Interplay of nuclear physics, effective field theories, phenomenology, and lattice QCD in neutrino physics

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Motivation

EFT decomposition of cross section/rate

$$\sigma \simeq (\text{short-distance/BSM}) \otimes (\text{hadronic}) \otimes (\text{nuclear})$$

- Neutrinos provide window into **physics beyond the Standard Model** (BSM)

  $\hookrightarrow$ origin of neutrino mass, PMNS mixing matrix, *CP* violation, ...
- Yet: notoriously hard to detect, need large-scale detectors

  $\hookrightarrow$ measurements in **nuclear/hadronic environments**
- To predict cross sections/rates need to control hadronic and nuclear physics:
  - Decomposition using effective field theories
  - (Nuclear) matrix elements to be determined from phenomenology and/or lattice QCD

Outline

Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{\mathrm{A}}(Z+1)^*(\to {}^{\mathrm{A}}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\to N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

TABLE I. Main neutrino interaction channels in different energy ranges.

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- Many different aspects, see, e.g., Snowmass white papers:
  - "**Theoretical tools for neutrino scattering**: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators" 2203.09030
    $\hookrightarrow$ covers theory requirements over wide energy range
  - "Coherent elastic neutrino-nucleus scattering (**CEνNS**): Terrestrial and astrophysical applications" 2203.07361
  - "Neutrinoless Double-Beta Decay (**$0\nu\beta\beta$**): A Roadmap for Matching Theory to Experiment" 2203.12169
- Here: will focus on two examples, CEνNS and $0\nu\beta\beta$

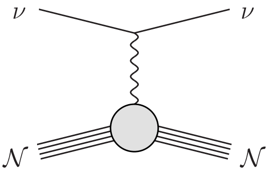
Coherent neutrino–nucleus scattering: EFT approach

EFT approach to CEνNS

Rate $=$ (B)SM couplings $\otimes$ hadronic matrix elements $\otimes$ nuclear structure $\otimes$ neutrino flux

$\hookrightarrow$ most efficiently addressed in **effective field theory** (will assume heavy mediator)

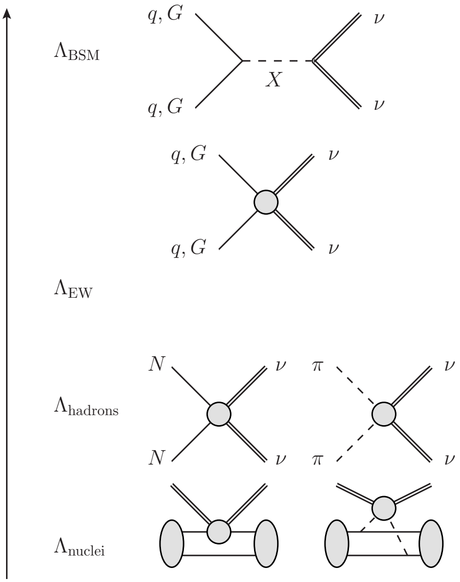
kinematics	elastic, ν relativistic
mediator	Z, BSM?
quantum numbers	$V - A$, others?
momentum transfer q	$\lesssim 50\,\text{MeV}$



- Predicted in 1974 Freedman, first observation 2017 COHERENT off CsI
- Light BSM physics can be added to set of BSM operators, e.g. light Z'

  $\hookrightarrow$ requires same hadronic/nuclear input

Scales



1. **BSM scale** Λ_{BSM}: $\mathcal{L}_{\text{BSM}}$
2. **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda^i_{\text{BSM}}} \mathcal{O}_{i,k}$
3. Integrate out **EW physics**

   (start here if only SM)
4. **Hadronic scale**: nucleons and pions

   $\hookrightarrow$ effective interaction Hamiltonian H_I
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

   $\hookrightarrow$ nuclear wave function

Hadronic matrix elements

- **Effective operators** defined at level of quarks and gluons

$$\mathcal{L}^{\text{SM}} = \sum_q \Big(C_q^V \bar{\nu}\gamma^\mu P_L \nu\, \bar{q}\gamma_\mu q + C_q^A \bar{\nu}\gamma^\mu P_L \nu\, \bar{q}\gamma_\mu \gamma_5 q \Big)$$

$$\mathcal{L}^{\text{BSM}} = C_F \bar{\nu}\sigma^{\mu\nu} P_L \nu F_{\mu\nu} + \sum_q \Big(C_q^T \bar{\nu}\sigma^{\mu\nu} P_L \nu\, \bar{q}\sigma_{\mu\nu} q + C_q^S \bar{\nu} P_L \nu\, m_q \bar{q} q + C_q^P \bar{\nu} P_L \nu\, m_q \bar{q} i\gamma_5 q \Big) + \cdots$$

  but **Confinement**!

- Need **hadronic matrix elements** to convert to observables

  $\hookrightarrow$ from phenomenology and/or lattice QCD (with further EFTs constraints)

- Strategies:
  - **Phenomenology**: possible for physical flavor combinations of (axial-) vector currents
  - **EFT constraints**: e.g., Cheng–Dashen theorem for scalar–isoscalar operator
  - **Ward identities**: scalar/pseudoscalar matrix elements from vector/axial-vector ones
  - **Unitarity constraints**: e.g., momentum dependence of tensor matrix elements
  - **Lattice QCD**: gives access to all of them (in principle), benchmarking!

First example: axial-vector case

Axial-vector and pseudoscalar matrix elements of the nucleon

$$\langle N(p')|\bar{q}\gamma^\mu\gamma_5 q|N(p)\rangle = \bar{u}(p')\Big[\gamma^\mu\gamma_5 G_A^{q,N}(t) + \gamma_5\frac{q^\mu}{2m_N}G_P^{q,N}(t) + \frac{i\sigma^{\mu\nu}}{2m_N}q_\nu\gamma_5 G_T^{q,N}(t)\Big]u(p)$$

$$\langle N(p')|m_q\bar{q}i\gamma_5 q|N(p)\rangle = m_N\bar{u}(p')i\gamma_5 G_5^{q,N}(t)u(p) \qquad \langle N(p')|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}|N(p)\rangle = 2m_N\bar{u}(p')i\gamma_5 G^N_{G\tilde{G}}(t)u(p)$$

- **Axial-vector form factors** ($q = p' - p$, $t = q^2$)
  - Direct axial-vector $G_A^{q,N}(t)$, defines axial-vector charges $G_A^{q,N}(0) \equiv g_A^{q,N} \equiv \Delta q^N$
  - Induced pseudoscalar $G_P^{q,N}(t)$
  - Tensor $G_T^{q,N}(t)$, second-class current Weinberg 1958, can induce G-parity-breaking corrections for β decays, neglect here
- Related by axial **Ward identity**

$$\partial_\mu\bar{q}\gamma^\mu\gamma_5 q = 2im_q\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}_a^{\mu\nu} \Leftrightarrow G_A^{q,N}(t) + \frac{t}{4m_N^2}G_P^{q,N} = G_5^{q,N}(t) - G^N_{G\tilde{G}}(t)$$

$\hookrightarrow$ for the charges at $t=0$: $g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}$

First example: axial-vector case

	$g_A^{u,p}$	$g_A^{d,p}$	$g_A^{s,p}$
HERMES 2006	0.842(12)	−0.427(13)	−0.085(18)
χQCD 2018	0.847(37)	−0.407(24)	−0.035(9)
PNDME 2018	0.777(39)	−0.438(35)	−0.053(8)

- What do we know about the charges?
  - **Isospin symmetry**: $g_A^{u,p} = g_A^{d,n}$, $g_A^{d,p} = g_A^{u,n}$, $g_A^{s,p} = g_A^{s,n}$
  - Triplet from neutron decay: $g_A^{u,p} - g_A^{d,p} = g_A = 1.27641(56)$ PERKEO III 2018
  - Octet from hyperon decays
  - Singlet: spin structure functions (scale dependent) HERMES 2006

$\hookrightarrow$ comparison to lattice QCD gives some idea of current uncertainties

- For singlet $g_5^{q,N}$ need $\tilde{a}_N$, so far only large-N_c estimate available

($\tilde{a}_N = -0.39(12)$ GeV MH, Menéndez, Noël 2022)

$\hookrightarrow$ lattice-QCD calculation of $\tilde{a}_N$ would be most welcome! talk by A. Shindler

Second example: scalar case

- Definitions of **scalar matrix elements**:

$$\langle N(p')|m_q\bar{q}q|N(p)\rangle = m_N f_q^N(t)\bar{u}(p')u(p) \qquad f_q^N \equiv f_q^N(0)$$

$\hookrightarrow$ often expressed via σ-terms, $\sigma_{\pi N} = m_N(f_u^N + f_d^N)$, $\sigma_s = m_N f_s^N$

- Ward identity González-Alonso, Martin Camalich 2014

$$\partial_\mu \bar{q}_f\gamma^\mu q_i = i(m_f - m_i)\bar{q}_f q_i$$

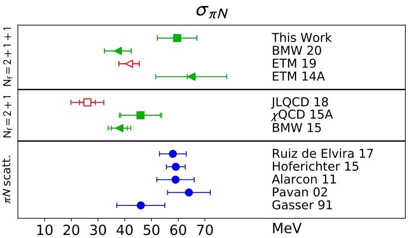
$\hookrightarrow$ only off-diagonal scalar charges accessible from the vector matrix elements

- Alternative strategies:
    - **Cheng–Dashen low-energy theorem** Cheng, Dashen 1971: relates $\sigma_{\pi N}$ to subthreshold pion–nucleon scattering

      $\hookrightarrow$ requires analytic continuation
    - **Chiral perturbation theory** Gasser, Leutwyler 1982: relates σ_s to $\sigma_{\pi N}$ and baryon masses

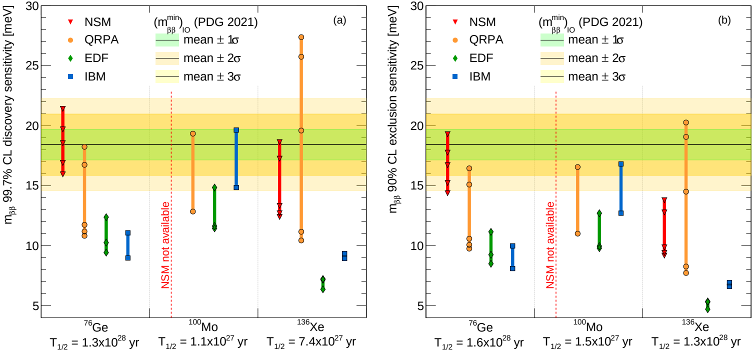
      $\hookrightarrow$ requires $SU(3)$ assumptions

Second example: scalar case



- Comparison to lattice QCD:
  - σ_s much smaller than expected, $SU(3)$ corrections too large to be predictive
  - $\sigma_{\pi N}$: unresolved tension between lattice and phenomenology
- Phenomenology: data input from **pionic atoms** or **πN cross sections** + dispersion relations for analytic continuation
- Lattice QCD: **direct method** or **Feynman–Hellmann theorem**
- Large **excited-state contamination** in direct method Gupta et al. 2021
- Important benchmark to be resolved!

Neutrinoless double-β decay



Agostini et al. 2021

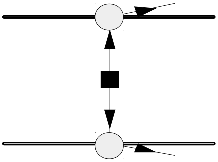
- Search for $0\nu\beta\beta$ decay

  $\hookrightarrow$ nature of neutrino masses, neutrino mass ordering

- Can derive a similar EFT decomposition as for CEνNS

  $\hookrightarrow$ **light Majorana exchange** one possible mechanism

- Dominant uncertainty from **nuclear matrix elements**



- If $0\nu\beta\beta$ is mediated by light Majorana exchange, compare experiments via

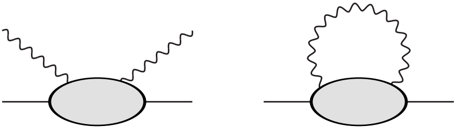
$$m_{\beta\beta} = \left| \sum_k m_k U_{ek}^2 \right| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i(\alpha_2 - \alpha_1)} + m_3 |U_{e3}|^2 e^{-i(\alpha_1 + 2\delta)} \right|$$

- How to get from $T_{1/2}$ to $m_{\beta\beta}$?
  - **Nuclear matrix elements**: complicated, but a lot of recent progress in ab-initio theory

    Yao et al. 2020, Belley et al. 2021, Novario et al. 2021
  - **Single-nucleon matrix** (vector and axial-vector) reasonably well known
  - **Few-nucleon amplitudes**: renormalizability in chiral EFT requires $nn \to ppee$ contact term at leading order, diagram (D) Cirigliano et al. 2020

    $\hookrightarrow$ coefficient $\tilde{\mathcal{C}}_1$ a priori unknown

How to determine the $0\nu\beta\beta$ contact?



- Strategies:
  - Lattice QCD: ongoing Davoudi, Kadam 2021, 2022, talk by A. Grebe, Fr., 15:40
  - Large-N_c: how well does it work? Richardson et al. 2021
  - **Cottingham approach** Cirigliano et al. 2020, 2021
- Basic idea:
  - **(Forward) Compton amplitude** $T^{\mu\nu}$ can be measured
  - "Close the loop" and Wick-rotate into the space-like region
  - **Elastic intermediate states** simple, and usually dominant!

    $\hookrightarrow$ pion, nucleon self energies Cottingham 1963
  - Try the same thing for weak currents

    $\hookrightarrow$ capture known momentum dependence from form factors and *NN* amplitude

Validation with charge independence breaking

- Similar contact term $\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2$ accessible in charge independence breaking (CIB) in *NN* scattering
- Cottingham strategy gives (in $\overline{\text{MS}}$):

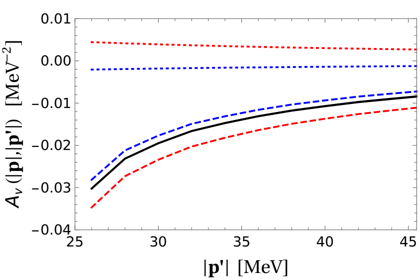
$$(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2)(\mu_\chi = M_\pi) = 2.9(1.1)_{\text{inel}}(0.3)_r(0.3)_{\text{par}} = 2.9(1.2)$$

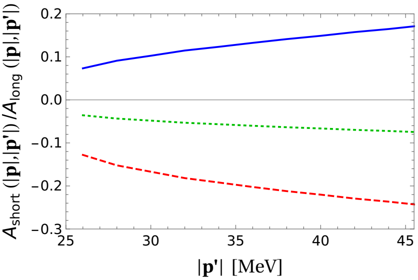
- Compares well to phenomenology $(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2)(\mu_\chi = M_\pi) = 5.0$
- Should really **compare observables**, e.g.,

$$a_{\text{CIB}} = \frac{a_{nn} + a_{pp}^C}{2} - a_{np} \overset{\text{exp}}{=} 10.4(2)\,\text{fm} \qquad \text{vs.} \qquad a_{\text{CIB}}\big|_{\text{Cottingham}} = 15.5^{+4.5}_{-4.0}\,\text{fm}$$

$\hookrightarrow$ works at the quoted level of accuracy

Result for the $0\nu\beta\beta$ contact





- Result in $\overline{\text{MS}}$:

$$\tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) = 1.32(50)_{\text{inel}}(20)_r(5)_{\text{par}} = 1.3(6)$$

  but: how to make this available in a useful form?

- Provide **renormalized *nn* $\rightarrow$ *pp* amplitude on a grid** of kinematic points

  $\hookrightarrow$ matching to ab-initio nuclear structure

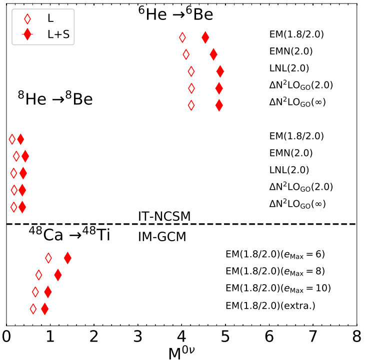
- Similar strategy should also apply to lattice QCD

A first application to $0\nu\beta\beta$ nuclear matrix elements

- Impact enhanced by node in wave function
- First ab-initio implementation in ^{48}Ca

  ↪ increases matrix element by 43(7)%

  ↪ **factor 2 in the rate**!
- Expect same pattern also for heavier nuclei



Wirth, Yao, Hergert 2021

Beyond light neutrino exchange

$\mu_{\overline{\text{MS}}} = 3\,\text{GeV}$	$\langle\pi^+\|O_1\|\pi^-\rangle$ $[10^{-4}\,\text{GeV}^4]$	$\langle\pi^+\|O_2\|\pi^-\rangle$ $[10^{-2}\,\text{GeV}^4]$	$\langle\pi^+\|O_3\|\pi^-\rangle$ $[10^{-2}\,\text{GeV}^4]$	$\langle\pi^+\|O_4\|\pi^-\rangle$ $[10^{-2}\,\text{GeV}^4]$	$\langle\pi^+\|O_5\|\pi^-\rangle$ $[10^{-2}\,\text{GeV}^4]$
Cirigliano et al. 2017	1.0(1)(2)	−2.7(3)(5)	0.9(1)(2)	−2.6(8)(8)	−11(2)(3)
Nicholson et al. 2018	0.93(5)	−1.89(16)	0.62(6)	−1.89(13)	−7.81(54)

- New matrix elements/contact terms for heavy mechanism, e.g.,

  $O_1 = \bar{q}_L^\alpha \gamma^\mu \tau^+ q_L^\alpha \, \bar{q}_L^\beta \gamma_\mu \tau^+ q_L^\beta$, $O_2 = \bar{q}_R^\alpha \tau^+ q_L^\alpha \, \bar{q}_R^\beta \tau^+ q_L^\beta, \ldots$

- In progress for $nn \to pp$, but results already available for $\pi^- \to \pi^+$
  - Related by $SU(3)$ symmetry to K^0–$\bar{K}^0$ and $K \to \pi\pi$ matrix elements Cirigliano et al. 2017, which were already known from lattice QCD
  - Direct lattice calculation Nicholson et al. 2018

$\hookrightarrow$ good agreement between lattice and EFT in this case!

Conclusions

- Short version of the title "**Theory input for neutrino experiments**"

  $\hookrightarrow$ nuclear physics, EFTs, phenomenology, lattice QCD

- **Coherent elastic neutrino–nucleus scattering**
  - EFT decomposition requires hadronic and nuclear input
  - Two examples: axial-vector and scalar matrix elements
  - Benchmarking of lattice calculations wherever possible

- **Neutrinoless double-β decay**
  - Similar EFT decomposition, less known about short-range matrix elements
  - Accessible in lattice QCD, but $nn \to pp$ hard
  - Estimate from phenomenology via Cottingham approach

Cottingham approach: pion and nucleon mass difference

- Starting point:

$$\delta M_\gamma^2 = \frac{ie^2}{2}\int \frac{d^4k}{(2\pi)^4}\frac{T_\mu^\mu}{k^2+i\epsilon} \qquad \text{with} \qquad T_\mu^\mu\big|_{\text{el}} = \frac{2k^2(3k^2-4M_\pi^2)-16(k\cdot p)^2}{(k^2)^2-4(k\cdot p)^2}\big[F_\pi^V(k^2)\big]^2$$

- Dispersive analysis gives same thing as **scalar QED + pion form factor**!
- After Wick rotation

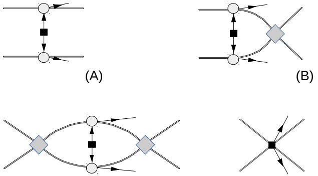
$$\delta M_\gamma^2 = \frac{\alpha}{8\pi}\int_0^\infty ds\,\big[F_\pi^V(-s)\big]^2 \times \bigg(4W+\frac{s}{M_\pi^2}(W-1)\bigg) \qquad W=\sqrt{1+\frac{4M_\pi^2}{s}}$$

$\hookrightarrow$ saturates 99.5(9.0)% of experimental pion mass difference!

- For the nucleon mass difference
  - Dispersive analysis matters

    $\hookrightarrow$ nucleon pole and Born terms differ
  - Inelastic effects more important, but elastic estimate still accurate at 30% level

    $\hookrightarrow$ **information contained in nucleon form factors**
  - Assumptions on high-energy behavior matter: tension with lattice due to fixed poles?
- Strategy for $0\nu\beta\beta$: try to capture the main effects along the same lines



- **Matching procedure**:

$$\mathcal{A}_\nu^{\text{full}} = \int_0^\infty d|\mathbf{k}|\; a^{\text{full}}(|\mathbf{k}|) = \mathcal{A}^< + \mathcal{A}^> \stackrel{!}{=} \mathcal{A}_\nu^{\text{EFT}}$$

$$\mathcal{A}^< = \int_0^\Lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|) \qquad \mathcal{A}^> = \int_\Lambda^\infty d|\mathbf{k}|\; a_>(|\mathbf{k}|)$$

  with
  - $\boldsymbol{\mathcal{A}^<}$: low- and intermediate energies, keep momentum dependence of form factors and *NN* amplitude $\leftrightarrow$ **"elastic states"**
  - $\boldsymbol{\mathcal{A}^>}$: high-energy region $\leftrightarrow$ **OPE**
  - Model dependence from **interpolation**: explicit estimate of inelastic diagrams, variation of scales and OPE coefficients

- Reformulated pion Cottingham result along the same lines, works!