Soft pions and the dynamics of the chiral phase transition

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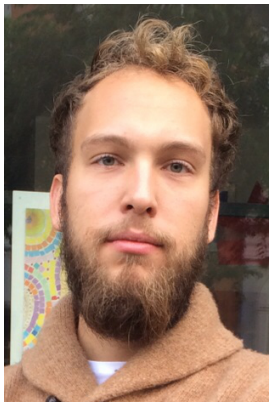
Presented by Alexander Soloviev

Stony Brook University

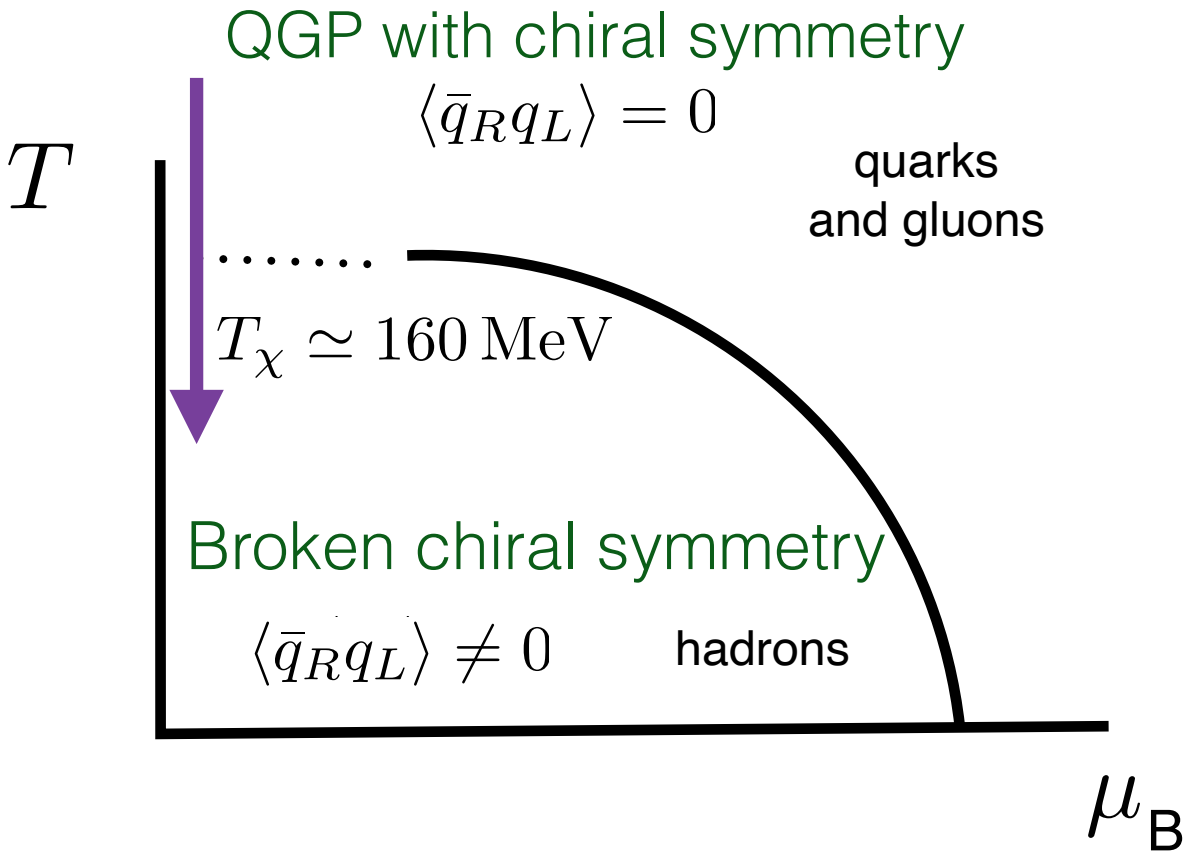
- Eduardo Grossi, AS, DT, Fanglida Yan: PRD, arXiv:2005.02885
- Eduardo Grossi, AS, DT, Fanglida Yan: PRD, arXiv:2101.10847
- Adrien Florio, Eduardo Grossi, AS, DT, PRD, arXiv:2111.03640






Chiral symmetry breaking and heavy ion collisions

Pisarski, Wilczek



For two massless quarks the chiral symmetry group is

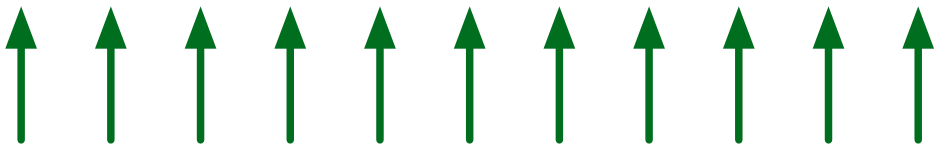
$$SU_L(2) \times SU_R(2) \simeq O(4)$$

This is broken, and the transition is 2nd order.

The mass smooths the transition to a crossover, like a magnetic field in the Ising model

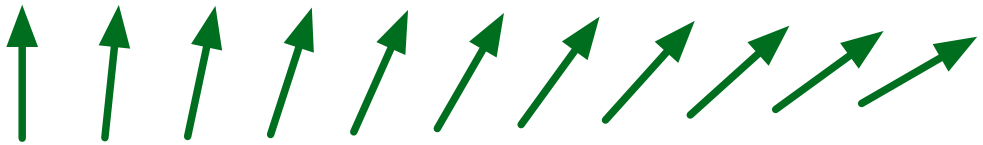
Chiral symmetry plays no role in the hydro model . . .

Our cold world: T< Tcritical



$$\langle \bar{q}_R q_L \rangle = \bar{\sigma}\, \mathbb{I}_{2\times 2}$$

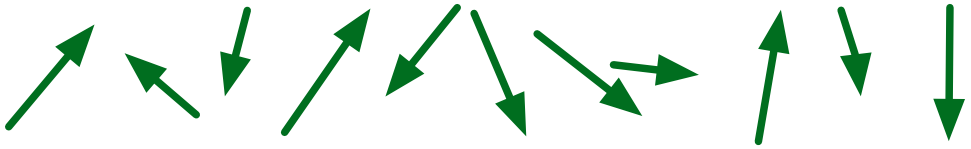
State is ordered. $\langle \bar{q}_R q_L \rangle$ is like the magnetization. $q = u, d$



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

The slow modulation of the $SU_A(2)$ phase of $\bar{q}_R q_L$ is a pion, $\vec{\pi}{=}\bar{\sigma}\vec{\varphi}$

The hot world: T> Tcritical



State is disordered: pion propagation is frustrated

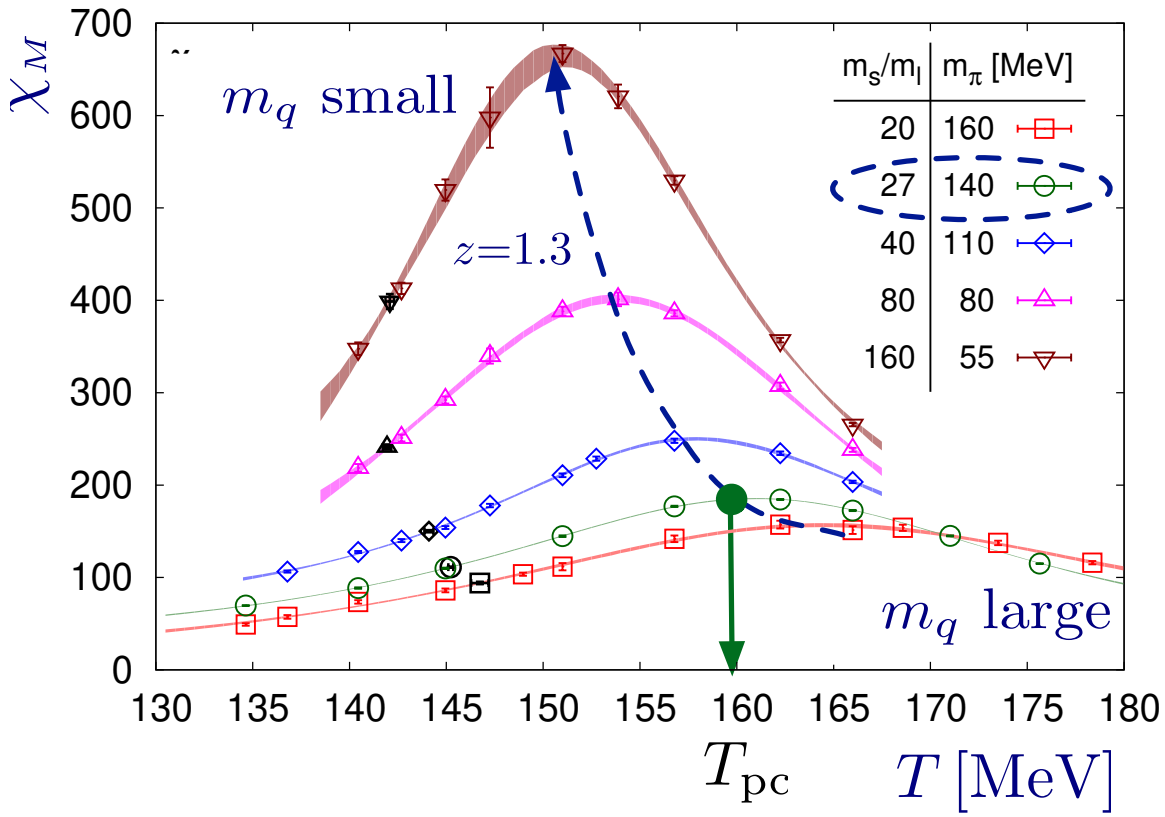
This talk will describe pion propagation during the $O(4)$ phase transition which is a model for the real world

Real world lattice QCD and the $O(4)$ critical point:

Hot QCD, PRL 2019

Fluctuations of order parameter, $\sigma \propto \bar{u}u + \bar{d}d$, vs temperature and m_q

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$



O(4) Scaling predictions

$$\chi_M = m_q^{1/\delta - 1} f_\chi(z)$$

$$z \equiv z_0 \frac{(T - T_c)}{T_c} m_q^{-1/\beta\delta}$$

The QCD lattice knows about the $O(4)$ critical point!

Static Universality and the Chiral Phase Transition

- The $O(4)$ order parameter fluctuates in amplitude and phase:

$$\phi_a = (\phi_0, \phi_1, \phi_2, \phi_3) = (\sigma, \vec{\pi})$$

  The quark condensate scales as

$$\bar{q}_R q_L \sim \sigma e^{i\vec{\tau}\cdot\vec{\varphi}} \simeq \sigma + i\vec{\tau}\cdot\vec{\pi}$$

- The Landau Ginzburg function for the $O(4)$ order parameter is: $\phi^2 \equiv \phi_a \phi_a$

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\,\phi^2 + \frac{\lambda}{4}\,\phi^4 - \underbrace{H}_{\propto m_q} \, \sigma$$

- The model has a critical mass, $m_0 - m_c \propto (T - T_c)$

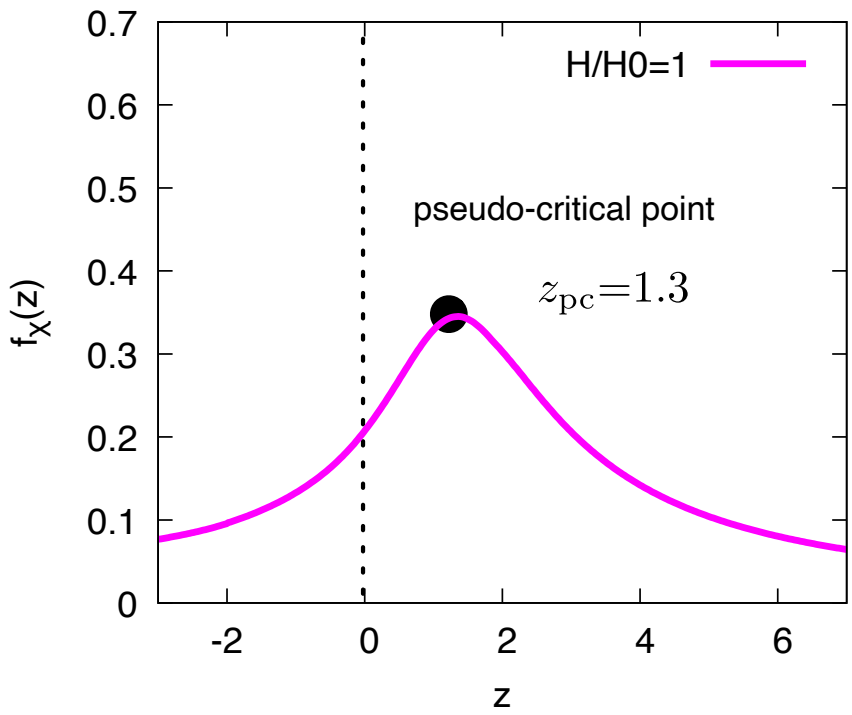
The critical model makes a definite prediction for the susceptibility:

Scaling predictions from the $O(4)$ model

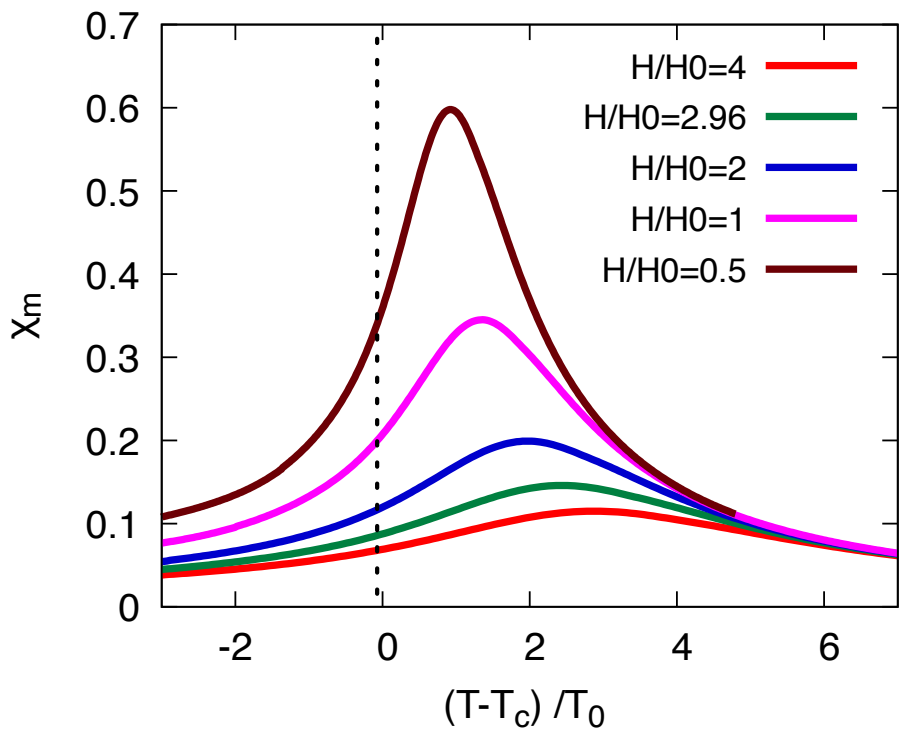
Simulations at different magnetic field are related to each other

$$\chi_M = h^{1/\delta - 1} f_\chi(z) \qquad z = z_0\, t_\mathrm{r} h^{-1/\beta\delta}$$

Here $h \propto H$ and $t_\mathrm{r} \propto (T - T_C)$ are the reduced field and temperature

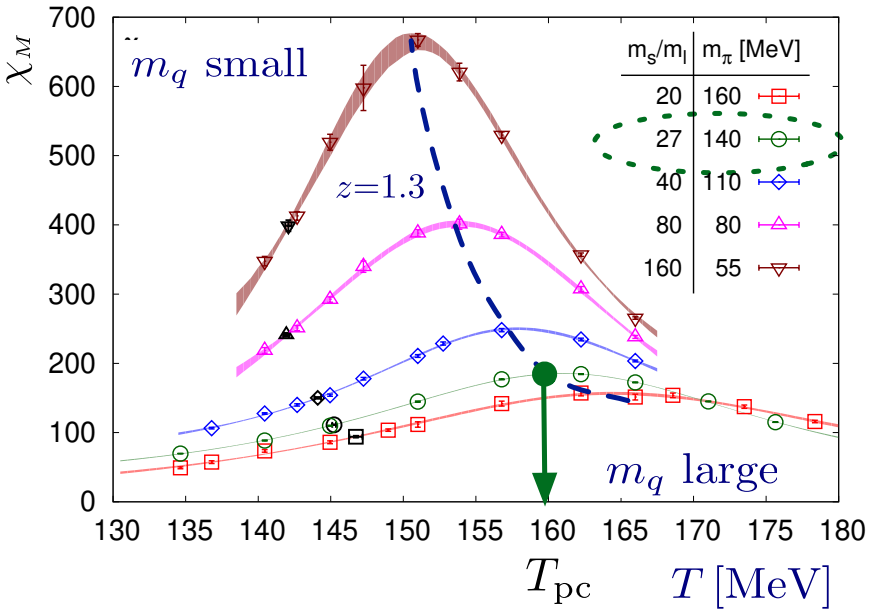


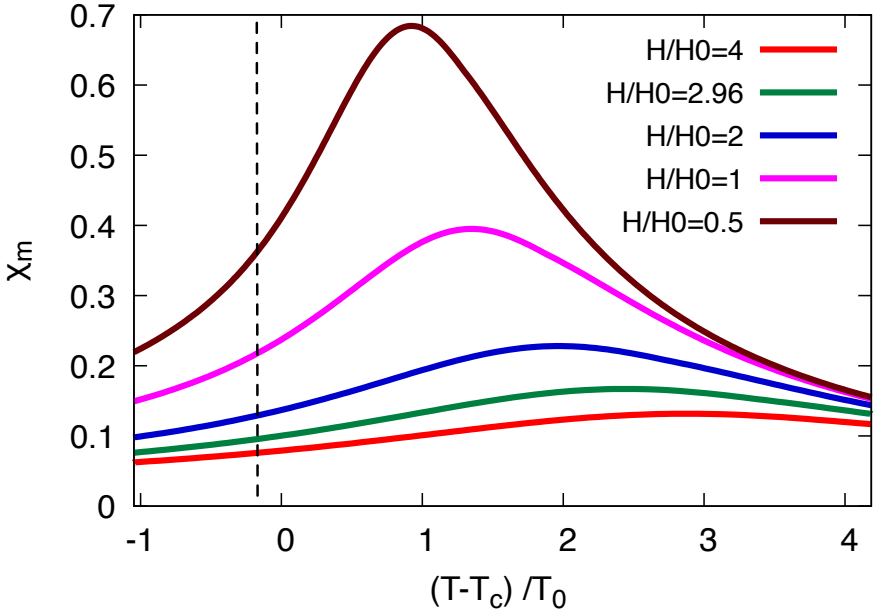
numerical data from
Engels, Seniuch, Fromme, Karsch



Scaling predictions and QCD

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$





Scaling predictions reasonably describe how the peak rises and shifts.

$$\chi_M \propto m_q^{1/\delta - 1} f_\chi(z) \qquad z = z_0 \left(\frac{T - T_C}{T_C} \right) m_q^{-1/\beta\delta}$$

From Thermodynamics to Hydrodynamics

Hydrodynamics of the $O(4)$ transition:

Rajagopal and Wilczek '92, Son '99, Son and Stephanov '01, and finally us, arxiv:2101.10847.

1. The order parameter

$$\phi_a = (\sigma, \vec{\pi})$$

2. The approximately conserved charges quantities:

$$\vec{n}_V = \underbrace{\bar{\psi}\gamma^0\vec{\tau}\psi}_{\text{isovect chrg}} \quad \textit{and} \quad \vec{n}_A = \underbrace{\bar{\psi}\gamma^0\gamma^5\vec{\tau}\psi}_{\text{isoaxial-vect chrg}}$$

which are combined into an anti-symmetric $O(4)$ tensor n_{ab}

$$n_{ab} = (\vec{n}_A, \vec{n}_V)$$

The charge n_{ab} generates $O(4)$ rotations, $\phi \to \phi_c + \frac{i}{\hbar}\theta_{ab}[n_{ab}, \phi_c]$, implying a Poisson bracket between the hydrodynamic fields:

$$\{n_{ab}(\boldsymbol{x}), \phi_c(\boldsymbol{y})\} = \epsilon_{abcd}\, \phi_d(\boldsymbol{x})\, \delta(\boldsymbol{x} - \boldsymbol{y})$$

The Landau-Ginzburg Hamiltonian for the $O(4)$ transition:

The Hamiltonian is tuned to the crit. point with $m_0^2(T) < 0$ and $H \propto m_q$:

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\phi^2 + \frac{\lambda}{4}\phi^4 - H\sigma + \frac{n_{ab}^2}{4\chi_0}$$

and gives the equilibrium distribution with the correct critical EOS:

$$Z = \int D\phi \, Dn \, e^{-\mathcal{H}[\phi,n]/T_c}$$

The hydro equations of motion take the form

$$\frac{\partial\phi}{\partial t} + \{\phi, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

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The hydro equations of motion take the form

$$\frac{\partial\phi}{\partial t} + \{\phi, \mathcal{H}\} = -\Gamma\frac{\delta\mathcal{H}}{\delta\phi_a} + \xi_a$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = \underbrace{\sigma_0\nabla^2\frac{\delta\mathcal{H}}{\delta n_{ab}}}_{\text{dissipation}} + \underbrace{\nabla\cdot\xi_{ab}}_{\text{noise}}$$

The equations and the simulations:

see also Schlichting, Smekal

We have a charge diffusion equation coupled to order parameter:

$$\partial_t n_{ab} + \underbrace{\nabla \cdot (\nabla \phi_{[a} \phi_{b]})}_{\text{poisson bracket}} + H_{[a}\phi_{b]} = \underbrace{D_0 \nabla^2 n_{ab}}_{\text{diffusion}} + \underbrace{\nabla \cdot \xi_{ab}}_{\text{noise}}$$

and a rotation of the order parameter induced by the charge:

$$\partial_t \phi_a + \underbrace{\frac{n_{ab}}{\chi_0}\phi_b}_{\text{poisson bracket}} = \underbrace{\Gamma_0 \frac{\delta H}{\delta \phi_a}}_{\text{dissipation}} + \underbrace{\xi_a}_{\text{noise}}$$

Numerical scheme based operator splitting:

1. Evolve the Hamiltonian evolution with a position Verlet type stepper
2. Treat the dissipative Langevin steps as Metropolis-Hastings updates

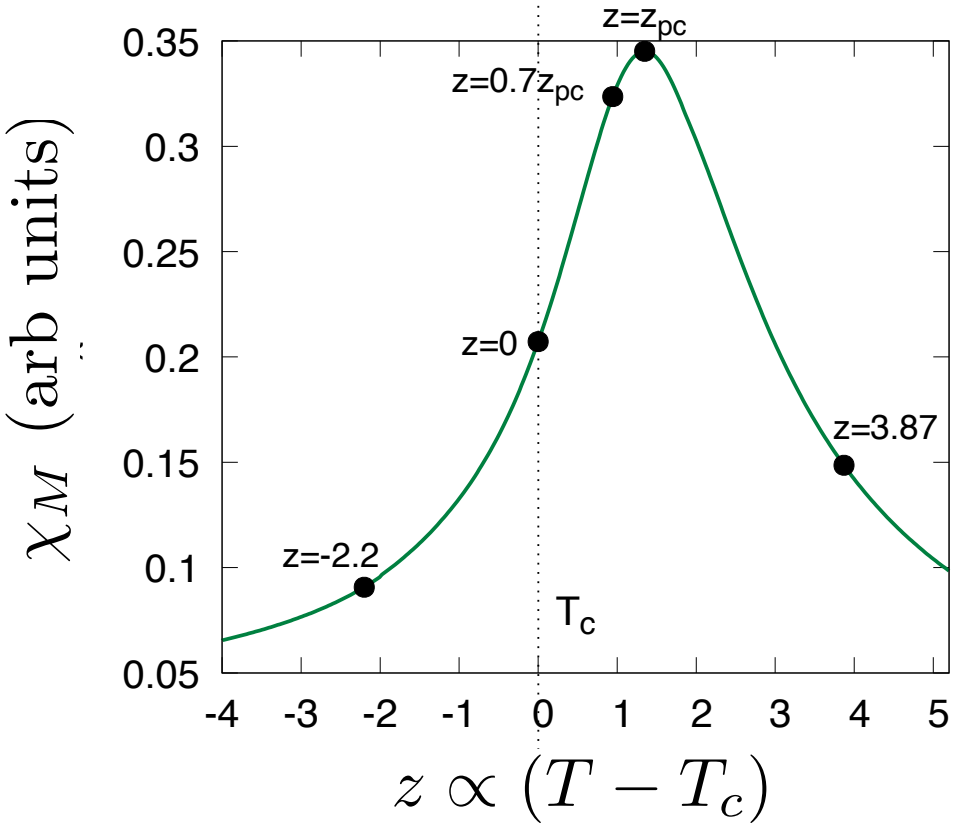
Scan the phase transition:

Engels, Fromme, Seniuch, Karsch; Hasenbuch

After measuring order parameter, susceptibility, etc

$$\bar{\sigma} = h^{1/\delta} f_G(z) \qquad z = t_{\rm r} h^{-1/\beta\delta}$$

we have fixed the scaling parameters, $h = H/H_0$, and $t_{\rm r} = (m_0^2 - m_c^2)/\mathfrak{m}^2$

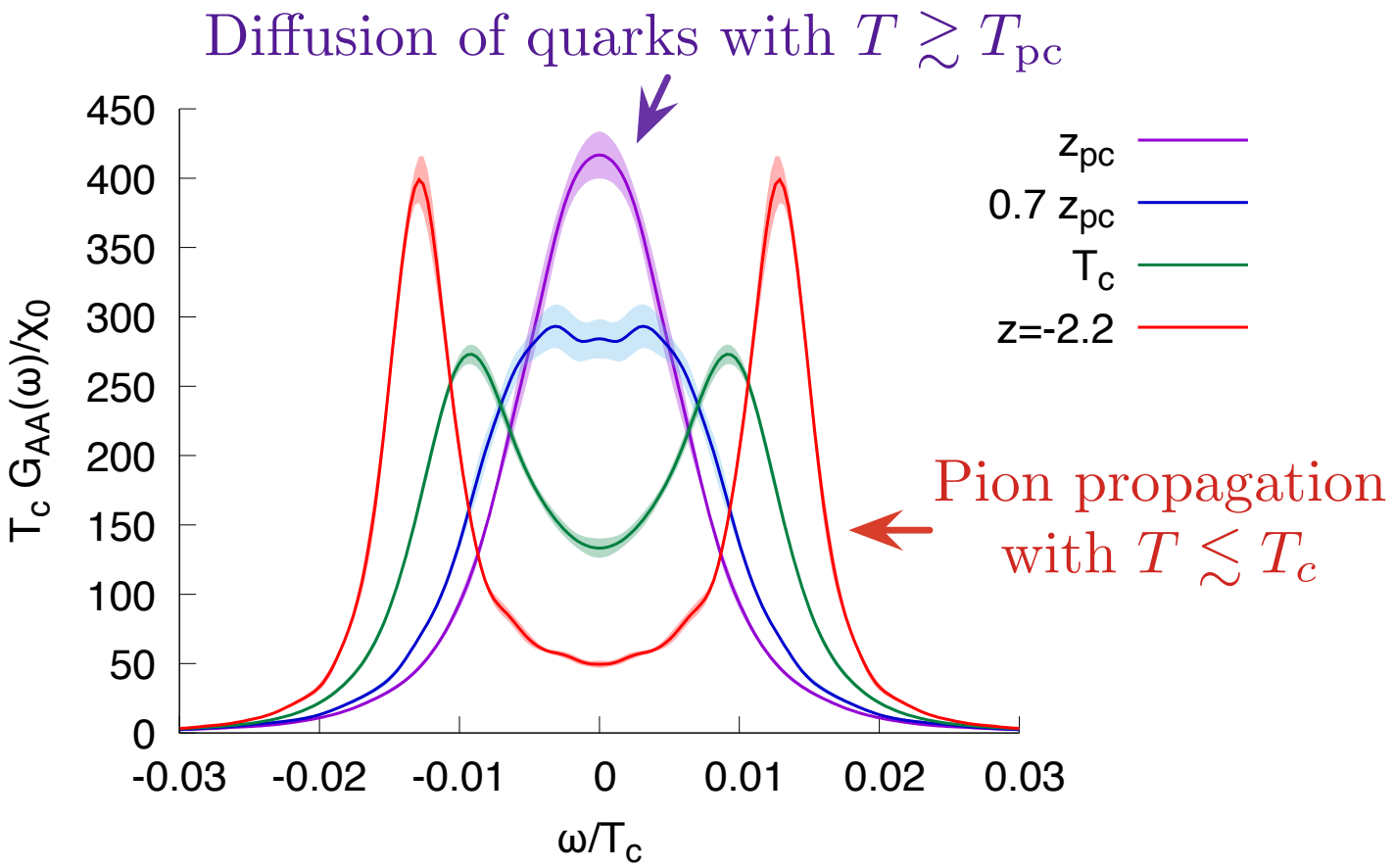


$$\chi_M = \frac{\partial \bar{\sigma}}{\partial H}$$

now scan the transition

Features of the phase transition in the axial charge correlations:

$$G_{AA}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\, \langle \vec{n}_A(t, \boldsymbol{x}) \cdot \vec{n}_A(0, \mathbf{0}) \rangle$$

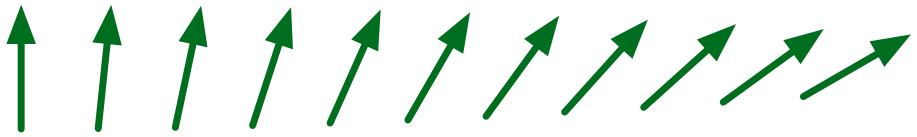


Can see the transition from diffusion of quarks to propagation of pions!

The Pion EFT for $T \ll T_c$

Son and Stephanov '01+us

- Below T_C the condensate is frozen up to phase fluctuations



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

- The ideal equations of motion the phase is (with $\mu_A = n_A/\chi_0$):

$$\partial_t \varphi = \mu_A \qquad \text{Josephson Constraint}$$

while the axial charge EOM is:

$$\partial_t n_A + \nabla \cdot \boldsymbol{J}_A = f^2 m^2\, \varphi \qquad \text{Axial Current}$$

where the current is the gradient of the phase: $\boldsymbol{J}_A = f^2 \nabla \varphi$

- The pion EFT is written with $f^2 \simeq \bar{\sigma}^2$ and $f^2 m^2 = H\bar{\sigma}$

We can use the EFT to find the dispersion curve of soft pions, including dissipative corrections

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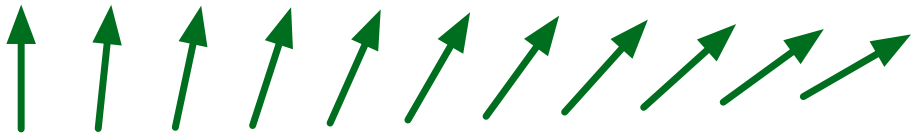


$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

- The ideal equations of motion the phase is (with $\mu_A = n_A/\chi_0$):

$$\partial_t \varphi = \mu_A + \mathcal{O}(\Gamma \nabla^2 \varphi) \qquad \text{Josephson Constraint}$$

while the axial charge EOM is:

$$\partial_t n_A + \nabla \cdot \boldsymbol{J}_A = f^2 m^2\, \varphi + \mathcal{O}(D \nabla^2 n_A) \qquad \text{Axial Current}$$

where the current is the gradient of the phase: $\boldsymbol{J}_A = f^2 \nabla \varphi$

- The pion EFT is written with $f^2 \simeq \bar{\sigma}^2$ and $f^2 m^2 = H\bar{\sigma}$

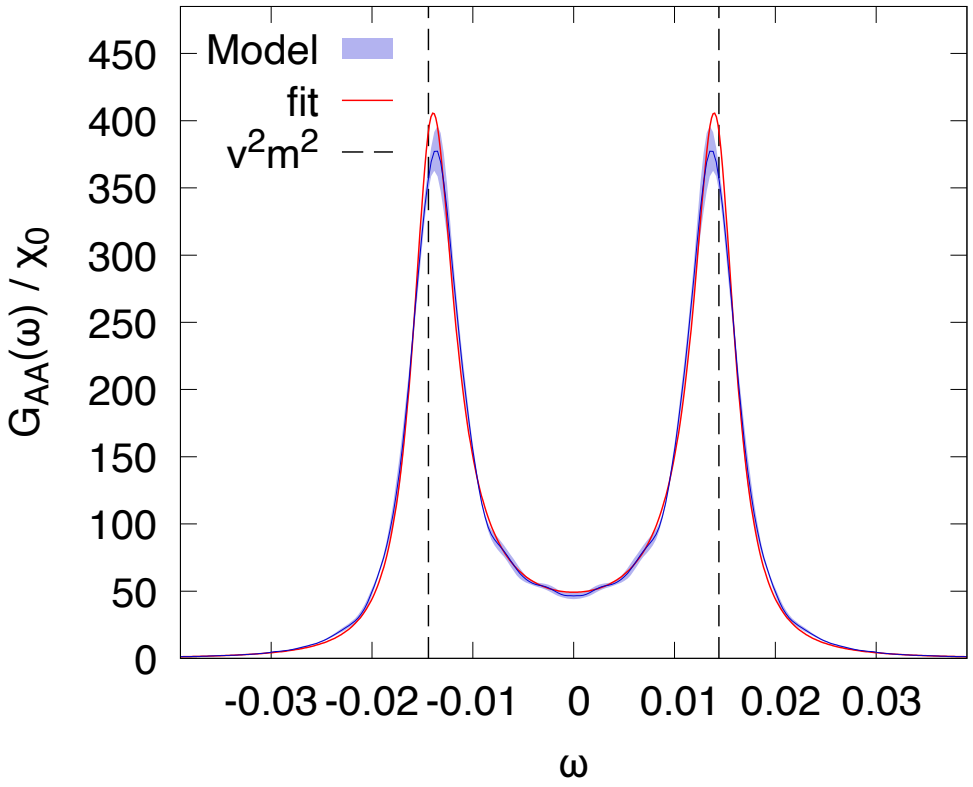
We can use the EFT to find the dispersion curve of soft pions, including dissipative corrections

Quantitative analysis of a pion EFT well below T_c, $z = -2.2$:

The predicted pole position m_p^2 of pion waves is given by static quantities:

$$m_p^2 = v^2 m^2 = \frac{H\bar{\sigma}}{\chi_0}$$

This is the finite temperature Gell-Mann Oakes Rener relation:



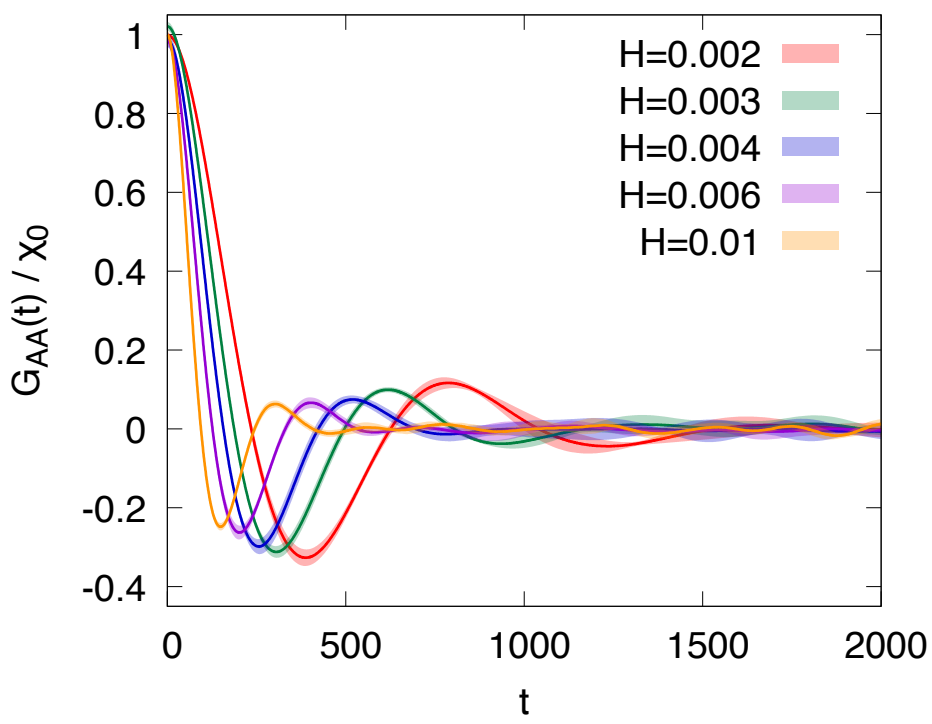
These are static inputs to fit:

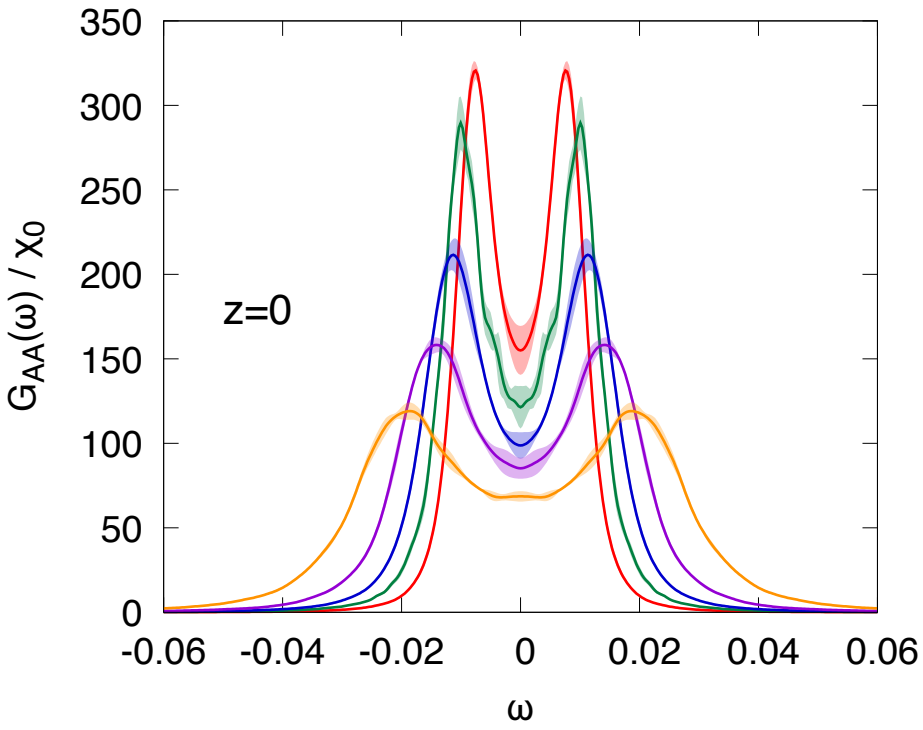
$$v^2 = \frac{f^2}{\chi_0}$$

$$m^2 = \text{screening mass}$$

Scaling of simulations at T_c:

At $T = T_c$, we varied the magnetic field, finding the response functions:





See a scaling behavior of the real time correlations, with quark mass, which tunes the correlation length

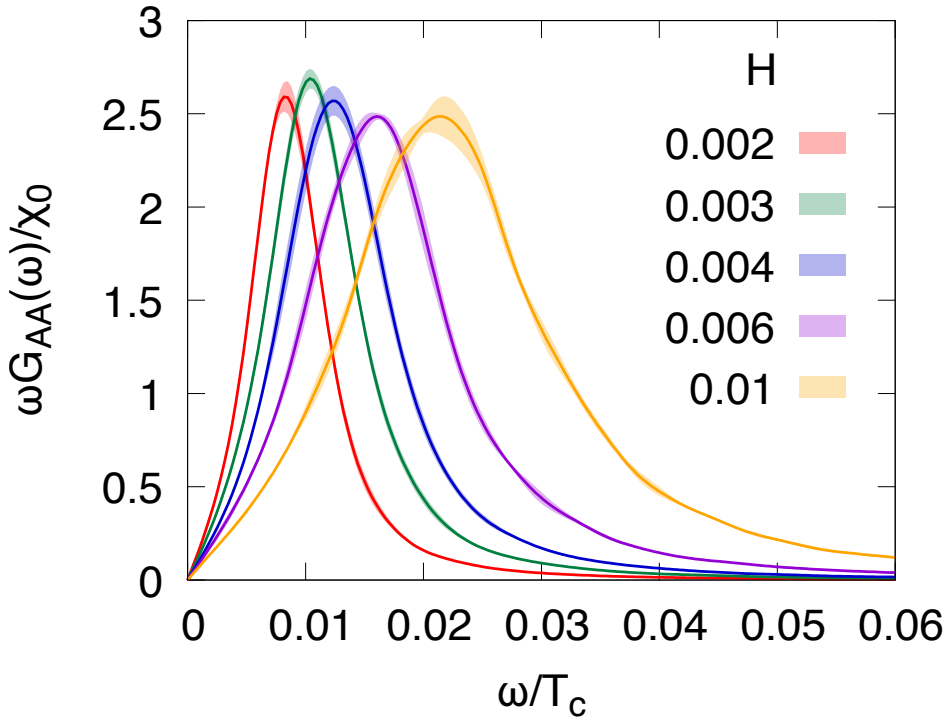
Dynamical critical exponent of the $O(4)$ transition:

Rajagopal, Wilczek

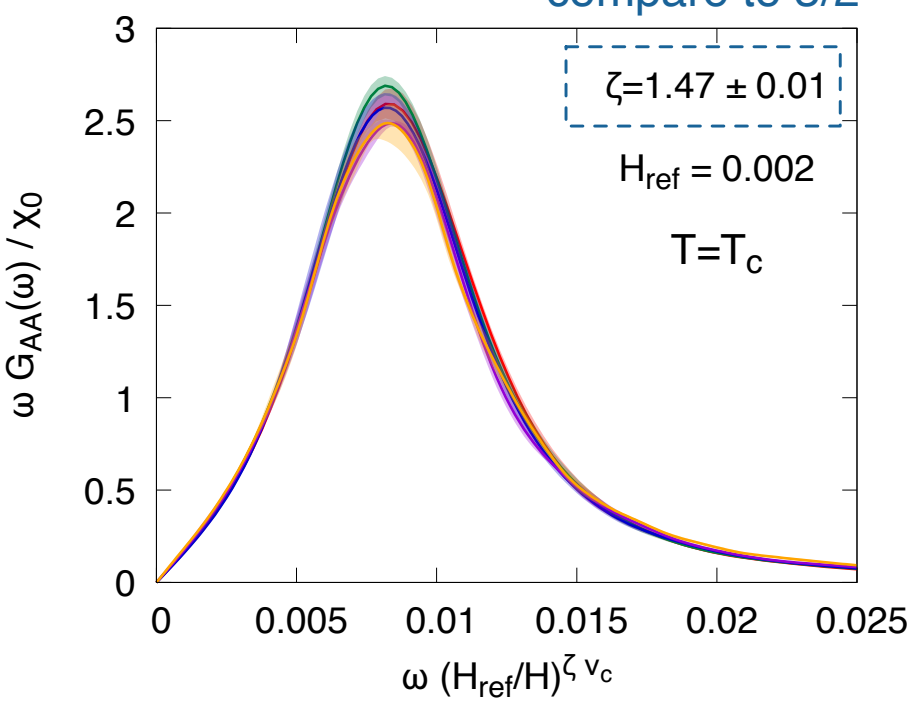
The relaxation time and correlations *scale* with the correlation length ξ:

$$\omega\, G_{AA}(\omega,\xi) = \underbrace{f(\omega\,\tau_R)}_{\text{universal fcn}} \quad \text{with} \quad \underbrace{\tau_R \propto \xi^{\zeta}}_{\text{relaxation time}}$$

The correlation length scales as $\xi \propto H^{-\nu_c}$ and the time as $\tau_R \propto H^{-\zeta\nu_c}$:



compare to 3/2



Summary and Outlook:

1. We are simulating the real-time dynamics of the chiral critical point
   - The numerical method may be useful for stochastic hydro generally
2. We reproduced the expected dynamical scaling laws:

$$\tau_R \propto \xi^{\zeta} \qquad \zeta = \frac{d}{2} \simeq 1.47 \pm 0.01$$

3. The pion waves are well calibrated.
4. The next step is to study the expanding case:
   - This will predict soft pions and their correlations with expansion for heavy ion collisions

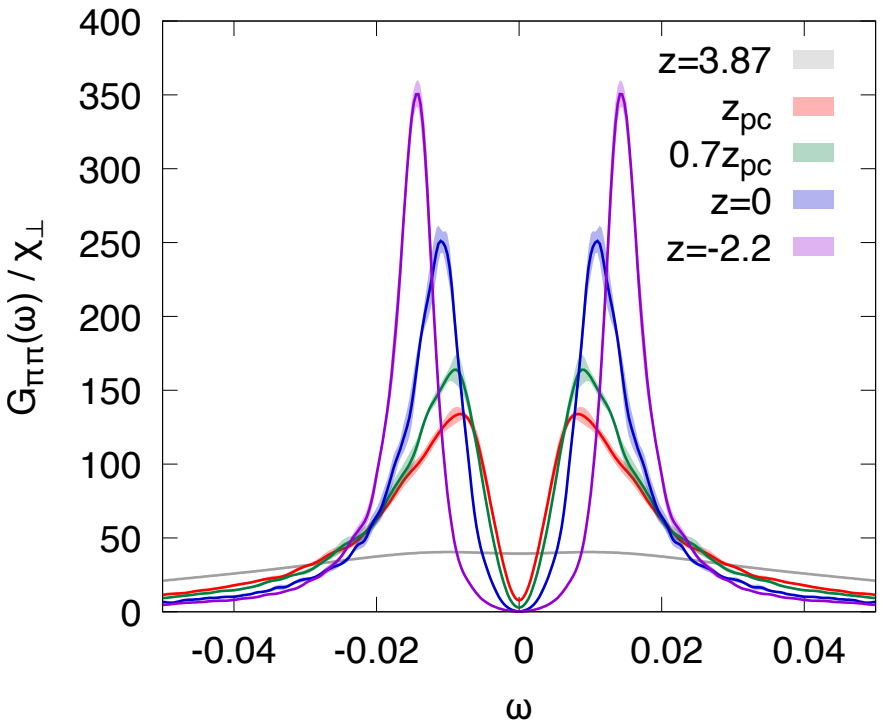
The hadronization of the pion is the (only) hadronization process that can be studied rigorously, *and only with hydrodynamics!*

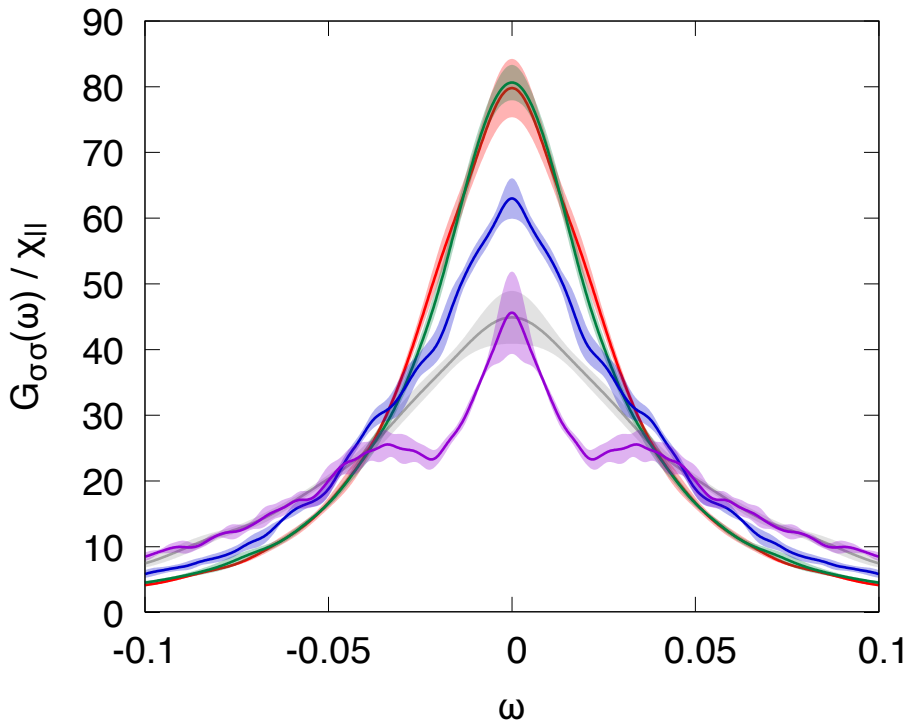
Backup

Comparison of π and σ

$$G_{\sigma\sigma}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\, \langle \sigma(t, \boldsymbol{x}) \cdot \sigma(0, \mathbf{0}) \rangle$$

$$G_{\pi\pi}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\, \langle \vec{\pi}(t, \boldsymbol{x}) \cdot \vec{\pi}(0, \mathbf{0}) \rangle$$

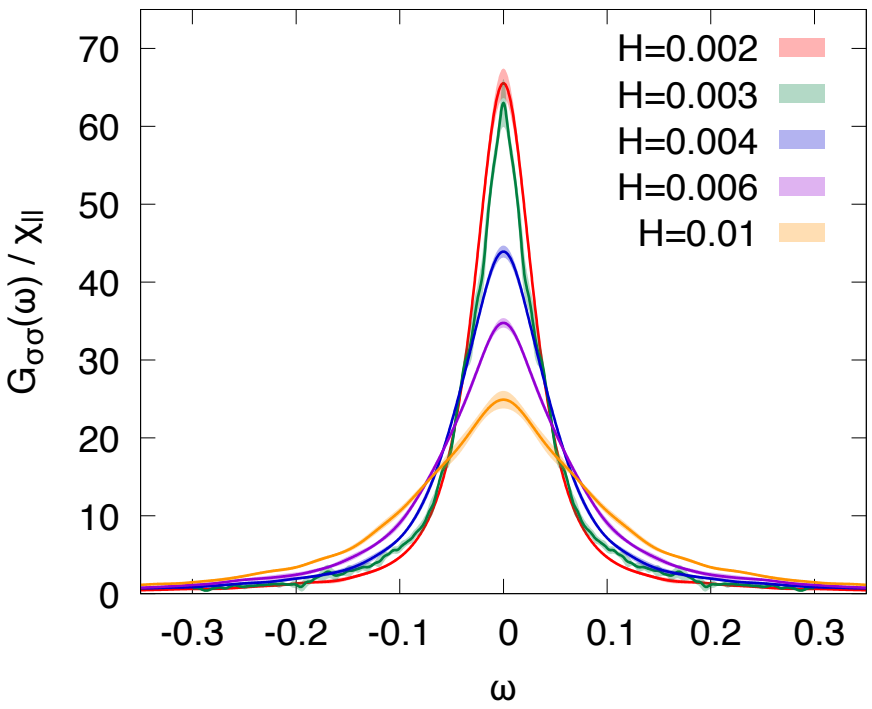


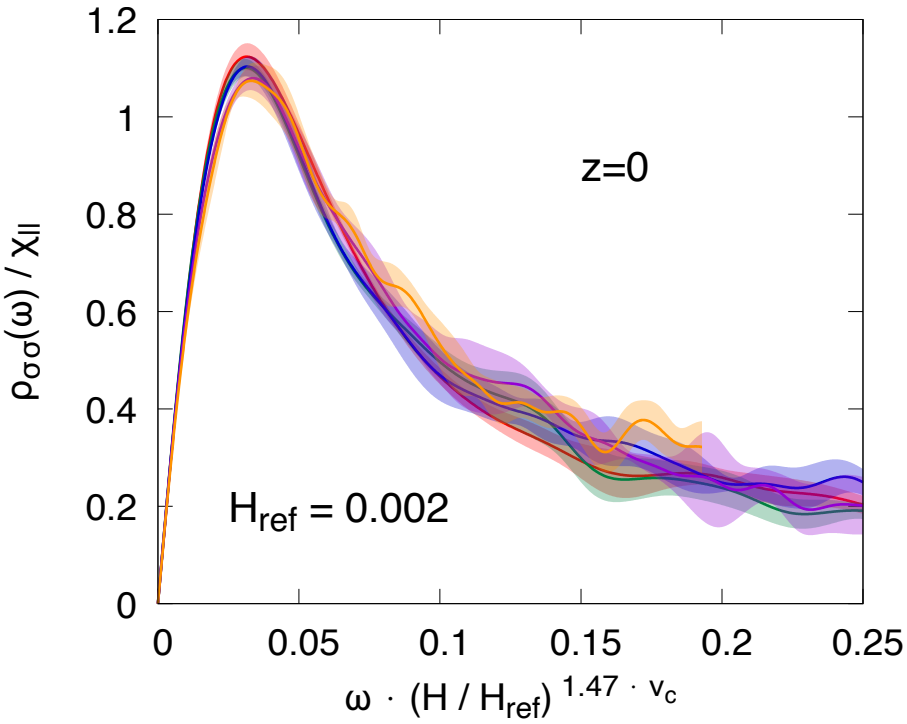


The grey curves are essentially the same in both figures but with different scales.

Dynamical scaling of σ correlation functions:

$$G_{\sigma\sigma}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\, \langle \sigma(t, \boldsymbol{x}) \cdot \sigma(0, \mathbf{0}) \rangle$$





Preliminaries: Statics

$$M_a(t) \equiv \frac{1}{V} \sum_{\boldsymbol{x}} \phi_a(t, \boldsymbol{x}) \equiv \text{Order parameter}$$

with time average:

$$\bar{\sigma} = \langle M_0(t) \rangle \qquad \text{and} \qquad \Sigma = \lim_{H \to 0} \lim_{V \to \infty} \bar{\sigma}$$