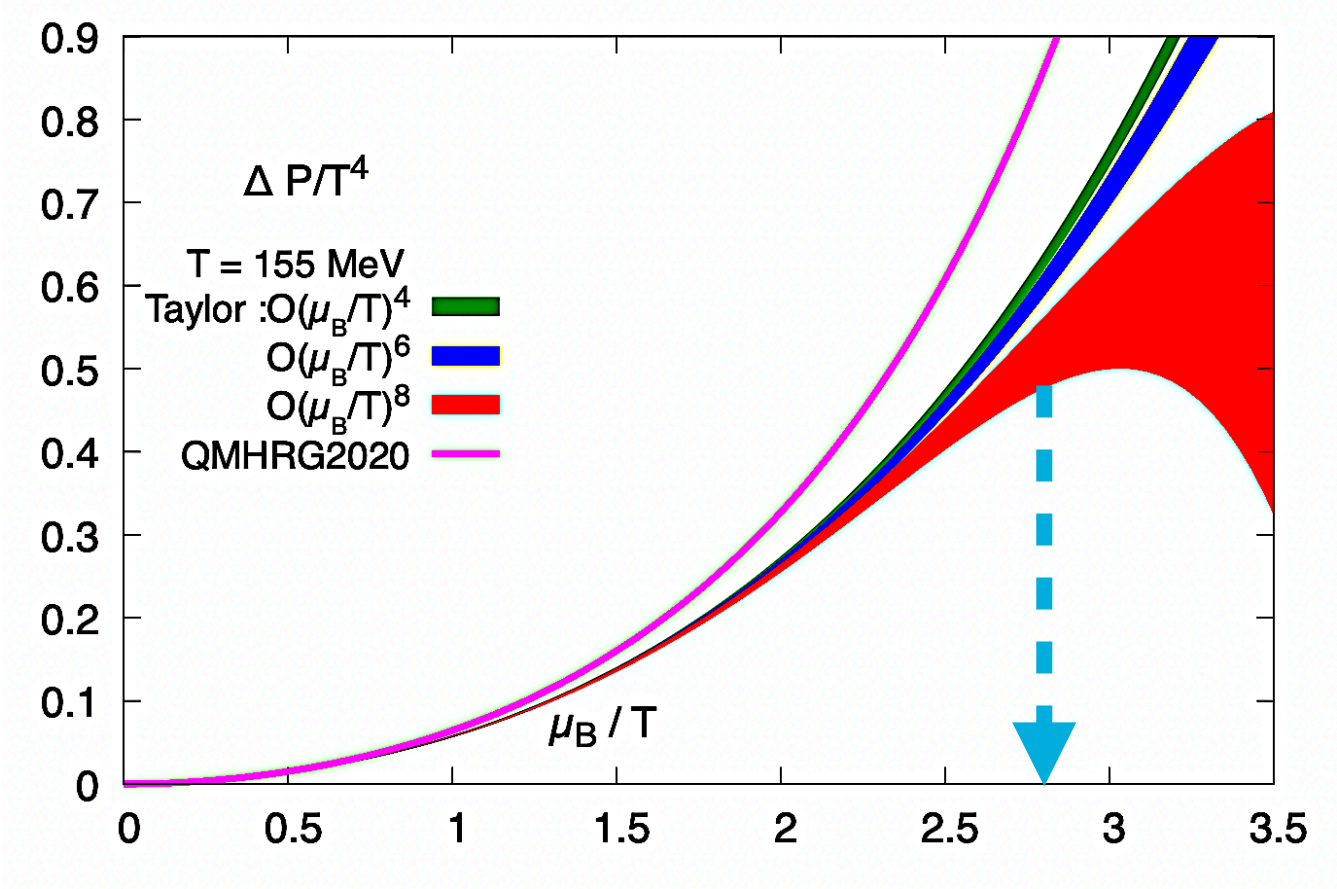
- Taylor expansion of pressure:

$$\frac{\Delta P_N^E}{T^4} = \sum_{n=1}^{N} \frac{\chi_n^B}{n!} \left(\frac{\mu_B}{T}\right)^n$$

$$\Delta P = P(T, \mu_B) - P(T,0)$$

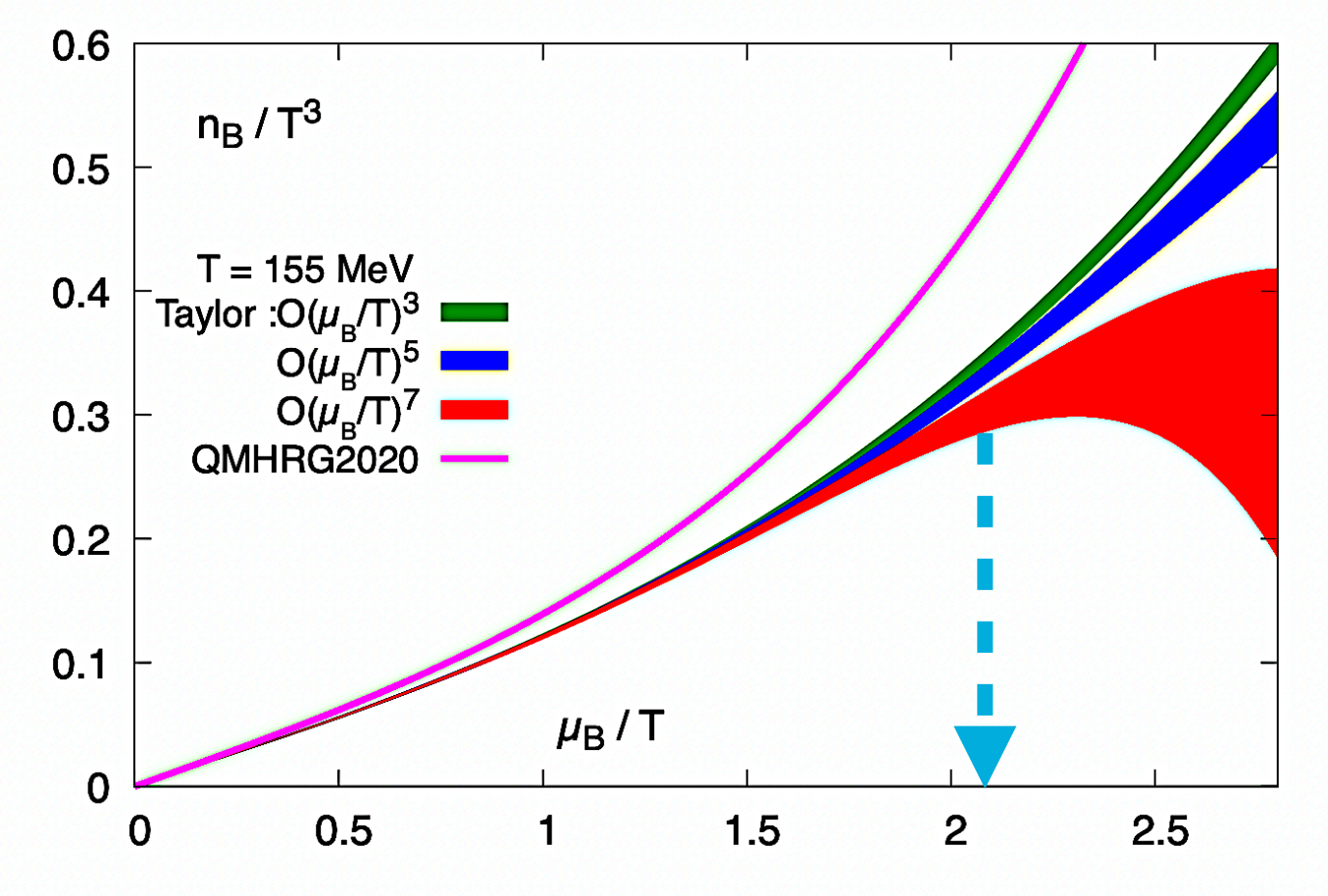
- expansion coefficients:

$$\chi_n^B(T) = \frac{1}{VT^3} \left. \frac{\partial^n \ln Z(T, \mu_B)}{\partial(\mu_B/T)^n} \right|_{\mu_B=0}$$



- Taylor expansion of net-baryon number density:

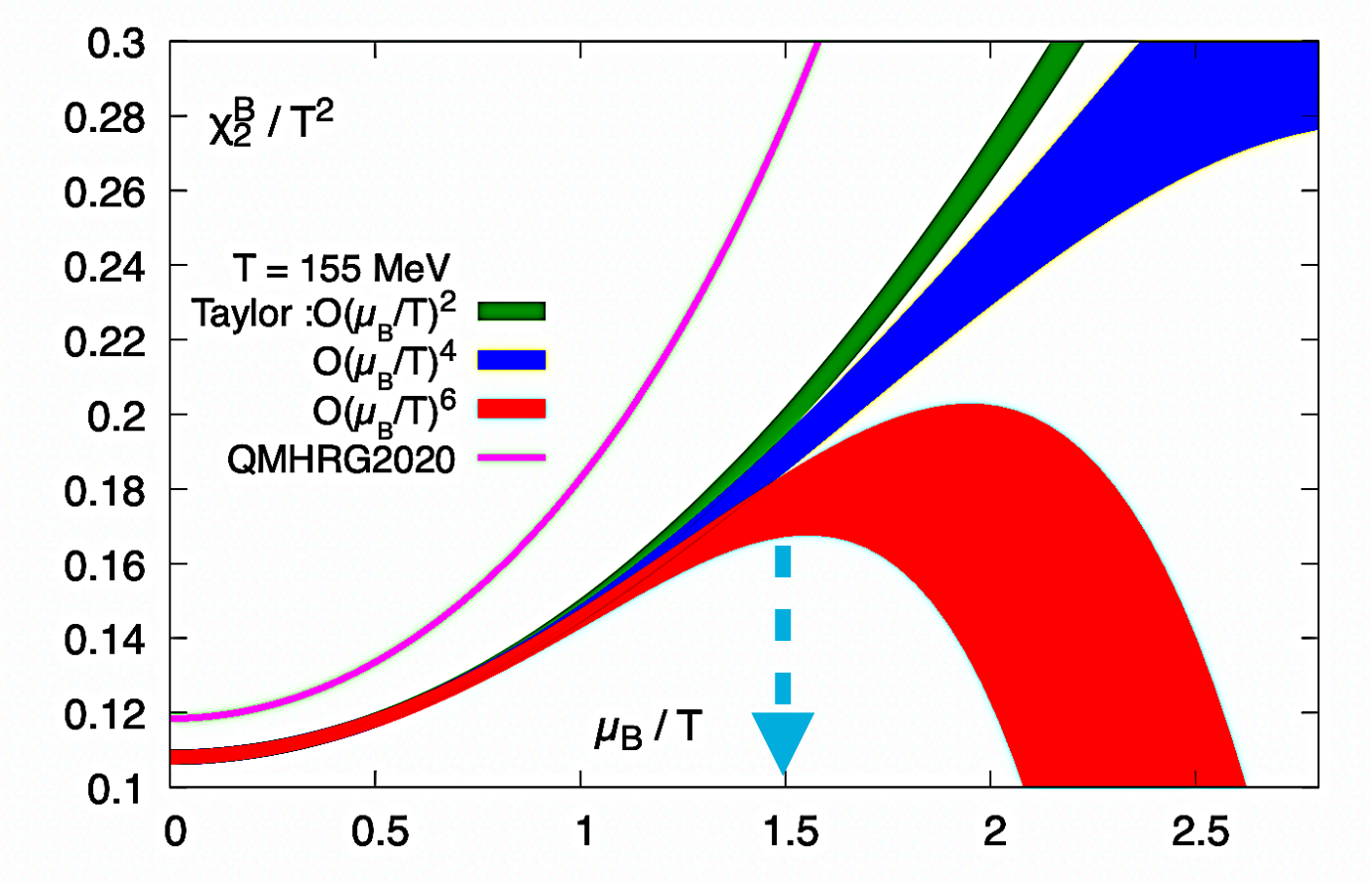
$$\mathcal{N}_N^E(T,\mu_B) = \frac{\partial P_N^E(T,\mu_B)}{\partial \mu_B}$$



- Taylor expansion baryon number susceptibility:

$$\chi_2^E(T,\mu_B) = \frac{\partial^2 P_N^E(T,\mu_B)}{\partial \mu_B^2}$$

reliable range of μ_B from empirical observations, non-monotonic behaviors



Jishnu Goswami, Phys. Rev. D105, 074511 (2022)

Taylor expansion: a new variant

update: expansions in T, μ_B around $T_0(\mu_B)$, 0

$$\Delta\left[\frac{P}{T^4}\right] \equiv \frac{P(T,\mu_B)}{T^4} - \frac{P(T_0,0)}{T_0^4}$$

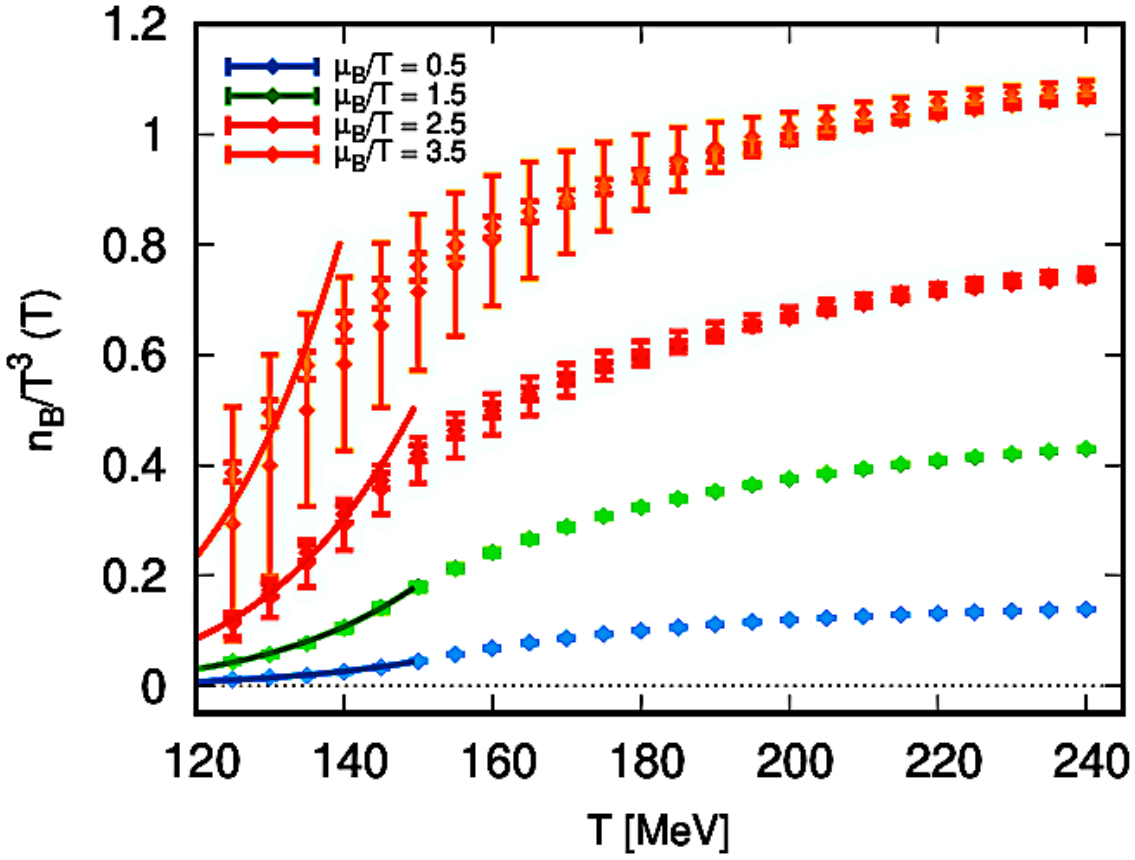
$$\Delta\left[\frac{P}{T^4}\right] = \left.\frac{d[P(T,0)/T^4]}{dT}\right|_{T_0} \Delta T + \frac{1}{2!}\chi_2^B(T_0)\hat{\mu}_B^2 + \frac{1}{2!}\left.\frac{d\chi_2^B(T)}{dT}\right|_{T_0} \hat{\mu}_B^2 \Delta T + \mathcal{O}(\hat{\mu}_B^4, (\Delta T)^2)$$

$$\Delta T = T - T_0(\mu_B)$$

choose $T_0(\mu_B)$ along a physics-motivated path in $T - \mu_B$ plane

empirically observed to work till larger value of μ_B

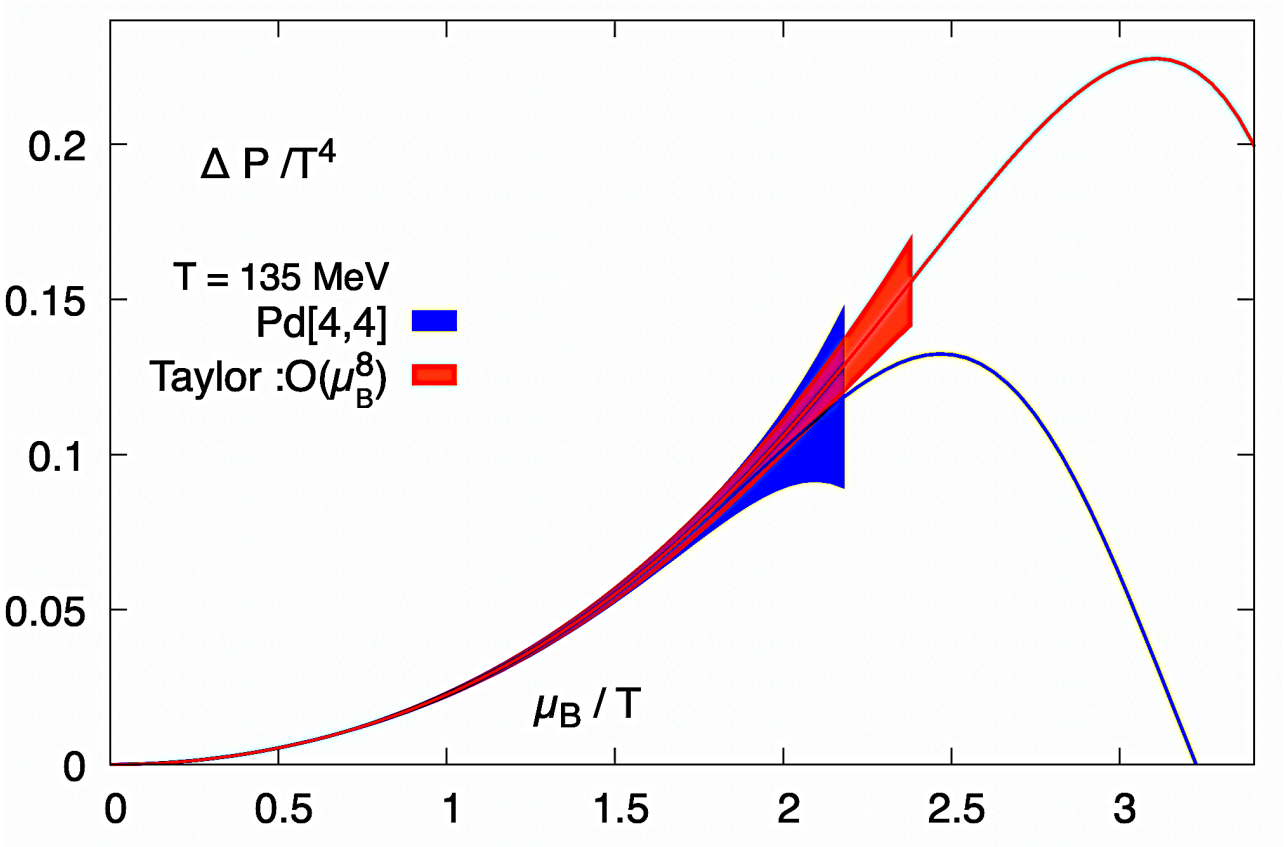
Borsanyi et.al.,Phys. Rev. Lett. 126, 232001 (2021)



… still an expansion up to $\mathcal{O}(\mu_B^4)$

Padé resummation of Taylor expansion

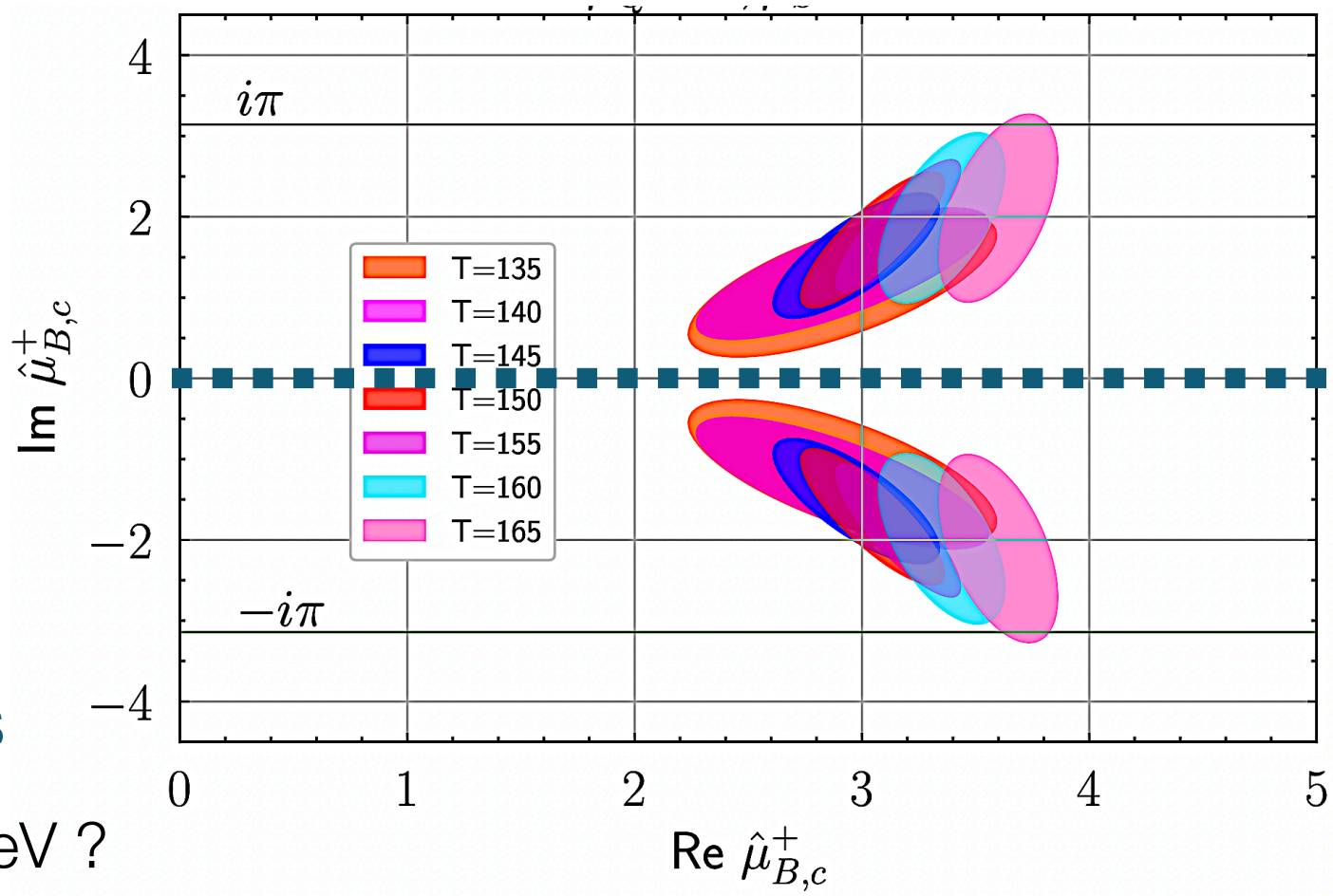
$$\sum_n \chi_n^B \mu_B^n \longrightarrow \sum_i \frac{a_j}{\mu_B - b_i}$$



Jishnu Goswami, Phys. Rev. D105, 074511 (2022)

$$\sum_n \chi_n^B \mu_B^n \longrightarrow \sum_i \frac{a_j}{\mu_B - b_i}$$

zeros of QCD partition function



no zero on the real-μ_B axis

no QCD critical point for T > 135 MeV ?

very sensitive to orders & accuracies of Taylor coefficients

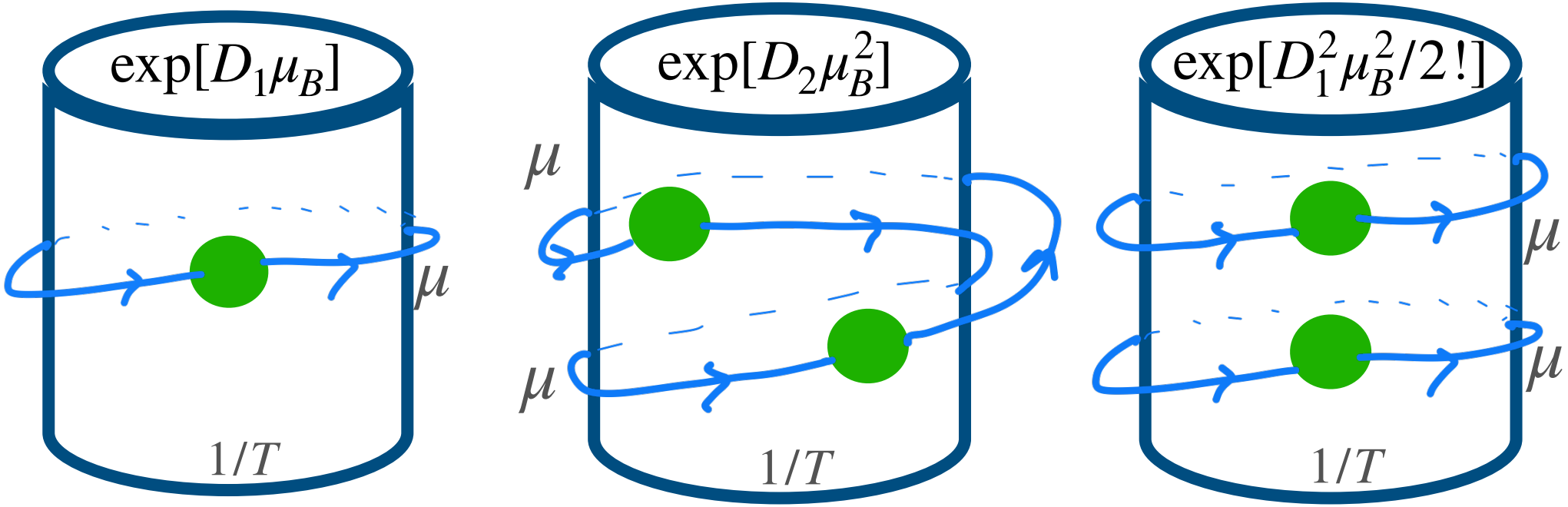
expansion in current-current correlation: all orders in μ_B

$$D_n = \int \mathrm{d}\mathbf{x_1} \cdots \mathrm{d}\mathbf{x_n} J_0(\mathbf{x_1}) \cdots J_0(\mathbf{x_n})$$

$J_0 \equiv \gamma_0$

$\vec{q}_f$

q_f



for a given gauge field background

- resummed partition function: contributions of up to N-pt current-current correlation to all orders in μ_B

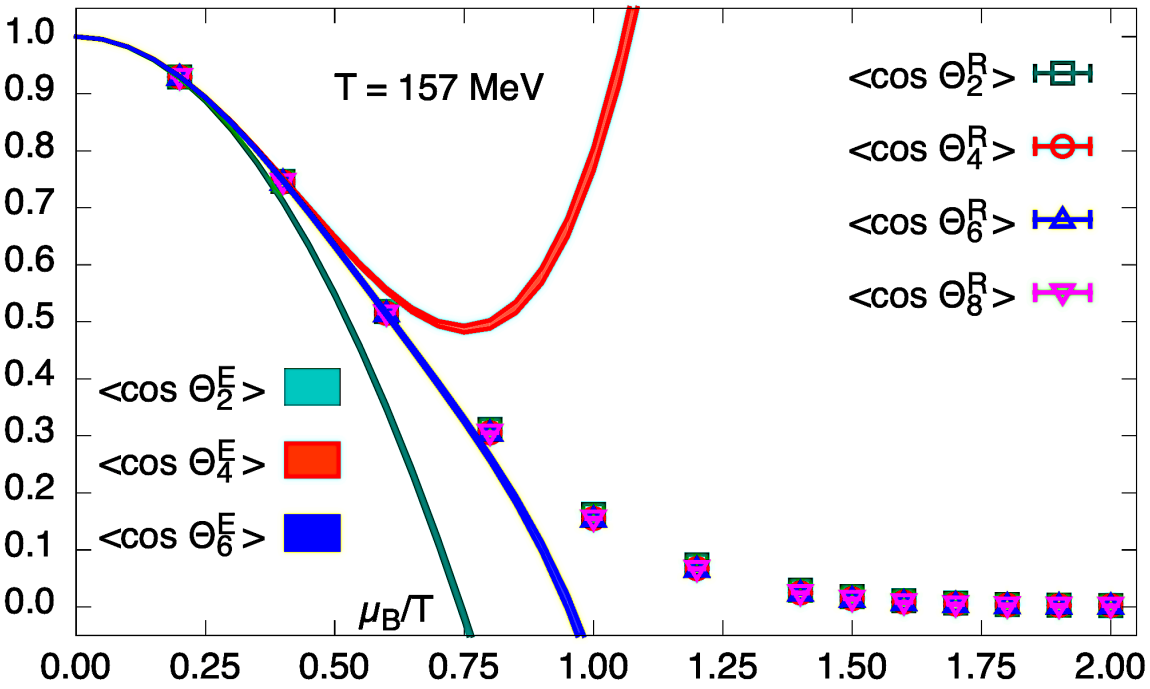
$$\frac{Z_N^R(T,\mu_B)}{Z(T,0)} = \left\langle \exp\left[\sum_{n=1}^{N} \bar{D}_n \left(\frac{\mu_B}{T}\right)^n\right]\right\rangle \qquad \bar{D}_n = D_n/n!$$

zeros of Z_N^R provides self-consistent check of reliability of results

not expansion of Z, but of fermion action on a fixed background

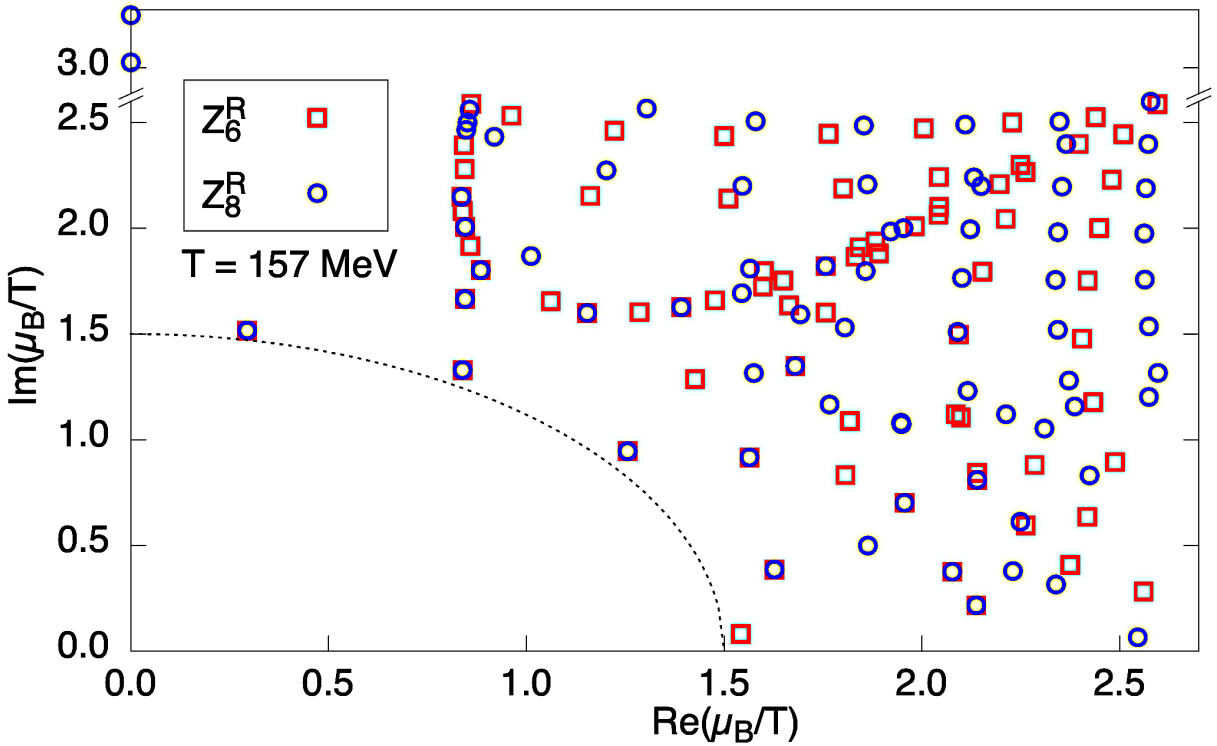
- has been generalized to resum all order in T, μ_B
  - generalization of Borsanyi et.al., Phys. Rev. Lett. 126, 232001 (2021)

average phase of partition function



excellent convergence

zeros of partition function



stable while increased order

Mondal, Mukherjee, Hegde, Phys. Rev. Lett. 128, 022001 (2022)

summary

- first results for up to 3S & 2P bottomonia in QGP
  - heavy-quark potential from these results
- QGP static-quark potential for full QCD
- heavy-quark diffusion constant for full QCD
- resummed QCD EoS to all orders in chemical potential
  - resumming contributions of up to N-point current-current correlations
  - self-consistent check of reliability