Dynamical fermions, center vortices, and emergent phenomena

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In collaboration with:

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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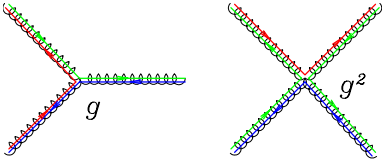
Introduction

- The self interactions of gluons in QCD make the empty vacuum unstable to the formation of quark and gluon condensates.



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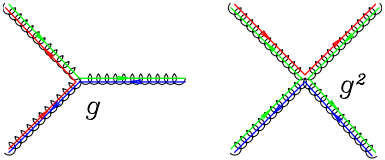
- These ground-state QCD-vacuum fields permeate spacetime and form the foundation of matter.



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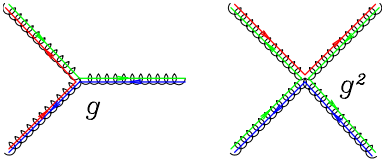

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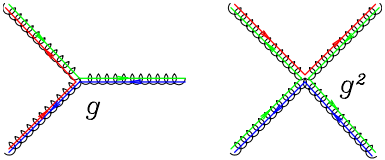
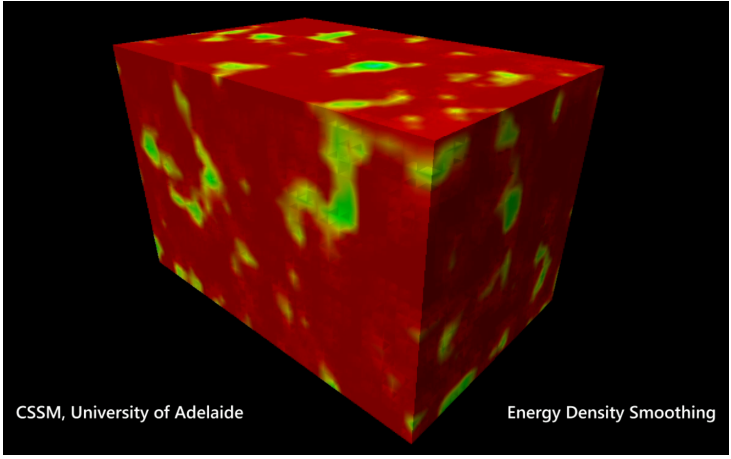
- These ground-state QCD-vacuum fields permeate spacetime and form the foundation of matter.
- Quarks carrying the quantum numbers of hadrons are embedded in these fields.
- Commence exploring the ground-state field structure by examining the energy density

$$\sum_{\alpha=1}^{8} \left(E_\alpha^2(\vec{x},t) + B_\alpha^2(\vec{x},t) \right) .$$

of the chromo-electric and chromo-magnetic fields.



Introduction: CSSM Visualisations on YouTube



Introduction

- Is there something more fundamental that captures the salient features of QCD?
  - Confinement.
  - Dynamical generation of mass via Chiral Symmetry breaking.

Centre-Vortices in the Ground-State QCD-Vacuum Fields

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  - Visualisations of centre vortices in Monte-Carlo generated lattice QCD configurations.
  - Interesting features: percolation, monopoles, secondary loops, branching points, ...

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- How does this manifest in the emergent properties of QCD?
  - Static Quark Confinement
  - Gluon Propagator
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- What is the origin of dynamical mass generation and confinement in QCD?

Foundations

- Early ideas...
  - Spaghetti Vacuum, a condensate of vortices with finite thickness.
    - H. B. Nielsen and P. Olesen, Nucl. Phys. B **160** (1979), 380-396
  - Importance of the Z_N centre of the $SU(N)$ gauge group.
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- Renewed Excitement...
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- Review: "The Confinement problem in lattice gauge theory,"
  J. Greensite, Prog. Part. Nucl. Phys. **51** (2003), 1. 500+ citations

Vortex Structure in the Colour Fields of the QCD Vacuum

What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the 'axis of rotation' of the vortex.

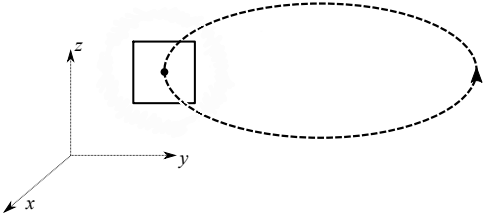


Figure: A centre vortex (dashed line) intersecting a lattice plaquette (solid square).

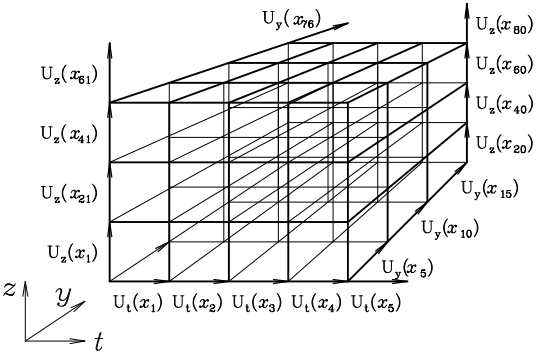
How do you find centre vortices?

Lattice Links

- On the lattice, the gluon-field is encoded in terms of the link variable

$$U_\mu^{ab}(x) \simeq \exp\left(i\, a\, g\, A_\mu^{ab}(x)\right) ,$$

a 3×3 complex special-unitary matrix.

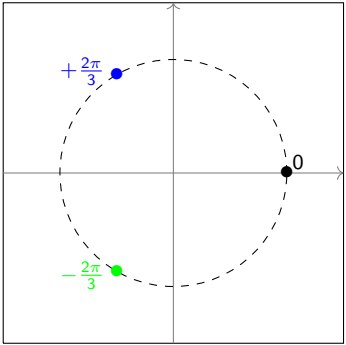


Centre Group of SU(3)

- Centre elements commute with every group element,

$$z = \exp\left(\frac{2\pi i}{3} m\right) I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3 \,.$$

- Each of the three centre phases corresponds to a centre element of SU(3).



1. Maximal Centre Gauge

- Gauge transformations bring the links close to an element of the group centre

$$z = \exp\left(\frac{2\pi i}{3}\, m\right) I\,, \;\; m \in \{-1,\, 0,\, +1\}\,.$$

- This is done by maximising the functional

$$R = \sum_x \sum_\mu |\, \mathrm{tr}[U_\mu(x)]\,|^2$$

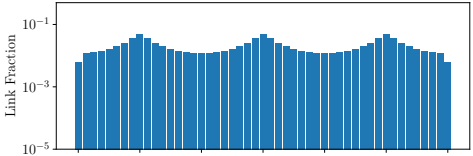
- This is called **Maximal Centre Gauge**

1. Maximal Centre Gauge

- Distribution of link phases.

$$\mathrm{tr}\, U_\mu^{\mathrm{MCG}}(x) \;=\; \underbrace{r_\mu(x)}_{\text{real}}\; \exp\underbrace{\left(\frac{2\pi i}{3}\,\phi_\mu(x)\right)}_{-\pi\,<\,\text{phase}\,\leq\,\pi}\,, \qquad -\frac{3}{2} < \phi_\mu(x) \leq \frac{3}{2}\,.$$

- $\phi_\mu(x)$ before gauge fixing.



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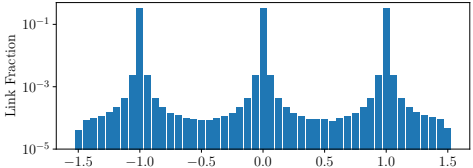

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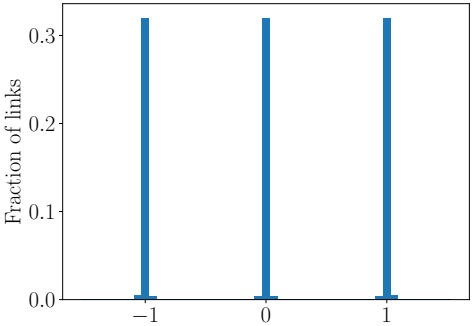
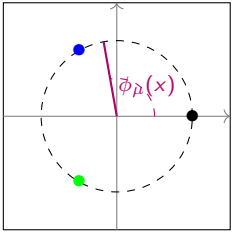


2. Centre Projection

- Project onto $Z(3)$

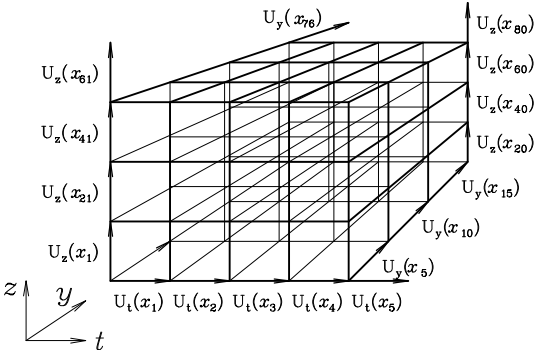
$$U_\mu^{\mathrm{MCG}}(x) \to Z_\mu(x) = \exp\left(\frac{2\pi i}{3}\, m_\mu(x)\right) I\,,\;\; m_\mu(x) \in \{-1,\, 0,\, +1\}\,.$$



- Eight degrees of freedom are replaced by one of the three cube-roots of 1.

3. Identifying Vortices

- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).
- Each plaquette takes a value from $Z(3)$.



3. Identifying Vortices

- Non-trivial plaquettes with values

$$\exp\left(\frac{2\pi i}{3}\, m\right) \neq 1\,, \quad i.e.\ m \in \{-1,\, +1\}\,,$$

  identify our thin vortices.

- Thin vortices locate the centre of the physical thick vortices

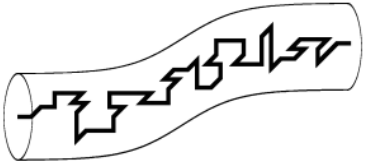


Figure: An example of a vortex path embedded within a thick vortex.
M. Engelhardt, H. Reinhardt, Nuclear Physics B **585** (2000) 597

Configurations

- This projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu(x)$
  - Vortex Only - $Z_\mu(x)$
  - Vortex Removed - $R_\mu(x) = Z_\mu^\dagger(x)\, U_\mu(x)$

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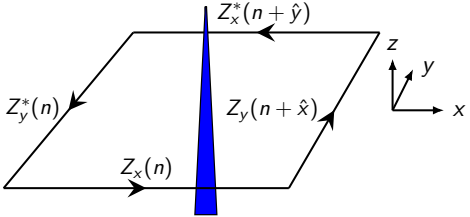
- 4 ensembles
  - $20^3 \times 40$ pure gauge (PG), spacing $a = 0.125$ fm
  - $32^3 \times 64$ pure gauge (PG), spacing $a = 0.100$ fm
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.1022$ fm, $m_\pi = 701$ MeV
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.0933$ fm, $m_\pi = 156$ MeV
  - S. Aoki, *et al.* (PACS-CS), Phys. Rev. D **79**, 034503.

What do centre vortices look like?

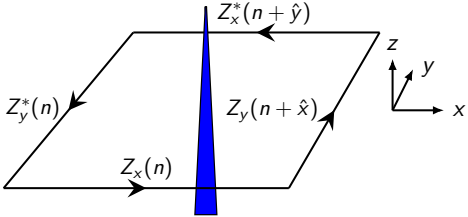
Rendering Projected Vortices

- Vortex sheets are sliced to vortex lines in a 3D slice of the 4D lattice.



Rendering Projected Vortices

- Vortex sheets are sliced to vortex lines in a 3D slice of the 4D lattice.
- Flow of centre charge $+1$ is indicated using a right-handed coordinate system.
- For example,
  - An $m = +1$ vortex in the x-y plane is plotted in the $+\hat{z}$ direction as a blue jet.



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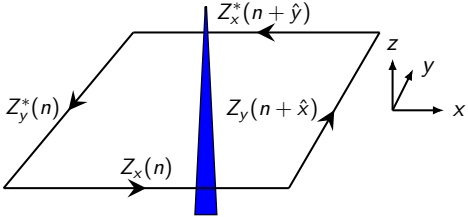
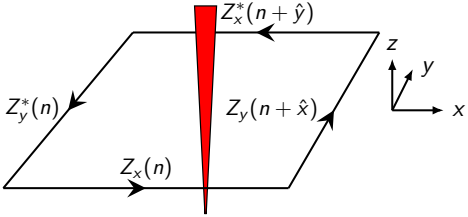
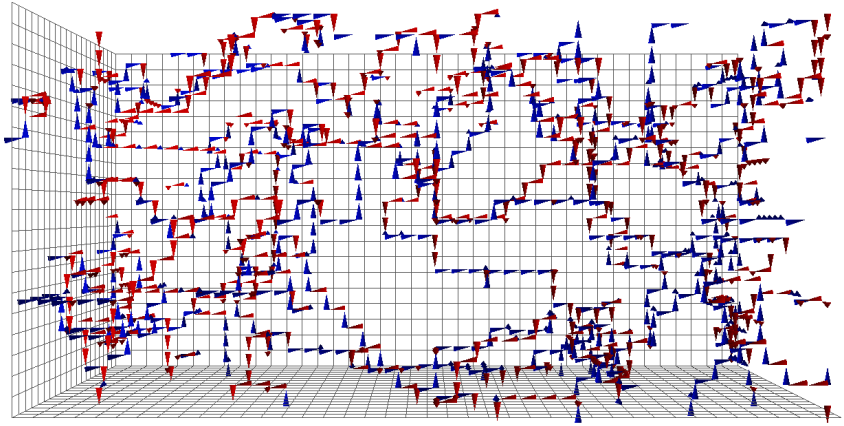
  - An $m = -1$ vortex in the x-y plane is plotted in the $-\hat{z}$ direction as a red jet.

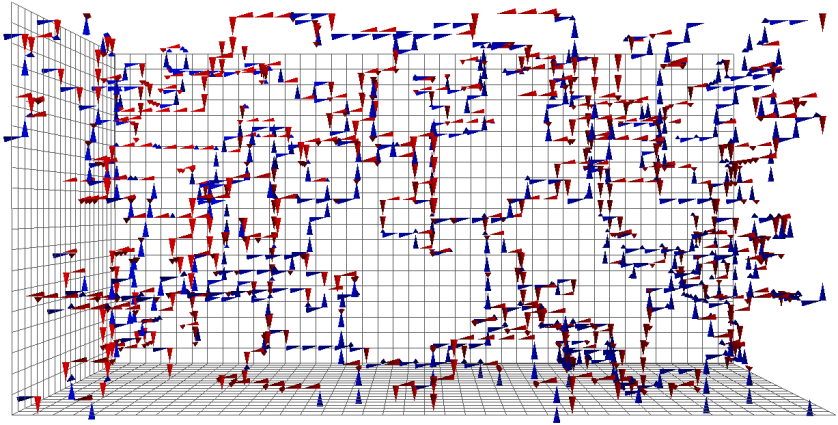




$t = 1$ J. Biddle, W. Kamleh and DBL, Phys. Rev. D **102**, 034504 [arXiv:1912.09531 [hep-lat]]

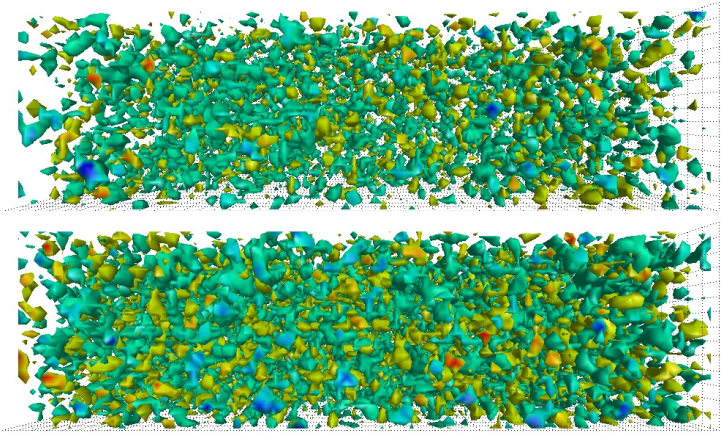


$t = 2$ J. Biddle, W. Kamleh and DBL, Phys. Rev. D **102**, 034504 [arXiv:1912.09531 [hep-lat]]



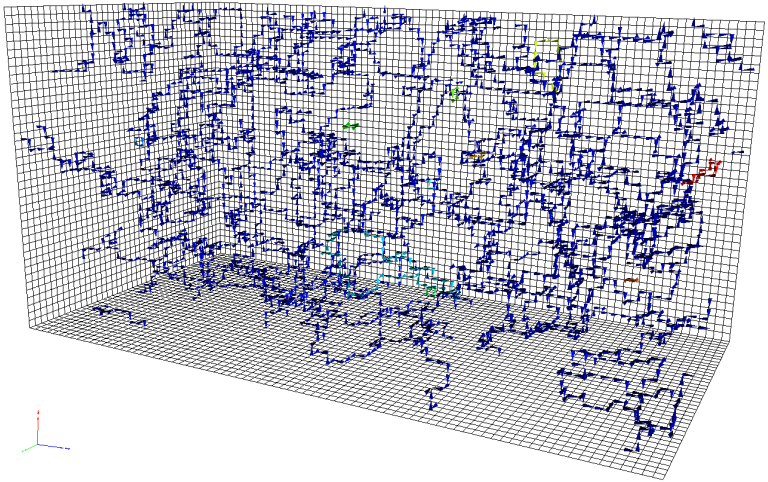
Impact of Dynamical Fermions on Centre Vortex Structure

Pure Gauge versus MILC $2+1$-Flavour QCD: $m_{u,d} = 27.1$ MeV

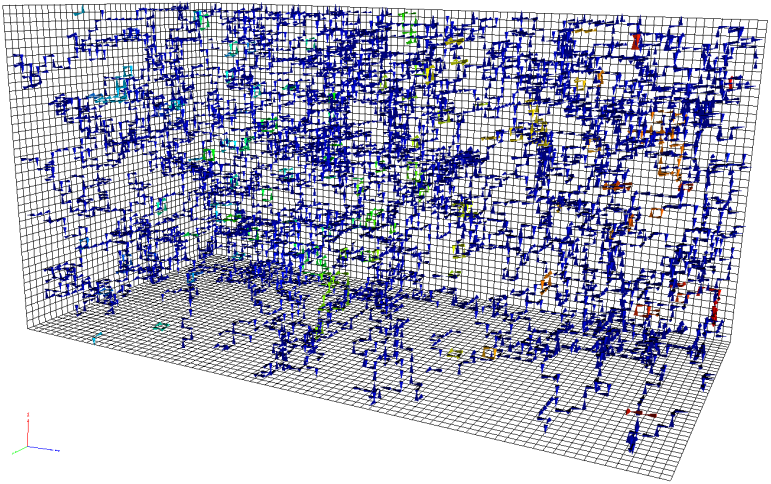


- "Impact of Dynamical Fermions on QCD Vacuum Structure," P. J. Moran and DBL, Phys. Rev. D **78** (2008) 054506 [arXiv:0801.2016 [hep-lat]].

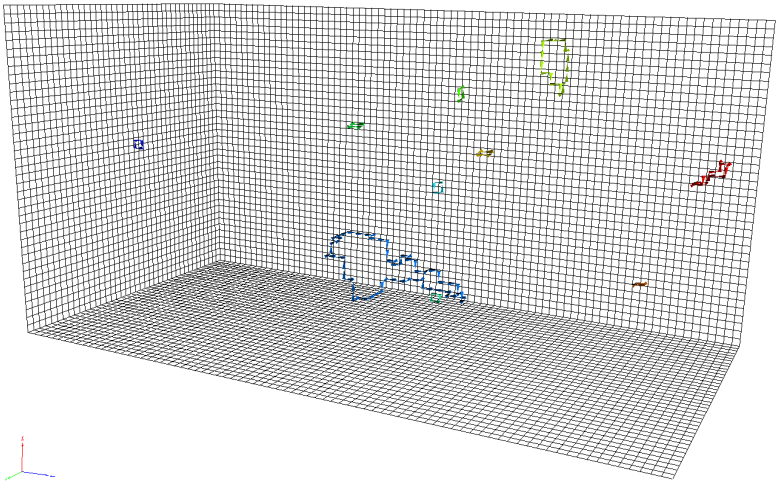
Vortices on a Pure-Gauge $32^3 \times 64$ Lattice



$2+1$ Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



Secondary Loops on a Pure-Gauge $32^3 \times 64$ Lattice



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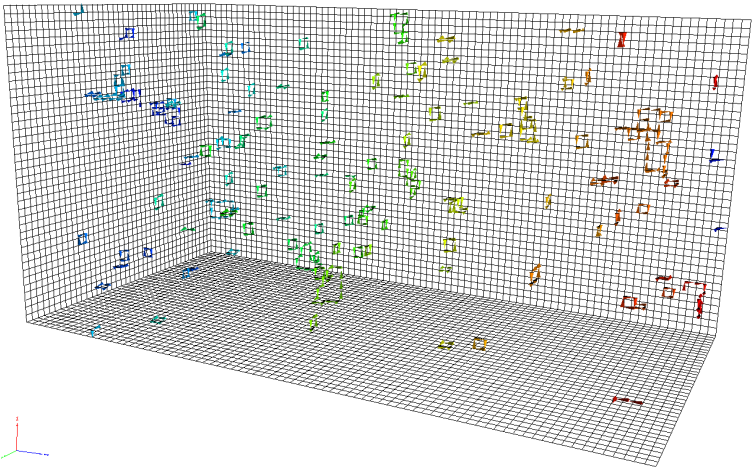

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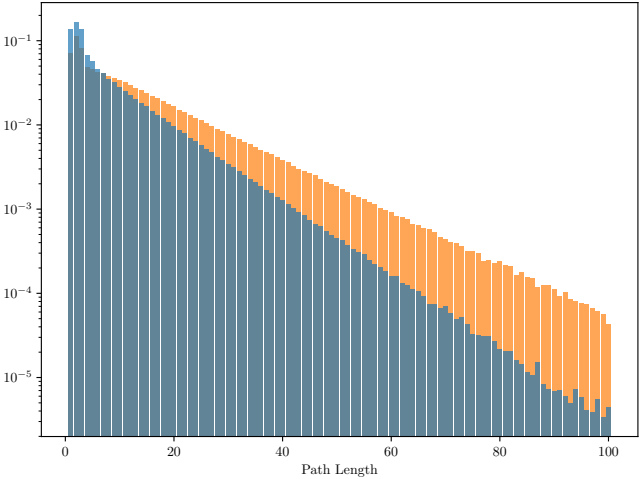
- The number of vortices composing the primary cluster is
  - $3,277 \pm 156$ vortices in the Pure Gauge theory.
  - $5,924 \pm 239$ vortices in Full QCD.
- There is further enhancement of the secondary loop structure,
  - Both in number and in complexity via monopoles.

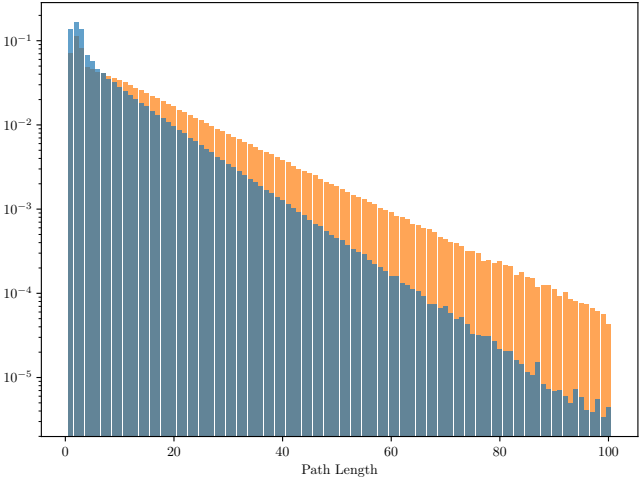
Impact of Dynamical Fermions: Vortex Path Lengths

- Histogram of vortex path lengths in the percolating cluster.
- Pure Gauge and Dynamical Fermion ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.



Impact of Dynamical Fermions: Vortex Path Lengths

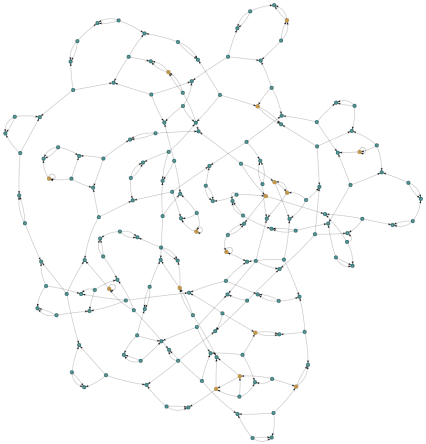
- Histogram of vortex path lengths in the percolating cluster.
- Pure Gauge and Dynamical Fermion ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.
- Moderate size loops are exponentially distributed.
  - Fixed probability of branching .
  - Branching is independent of length.
  - Branching probability:
    - PG: $\sim 2/3$ fm^{-1}. DF: ~ 1 fm^{-1}.



Directed Graphs

- Abstract the vortex clusters as a directed graph, independent of 3D coordinates.
- Points of vortex branching/intersection are nodes.
- Edges are weighted by the vortex path length connecting them.
- Visualisations are generated with the Pyvis package.

Directed graphs: Pure Gauge versus Dynamical Fermions



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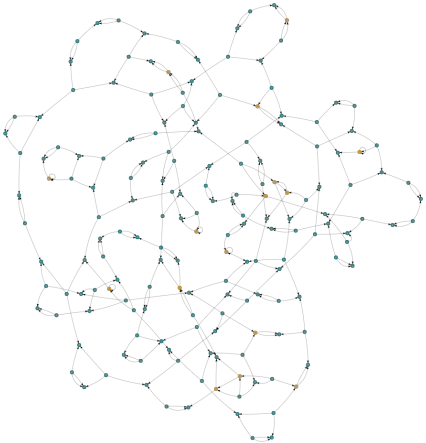


Impact of Dynamical Fermions on the Centre-Vortex Structure of the Static Quark Potential

Centre Vortices and Confinement – Pure Gauge Sector

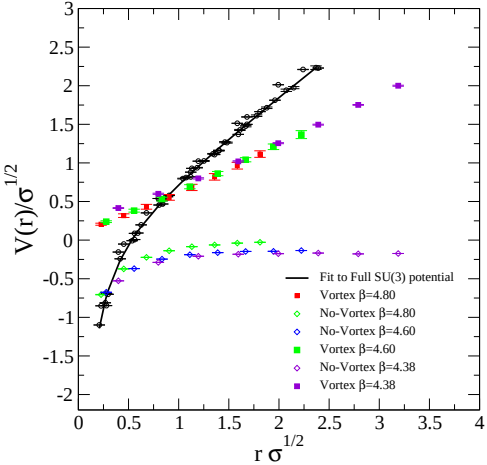
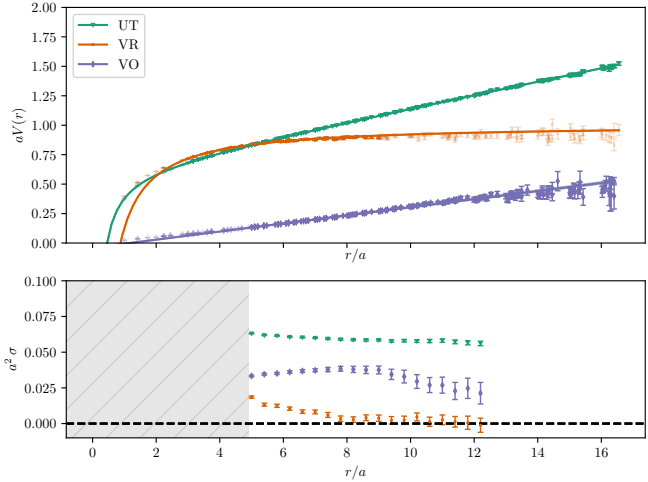


Figure from

P. O. Bowman, K. Langfeld, D. B. Leinweber, A. Sternbeck, L. von Smekal and A. G. Williams, Phys. Rev. D ***84*** *(2011), 034501 [arXiv:1010.4624 [hep-lat]].*

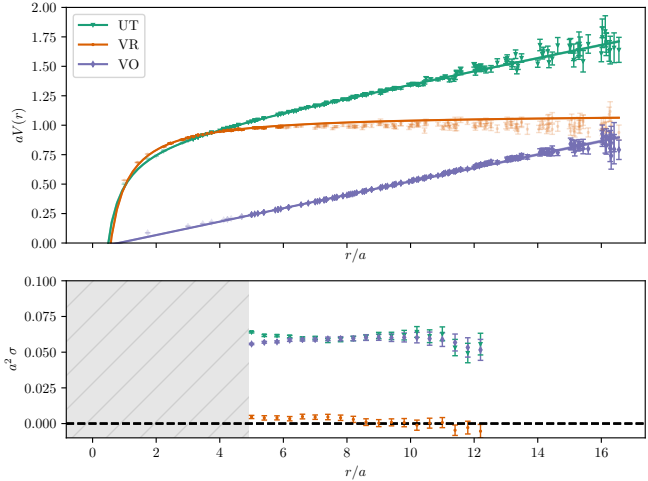
Static Quark Potential – Pure Gauge Sector

- Lower plot reports the local slope from fits to $V(r)$ over a forward-looking window from r to $r + 4a$.
- Vortex removal (VR) leaves no residual confining potential.
- Vortex-only (VO) reproduces only 62% of the original (UT) static quark potential.



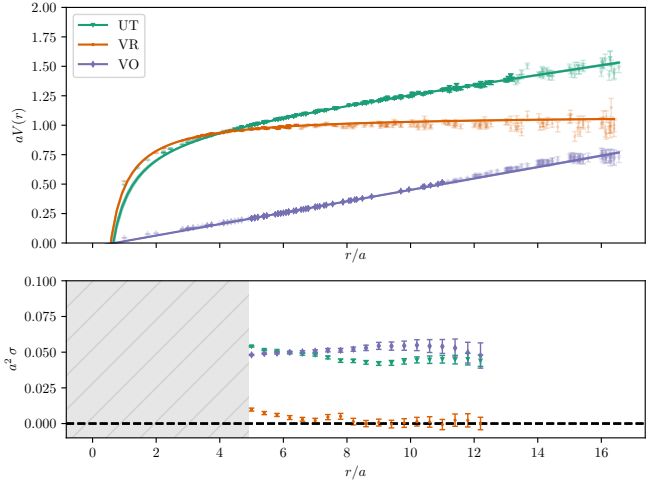
Introducing Dynamical Fermions ($m_\pi = 701$MeV)

- In the presence of dynamical fermions, vortices capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Lighter dynamical fermions ($m_\pi = 156$MeV)

- Lighter quark masses screen the confining potential.
- Vortices continue to capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Impact of Dynamical Fermions on the Centre-Vortex Structure of the Gluon Propagator

Centre Vortices and the Landau-Gauge Gluon Propagator

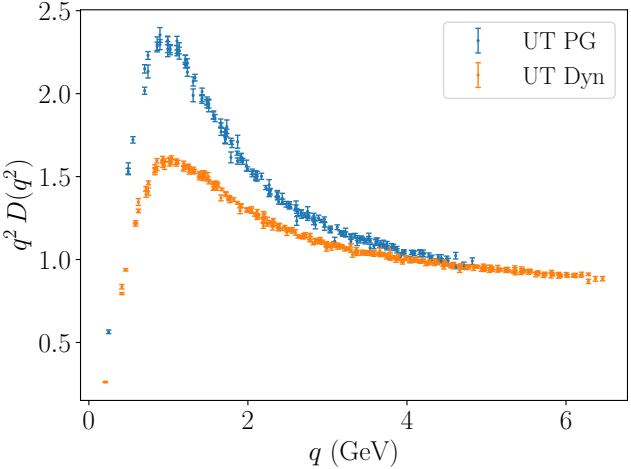
- The nonperturbative scalar gluon propagator in momentum space is

$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \to \frac{1}{q^2} \text{ at tree level.}$$

- Consider the renormalisation function

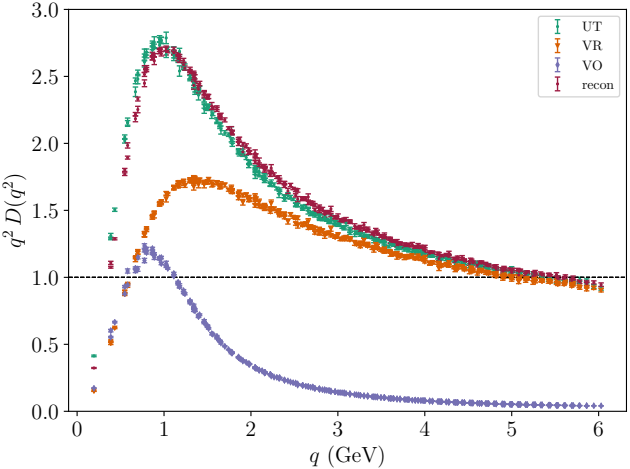
$$Z(q^2) = q^2\, D(q^2)\,.$$

- UT Dynamical has $m_\pi = 156$ MeV.



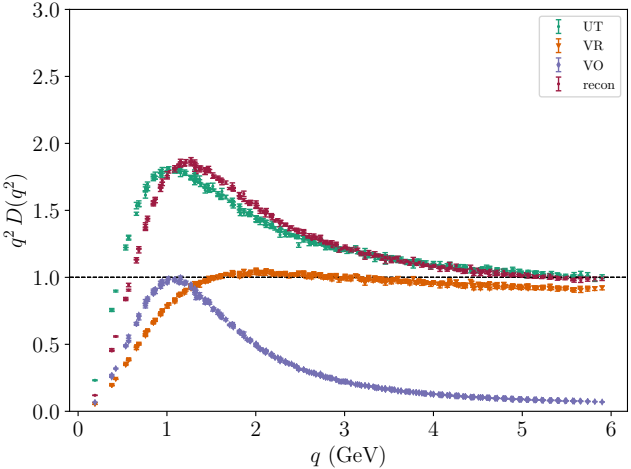
Gluon Propagator – Pure Gauge Sector

- Vortex Removal (VR) suppresses infrared enhancement whilst preserving UV perturbative behaviour.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction of the propagator as a linear combination of the vortex-modified parts recovers full propagator.
- Renormalisation maintains $Z(q^2) = 1$ at $q = 5.5$ GeV for UT and Recon.



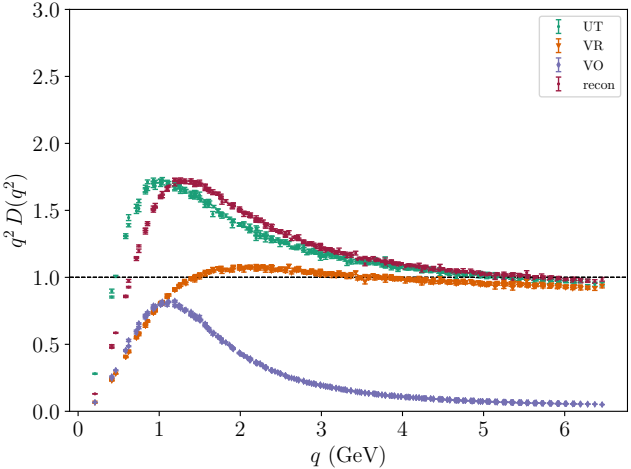
Gluon Propagator – Dynamical Fermions $m_\pi = 701$ MeV

- Dynamical fermions (UT) suppress the overall infrared strength.
- Vortex Removal (VR) almost eliminates infrared enhancement.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction is less perfect.

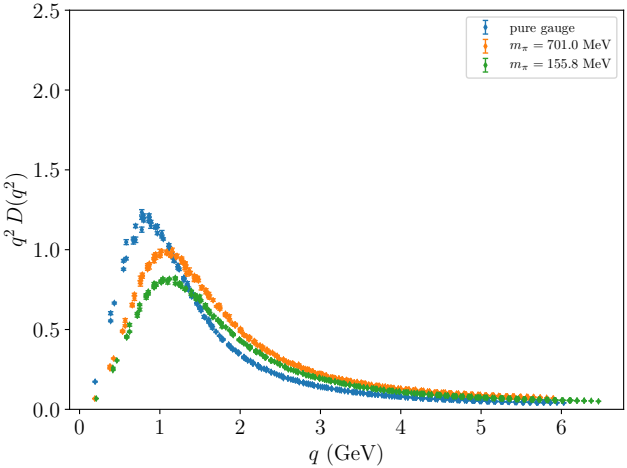


Gluon Propagator – Dynamical Fermions $m_\pi = 156$ MeV

- Lighter dynamical u and d quarks further suppress the infrared enhancement.
- Centre Vortex degrees of freedom are able to capture the screening effects of dynamical fermions in QCD.



Dynamical-Fermion Screening Effects in Vortex-Only Configurations



The Gluon Propagator and Confinement

- The 1-dimensional Fourier transform of the gluon propagator at zero spatial momentum defines the wall-to-wall correlator

$$C(t) = \int_0^\infty dm\, \rho(m^2)\, e^{-mt}\,.$$

  - C(t) is negative in QCD.

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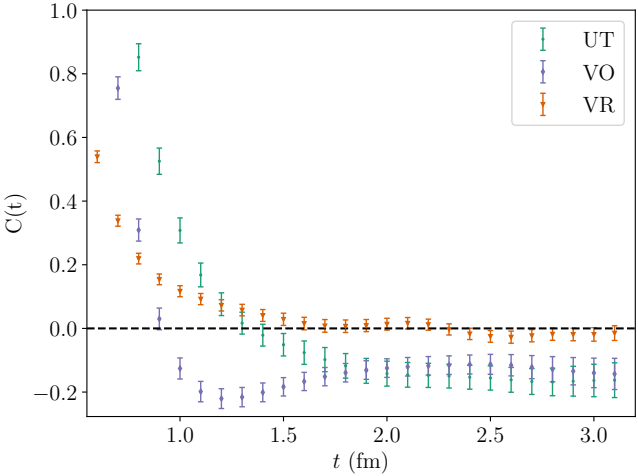
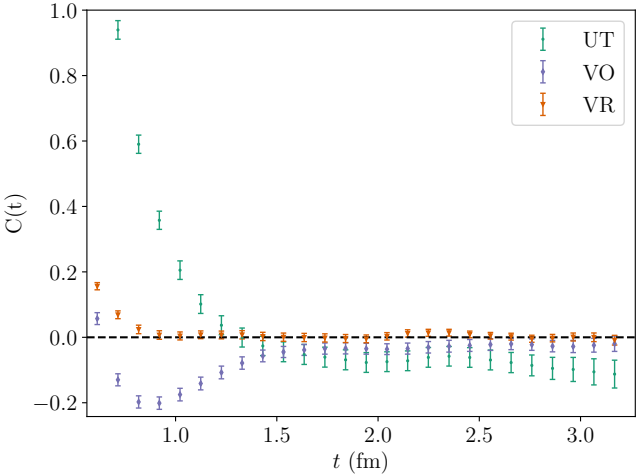
- The states are confined from the physical world.
  - J. E. Mandula and M. Ogilvie, Phys. Lett. **B185**, 127 (1987).
  - C. A. Aubin and M. C. Ogilvie, Phys. Lett. **B570**, 59 (2003), hep-lat/0306012.

Positivity Violation in the Pure-Gauge Sector



Positivity Violation in QCD: $m_\pi = 701$ MeV



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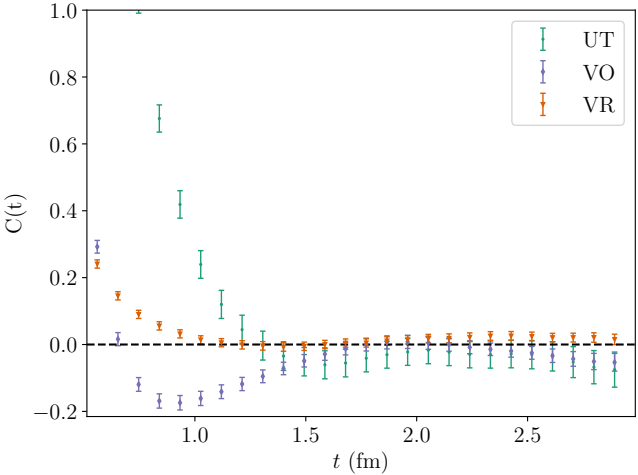


Impact of Dynamical Fermions on the Centre-Vortex Structure of the Quark Propagator

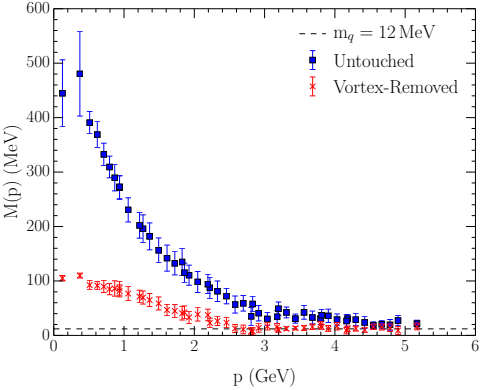
Centre Vortices and the Landau-Gauge Quark Propagator

- Probe dynamical mass generation using the quark propagator

$$S(p) = \frac{Z(p)}{i\not{p} + M(p)},$$

- Enhancement of the mass function, $M(p)$, at low momenta indicates dynamical mass generation.
- Renormalisation function, $Z(p)$, is typically infrared suppressed.
- Consider the Overlap-Dirac fermion action
  - Provides a lattice implementation of chiral symmetry,
  - No additive mass renormalisation,
  - Sensitive to the topological structure of the gauge fields.

Vortex-Removed Quark Mass Function: Pure Gauge vs Dynamical (preliminary)



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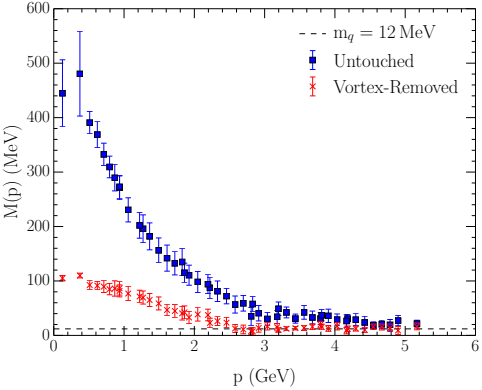
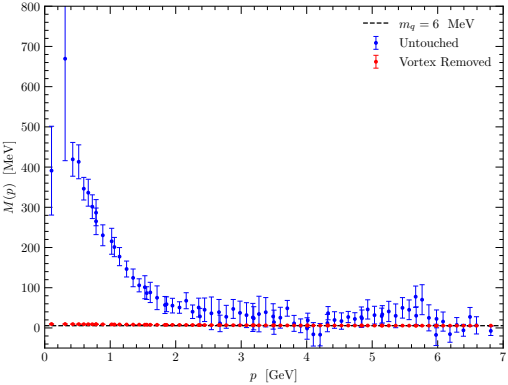
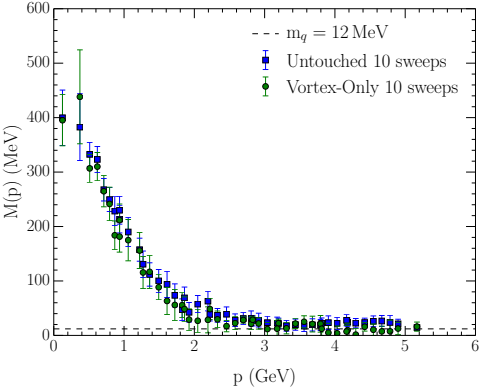
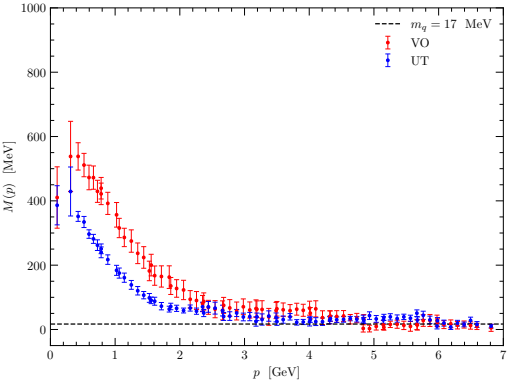




Vortex-Only Mass Function: Pure Gauge vs Dynamical (preliminary)





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  J. Biddle, W. Kamleh and D. Leinweber, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].
- "Impact of centre vortex removal on the Landau-gauge quark propagator in dynamical QCD,"
  A. Virgili, W. Kamleh and D. Leinweber, PoS **LATTICE2021** (2021), 082 [arXiv:2204.06978 [hep-lat]].
- "Centre vortex structure of QCD-vacuum fields and confinement,"
  D. Leinweber, J. Biddle and W. Kamleh, SciPost Phys. Proc. **6** (2022), 004 [arXiv:2205.12518 [hep-lat]].
- "Impact of dynamical fermions on the center vortex gluon propagator,"
  J. Biddle, W. Kamleh and D. Leinweber, Phys. Rev. D **106** (2022) 014506 [arXiv:2206.02320 [hep-lat]].
- "Smoothing algorithms for projected center-vortex gauge fields,"
  A. Virgili, W. Kamleh and D. Leinweber, Phys. Rev. D **106** (2022) 014505 [arXiv:2203.09764 [hep-lat]].
- "Static quark potential from centre vortices in the presence of dynamical fermions,"
  J. Biddle, W. Kamleh and D. Leinweber, [arXiv:2206.00844 [hep-lat]].
- "Centre vortex structure in the presence of dynamical fermions,"
  J. Biddle, W. Kamleh and D. Leinweber, In preparation.

Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition.
  `http://www.avs.com/solutions/express/`
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.
  `https://www.pdf3d.com/products/pdf3d-reportgen/`
- Imported into LaTeX via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).
  `ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/`

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- Dynamical fermions radically alter the centre-vortex structure of the vacuum fields.

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- In QCD, vortex removal:
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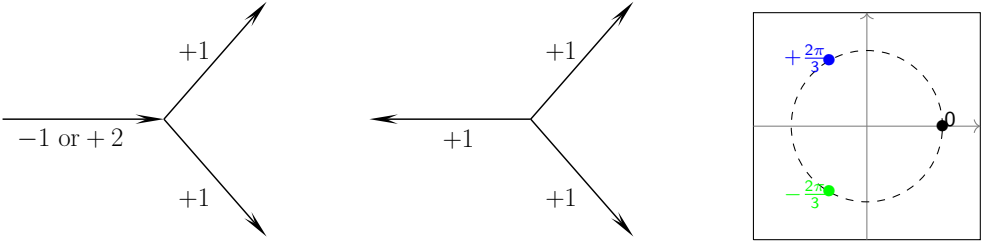
- Centre vortices in the foundation of matter capture the essence of nonperturbative QCD.

Additional Information

Branching Points versus Monopoles



- Our convention illustrates the directed flow of charge $m = +1$.
- Arrows indicate the direction of flow for the labelled charge.
- However, a vortex monopole with charge $+1$ flowing out of the vertex (centre) is equivalent to
  a vortex branching point with centre charge $+2$ flowing into a vertex (left).

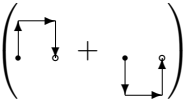
Centrifuge preconditioned smoothing

- Work with centre phase field

$$e^{i\lambda_\mu(x)} I \to \mathrm{diag}[\lambda_\mu(x),\, \lambda_\mu(x),\, \lambda_\mu(x)]$$

- Define the staple phase

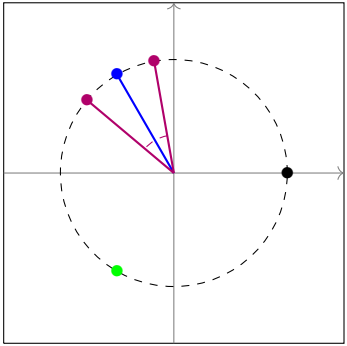
$$\sigma_\mu(x) = \frac{1}{6} \sum_{\nu \neq \mu}$$



- Select a pair of indices randomly for each link,

$$\lambda_\mu(x) \to (1 \mp \omega)\, \lambda_\mu(x) \pm \omega\, \sigma_\mu(x)$$

- Update corresponds to a phase rotation by $\mp\omega(\lambda - \sigma)$



Restoration of Chiral Symmetry

- If vortices are responsible for $D\chi SB$, then their removal should restore chiral symmetry

$$\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{A}}$$

- Expect hadrons related by chiral transformations to become degenerate

$$\begin{array}{ccc} \pi & \xleftrightarrow{\mathrm{U}(1)_{\mathrm{A}}} & a_0 \\ \rho & \xleftrightarrow{\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}}} & a_1 \\ N & \xleftrightarrow{\mathrm{SU}\,2_{\mathrm{L}} \times \mathrm{SU}\,2_{\mathrm{R}}} & \Delta \end{array}$$

- At light quark masses, all symmetries are observed to be restored.
- A. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) 125002 [arXiv:1708.06789 [hep-lat]].

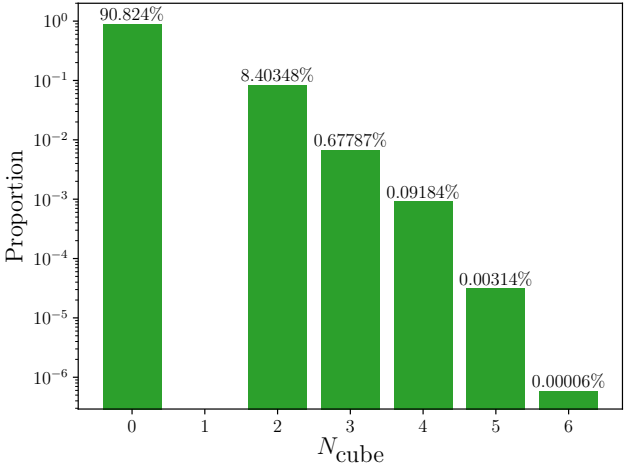
Visualising Centre Vortices

- Consider the number of vortices entering a 3D cube on the dual lattice.

$N_{\text{cube}}(\tilde{x})$	Interpretation
0	No vortices present.
1	Terminating vortex, forbidden by Bianchi*.
2	Vortex line flowing through the cube.
3	Simple three-way vortex monopole.
4	Vortex intersection.
5	Complex five-way monopole path.
6	Vortex intersections or double monopoles.

*Bianchi identity implies a continuous flow of centre vortex flux through a spatial cube.

Visualising Centre Vortices



Smoothed Vortices vs Instantons

Quark propagator on vortex only fields

- Vortex only configurations consist only of centre elements

$\Rightarrow$ very rough

- The Overlap operator has a smoothness condition

$\Rightarrow$ 10 sweeps of cooling on vortex only configurations.

Does 10 sweeps of cooling recreate instantons?

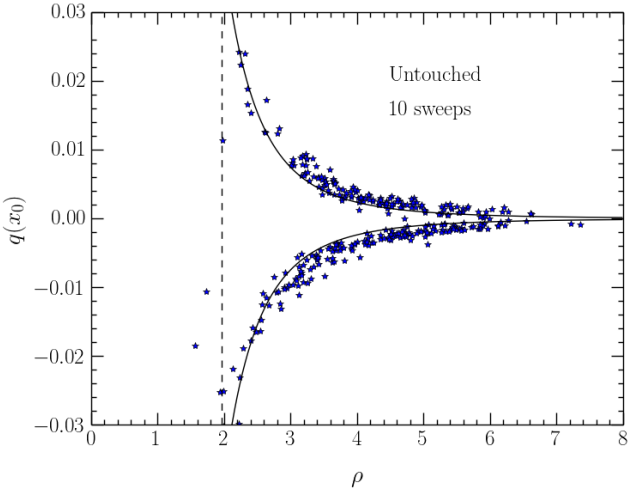
- Scan each configuration for local maxima of the action.
- Fit the instanton profile around them to learn the radius ρ

$$S_0(x) = \xi \frac{6}{\pi^2} \frac{\rho^4}{((x - x_0)^2 + \rho^2)^4} \,.$$

- Compare lattice value for $q(x_0)$ to the theoretical relationship

$$q(x_0) = Q \frac{6}{\pi^2 \rho^4} \,.$$

Instanton ρ vs $q(x_0)$



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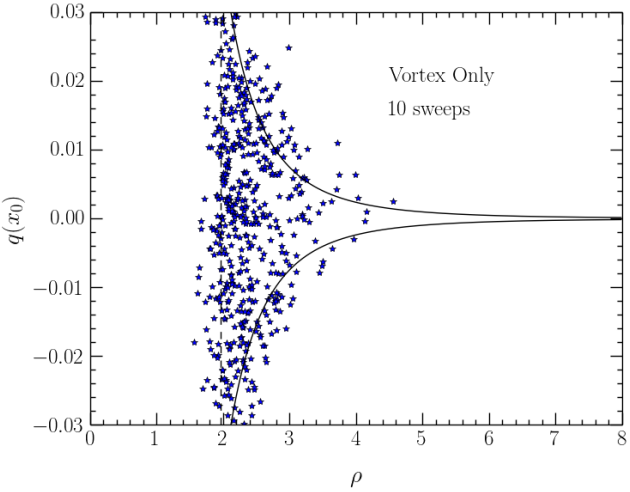

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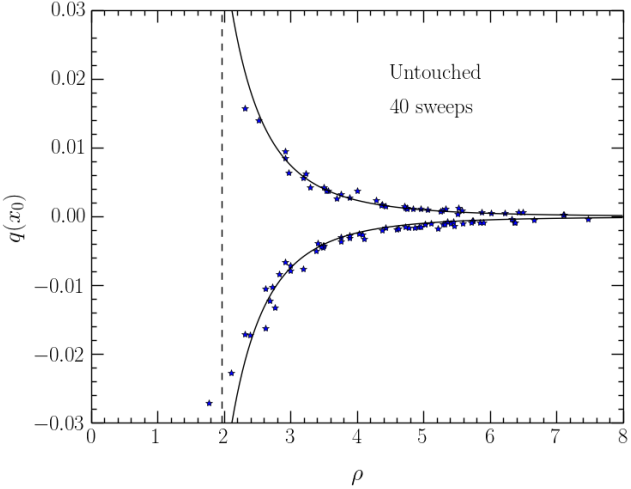

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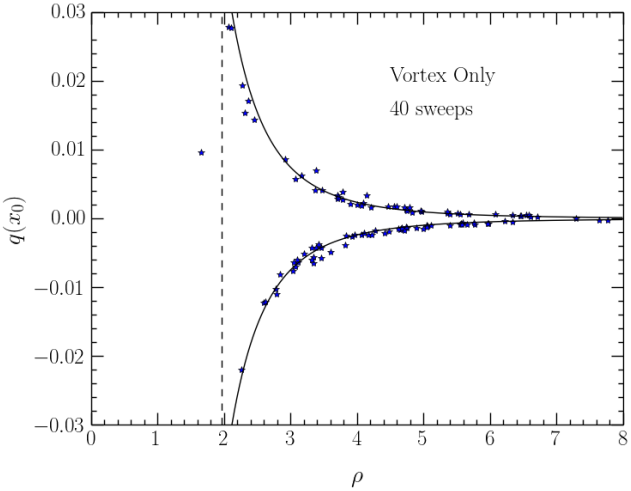
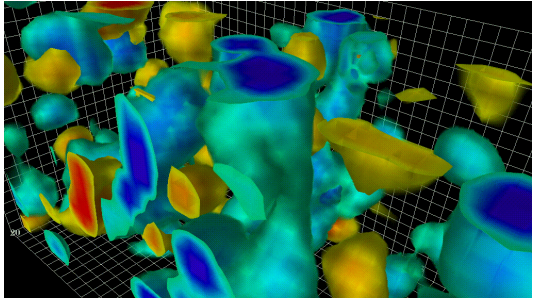
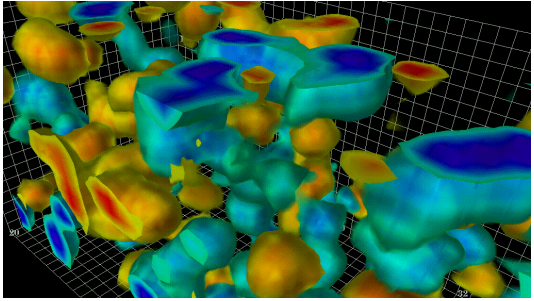


Untouched (left) and Vortex-Only comparison (10 sweeps)





Space-Time Oriented Vortices

Rendering Space-Time Oriented Projected Vortices

- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.

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- The three jets associated with the spatial x-y, y-z and z-x plaquettes, are complemented by
  - Jets in the three forward time x-t, y-t and z-t plaquettes, and
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- See "Visualization of center vortex structure," to link vortices to topological charge.

  J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].

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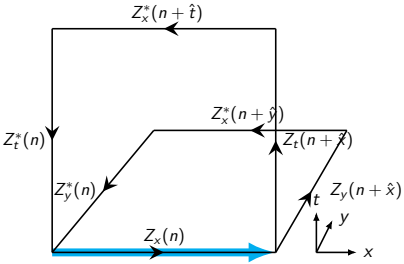
  - The link is rendered in cyan for an $m = +1$ vortex.



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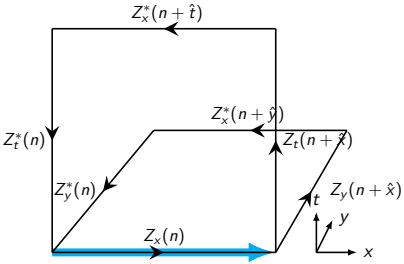
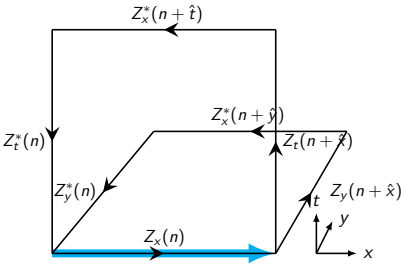
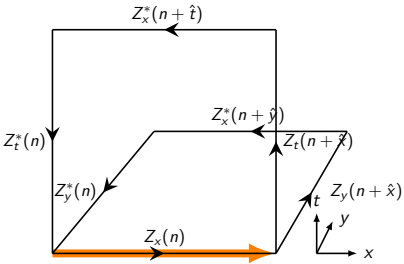


Rendering Space-Time Oriented Projected Vortices

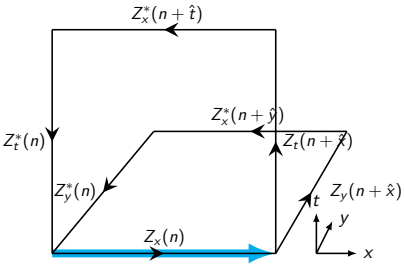
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.

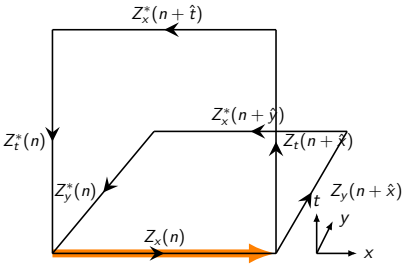




Rendering Space-Time Oriented Projected Vortices

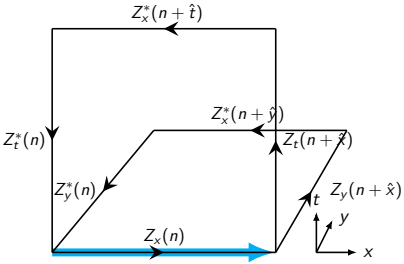
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex, and in orange for $m = -1$.
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.

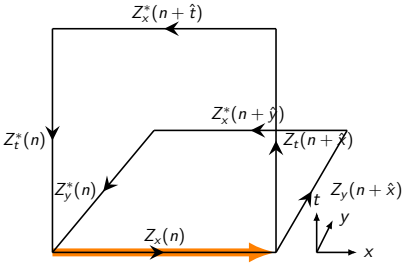




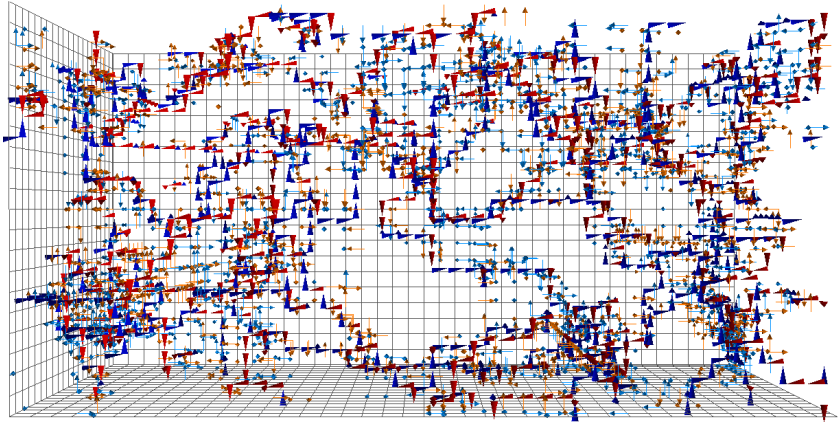
Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an $m = +1$ vortex
  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.





Time slice $t = 1$



Animation of Centre Vortex Structure Google: YouTube CSSM Visualisations