Higher Twists

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University of Regensburg

The XVth Quark confinement
and the Hadron spectrum conference

Stavanger, August 2022

At the beginning was . . .

PHYSICAL REVIEW D VOLUME 4, NUMBER 4 15 AUGUST 1971

Light-Cone Structure of Current Commutators in the Gluon–Quark Model*

David J. Gross† and S. B. Treiman
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 12 May 1971)

of the operator $\Theta^{(n)}_{\mu_1\ldots\mu_{Jn}}$. Evidently it is no longer the dimension alone that determines the importance of an operator near the light cone, but rather the difference between dimension and spin. We shall call this quantity the "*twist*" (τ) of an operator:

$$\tau_n \equiv d_n - J_n .$$

$$O_{\mu\mu_1\ldots\mu_n} = \bar{q}\gamma_\mu D_{\mu_1}\ldots D_{\mu_n} q$$

$$\tau = (n+3) - (n+1) = 2$$

Today . . .

What do we mean by leading and higher twist?

- "Geometric twist" $\longleftarrow$ dimension - spin
- "Collinear twist" $\longleftarrow$ dimension - spin projection on a light-like direction
- Generic usage: Power counting in large momentum;
  "higher twist" = power corrections

Why bother?

- New quality in experimental hadron physics on the 10 years scale
- Strive for high precision
- Want to learn more about hadrons:
  quark-gluon correlations and quantum inteference effects (H.D.Politzer, '80)

Partonic degrees of freedom

- In light-cone dominated processes, HT corrections are generated by
  - "minus" components of the spinors and gauge fields
  - parton transverse momenta/off-shellness
  - coherent scattering from a parton pair, higher Fock states

— These effects are related by exact QCD equations of motion (EOM)

Example: next slide

- Independent degrees of freedom $\longrightarrow$ operator basis
  - multipartonic (longitudinal) basis —simpler interpretation and evolution equations
    Politzer, Lipatov, Jaffe, '80-'83...
  - transverse derivatives — simpler coefficient function
    Ellis, Furmanski, Petronzio '83
  - $SL(2)$-covariant basis — symmetries made explicit
    VB, Manashov. Rohrwild, '09

Example: twist-three pion light-cone distribution amplitudes

V.B., Filyanov '90

$$\langle 0|\bar{q}(z)\gamma_5 q(0)|\pi(q)\rangle \mapsto \phi_p(x)$$

$$\langle 0|\bar{q}(z)\sigma_{+-}\gamma_5 q(0)|\pi(q)\rangle \mapsto \phi_\sigma(x)$$

$$\langle 0|\bar{q}(z)\sigma_{+\perp} gF_{+\perp}(\alpha z)\gamma_5 q(0)|\pi(q)\rangle \mapsto \Phi_3(x_1,x_2,x_3)$$

$$\Phi_3 = 360 x_1 x_2 x_3^2 \big[\omega_{00} + \frac{1}{2}\omega_{10}(7x_3-3) + \omega_{20}(2-4x_1x_2-8x_3(1-x_3)) + \omega_{11}(3x_1x_2-2x_3+3x_3^2)+\ldots\big]$$

$$\phi_p = 1 + 30\omega_{00}C_2^{1/2}(2x-1) + \frac{3}{2}(4\omega_{20}-\omega_{11}-2\omega_{10})C_2^{1/2}(2x-1)+\ldots$$

$$\phi_\sigma = 6x(1-x)\big[1 + (5\omega_{00}-\tfrac{1}{2}\omega_{10})C_2^{3/2}(2x-1) + \frac{1}{16}(4\omega_{10}-\omega_{11})C_4^{3/2}(2x-1)+\ldots$$

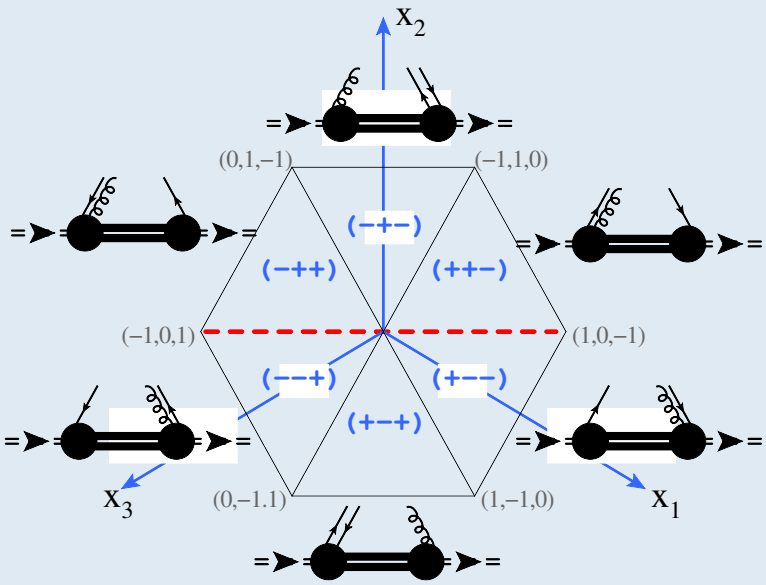
— These relations between Φ_3, ϕ_P and ϕ_σ are exact in QCD

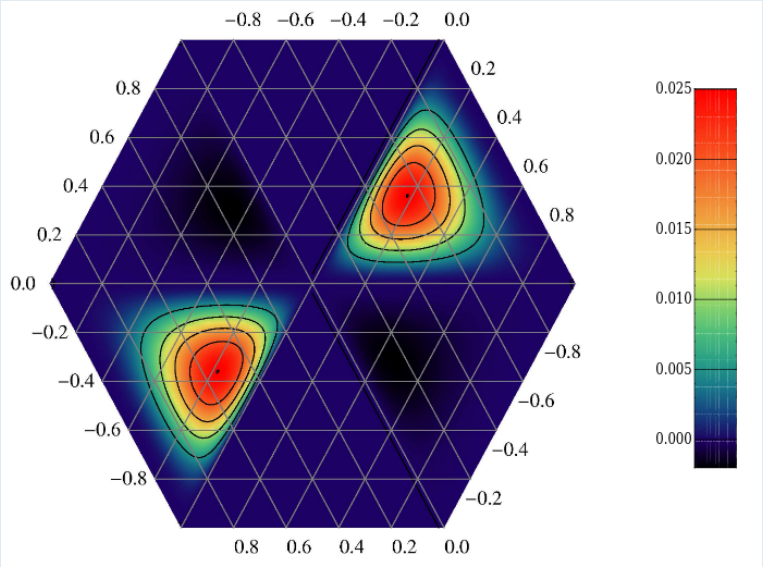
— ω_{ik} are nonperturbative parameters

- Taking into account only two-particle twist-3 LCDAs is not selfconsistent, see Anikin et al., arXiv:0909.4090 for $\gamma^* N \to \rho_T N$

Quark-antiquark-gluon correlation function in the proton

— Complete information on twist-three observables encoded in a function of two variables





— Domain interpretation (left) and the light-front wave function overlap model arXiv:1103.1269 (right) for the twist-three u-quark-antiquark-gluon correlation function in the proton

— Scale dependence well understood, summary in arXiv:0909.3410

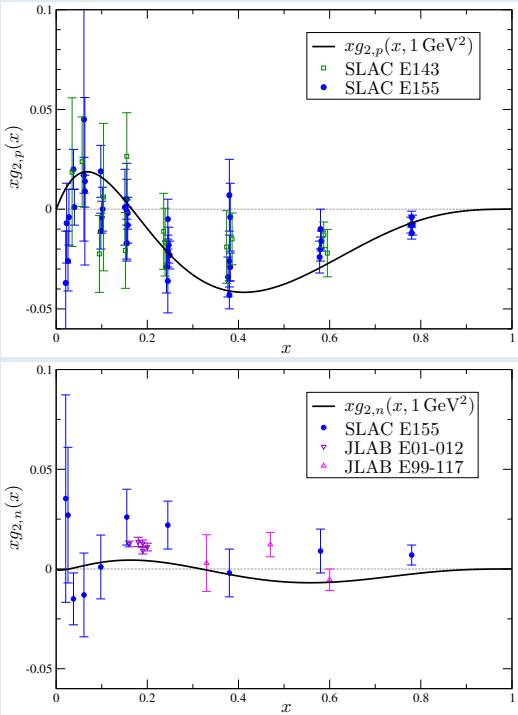
1. $\leftrightarrow$ The structure function $g_2(x, Q^2)$ — certain integral over the hexagon
2. $\leftrightarrow$ Qiu-Sterman function (spin asymmetries in SIDIS) — horizontal red line
3. $\leftrightarrow$ Lattice

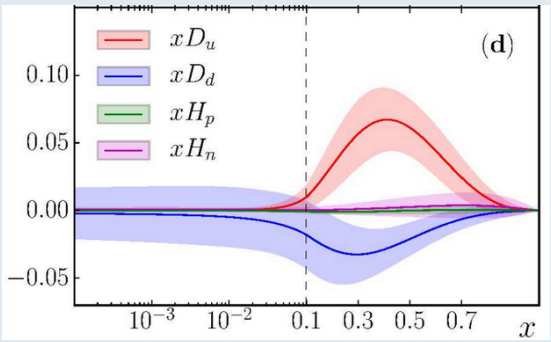
The structure function $g_2(x, Q^2)$

- $g_2(x, Q^2)$ has *collinear* twist three: $1/Q$ suppressed
- $g_2(x, Q^2)$ contains contributions of *geometric* twist two and twist three

$$g_2^{(\tau 2)}(x, Q^2) = g_1(x, Q^2) - \int_x^1 \frac{dy}{y} g_1(y, Q^2) \qquad \Leftarrow \quad \text{Wandzura-Wilczek}$$

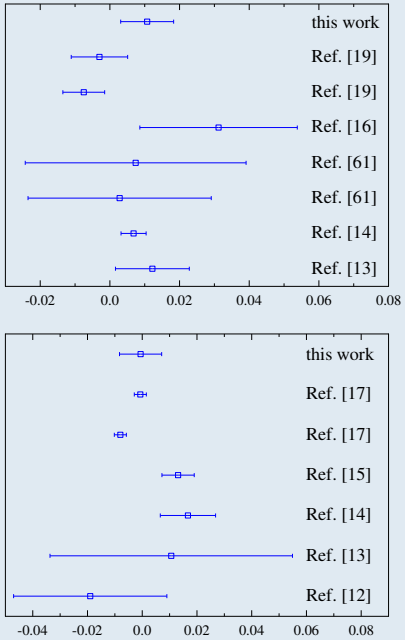
$$g_2^{(\tau 3)}(x, Q^2) \stackrel{!}{=} D(x, Q^2) - \int_x^1 \frac{dy}{y} D(y, Q^2) \qquad \Leftarrow \quad \text{convenient parametrization}$$





JAM arXiv:1601.07782

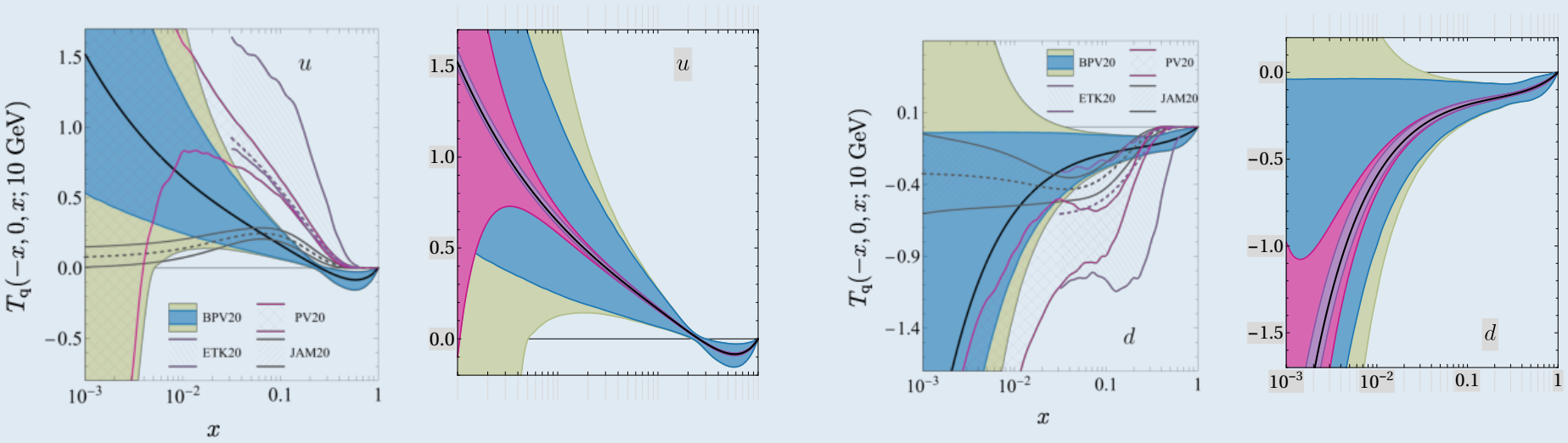
$d_2 = 6 \int dx x^2 \, g_2^{(\tau 3)}(x)$



RQCD arXiv:2111.08306

Qiu-Sterman function

— Extractions from Sivers function at small $\mathbf{b}$:



Left panels: BPVV, arXiv:2012.05135

Right panels: EIC impact studies (A. Vladimirov)

Twist four

— Seven parton distribution functions of two and three variables

nonperturbative input complicated

— Coefficient functions for DIS available since 1981–1983

R.L. Jaffe, M. Soldate '81-'82, R.L. Jaffe '83

— One-loop evolution equations known

quasipartonic operators A. Bukhvostov *et al.* '85

complete results V.B., A. Manashov, J. Rohrwild '10

— Phenomenology inconclusive (effects are "small")

— Extension to fragmentation functions available

I. Balitsky, V.B. '91

Concept

M. Beneke, Phys.Rept. 317 (1999)
M. Beneke, V.B., hep-ph/0010208

- Leading twist calculation "knows" about the necessity to add a power correction

Example:

$$F_2(x,Q^2) = 2x\int\limits_x^1 \frac{dy}{y} C(y,Q^2/\mu^2) q(\tfrac{x}{y},\mu^2) + \frac{1}{Q^2} D_2(x)$$

$$C(y) = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1}, \qquad \alpha_s = \alpha_s(\mu)$$

Cut-off scheme

Imagine the separation between CFs and MEs is done using exlicit cutoff at $|k| = \mu$.
CFs will be modified compared to usual calculation by terms $\sim \mu^2/Q^2$

$$C(y)|^{\rm cut} = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} - \frac{\mu^2}{Q^2} d(x) + \mathcal{O}\left(\frac{\mu^4}{Q^4}\right)$$

The dependence on μ must cancel:

- Logarithmic terms $\ln Q^2/\mu^2$ in CFs against μ-dependence in PDFs
- Power-terms μ^2/Q^2 against the higher-twist contributions

This means that $D_2(x)$ *in the cutoff scheme* must have the form

$$D_2(x) = \mu^2 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y}) + \delta D_2(x)$$

— related to quadratic UV divergences in matrix elements of twist-4 operators (in this scheme!)

Dimensional regularization

- In dim.reg. power-like terms in the CFs do not appear.
  Instead, the coefficients c_k diverge factorially with the order k

— The sum of the pert. series is only defined to a power accuracy and this ambiguity (renormalon ambiguity) must be compensated by adding a non-perturbative higher-twist correction

— Detailed analysis [Beneke:2000kc]: large-order behavior of the coefficients (the renormalons) is in one-to-one correspondence with the sensitivity to extreme (small or large) loop momenta

— IR renormalons in twist-two CFs are compensated by UV renormalons in MEs of twist-four operators. At the end the same picture re-appears: only the details depend on the factorization method

Deep inelastic scattering

Quadratic term in μ is spurious since its sole purpose is to cancel the similar contribution to the CF $\Rightarrow$ does not contribute to any physical observable.

— Assume that the "true" twist four term is of the same order, get a **renormalon model**

$$D_2(x) = \varkappa \Lambda^2_{\rm QCD} 2x \int\limits_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y}), \qquad \varkappa = \mathcal{O}(1)$$

One-loop result:

$$\boxed{\begin{aligned} d_2^{(q)} &= -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x) \\ d_L^{(q)} &= 8x^2 - 4\delta(1-x) \end{aligned}}$$

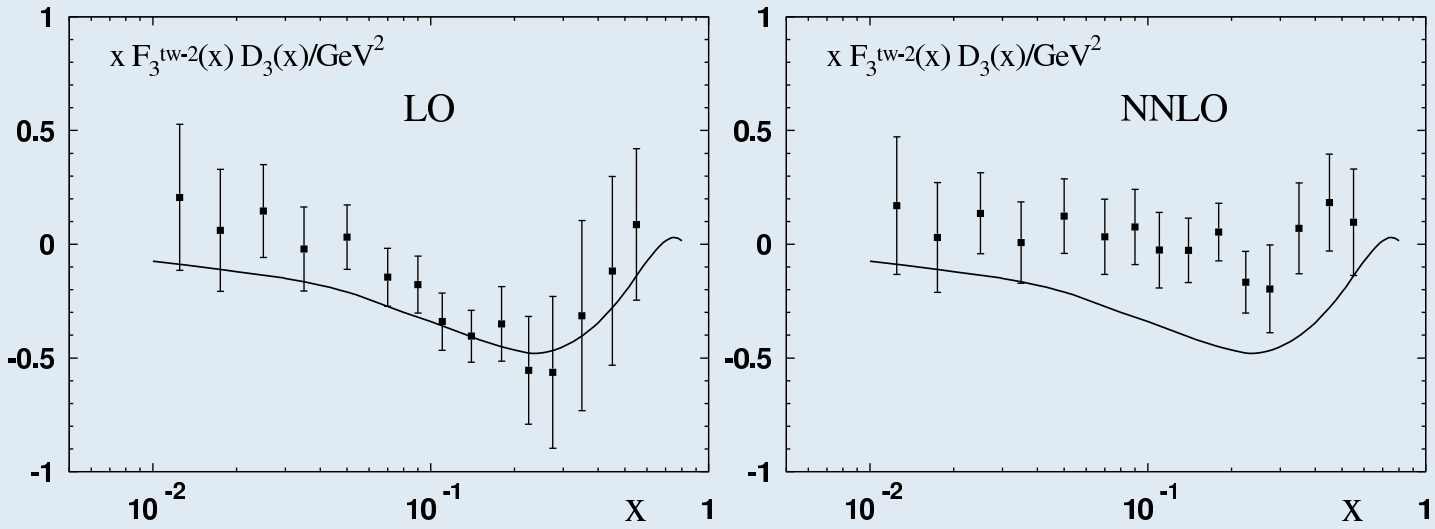
Main conclusions:

- ratio twist-4/twist-2 is target-independent (if assumption correct)
- enhancement at $x \to 1$:

$$\left[\frac{\Lambda^2}{Q^2(1-x)}\right]^n$$

Is $\varkappa$ universal? E.g. the same for F_L and F_2?
Dokshitzer, Webber: $\varkappa \to \bar{\alpha}_0$ universal nonperturbative coupling

Example: CCFR data on $F_3(x, Q^2)$:



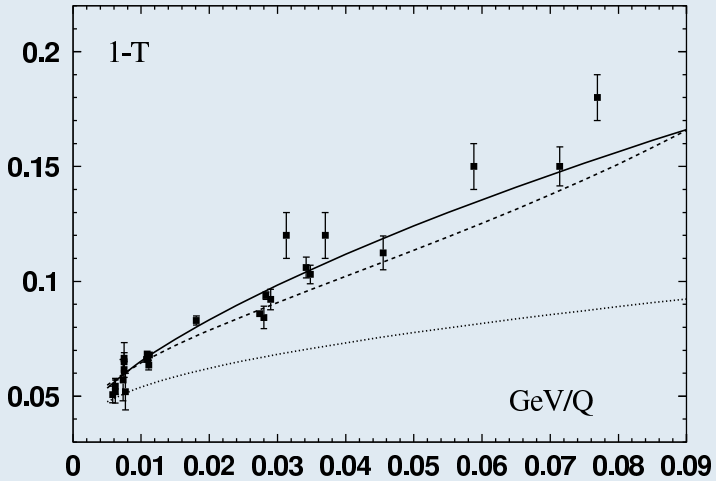
based on: Kataev, Parente, Sidorov, arXiv:hep-ph/021115

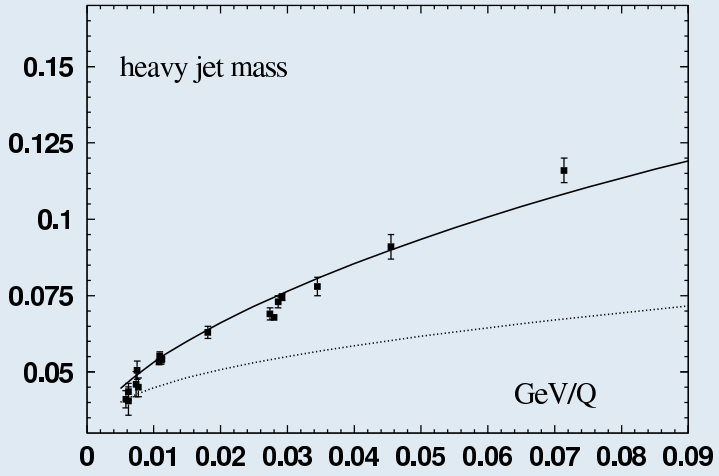
Hadronic event shape variables in e^+e^- annihilation

$$\langle S\rangle = \int d\mathsf{PS}[p_i]|\mathcal{M}_{q\bar{q}g}|^2 S(p_i)$$

e.g. thrust $S = 1 - T$

$$\langle 1-T\rangle = t_1\alpha_s(\mu) + \left[t_2^{(0)} + t_2^{(1)}\ln\frac{\mu^2}{Q^2}\right]\alpha_s^2(\mu) + \varkappa_T\frac{\Lambda_{\rm T}}{Q} + \mathcal{O}(1/Q^2)$$



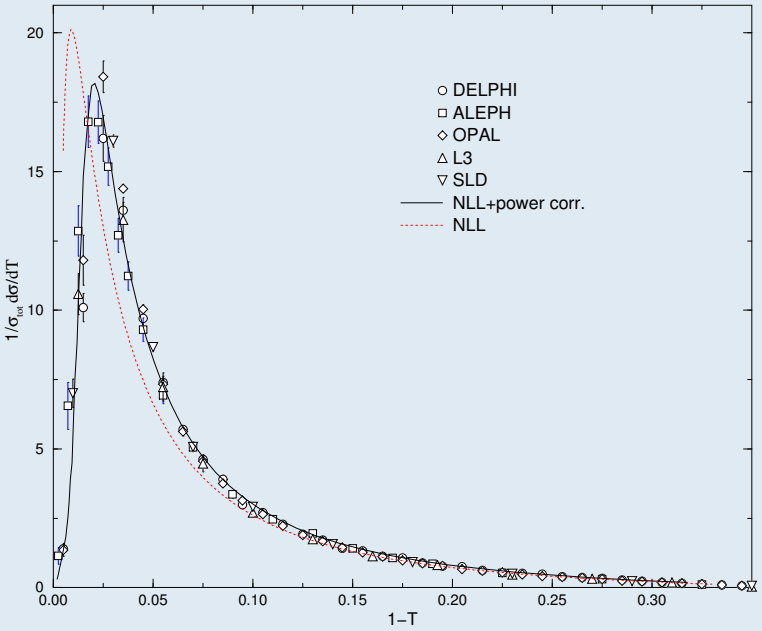


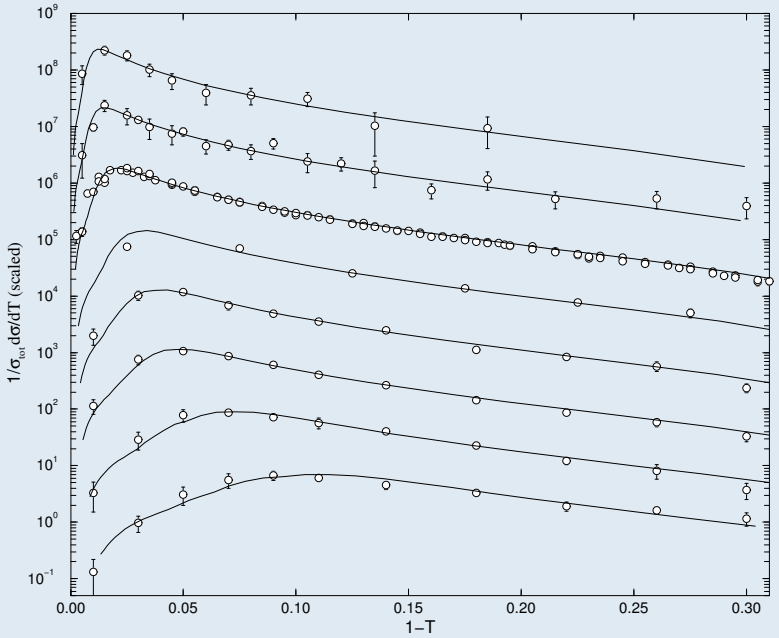
Dotted lines: NLO, solid: with $\frac{\Lambda}{Q}$, dashed: NLO with $\mu = 0.07Q$.
Universality $\Lambda_{\rm thrust} \simeq \Lambda_{\rm jet\,mass}$ works to 20%.

Event shape distributions

$$\frac{d\sigma}{dt}(t) \;\mapsto\; \frac{d\sigma}{dt}(t-\Lambda/Q) + \mathcal{O}\left(1/(tQ)^2\right)$$

If $t \sim \Lambda/Q$ all terms in $1/(tQ)^k$ have to be resummed — "shape function"





— fitted for $Q = 91.2$ GeV, used for $Q = 14, 22, 35, 44, 55, 91, 133, 161$ GeV:

Beyond the twist expansion

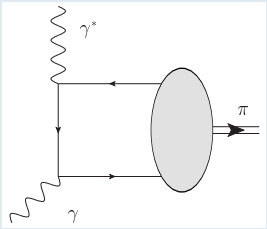
- Not all power corrections $1/Q^k$ can be obtained from the expansion near the light cone

Example:

I. Musatov, A. Radyushkin, arXiv:hep-ph/9702443

In Brodsky-Lepage formalism, starting from the light-front WF

$$\Psi^{\bar q q}(x, k_\perp^2) \sim \phi_\pi(x) \exp\Big[-\frac{k_\perp^2}{2\sigma x \bar x}\Big] \qquad \bar x = 1 - x$$



one obtains

$$F^{\bar q q}_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3Q^2}\int_0^1 \frac{dx}{x}\phi_\pi(x)\Big[1 - \exp\Big(-\frac{xQ^2}{2\bar x\sigma}\Big)\Big]$$

so that for $Q^2 \to \infty$

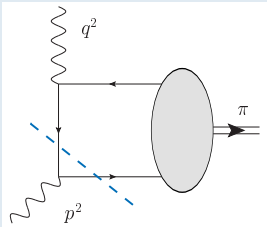
$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3Q^2}\int_0^1 \frac{dx}{x}\phi_\pi(x) + \underbrace{\frac{1}{Q^4}[\text{higher Fock states}]}_{\text{higher twists}} \underbrace{-\frac{4\sqrt{2} f_\pi \sigma}{Q^4}}_{\text{end-point (Feynman)}}$$

— The last term comes from large distances between the quark and the antiquark in transverse plane

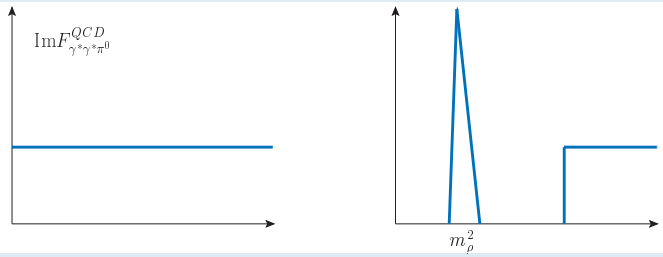
— Such terms are generated naturally in models

Dispersion relations + duality: Light-cone sum rules

I. Balitsky, V.B., A. Kolesnichenko '89



1. Consider a more general process with two virtual photons
2. Write a dispersion relation in one virtuality
3. Correct the spectral density to account for confinement
4. Use duality to fix the coefficients

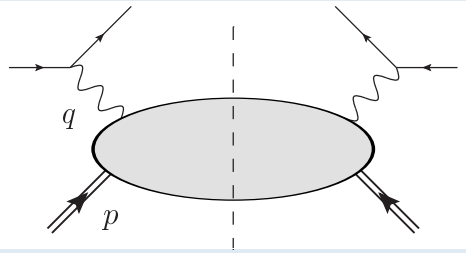


this example: A. Khodjamirian '97

- Many applications to meson and baryon form factors, e.g. to B meson decays

Planar vs. non-planar kinematics

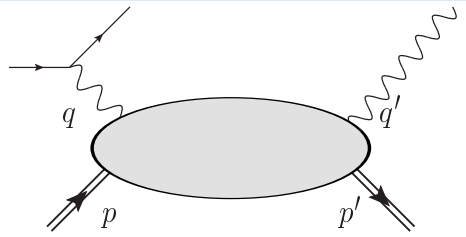
- "Natural" separation of longitudinal and transverse d.o.f. in DIS



$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x_B

- Many possible choices in DVCS



"Laboratory frame"

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow \Delta = p' - p$ transverse

- noncomplanarity makes separation of collinear directions ambiguous

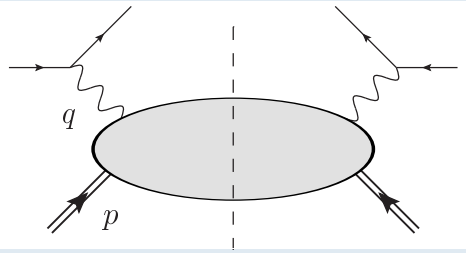
  — hence "leading twist approximation" ambiguous

  — related to violation of translation invariance and EM Ward identities

- have to be repaired by adding power corrections of special type, "kinematic" PC

Planar vs. non-planar kinematics

- "Natural" separation of longitudinal and transverse d.o.f. in DIS



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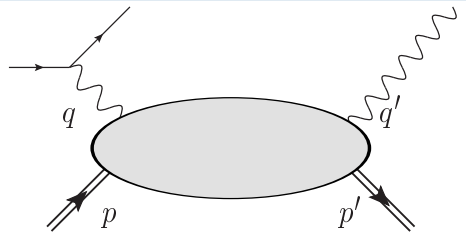


"Photon frame"

$$q' = (q'_0, \vec{0}_\perp, q'_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow \Delta = p' - p$ longitudinal

- noncomplanarity makes separation of collinear directions ambiguous

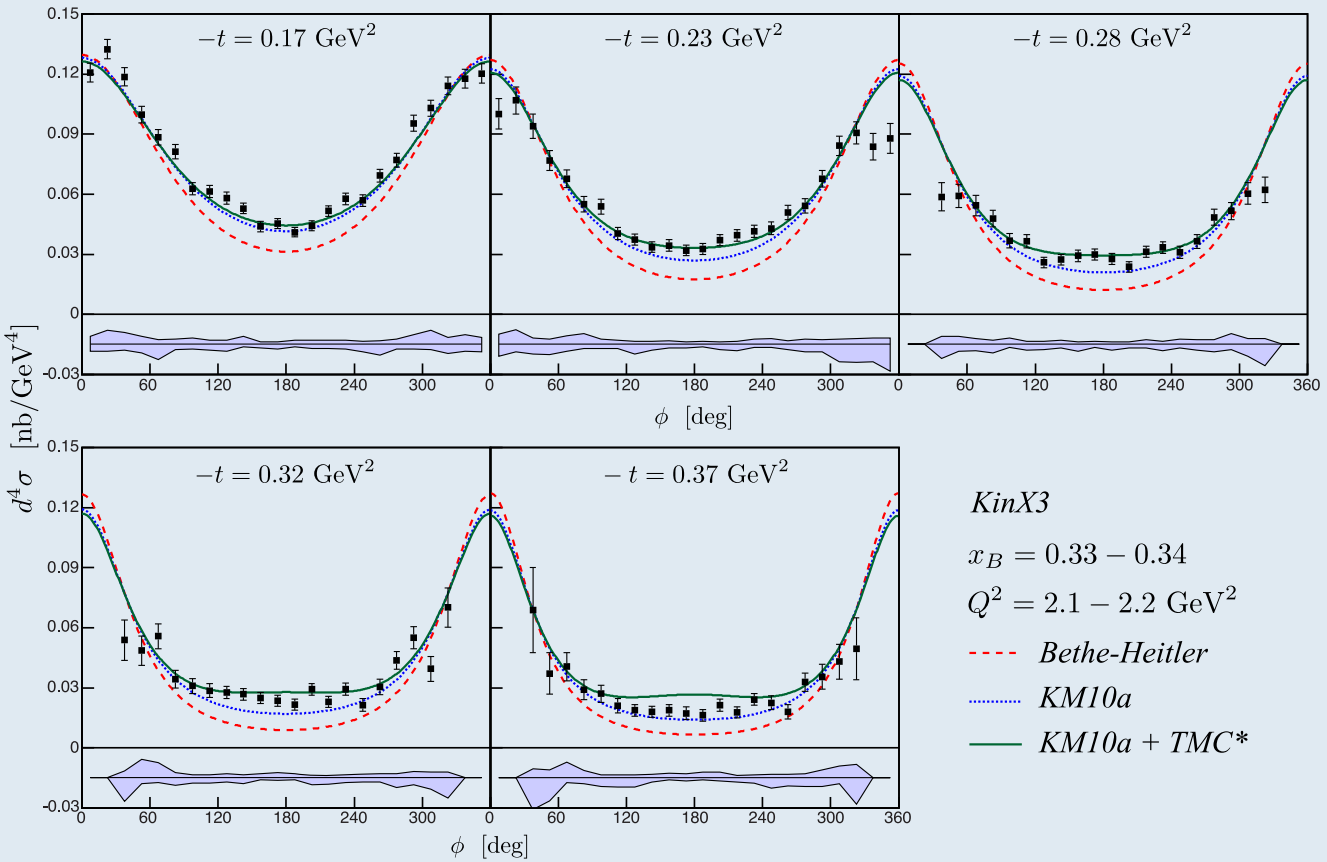
  — hence "leading twist approximation" ambiguous

  — related to violation of translation invariance and EM Ward identities

- have to be repaired by adding power corrections of special type, "kinematic" PC

Large uncertainty in the cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

$(\sqrt{-t}/Q)^k$ and $(m/Q)^k$ power corrections to DVCS

- power corrections that repair the frame dependence and Ward identities come from
  (1) corrections m/Q and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)
  (2) higher-twist operators that are obtained from twist-two by adding total derivatives

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\cdots\mu_N}}_{\text{descendants of twist 2}}$$
$$+ C_N^{\mu_1\cdots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\cdots\mu_N}}_{\text{descendants}} + \dots \Big\}$$
$$+ \text{ quark-gluon operators}$$

- Problem: Descendant operators are related to $\bar{q}Fq$ operators by EOM

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu\leftrightarrow\nu)]$$

- Solution: The CFs of descendants are related to the CFs of twist-2 operators by conformal symmetry

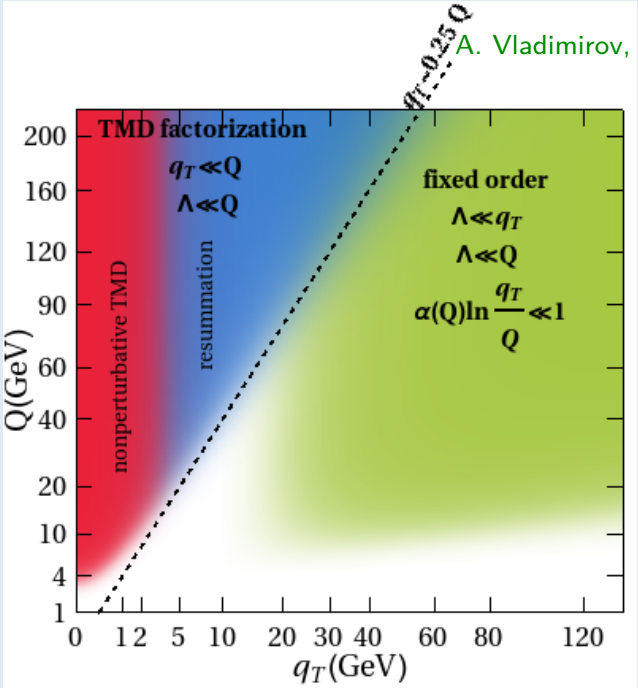
$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} \{B_N, C_N, D_N, \dots\}^{\mu_1\cdots\mu_N}$$

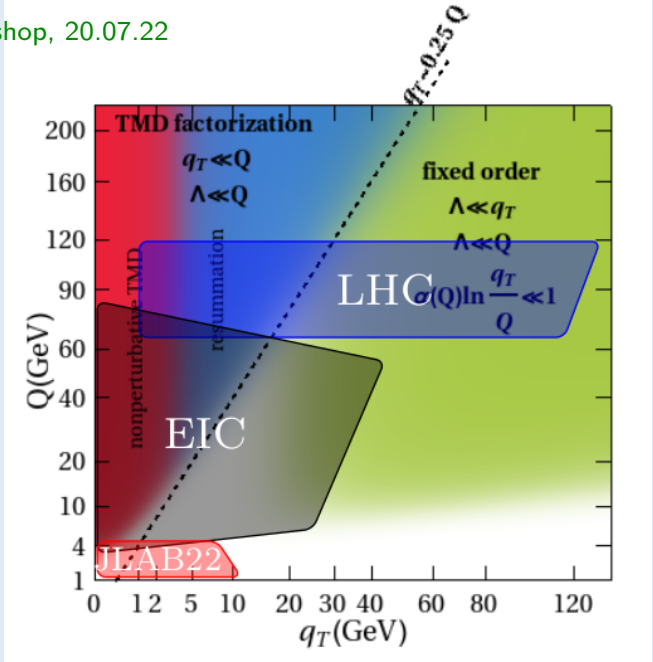
status: twist-4 completed, V.B., Manashov, Pirnay, arXiv:1209.2559
twist-5,6 in progress, V.B., Yao Ji, Manashov, arXiv:2011.04533

Semiinclusive reactions

- Regions of applicability of collinear and TMD factorization in the EIC range (and below) do not overlap:

A. Vladimirov, JLAB22 workshop, 20.07.22





recent work:
Balitsky, Tarasov, arXiv:1712.09389
Vladimirov, Moos, Scimeni, arXiv:2109.09771
Ebert, Gao, Stuart, arXiv:2112.07680

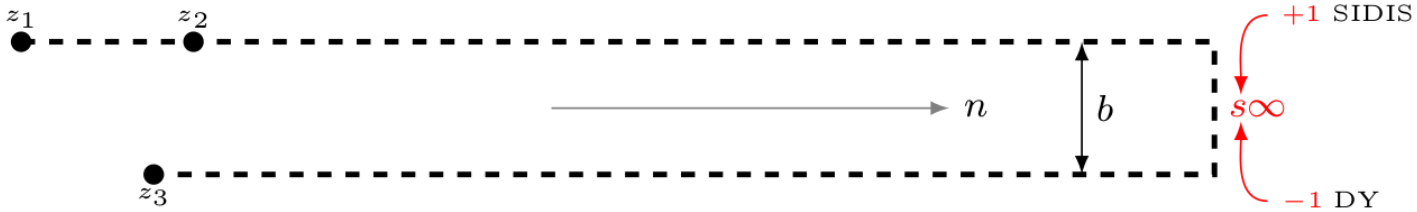
TMD factorization at twist three

TMD-twist-(2,1)

Appear at NLP

$U_1 = [..]\xi$ = good-component of quark field (twist-1)
$U_2 = [..]F_{\mu+}[..]\xi$ = good-components of gluon and quark fields (twist-2)

$$\widetilde{\Phi}_{21}^{[\Gamma]}(\{z_1,z_2,z_3\},b) = \langle p,s|\bar{\xi}(z_1 n+b)..F_{\mu+}(z_2 n+b)..\tfrac{\Gamma}{2}..\xi(z_3 n)|p,s\rangle$$



$$\mu^2\frac{d}{d\mu^2}\widetilde{\Phi}_{21}(\{z_{1,2,3}\},b;\mu,\zeta) = \left(\widetilde{\gamma}_2(z_1,z_2,\mu,\zeta)+\widetilde{\gamma}_1(z_3,\mu,\zeta)\right)\widetilde{\Phi}_{21}(\{z_{1,2,3}\},b;\mu,\zeta)$$

$$\zeta\frac{d}{d\zeta}\widetilde{\Phi}_{21}(\{z_{1,2,3}\},b;\mu,\zeta) = -\mathcal{D}(b,\mu)\widetilde{\Phi}_{21}(\{z_{1,2,3}\},b;\mu,\zeta)$$

- γ_1 = anomalous dimension of U_1
- γ_2 = anomalous dimension of U_2
- $\mathcal{D}$ = CS kernel

Similar for TMD-twist-(1,2)

S. Rodini, A. Vladimirov, arXiv:2204.03856

Summary

- Twist expansion in collinear factorization is well understood

  Twist-3: a relatively compact description in terms of $\bar{q}Fq$ correlation functions

  — physics: quantum interference

  — can be at reach in EIC era, experiment + lattice

  Twist-4 and beyond: Nonperturbative input too complicated

  — careful distinction to higher orders in PT necessary

  — renormalon model offers first approximation

- Not all power corrections are related to twist expansion

  — Example: end-point (Feynman) contributions in form factors

  — Dispersion relations + duality as approximation

- Kinematic power corrections in off-forward processes restore Lorentz symmetry and Ward identities

  — well understood, constrained by conformal symmetry

- TMD factorization beyond leading twist — important new topic

Other topics (not covered):

- Power corrections to jet observables
- SCET beyond leading twist
- Heavy-quark expansions beyond leading power