The Non-perturbative Quark-Gluon Interaction and its Implications

A.Kizilersu, J.Skullerud, O.Oliveira, A.Sternbeck, P. Silva

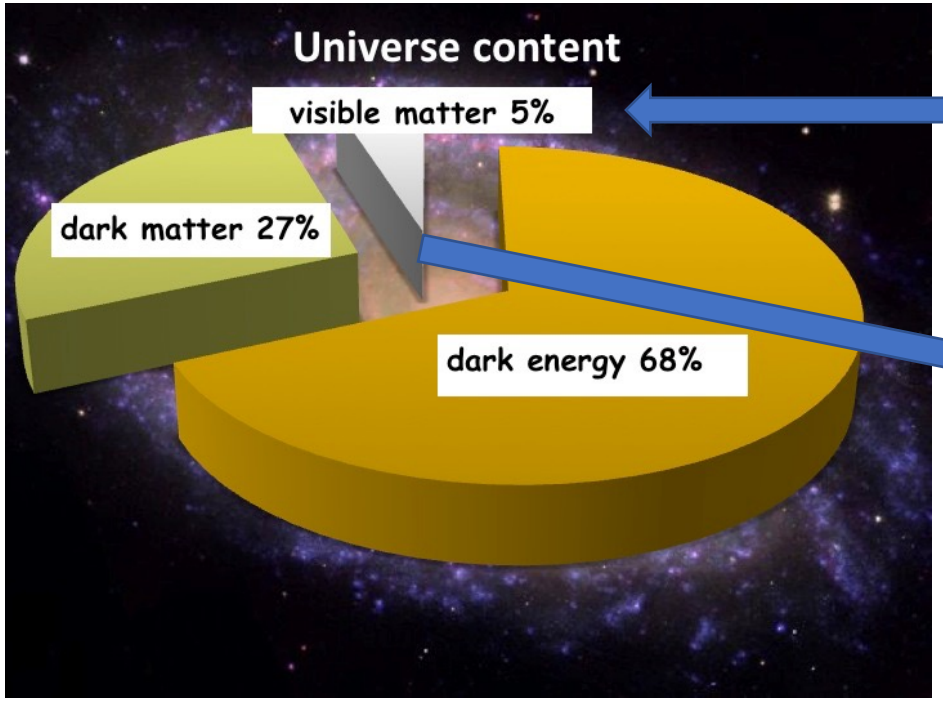
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

XVth Quark Confinement and the Hadron spectrum

1-6 August 2022

THE UNIVERSITY of ADELAIDE

MASS is a MYSTERIOUS CONCEPT!



Protons , neutrons, electrons … : it is us

0.1% of visible matter is due to the "HIGGS" mechanism

What about the remaining of the visible matter?

The rest emerges from the interactions to keep quarks together inside hadrons

Visible world: mainly made of light quarks

Existence of our Universe:

Proton (uud) : massive and stable

Proton **mass** ~ 940 MeV (~1 GeV)

QCD

Gluon Field — Quark Field — Quark Mass

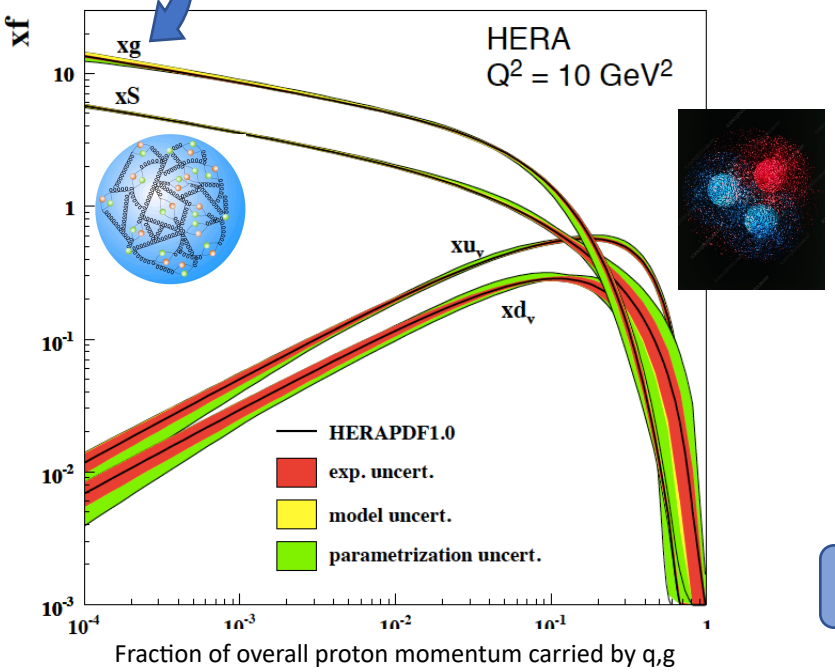
$$\mathcal{L}_{QCD} = -g\overline{\Psi}_i\gamma^\mu A^a_\mu T^a_{ij}\Psi_j + \overline{\Psi}_i(i\ \partial\!\!\!/ - m)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \boxed{g f^{abc} A^b_\mu A^c_\nu}$$

Non-Abelian

Gluons and sea quarks dominate the proton structure at x<0.1

Density of quarks or gluons inside the proton



Confinement and Mass Generation are IR dynamics of QCD NONPERTURBATIVE

Asymptotic Freedom is UV dynamics of QCD PERTURBATIVE

SDE,FRG, Lattice, , Effective Field theories, ...

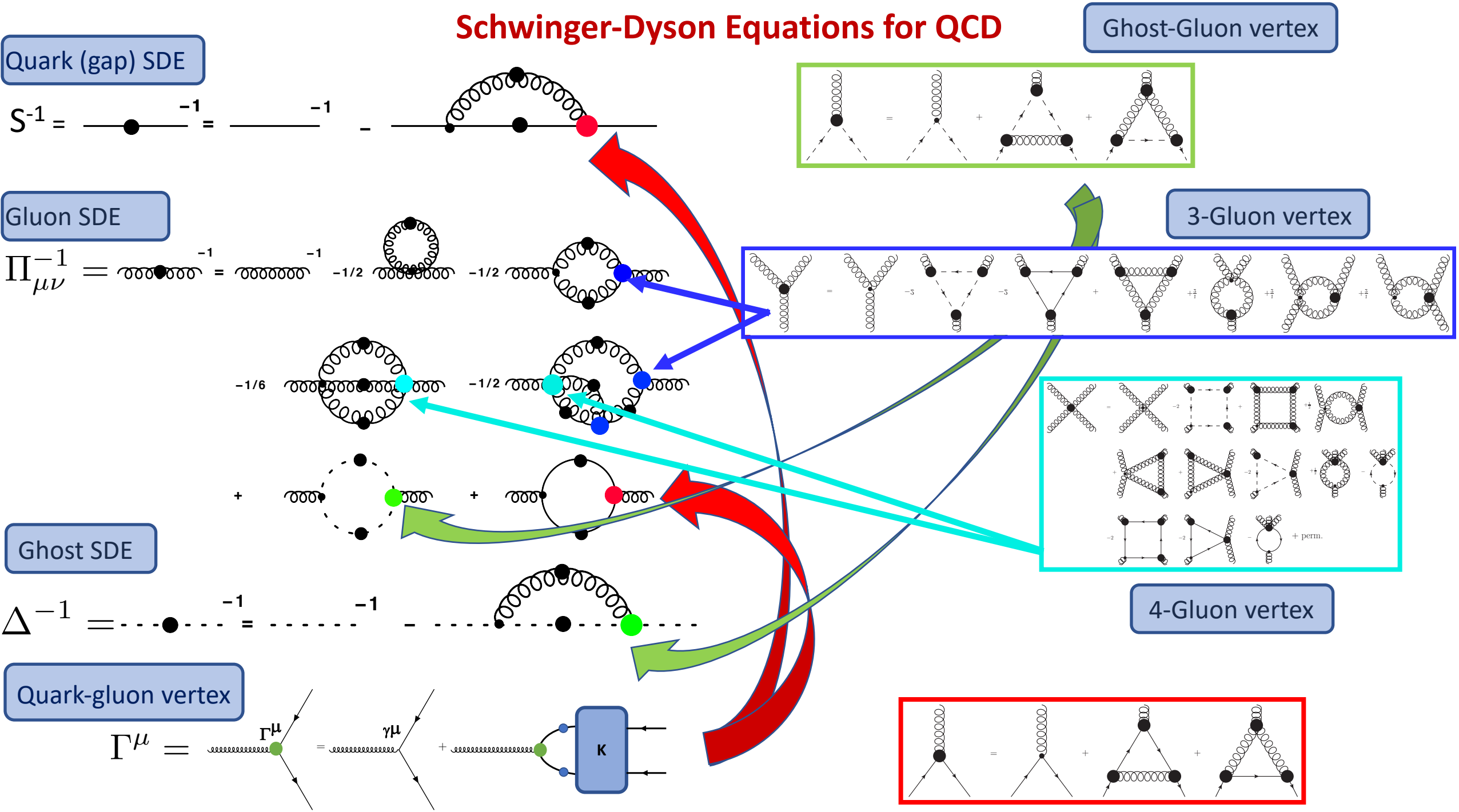
Proton Size ~10^-15m~ 1fm

Confinement ~ 1fm

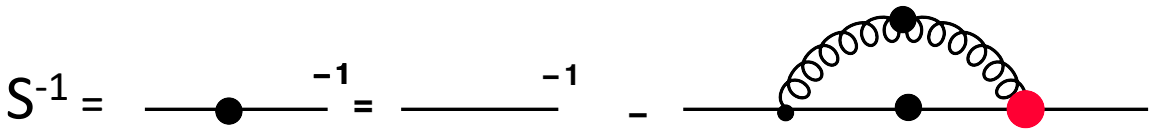
Quark and Gluon Size ~10^-17m

Asymptotic freedom ~ 1/10 fm

Schwinger-Dyson Equations for QCD



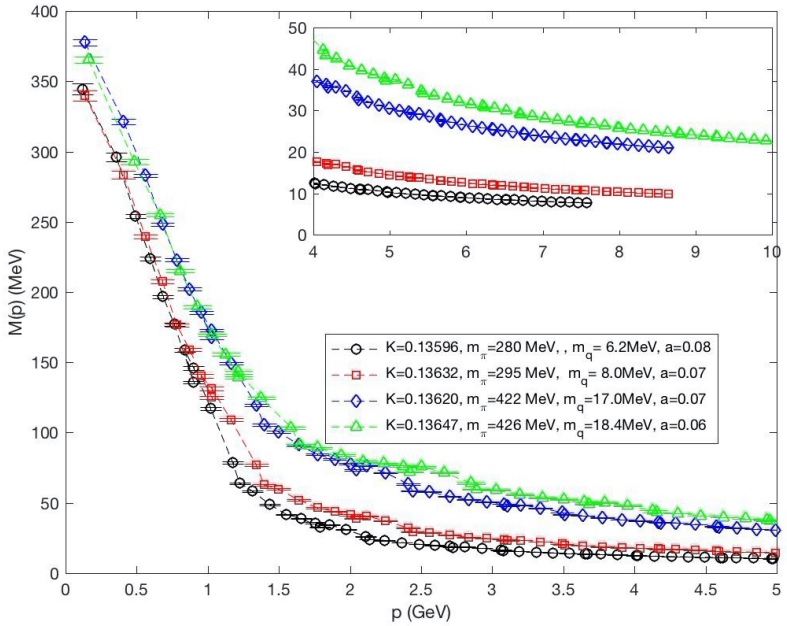
Quark Propagator (MATTER SECTOR of QCD)



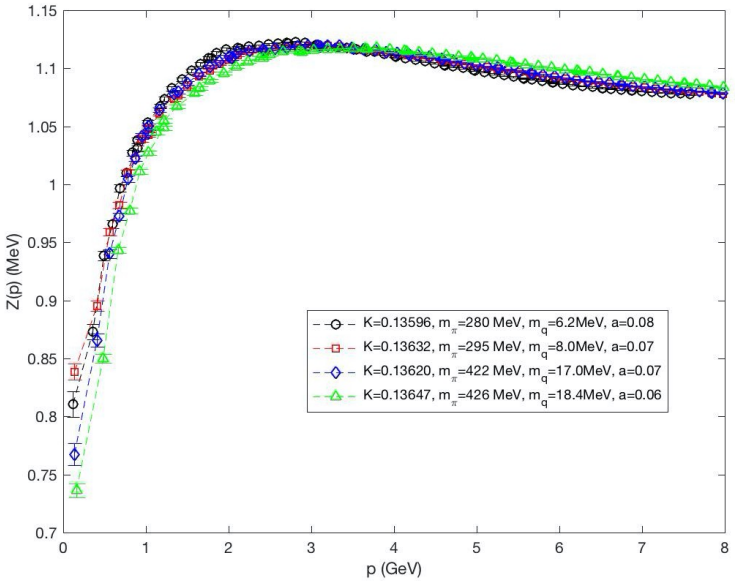
$$S_F(p) = \frac{Z(p^2)}{i\ \not{p} + M(p^2)} = \frac{1}{i\ \not{p}A(p^2) + B(p^2)}$$

$$S_0(p) = \frac{1}{i\ \not{p} + m_0}$$

All 4 ensembles on the 32^3 x 64 volume



All 4 ensembles on the 32^3 x 64 volume, using the tree-level corrected Z(p)



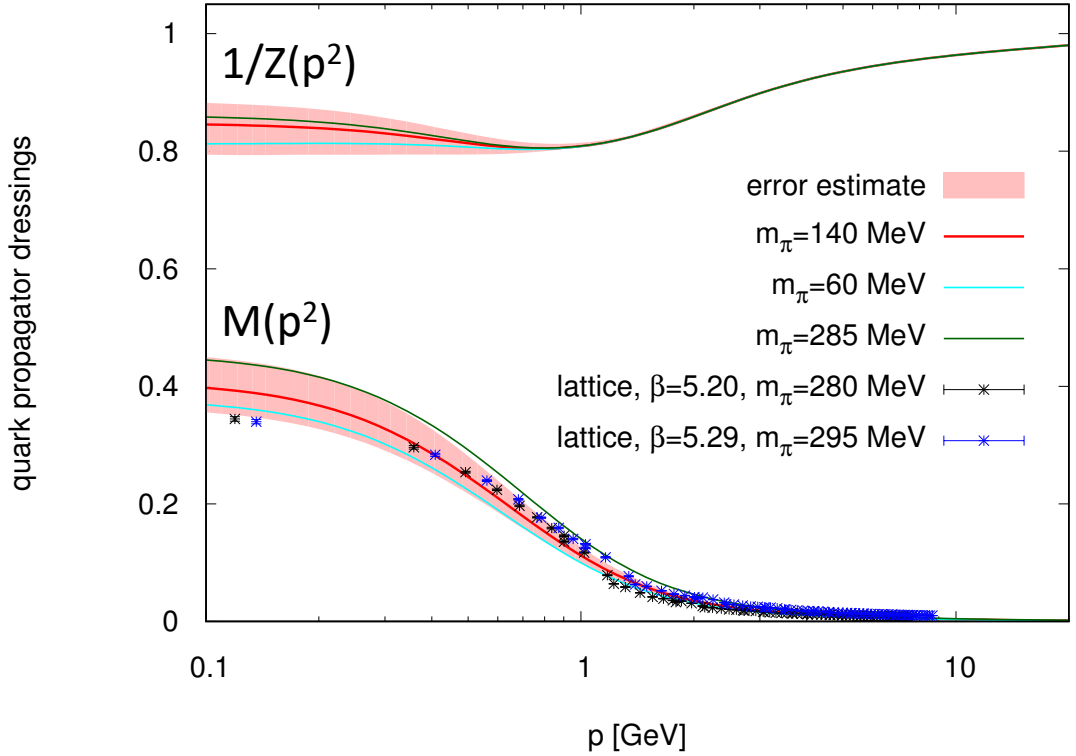
O. Oliveira, A. Kizilersu, P. J. Silva, J.-I. Skullerud, A.Sternbeck, and A. G. Williams, APPB Proc.Sup. Vol9 (2016) 63 (LATTICE)

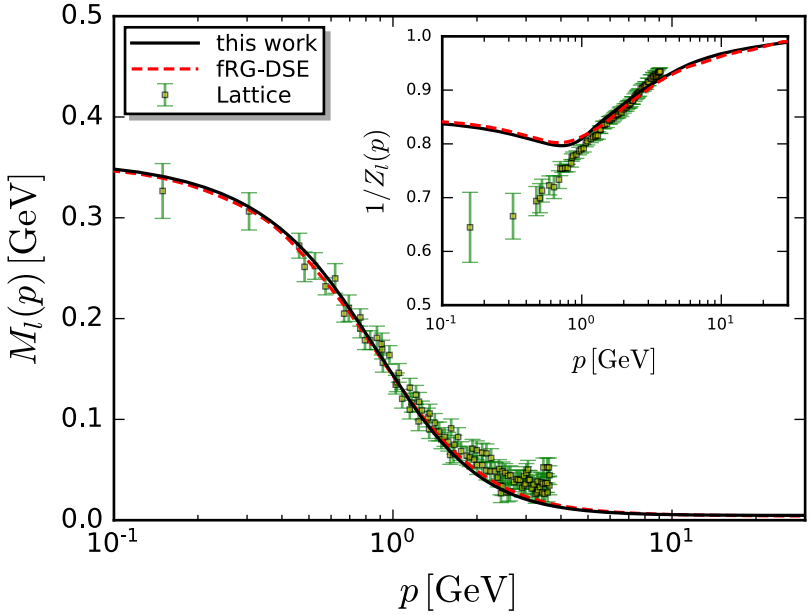
Quark Propagator

$S_F^{-1}(p) =$ 

$$S_F(p) = \frac{Z(p^2)}{i\,\not{p} + M(p^2)} = \frac{1}{i\,\not{p}A(p^2) + B(p^2)}$$

$$S_0(p) = \frac{1}{i\,\not{p} + m_0}$$





F. Gao, J. Papavassiliou, J. M. Pawlowski, Phys. Rev. D 103 (9) (2021) 094013

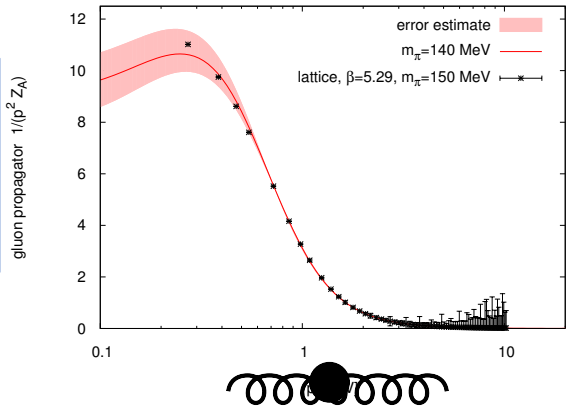
P. O. Bowman, et al., Phys. Rev. D71 (2005)054507 (LATTICE)

A. K. Cyrol, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D97 (5) (2018) 054006 (FRG)

O. Oliveira, A. Kizilersu, P. J. Silva, J.-I. Skullerud, A.Sternbeck, and A. G. Williams, APPB Proc.Sup. Vol9 (2016) 63 (LATTICE)

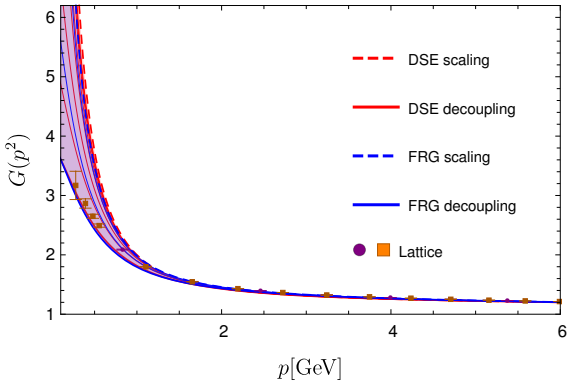
Gluon Prop. (N_F=2, Dynamical)

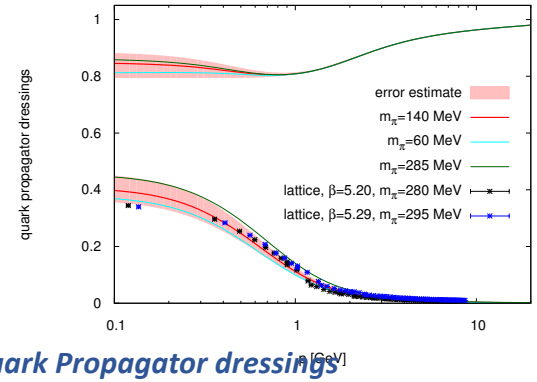
K. Cyrol, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D97 (5) (2018) 054006 (FRG)
Sternbeck, K. Maltman, M. Muller-Preussker, and L.von Smekal, Proc. Sci., LATTICE2012 (2012) 243 (Lattice)



Ghost Dressing function

A. Sternback, PhD thesis,(2006) (Lattice SU(2))
M. Q. Huber, Phys. Rev. D 101 (2020)114009 (SDE)
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D 94 (5) (2016) (FRG)



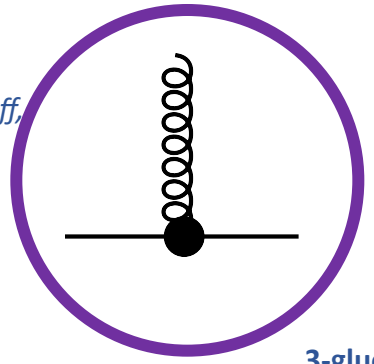


Quark Propagator dressings

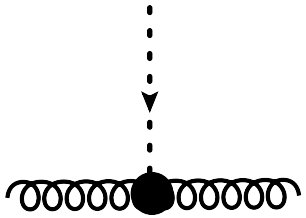
K. Cyrol, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D97 (5) (2018) 054006 (FRG)
O. Oliveira, A. Kizilersu, P. J. Silva, J.-I. Skullerud, A.Sternbeck, and A. G. Williams, APPB Proc.Sup. Vol9 (2016) 63 (LATTICE)

Quark Propagator

Ghost Propagator

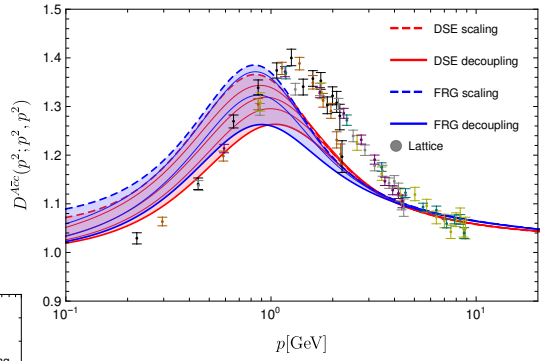


QCD

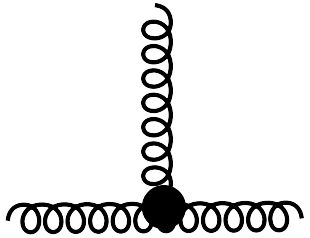


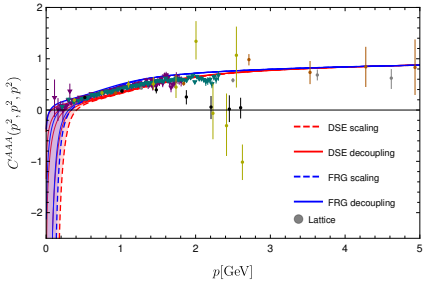
Ghost-gluon Vertex

A. Maas, SciPost Phys. 8 (5)(2020) 071 (Lattice)



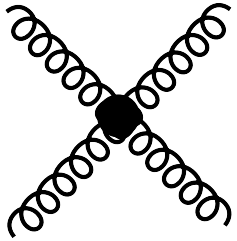
3-gluon Vertex

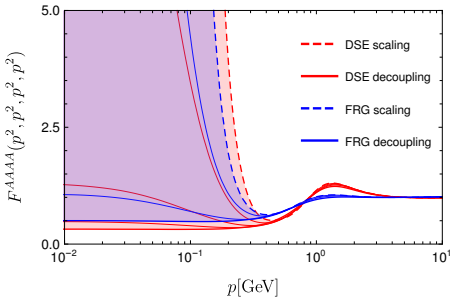




A.Sternbeck, P.-H. Balduf, A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, A. G. Williams, PoS LATTICE2016(2017) 349
A. Cucchieri, A. Maas, T. Mendes, Phys.Rev. D 77 (2008) 094510

4-gluon Vertex





M. Q. Huber, Phys. Rev. D 101 (2020)114009 (SDE)
K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D 94 (5) (2016) 054005(FRG)

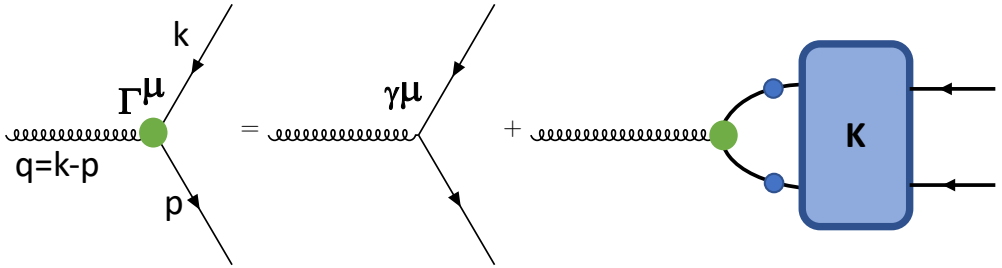
QUARK-GLUON VERTEX

We study the quark-gluon vertex in the limit of vanishing gluon momentum using lattice QCD with two flavors for several lattice spacings, volumes and quark masses

A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, A. Sternbeck, Phys. Rev. D 103 (11) (2021) 114515

Non-Perturbative Quark-Gluon Vertex:

$$\left(\Delta^a_\mu\right)^{ij}_{\beta\rho} = t^a_{ij}\ \left(\Delta_\mu\right)_{\beta\rho} =$$



$$\mathbf{\Lambda^\mu_F(p,k,q) = \sum_{i=1}^{4} \lambda^i(p^2,k^2,q^2,\xi,m)\, L^\mu_i(p,k) + \sum_{i=1}^{8} \tau^i(p^2,k^2,q^2,\xi,m)\, T^\mu_i(p,k)}$$

Non-Transverse Part

Transverse Part

$$q\cdot T_i(p,k,q) = 0 \qquad \Lambda^\mu_T(p,p,0) = 0$$

$$q_\mu\Lambda^\mu(p,q,k) \quad = \quad q_\mu\Lambda^\mu_{NT}(p,q,k)$$

Transverse Slavnov-Taylor- Identities

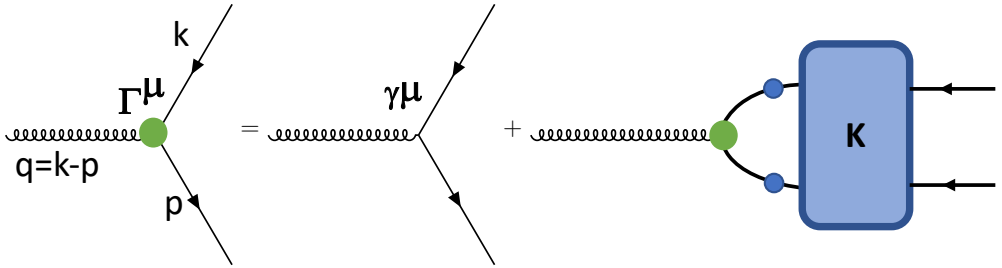
Ghost-Quark Scattering Kernel

Slavnov-Taylor Identity (QCD)

$$q_\mu\Lambda^\mu(p,q,k) = G_h(q^2)\ \left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]$$

Non-Perturbative Quark-Gluon Vertex:

$$\left(\Delta^a_\mu\right)^{ij}_{\beta\rho} = t^a_{ij}\ \left(\Delta_\mu\right)_{\beta\rho} =$$



Transverse Slavnov-Taylor- Identities

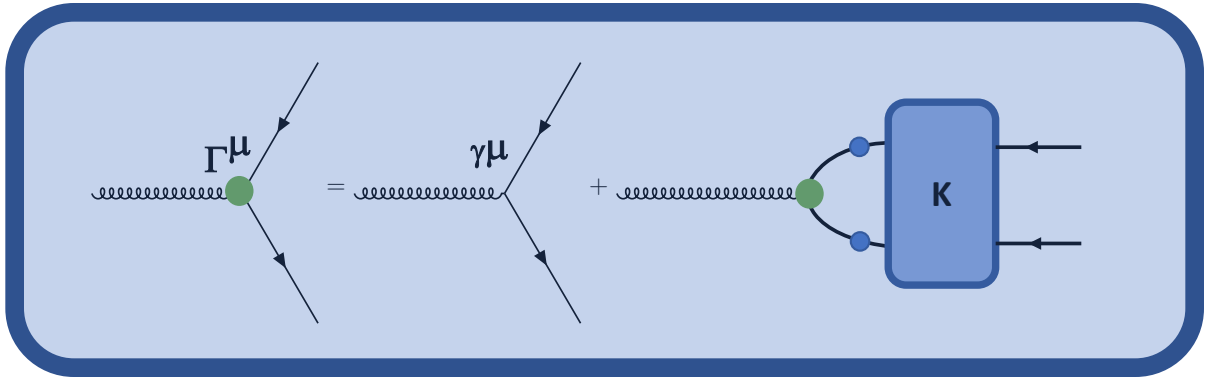
Normal STI

$$q_\mu \Lambda^\mu(p,q,k) = G_h(q^2)\ \left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]$$

Transverse STI

$$\begin{aligned} iq^\mu \Gamma^\nu_V(p_1,p_2) - iq^\nu \Gamma^\mu_V(p_1,p_2) &= S_F^{-1}(p_1)\sigma^{\mu\nu} + \sigma^{\mu\nu} S_F^{-1}(p_2) \\ &+ 2m\boxed{\Gamma^{\mu\nu}_T(p_1,p_2)} \\ &+ (p_{1\lambda} + p_{2\lambda})\,\epsilon^{\lambda\mu\nu\rho}\boxed{\Gamma_{A\rho}(p_1,p_2)} \\ &- \int \frac{d^4k}{(2\pi)^4} 2k_\lambda\,\epsilon^{\lambda\mu\nu\rho}\,\Gamma_{A\rho}(p_1,p_2;k) \end{aligned}$$

Quark-Gluon Vertex



Slavnov-Taylor Identity:

$$\mathbf{q_\mu \Lambda^{\mu}_{NT}(p,q,k) = G_h(q^2)\ \left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]}$$

Quark-Gluon Vertex

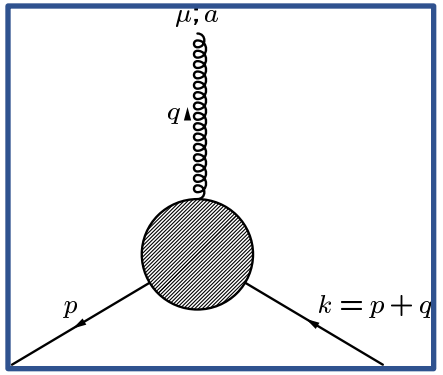
Ghost Dressing function

Ghost-Quark Scattering Kernel

Quark Propagator

- Free of kinematic singularity
- Multiplicatively Renormalisable
- Reduce tree level form in the free field limit
- Make sure local gauge covariance of the SDE's (LKT)
- Must have the same transformation properties as the bare vertex under charge conjugation and Lorenz transformation (P,T)
- Must satisfy STI's
- Must satisfy TSTI's

Non-Perturbative Vertex (from LATTICE)



Quark-gluon vertex on the lattice :

$$\Lambda_\mu^{a,lat.}(p,q) = S_R(p)^{-1} V_\nu^a(p,q) S_R(p+q)^{-1} D(q)^{-1}_{\nu\mu}$$

Unamputated vertex

Gauge dependent quantity

$$V_\mu^a(p,q) = << S_R(p;U) A_\mu^a(q) >>$$

- ***(Sheikholeslami-Wohlert)*** clover fermion action
  - $\mathcal{O}(\alpha)$ improved rotated propagator

Wilson gauge action

Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\tilde{\Lambda}_\mu^T(p,k,q) = P_{\mu\nu}^T(q)\Lambda_\nu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Lambda_\nu(p,k,q)$$

Some of the Form Factors in Special Kinematics in Landau gauge are calculated in quenched QCD

J.Skullerud, A.Kizilersu, JHEP09(2002)013
J. Skullerud,P. Bowman, A.Kizilersu, D.Leinweber,A.Williams, JHEP04(2003)047

Lattice Parameters of Gauge Ensembles in this Study (N_f=2)

Lattice action

- Wilson gauge action
- ***(Sheikholeslami-Wohlert)*** clover fermion action
- $\mathcal{O}(\alpha)$ improved rotated propagator
- Landau gauge $(\xi = 0)$

Name	β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	$N_{\rm cfg}$	$N_{\rm src}$
L08	5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900	4
H07	5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900	4
L07	5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908	4
L07-64	5.29	0.13632	0.071	$64^3 \times 64$	290	8.0	750	2
H06	5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900	2
Q07	6.16	0.13400	0.071	$32^3 \times 64$	1000	130	998	4

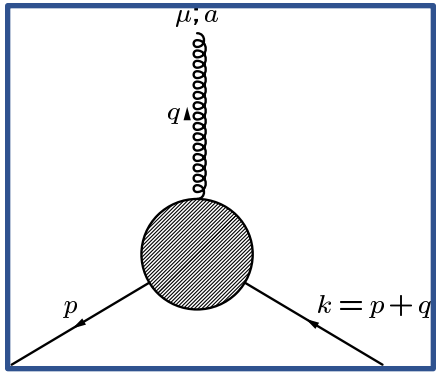
Acknowledgements

N_f = 2 gauge ensambles are provided by RQCD collaboration (Regensburg), *S. Bali et all, Phys Rev D91, 054501 (2014)*

A. Kizilersu, O. Oliveira, P.J. Silva, J. Skullerud and A. Sternbeck, Phys.Rev.D103 (2021)114515

Form Factor Extraction

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$



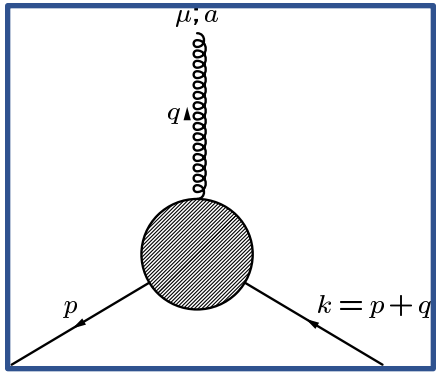
$$(\overline{\Lambda}^a_\mu) = -ig_0\,(\lambda_1\,[\gamma_\mu] + \lambda_2\,[-4\,\not{p}p_\mu] + \lambda_3\,[-2ip_\mu]\,)$$

Covariant Non-Transverse Form factors:

- $$\lambda_1 = \frac{1}{(-ig_0)}\left\{\frac{1}{3}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - \frac{p_\mu p_\nu}{p^2}\,\mathrm{Tr}_4(\gamma_\nu\overline{\Lambda}_\mu)\right]\right\}$$
- $$\lambda_2 = \frac{1}{(-ig_0)}\left\{\frac{1}{3p^2}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - 4\frac{p_\mu p_\nu}{p^2}\,\mathrm{Tr}_4(\gamma_\nu\overline{\Lambda}_\mu)\right]\right\}$$
- $$\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\,\frac{p_\mu}{p^2}\,\mathrm{Tr}_4(I\,\overline{\Lambda}_\mu)\right\}$$

Form Factor Extraction

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$



$$(\overline{\Lambda}^a_\mu) = -ig_0\left(\lambda_1\,[\gamma_\mu] + \lambda_2\,[-4\,\not{p}p_\mu] + \lambda_3\,[-2ip_\mu]\right)$$

Covariant Form factors in Continuum:

- $\lambda_1 = \frac{1}{(-ig_0)}\left\{\frac{1}{3}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - \frac{p_\mu p_\nu}{p^2}\mathrm{Tr}_4(\gamma_\nu\overline{\Lambda}_\mu)\right]\right\}$
- $\lambda_2 = \frac{1}{(-ig_0)}\left\{\frac{1}{3p^2}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - 4\frac{p_\mu p_\nu}{p^2}\mathrm{Tr}_4(\gamma_\nu\overline{\Lambda}_\mu)\right]\right\}$
- $\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\frac{p_\mu}{p^2}\mathrm{Tr}_4(I\,\overline{\Lambda}_\mu)\right\}$

$\nu=\mu$

$P_\mu=0$

$\nu \neq \mu$

Non-covariant Form factors in Continuum :

- $\lambda_1 = \frac{1}{(-ig_0)}\left\{\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu\\ p_\mu=0}}\right\}$
- $\lambda_2 = \frac{1}{(-ig_0)}\left\{-\frac{1}{4\,p^2}\frac{p_\alpha p_\mu}{p^2}\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right]\right\}$
- $\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\frac{p_\mu}{p^2}Tr_4(I\,\overline{\Lambda}_\mu)\right\}$

MOM Renormalisation: $\lambda_1^R(\mu^2, 0, \mu^2) = 1$ $\Rightarrow$ $\Gamma^{\mathrm{lat}}_\mu(p,k,q) = Z_1\Gamma^R_\mu(p,k,q)$

Lattice form factors and Tree-Level Corrections

Continuum form factors

$$\lambda_1 = \frac{1}{(-ig_0)}\left\{\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu\\ p_\mu=0}}\right\}$$

$$\lambda_2 = \frac{1}{(-ig_0)}\left\{-\frac{1}{4\,p^2}\frac{p_\alpha p_\mu}{p^2}\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right]\right\}$$

$$\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\frac{p_\mu}{p^2}Tr_4(I\,\overline{\Lambda}_\mu)\right\}$$

Tree-level corrected, lattice equivalents of the form factors

Lattice momentum variables: $\mathbf{p}_\mu \to \mathbf{K}_\mu(\mathbf{p}) \equiv \frac{\mathbf{1}}{\mathbf{a}}\sin(\mathbf{p}_\mu\mathbf{a}),\ \mathbf{C}_\mu(\mathbf{p}) = \cos(\mathbf{p}_\mu\mathbf{a})$

$$\lambda_1(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0}\left\{\left[\mathrm{Tr}_4(\gamma_\alpha\overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu\\ p_\mu=0}}\right\}\Big/\lambda_1^{(0)}$$

$$\lambda_2(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0}\left\{-\frac{1}{4\,K(p)^2}\frac{K_\alpha(p)K_\mu(p)}{K(p)^2}\left[\mathrm{Tr}_4(\gamma_\alpha\overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right]\right\} - \left(\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)}\right)$$

$$\lambda_3(p^2,0,p^2) = \frac{\mathrm{Re}}{(-g_0)}\left\{\frac{1}{2}\frac{K_\mu(p)}{K^2(p)}\mathrm{Tr}_4(I\,\overline{\Lambda}_\mu)\right\} - \left(\lambda_3^{(0)} + \overline{\lambda}_{3(\mu)}^{(0)}\right)$$

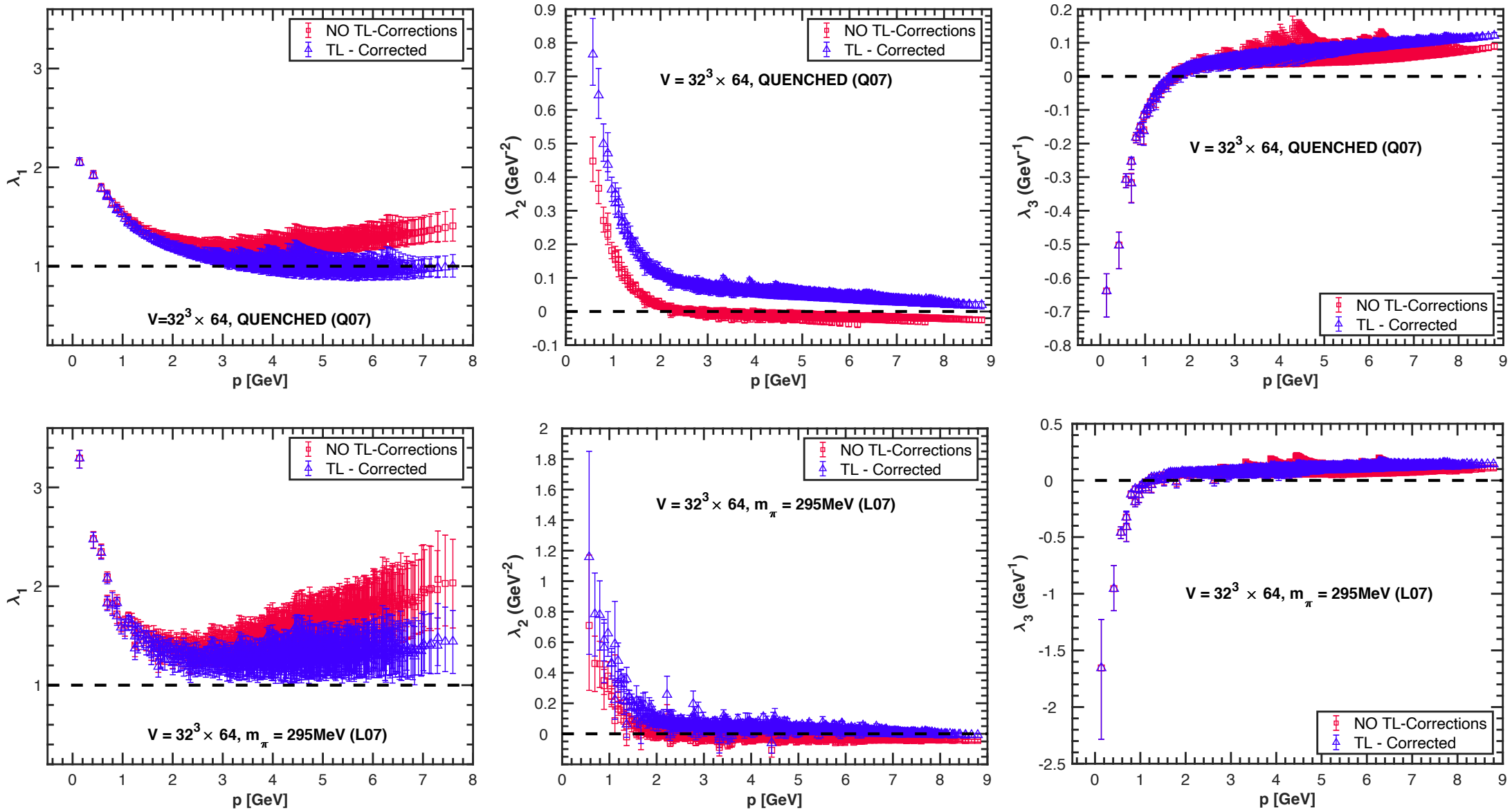
Lattice tree-level corrected form factors

$$\lambda_1^{(0)} = F(p)\left(1 + c_q^2 a^2 K^2(p)\right)^2$$

$$\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)} = a^2 F(p)\left[-c_q\left(1 - c_q^2 a^2 K^2(p)\right) + 2c_q^2 a\, C_\mu(p)\right]$$

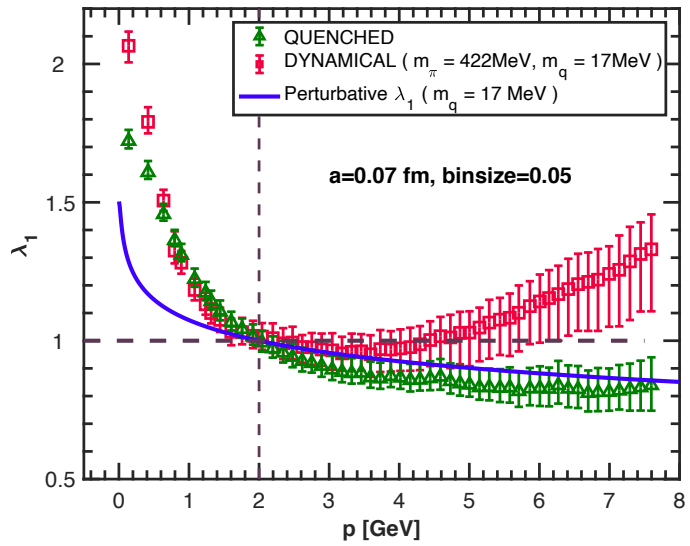
$$\lambda_3^{(0)} + \overline{\lambda}_{3,(\mu)}^{(0)} = \frac{a}{2}F(p)\left[\left(1 - c_q^2 a^2 K^2(p)\right)^2 - 4c_q^2 a^2 K^2(p) - 4c_q\left(1 - c_q^2 a^2 K^2(p)\right)C_\mu(p)\right]$$

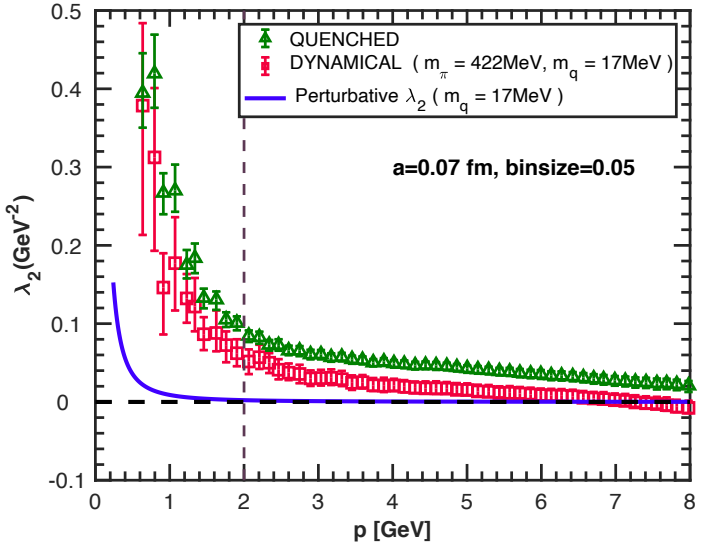
Tree Level Corrected vs Uncorrected Form Factors

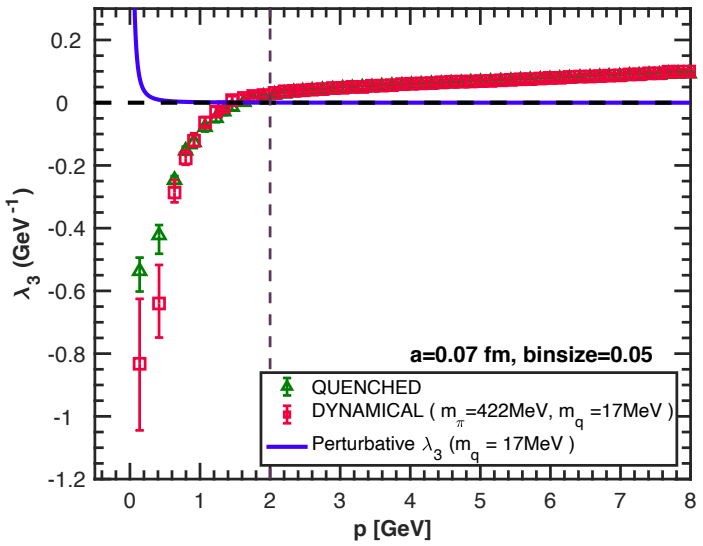


Quenched vs Dynamical

$N_f = 0$ vs $N_f = 2$ (a=0.07 fm)

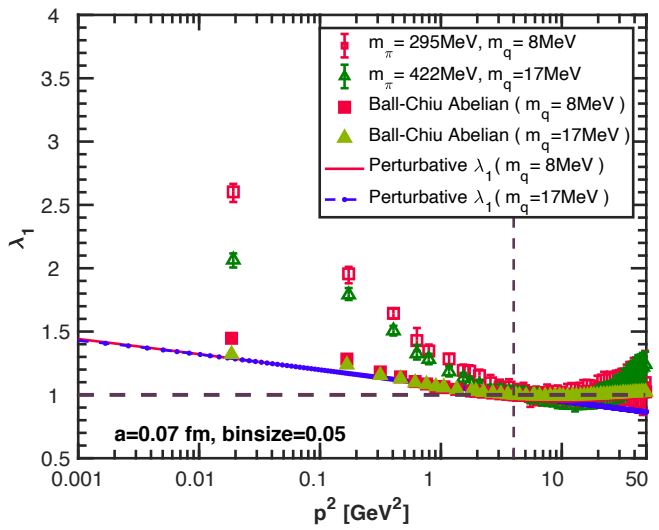




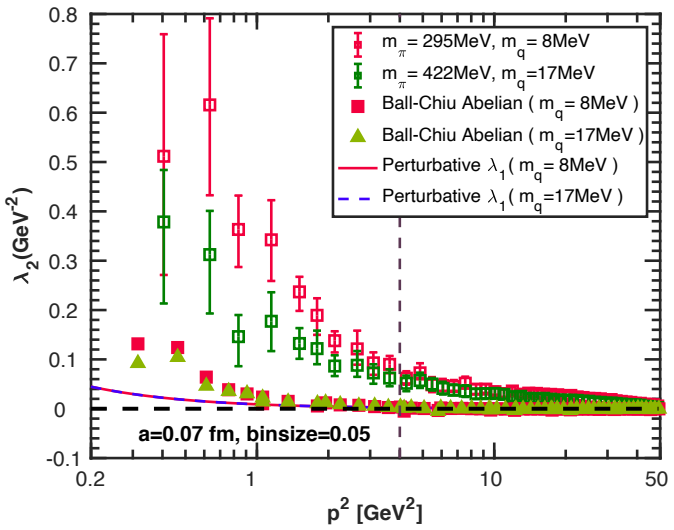


Quark Mass Dependence

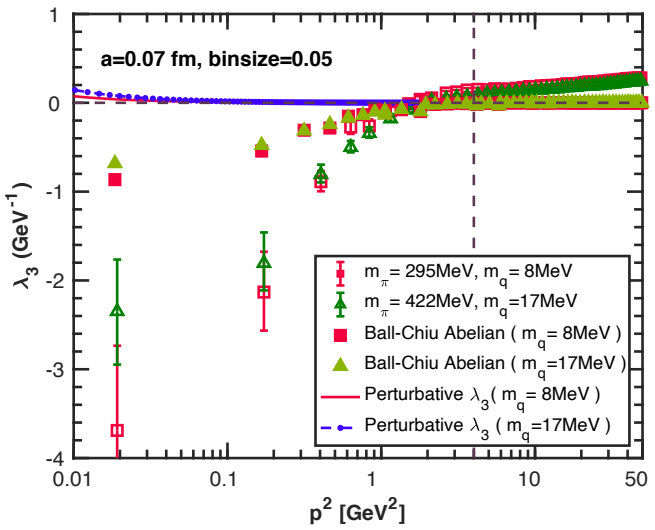
$$S_F(p) = \frac{F(p^2)}{\not{p} - M(p^2)} = \frac{1}{A(p^2)\ \not{p} - B(p^2)}$$



$$\lambda_1^{BC} = A(p^2)$$



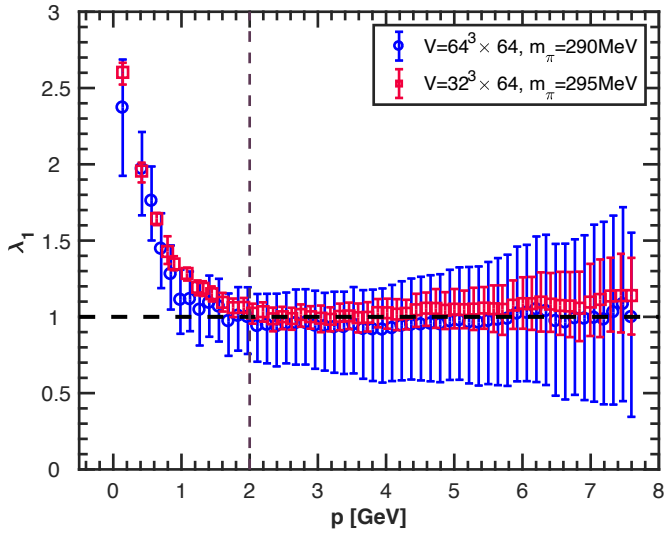
$$\lambda_2^{BC} = -\frac{1}{2}\frac{dA(p^2)}{dp^2}$$

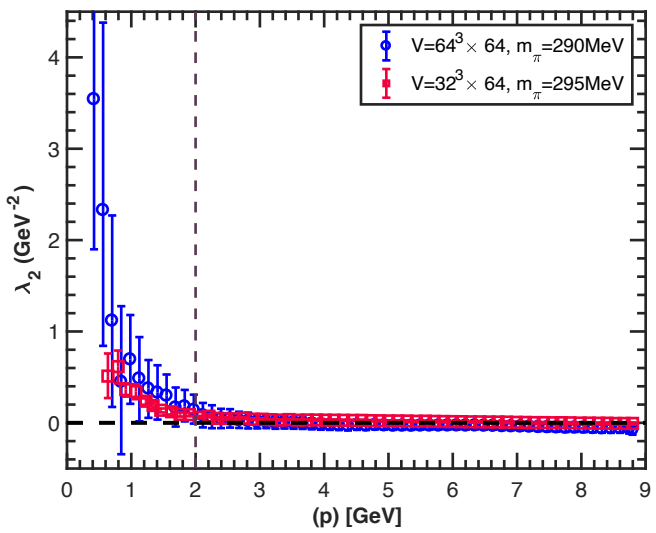


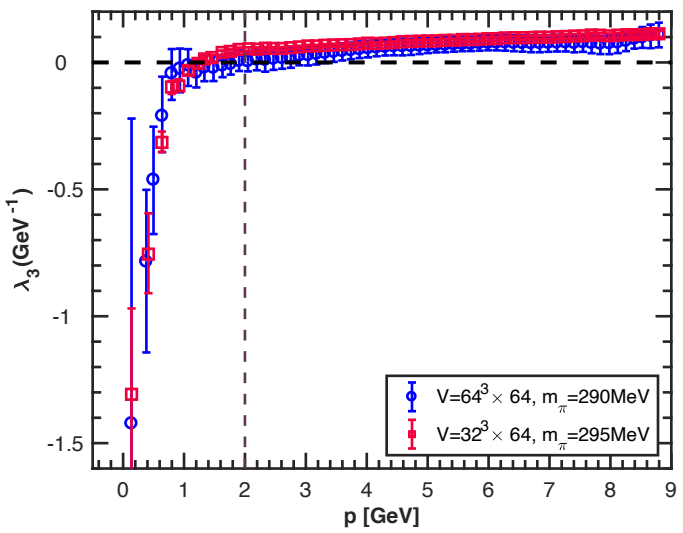
$$\lambda_3^{BC} = \frac{dB(p^2)}{dp^2}$$

discret./Volume Dependence

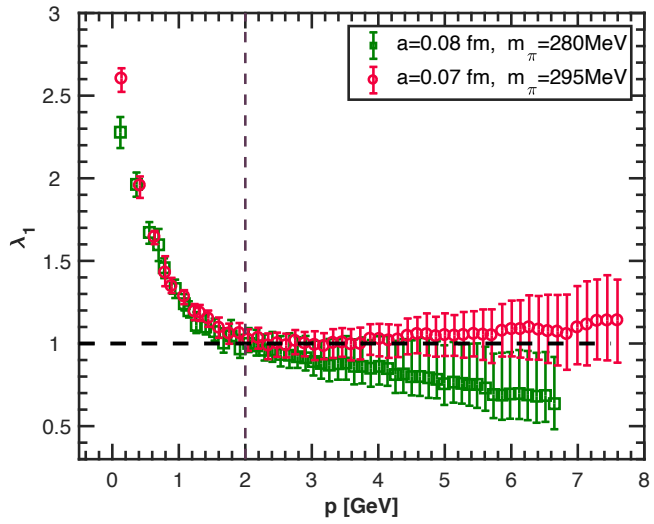
(a=0.07 fm)

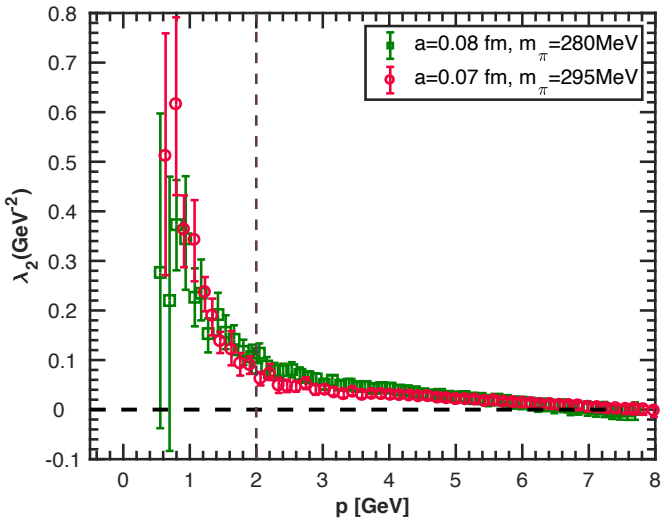


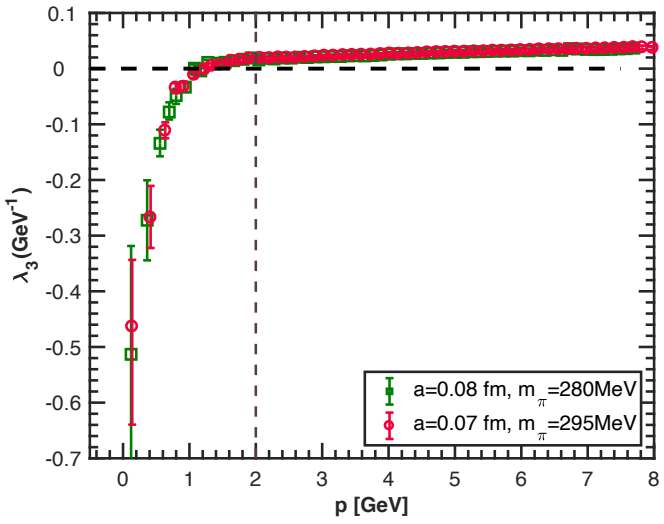


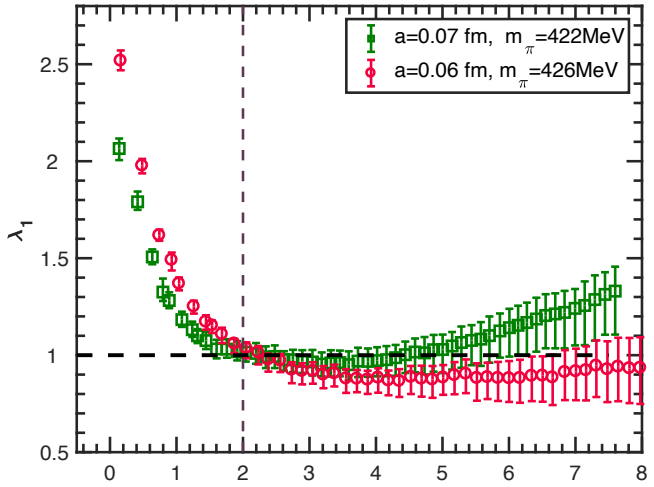


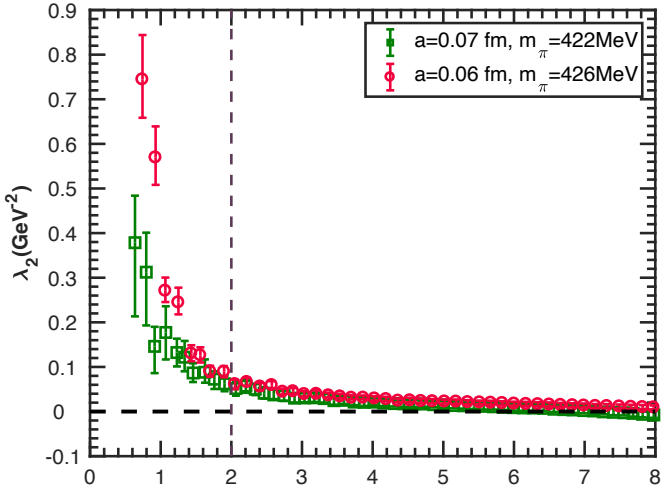
Lattice Spacing

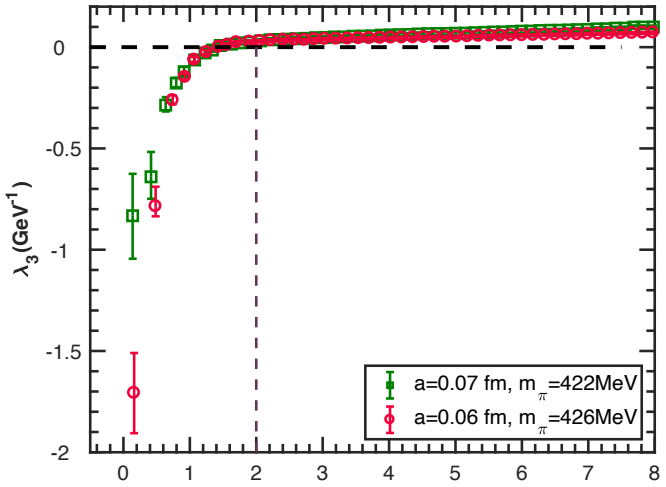






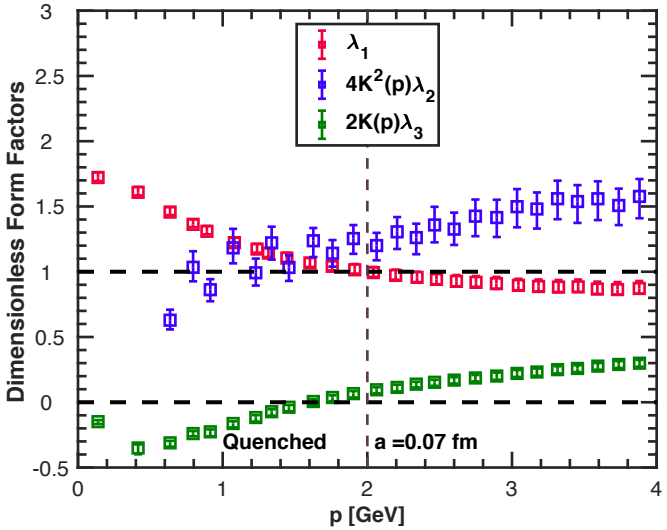


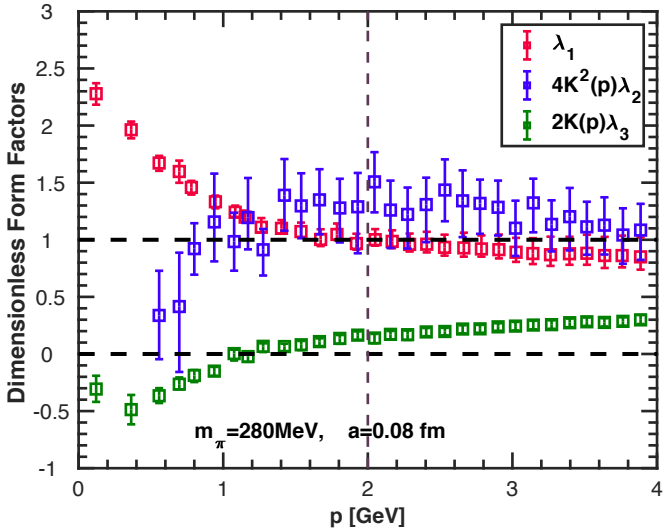


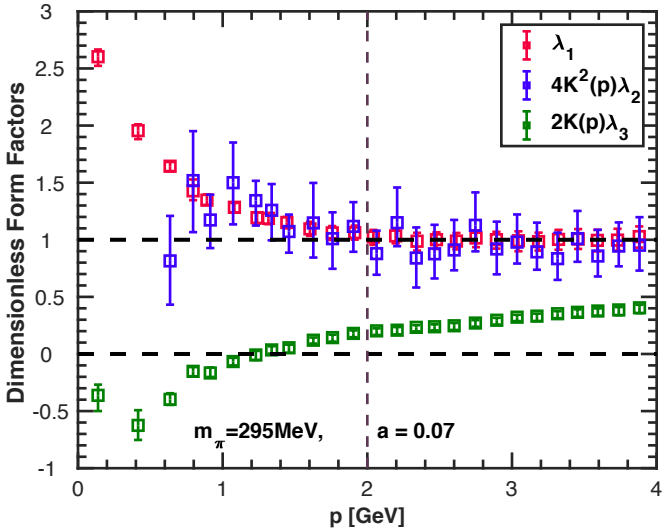


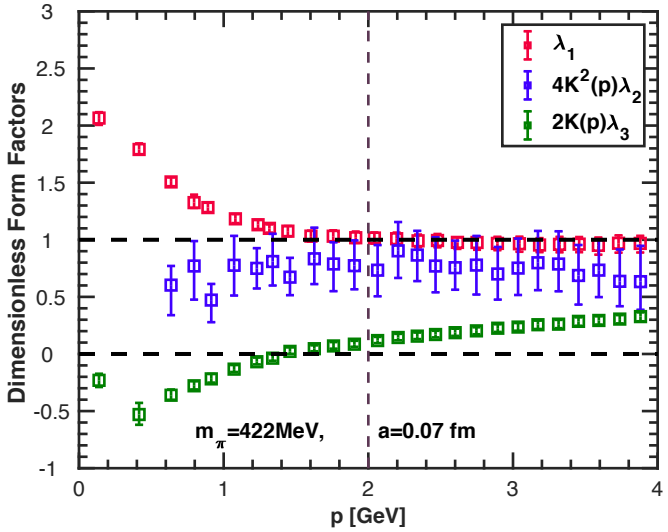
Dimensionless Form Factors

$$K_\mu(p) = \frac{1}{a}\, sin(p_\mu a)$$

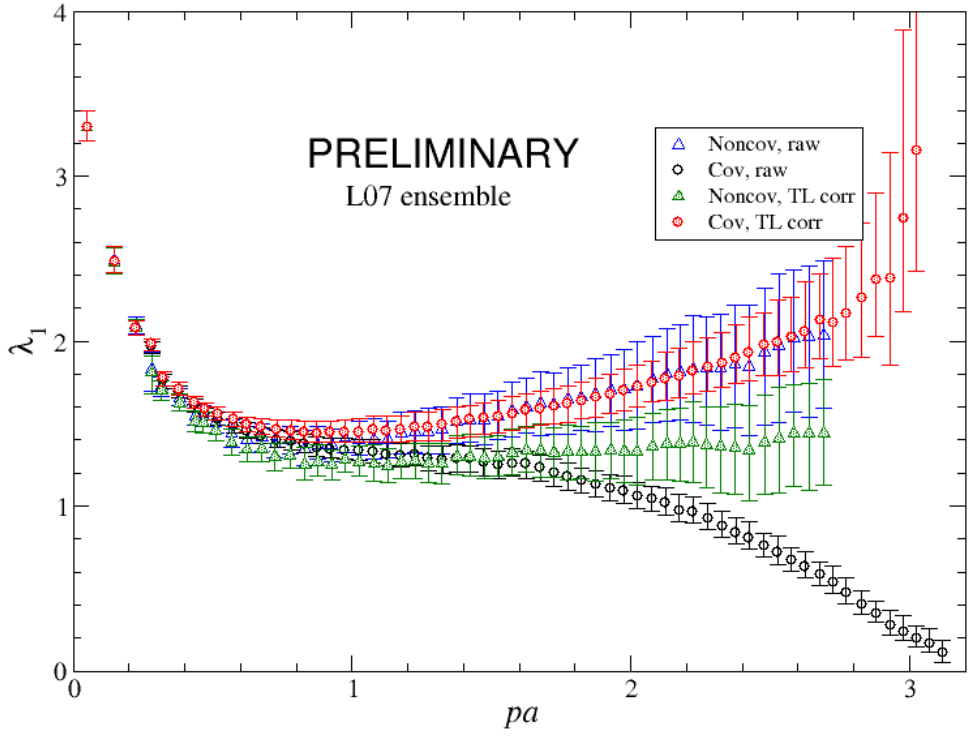


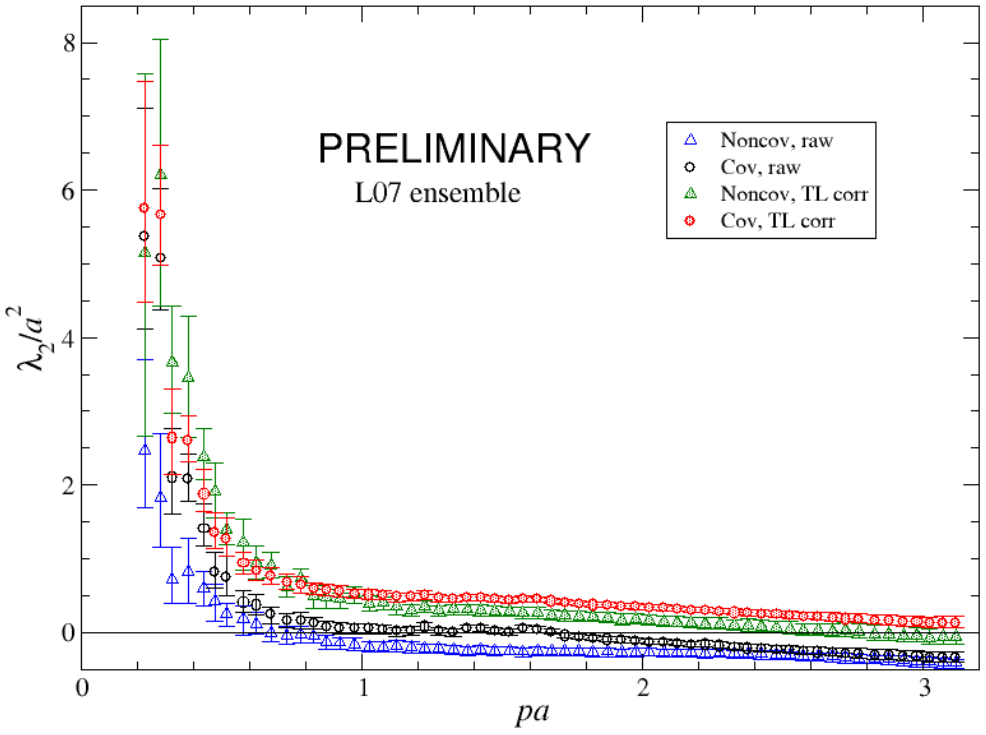






Covariant Form Factor





CONCLUSION

First ever study of Quark-Gluon Vertex in Soft Gluon Kinematics for Landau Gauge with N_f=2 dynamical fermions

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0$$

λ_1 is significantly enhanced in the IR that is stronger than in the quenched approximation and increases as the chiral limit is approached. No significant finite-volume effects

λ_2 exhibits an infrared strength smaller than λ_1, the enhancement increases as the continuum, infinite-volume, and chiral limits are approached

λ_3 shows considerably infrared strength larger than in the quenched approximation, increases as the continuum limit is approach. No significant volume effect

Orthogonal Kinematics : $q \cdot P = 0 \qquad k^2 = p^2$ $\qquad \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_6, \tau_5, \tau_7$