Compositeness and several applications to exotic hadronic states with heavy quarks

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Related references [1]Meißner,JAO,PLB751,59(2015); [2] JAO, ANP396,429(2018); [3]Guo,JAO,PRD93,096001(2016); [4]Kang,Guo,JAO,PRD94,014012(2016); [5]Guo,JAO,PRD103,054021(2021); [6]Guo,JAO,PRD103,034024(2021): [7]Du,Guo,JAO,PRD104,114034(2021)

More details on the basics: My talk today at 15:40h Session B

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Outline

1.- Basic formalism

Shallow bound state. Non-relativistic dynamics

S. Weinberg, PR130,776(1963); PR131,440(1963); PR137,B672(1964)

$$H = H_0 + V$$

H_0 Kinetic energy

Bare states. The same spectrum as H

$$H_0|\varphi_\alpha\rangle = E_\alpha|\varphi_\alpha\rangle \ , \quad \text{Continuum spectrum}$$
$$H_0|\phi_n\rangle = E_{B_n}|\phi_n\rangle \ , \quad \text{Discrete spectrum}$$

$|\varphi_\alpha\rangle$ is made up by free particles of the continuum spectrum
$|\phi_n\rangle$ bare "elementary" states by one particle

Physical spectrum of H:

$H|\psi_\alpha^\pm\rangle = E_\alpha|\psi_\alpha^\pm\rangle$, $|\psi_\alpha^\pm\rangle$ in/out states. Continuum spectrum
$H|\psi_{B_i}\rangle = E_{B_i}|\psi_{B_i}\rangle$, $E_{B_i} < 0$. Discrete spectrum

Elementariness: Z , **Composition**: X

$$\langle\psi_B|\psi_B\rangle = 1 = \underbrace{\sum_n |\langle\phi_n|\psi_B\rangle|^2}_{Z} + \underbrace{\int d\alpha |\langle\varphi_\alpha|\psi_B\rangle|^2}_{X}$$

$$1 = Z + X$$

Interpretation based on the number operator

Original developments in JAO, Ann.Phys. 396, 429 (2018)

Introducing bare "elementary" states as intermediate states are not needed.

Take two free particles of types A y B, $H_0|AB_\gamma\rangle = E_\gamma|AB_\gamma\rangle$
Creation and annihilation operators $a^\dagger_\alpha a_\alpha$, $b^\dagger_\beta b_\beta$

Number Operators: $N_D = \int d\alpha a^\dagger_\alpha a_\alpha + \int d\beta b^\dagger_\beta b_\beta = N_D^A + N_D^B$

$$N_D = \int d^3x \left[\psi^\dagger_A(x)\psi_A(x) + \psi^\dagger_B(x)\psi_B(x)\right]$$

New definition of X

$$X = \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle$$

Equivalence with the previous definition

$$|\psi_B\rangle = \int d\gamma C_\gamma |AB_\gamma\rangle + \sum_n C_n|\phi_n\rangle$$

$$X = \frac{1}{2}\langle\psi_B|N_D^A + N_D^B|\psi_B\rangle = \int d\gamma |C_\gamma|^2$$

This definition is specially suitable for Effective Field Theories (EFTs) (e.g. ChPT)
It is very usual not to have explicit bare "elementary" states (fields)

Calculation in QFT

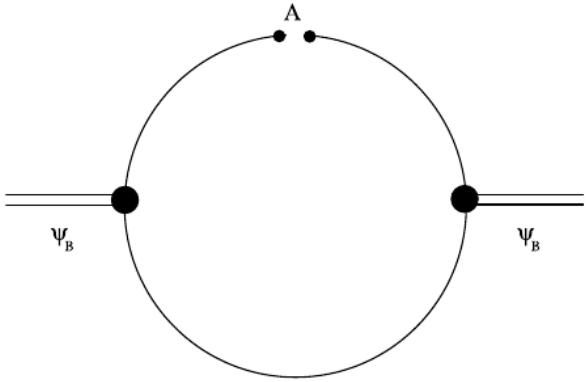
This new definition is the most adequate for the treatment in QFT

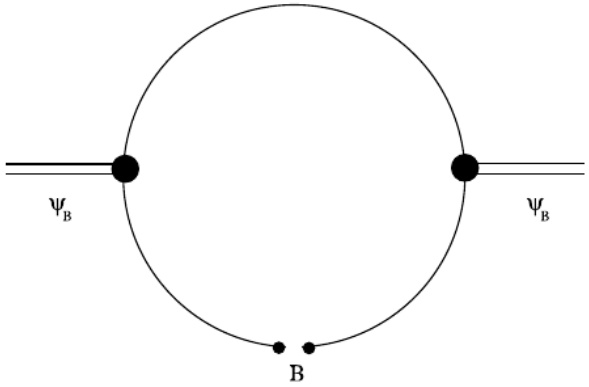
$$X = \frac{1}{2} \lim_{T \to +\infty} \frac{1}{T} \int d^4x \langle \varphi_B | P \left[e^{-i \int_{-\infty}^{+\infty} dt' V_D(t')} \sum_i \psi^\dagger_{A_i}(x) \psi_{A_i}(x) \right] | \varphi_B \rangle$$

S-matrix elements, with in/out states. LSZ formalism

$$X = \frac{1}{2} \lim_{E \to E_B} \frac{(E - E_B)^2}{g_\alpha(k_B)^2}$$
$$\times \lim_{T \to +\infty} \frac{1}{T} \int d^4x \langle \varphi_\alpha | P \left[e^{-i \int_{-\infty}^{+\infty} dt' V_D(t')} \sum_i \psi^\dagger_{A_i}(x) \psi_{A_i}(x) \right] | \varphi_\alpha \rangle$$

Explicit formulas





$$X_{\ell S} = \int \frac{d^3k}{(2\pi)^3} \frac{g^2_{\ell S}(k^2)}{(k^2/2\mu - E_B)^2}$$

$$X = \sum_{\ell S} X_{\ell S}$$

Weinberg's expression for $1 - Z$

Equation for $g_{\ell S}(k)$ from $T = V + VGT$

$$g_{\ell S}(k) = \frac{1}{2\pi^2} \int_0^\infty k'^2 dk' V_{\ell S}(k, k') \frac{1}{k'^2/2\mu - E_B} g_{\ell S}(k')$$

Energy-independent potentials: Pure potential scattering

As demonstrated in JAO, Ann.Phys. 396, 429 (2018)

$$\boxed{1 = \sum_{\alpha=1}^{n} X_\alpha}$$

Deuteron ChPT Energy-independent potentials worked up to N^4LO
Error $\lesssim 4\%$ for Deuteron properties
Rodríguez-Entem, Machleidt, Nosyk, Front.Phys.8:57(2022)

$$X = 0.96 - 1.0$$

2.- Resonances

Analysis based on Scattering Theory. Number operators

JAO, Ann.Phys.396,429(2018)

A resonance stems from the analytic continuation in energy of the in states with energy $E + i\varepsilon$, just above the real E axis

Calculation in QFT

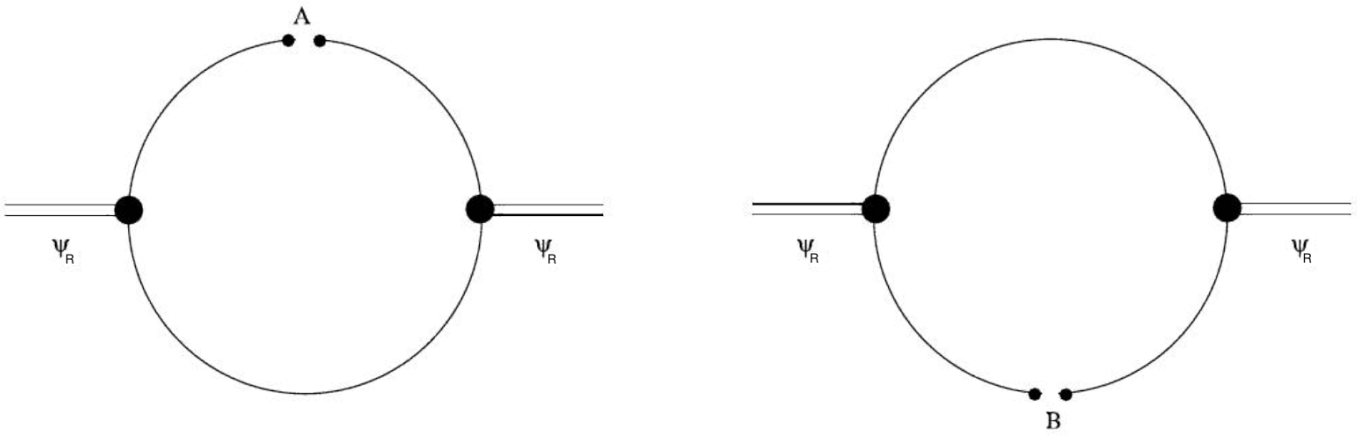
S-matrix element with an external source $\sum_i \psi^\dagger_{A_i}(x)\psi_{A_i}(x)$

$$X = \frac{1}{2}\lim_{E\to E_n}\frac{(E-E_n)^2}{g_\alpha(k_n)^2}$$

$$\times \lim_{T\to+\infty}\frac{1}{T}\int d^4x \langle\varphi_\alpha|P\left[e^{-i\int_{-\infty}^{+\infty}dt' V_D(t')}\sum_i \psi^\dagger_{A_i}(x)\psi_{A_i}(x)\right]|\varphi_\alpha\rangle$$

Pure potential scattering $\boxed{1 = X = \sum_{\alpha=1}^{n} X_\alpha}$

For instance, virtual state in the $^1S_0$ NN scattering



$$X_{\ell S} = \int \frac{d^3k}{(2\pi)^3} \frac{g^2_{\ell S}(k^2)}{(k^2/2\mu - E_n^2)^2} + \frac{i\mu^2}{\pi k_n} \left[\frac{\partial}{\partial k} k \, g^2_{\ell S}(k^2) \right]_{k=k_n}$$

$$X = \sum_{\ell S} X_{\ell S}$$

Z.H.Guo, JAO, PRD93,096001(2016)

Sum rule

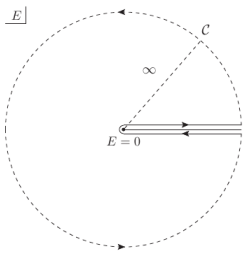
From two-body unitarity $\mathrm{Im}\ T^{-1}\big|_{ij} = \delta_{ij}\rho_i$ along the RC

$\rho_i = p_i/8\pi\sqrt{s}$

General expression for a PWA in coupled channel in matrix notation

$$T(s) = \left[\mathcal{K}(s)^{-1} + G(s)\right]^{-1}$$

$$G(s)_i = a(s_0)_i - \frac{s-s_0}{\pi}\int_0^\infty \frac{\rho_i(s')ds'}{(s'-s)(s'-s_0)}$$



Derivative with respect to s, $s \to s_R$, double pole

$$1 = \underbrace{-\sum_i g_i^2 \left.\frac{dG^{II}(s)_i}{ds}\right|_{s_R}}_{X_i} + \underbrace{g^T G^{II}(s_R) \left.\frac{dK(s)}{ds}\right|_{s_R} G^{II}(s_R) g}_{Z}$$

This is the same expression as for shallow bound states and separable potentials

Resonances: Take $|X_i|$.

Transformation of the S matrix: Phase redefinition of the couplings

Z.H.Guo, JAO, PRD93,096001(2016)

Let us consider a narrow resonance $\Gamma \ll M_R - m_{th}$

Laurent series around the resonance pole $s_P = (M_R - i\Gamma/2)^2$

$$S(s) = \frac{R}{s - s_P} + S_0(s)$$

$$S(s)S(s)^\dagger = I$$

Solution $S_0 = \mathcal{O}\mathcal{O}^T \ , \ \mathcal{O}\mathcal{O}^\dagger = I$

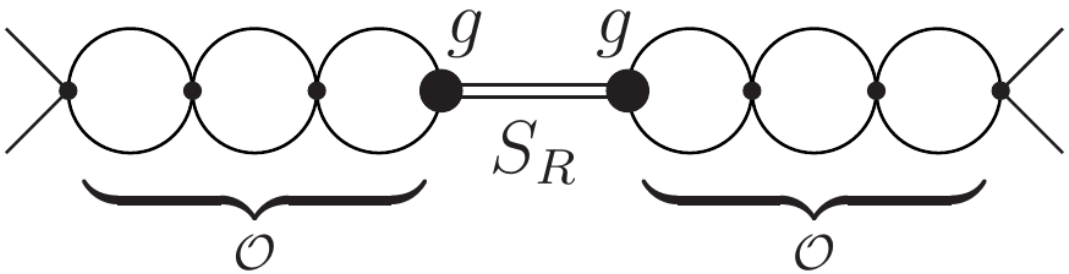
$$S(s) = \mathcal{O} \underbrace{\left(I + \frac{i\lambda\mathcal{A}}{s - s_R} \right)}_{S_R(s)} \mathcal{O}^T$$

$\mathcal{A}$ is a rank 1 symmetric projector operator

$S_R(s)$ is a purely resonant S matrix

$$S_{\alpha\beta}(s) = \mathcal{O}_{\alpha\mu}\mathcal{O}_{\beta\nu}\left(I + \frac{i\lambda\mathcal{A}}{s - s_R}\right)_{\mu\nu}$$

The nonresonant S matrix, S_0, dresses $S_R(s)$



Corrections due to initial- and final-state interactions from S_0

They typically modify the phases of the resonance couplings

Example: $\pi\pi - K\bar{K}$ scattering

JAO, Oset, NPA620,438(1997)

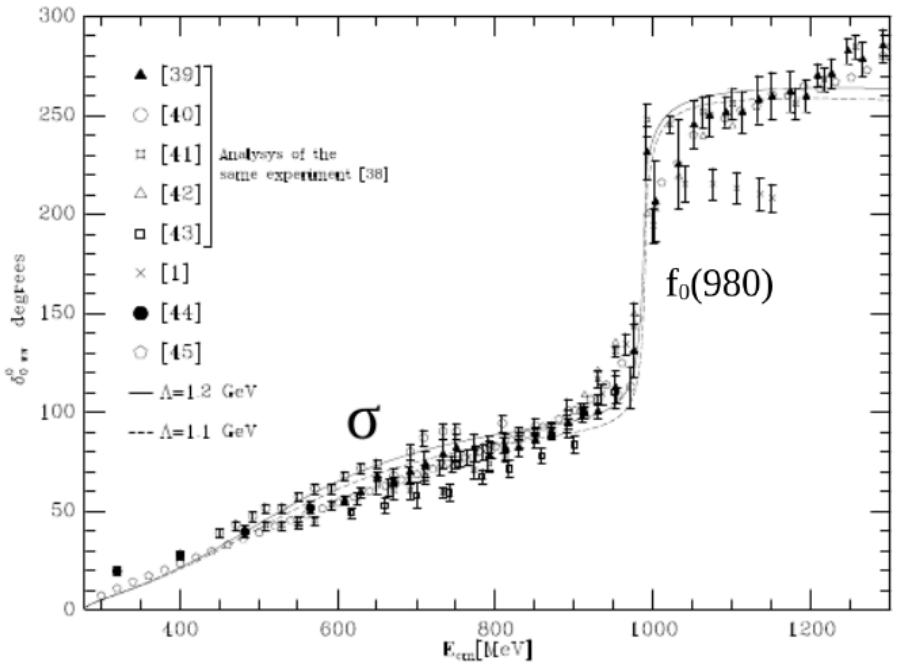


Figure: Isoscalar scalar $\pi\pi$ phase shifts. $J^{PC} = 0^{++}$

$$S = \begin{pmatrix} \eta e^{2i\delta_{11}} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

The coupling between channels implies a phase $\delta_1 \approx \pi/2$ just at the $f_0(980)$ rise $\rightarrow$ phase of the $f_0(980)$ coupling to $\pi\pi$

The moduli of the couplings g_α have physical meaning

$$\Gamma_\alpha = \frac{|g_\alpha|^2}{8\pi M_R^2}$$

The S-matrix phase transformation only change the phases of the resonance couplings

$$S_{\mathcal{O}}(s) \equiv \mathcal{O} S(s) \mathcal{O}^T$$
$$\mathcal{O} = \mathrm{diag}(e^{i\phi_1}, \ldots, e^{i\phi_n})$$
$$g_i^2 \to g_i^2 e^{2i\phi_i}$$

$$X_\alpha \to |X_\alpha| \geq 0$$

Condition $|X_\alpha|$

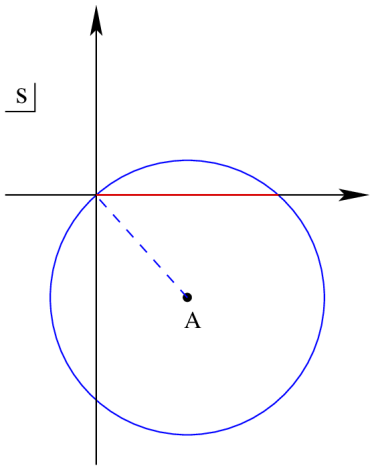
It makes sense if:

▷The Laurent expansion around s_P is valid in some interval of physical (real values above threshold) for s

$$S(s)S(s)^\dagger = I \quad \text{is meaningful}$$

Condition A: $s_n < \mathrm{Re} s_P < s_{n+1}$

s_n is the threshold of channel n



3.- S-wave Effective-Range Expansion

Kang,Guo,JAO,PRD94,014012(2016)

$$T(k) = \frac{1}{-\frac{1}{a} + \frac{1}{2} r k^2 - ik}$$

$$G(k) = -ik$$

$$E_R = M_R - i\Gamma/2$$

$$a = -\frac{2k_i}{|k_R|^2} \quad , \quad k_R = k_r - i\, k_i$$

$$r = -\frac{1}{k_i} \quad , \quad \frac{r}{a} > 2$$

$$X = -\gamma^2 \frac{dG}{ds} = -\gamma_k^2 \frac{dG}{dk} = i\frac{k_i}{k_r} = i \tan\frac{\phi}{2}$$

$|X| \leq 1 \leftrightarrow k_r \geq k_i \leftrightarrow M_R \geq 0 \quad \phi \in [0, \pi/2]$

$(|X| = 1$ for $M_R = 0$ and $\Gamma > 0)$

If the real part is taken then ALWAYS $X = 0$!

$$\tan\phi = \frac{\Gamma}{2M_R} \longrightarrow \phi \in [0, \frac{\pi}{2}] \text{ for } M_R \geq 0$$

$$X = \left(\frac{2r}{a} - 1\right)^{-1}$$

$Z_b(10610)$ and $Z_b(10650)$, or Z_b and Z_b'

$B^{(*)}\bar{B}^*$ system with $I^G(J^P) = 1^+(1^+)$ Bondar et al. (Belle Coll.) PRL108,122001(2012)

$$E_{Z_b} = 10607.2 \pm 2.0 - i(9.2 \pm 1.2)\ \mathrm{MeV}$$
$$E_{Z_b'} = 10652.2 \pm 1.5 - i(5.5 \pm 1.1)\ \mathrm{MeV}$$

M_R is around 3 MeV below $B^{(*)}\bar{B}^*$ threshold

	$Z_b(10610)$	$Z_b(10650)$
$a\,(\mathrm{fm})$	-1.03 ± 0.17	-1.18 ± 0.26
$r\,(\mathrm{fm})$	-1.49 ± 0.20	-2.03 ± 0.38
$X = \gamma_k^2$	0.75 ± 0.15	0.67 ± 0.16

Kang, Guo, JAO, Phys.Rev.D94,014012(2016)

Determining X by making use of the width of the resonance

Meißner,JAO,PLB751,59(2015)

$$\Gamma_1 = \frac{2X_1}{\mu} k(M_R)|k_R|$$

$$\Gamma_2 = \frac{X_2|k_R|M_R^2}{\pi\mu} \int_{M_{\rm th}}^{+\infty} dW \frac{k(W)}{W^2} \frac{\Gamma}{(M_R - W)^2 + \Gamma^2/4}$$

$$\Gamma = \Gamma_1 + \Gamma_2$$

For the Z_b, Z_b' it gave consistent results with the ERE-based method

Branching ratios are measured

	$Z_b(10610)$	$Z_b(10650)$
X	0.66 ± 0.11	0.51 ± 0.10

Kang,Guo,JAO, PRD94,014012(2016)

4.- $Z_{cs}(3985)$, $Z_c(3900)$, $X(4020)$

Guo,JAO,PRD103,054021(2021)

$Z_c(3900)$: $\bar{D}D^*/D\bar{D}^*, J/\psi\pi$

$X(4020)$: $D^*\bar{D}^*$, $h_c\pi$

$Z_{cs}(3985)$: $D_s^- D^{*0}/D_s^{*-}D^0$, $J/\psi K^-$

Elastic case: ERE study

Tetraquark Resonance	Mass (MeV)	Width (MeV)	Threshold (MeV)	a (fm)	r (fm)	X
$Z_c(3900)$	3888.4 ± 2.5	28.3 ± 2.5	$\bar{D}D^*$ (3875.5)	-0.84 ± 0.13	-2.52 ± 0.25	0.45 ± 0.06
$X(4020)$	4024.1 ± 1.9	13 ± 5	$\bar{D}^*D^*$ (4017.1)	-1.04 ± 0.30	-3.90 ± 1.35	0.39 ± 0.14
$Z_{cs}(3985)$	3982.5 ± 3.3	12.8 ± 6.1	$D_s^- D^{*0}$ (3975.2)	-1.00 ± 0.47	-4.04 ± 1.82	0.38 ± 0.18
			$D_s^{*-}D^0$ (3977.0)	-1.28 ± 0.60	-3.65 ± 1.60	0.46 ± 0.19

X, a, r ares similar for all the states$\rightarrow$ similar structure

r tends to be large and negative $\rightarrow$ significant elementariness

Still X is also sizeable

Coupled-channel study: Γ_i

$$(1)\ \Gamma_R = \Gamma_1 + \Gamma_2 = |g_1|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |g_2|^2 \int_{m_{\rm th}}^{M_R + n\Gamma_R} dE\, \frac{q_2(E^2)}{16\pi^2 E^2} \frac{\Gamma_R}{(M_R - E)^2 + \frac{\Gamma_R^2}{4}}$$

$$(2)\ Z_c(3900):\ \Gamma_{D\bar{D}^*}/\Gamma_{J/\psi\pi} = 6.2 \pm 2.9$$

Results

$$|g_1| = 1.46^{+0.43}_{-0.23}\,, \qquad |g_2| = 7.89^{+0.18}_{-0.44}$$

$|g_2| \ll |g_1|$

$$X_1 = 0.002 \pm 0.001\,, \qquad X_2 = 0.436^{+0.021}_{-0.047}$$

$$X = X_1 + X_2 = 0.438^{+0.021}_{-0.047}$$

X is almost identical to the ERE study

We then use X from ERE for the $X(4020)$, $Z_{cs}(3985)$ as 2nd input

Resonance	$\lvert g_1\rvert$ (GeV)	$\lvert g_2\rvert$ (GeV)	Γ_1 (MeV)	Γ_2 (MeV)	$X_1 \times 10^3$	X_2
$X(4020)$						
$X_{\rm ERE} = 0.39 \pm 0.14$	1.1 ± 0.2	6.5 ± 1.3	1.4 ± 0.5	11.6 ± 4.5	1 ± 1	0.39 ± 0.14
$Z_{cs}(3985)$						
Threshold $(D_s^- D^{*0})$						
$X_{\rm ERE} = 0.38 \pm 0.18$	0.8 ± 0.2	6.4 ± 1.7	1.2 ± 0.6	11.6 ± 5.3	0.8 ± 0.4	0.38 ± 0.18
Threshold $(D_s^{*-} D^0)$						
$X_{\rm ERE} = 0.46 \pm 0.19$	0.9 ± 0.2	6.8 ± 1.7	1.2 ± 0.6	11.6 ± 5.6	0.8 ± 0.4	0.46 ± 0.19

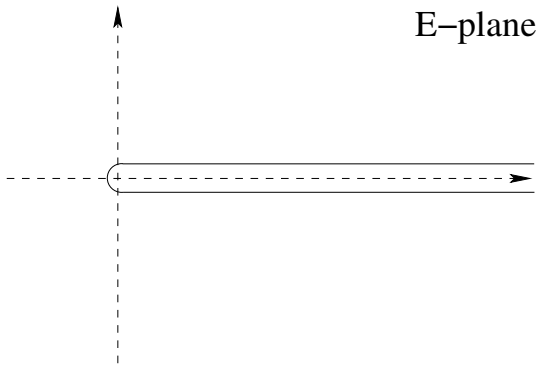
5.- Scattering Amplitude $t(E)$. CDD Poles

Dispersion Relation for the inverse of $t(E)$

$$\mathrm{Im}t(E)^{-1} = -ik$$

One subtraction is needed

$$\oint dz \frac{t(z)^{-1}}{(z-E)(z-C)}$$



The only other structure apart from the threshold that can give rise to a strong distortion in $t(E)^{-1}$ is a pole at M_Z

$$t(E) = \frac{1}{\frac{\lambda}{E-M_Z} + \beta - ik}$$

CDD pole Castillejo,Dalitz,Dyson, PR,101,453(1956)

The ERE or a Flatté parameterization break down for $|k| \gtrsim \sqrt{2\mu|M_Z|}$

The general formula for a partial-wave without crossed-channel dynamics was deduced in: JAO,Oset,PRD60,074023(1999)

A CDD contributes to r y a as

$$\delta a = \frac{M_Z}{\lambda}$$

$$\delta r = -\frac{\lambda}{m M_Z^2}$$

If $M_Z \to 0$ then $\delta a \to 0$ and $\delta r \to \infty$ (unless $\lambda = 0$)

$$X = \frac{k_i}{k_r} = \left(\frac{2r}{a} - 1\right)^{-1} \xrightarrow[M_Z \to 0]{} 0$$

This is a sufficient but not necessary condition

$\rho(770)$ the *CDD* moves to infinity because of the KSFR relation $[G_V = f_\pi/\sqrt{2}]$

$$\frac{2}{3}\frac{p^2}{f^2(M_\rho^2 - s)}\left[M_\rho^2 - s(1 - g_V^2)\right]$$

for $g_V = 1$ the zero at $M_\rho^2/(1 - g_V^2) \to \infty$

JAO,Oset,PRD60,074023(1999)

6.- $X(6900)$. Fits to data

Guo,JAO,PRD103,034024(2021)

$J/\psi J/\psi(\eta_c\eta_c)$, $\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$

S-wave scattering near threshold of the $\chi'_{c0,1}s$ $J^{PC}=0^{++}$

Aaij *et al.* (LHCb Coll.), Sci.Bull.**65**,1983(2020)

$$\text{Model I: } M = 6905 \pm 11 \pm 7 \text{ MeV}\,,\ \Gamma = 80 \pm 19 \pm 33 \text{ MeV}$$
$$\text{Model II: } M = 6886 \pm 11 \pm 11 \text{ MeV}\,,\ \Gamma = 168 \pm 33 \pm 69 \text{ MeV}$$

Channel	Threshold [MeV]
(1) $J/\psi J/\psi$	6193.8
(2) $\chi_{c0}\chi_{c0}$	6829.4
(3) $\chi_{c1}\chi_{c1}$	7021.3

$$\mathcal{T}(s) = [1 - \mathcal{V}(s)\cdot G(s)]^{-1}\cdot \mathcal{V}(s)\,,$$

$$\mathcal{V}(s) = \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{12} & \frac{b_{22}}{M^2_{J/\psi}}(s - M^2_{CDD}) & \frac{b_{23}}{M^2_{J/\psi}}(s - M^2_{CDD}) \\ b_{13} & \frac{b_{23}}{M^2_{J/\psi}}(s - M^2_{CDD}) & \frac{b_{33}}{M^2_{J/\psi}}(s - M^2_{CDD}) \end{pmatrix}$$

Heavy-quark symmetry $\quad b_{13} = \frac{b_{12}}{\sqrt{3}}\,,\quad b_{23} = \frac{b_{22}}{\sqrt{3}}\,,\quad b_{33} = \frac{b_{22}}{3}$

$$B(s) = [1 - \mathcal{V}(s) \cdot G(s)]^{-1} \cdot \mathcal{P}$$

$$\mathcal{P} = \begin{pmatrix} d_1 = 0 \\ d_2 \\ d_2/\sqrt{3} \end{pmatrix}$$

Fits are stable if releasing d_1

$$\frac{d\mathcal{N}(s)}{d\sqrt{s}} = |B_1(s)|^2 \frac{q_{J/\psi J/\psi}(s)}{M^2_{J/\psi}}$$

Free parameters: b_{12}, b_{22}, M^2_{CDD}, d_2

G function

$$G_j(s) = -\frac{1}{16\pi^2}\left[a(\mu^2) + \log\frac{m_2^2}{\mu^2} - x_+\log\frac{x_+-1}{x_+} - x_-\log\frac{x_--1}{x_-}\right],$$

$$x_\pm = \frac{s+m_1^2-m_2^2}{2s} \pm \frac{q_j(s)}{\sqrt{s}}.$$

Natural size estimate $a(\Lambda^2) = -2\log\left(1+\sqrt{1+\frac{m_X^2}{\Lambda^2}}\right) \simeq -3$

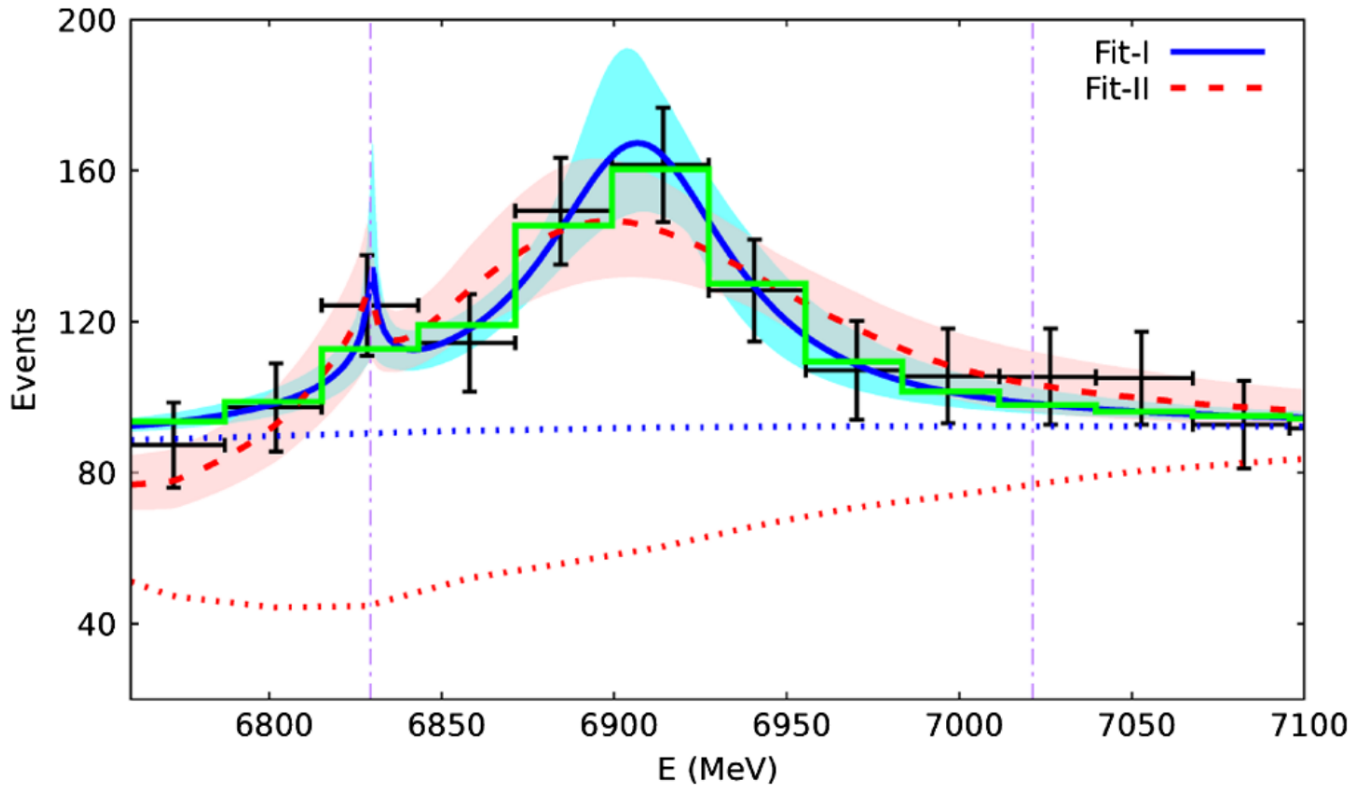
Matching at threshold with $G_\Lambda(s)$, $\Lambda \simeq 1$ GeV, a momentum cutoff

Change of Riemann sheet (RS)

$$G_j(s)^{\rm II} = G_j(s) - i\frac{q_j(s)}{4\pi\sqrt{s}}.$$

Riemann sheets:
1st $(+,+,+)$, 2nd $(-,+,+)$, 3rd $(-,-,+)$, 4th $(+,-,+)$, 5th $(-,-,-)$

Results of the fits



$J/\psi J/\psi$ event distribution. Green histogram averaging over the experimental width 27 MeV

	χ^2/d.o.f	$a(\mu)$	M_{CDD}	b_{22}	b_{12}	d_2
Fit-I	$1.6/(12-3)$	-3.0^*	6910^*	10817^{+8378}_{-2096}	151^{+153}_{-99}	2213^{+2106}_{-316}
Fit-II	$4.9/(12-3)$	-3.0^*	6885^*	21073^{+15141}_{-7359}	484^{+239}_{-112}	3645^{+1325}_{-714}

Residua and X_i

$$T_{ij} = -\frac{\gamma_i \gamma_j}{s - M^2_{\rm pole}} + \ldots$$

	Mass (MeV)	Width/2 (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)	$\lvert\gamma_3\rvert$ (GeV)	X_1	X_2	X_3	$X = \sum_{i=1}^{3} X_i$
Fit I	6907^{+5}_{-3}	33^{+14}_{-10}	$4.6^{+2.5}_{-2.8}$	$9.7^{+1.4}_{-2.6}$	$5.6^{+0.8}_{-1.5}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.04}_{-0.06}$	$0.03^{+0.01}_{-0.01}$	$0.17^{+0.04}_{-0.07}$
Fit II	6892^{+2}_{-2}	80^{+24}_{-17}	$10.3^{+1.8}_{-1.4}$	$6.9^{+1.4}_{-1.9}$	$4.0^{+0.8}_{-1.1}$	$0.05^{+0.02}_{-0.01}$	$0.06^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.03}_{-0.03}$

HQSS rule: $|\gamma_3| \approx |\gamma_2|/\sqrt{3}$

LHCb $X(6900)$

I: M_R $6905 \pm 11 \pm 7$ $\quad \Gamma_R = 80 \pm 19 \pm 33$ MeV

II: M_R $6886 \pm 11 \pm 11$ $\quad \Gamma_R = 168 \pm 33 \pm 69$ MeV

Total compositeness $X < 0.2$ Overwhelming bare component

In agreement with $M_{\rm CDD} \approx M_R \longrightarrow$ Morgan's pole-counting criterion

– Similar pole position in the 5th RS $(-,-,-)$

Distinction between Fits I and II

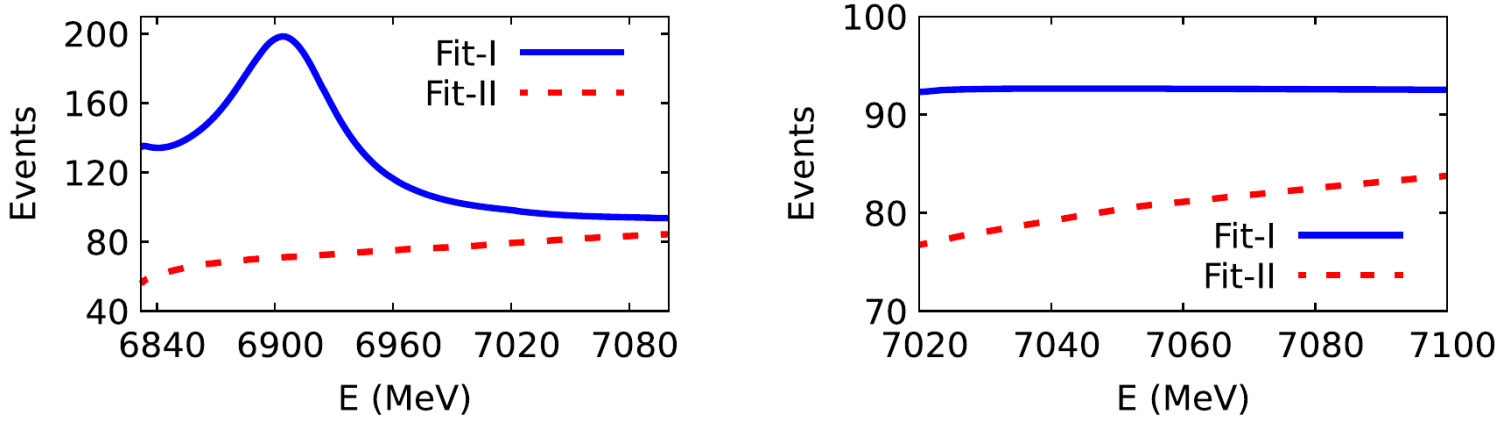


FIG. 3. Our predictions for the distributions of (left panel) $\chi_{c0}\chi_{c0}$ and (right panel) $\chi_{c1}\chi_{c1}$.

Prediction of the $X(6825)$

It lies in the 4th RS $(+,-,+)$

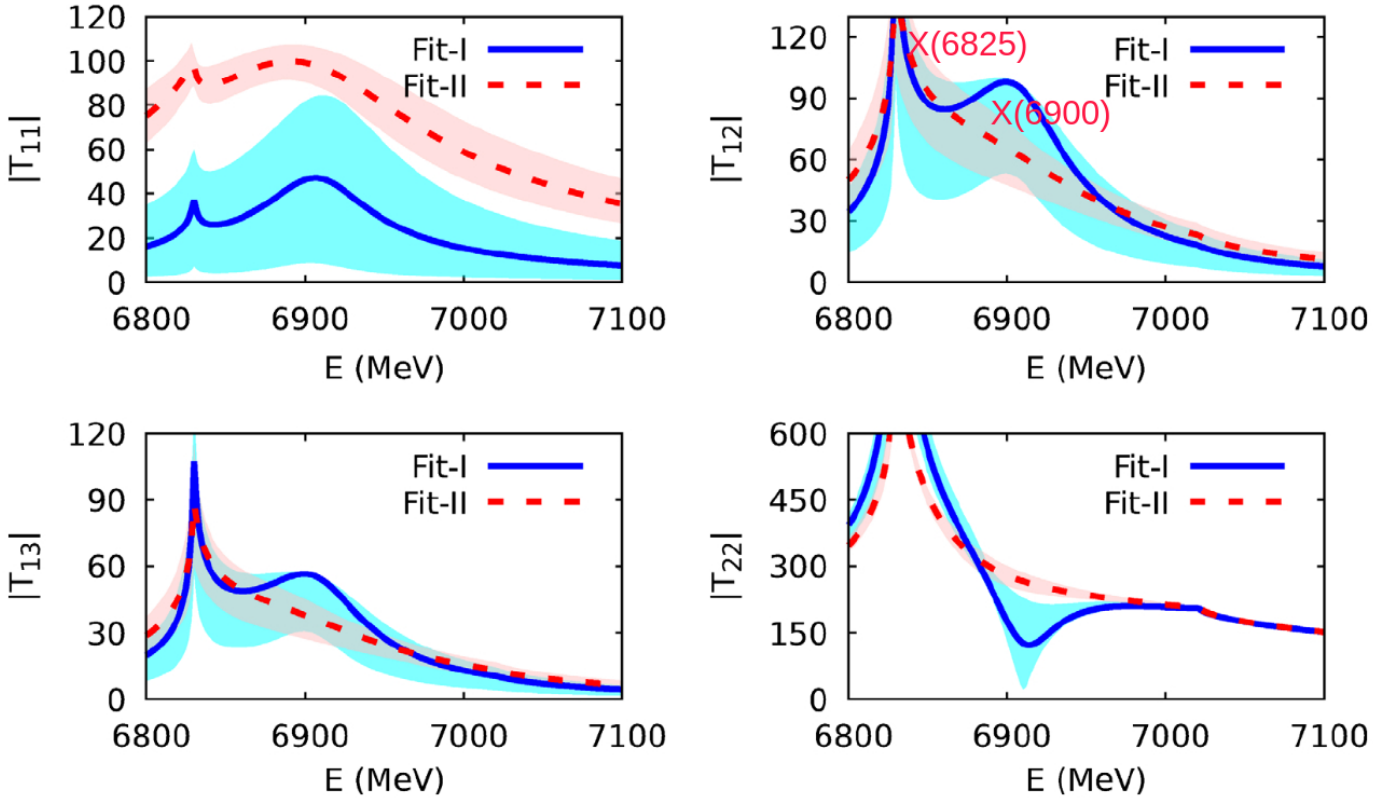
Fit	E'_R MeV	$\lvert\gamma'_1\rvert$	$\lvert\gamma'_2\rvert$	$\lvert\gamma'_3\rvert$
I	$6827.0^{+1.6}_{-4.8} - i\,1.1^{+1.3}_{-1.0}$	$1.4^{+0.6}_{-0.9}$	$11.9^{+3.2}_{-3.1}$	$6.8^{+1.8}_{-1.8}$
II	$6820.6^{+3.0}_{-2.7} - i\,4.0^{+1.7}_{-1.6}$	$2.5^{+0.5}_{-0.6}$	$15.8^{+0.7}_{-0.6}$	$9.1^{+0.4}_{-0.4}$

	Mass (MeV)	Width/2 (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)	$\lvert\gamma_3\rvert$ (GeV)	X_1	X_2	X_3	$X=\sum_{i=1}^{3} X_i$
Fit I	6907^{+5}_{-3}	33^{+14}_{-10}	$4.6^{+2.5}_{-2.8}$	$9.7^{+1.4}_{-2.6}$	$5.6^{+0.8}_{-1.5}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.04}_{-0.06}$	$0.03^{+0.01}_{-0.01}$	$0.17^{+0.04}_{-0.07}$
Fit II	6892^{+2}_{-2}	80^{+24}_{-17}	$10.3^{+1.8}_{-1.4}$	$6.9^{+1.4}_{-1.9}$	$4.0^{+0.8}_{-1.1}$	$0.05^{+0.02}_{-0.01}$	$0.06^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.03}_{-0.03}$

- $|\gamma'_1|$ are much smaller than for $X(6900)$ $\rightarrow$much smaller width
- $|\gamma'_{2,3}|$ are much larger than for $X(6900)$. HQSS rule $\gamma'_3 \approx \gamma'_2/\sqrt{3}$
- Virtual state present only at the 4th RS$\rightarrow$ dynamically generated (Morgan's pole counting rule)
- $b_{12}=0$. It becomes a pure bound state at 6825 (I) and 6827 (II) MeV

$X(6825)$

- $b_{12} = 0$ and $m_{\chi_{c0}} = m_{\chi_{c1}} \rightarrow$ bound state. When $b_{12} \neq 0$ resonance in the 2nd RS



$|\gamma_1'|$ is very small. $|\gamma_2'|$ is very large. HQSS rule $|\gamma_3'| \approx |\gamma_2'|/\sqrt{3}$

$\psi(3770)J/\psi$ extra channel

Channel	Threshold [MeV]
(1) $J/\psi J/\psi$	6193.8
(2) $\psi(3370)J/\psi$	6870.6

$$\hat{\mathcal{V}}(s) = \begin{pmatrix} 0 & \hat{b}_{12} \\ \hat{b}_{12} & \frac{\hat{b}_{22}}{M^2_{J/\psi}}(s - \hat{M}^2_{CDD}) \end{pmatrix} , \qquad (1)$$

	χ^2/d.o.f	$\hat{a}(\mu)$	$\hat{M}_{CDD}$	$\hat{b}_{22}$	$\hat{b}_{12}$	$\hat{d}_2$
Fit $\hat{\mathrm{I}}$	$2.8/(12-3)$	-3.2^*	6900^*	$(2.4^{+4.6}_{-1.7}) \times 10^5$	1303^{+1243}_{-597}	7825^{+6318}_{-3495}
Fit $\hat{\mathrm{II}}$	$2.4/(12-3)$	-3.2^*	6880^*	$(1.5^{+1.8}_{-0.6}) \times 10^5$	1356^{+741}_{-305}	9675^{+4674}_{-2043}

	Mass (MeV)	Width/2 (MeV)	$\lvert\hat{\gamma}_1\rvert$ (GeV)	$\lvert\hat{\gamma}_2\rvert$ (GeV)	$\hat{X}_1$	$\hat{X}_2$	$\hat{X} = \sum_{i=1}^2 \hat{X}_i$
Fit $\hat{\mathrm{I}}$	6900^{+2}_{-1}	44^{+20}_{-16}	$8.2^{+1.7}_{-1.6}$	$1.9^{+1.5}_{-0.9}$	$0.03^{+0.01}_{-0.01}$	$0.01^{+0.01}_{-0.00}$	$0.04^{+0.02}_{-0.01}$
Fit $\hat{\mathrm{II}}$	6877^{+1}_{-2}	78^{+21}_{-14}	$11.1^{+1.4}_{-1.1}$	$2.0^{+0.6}_{-0.7}$	$0.06^{+0.02}_{-0.01}$	$0.01^{+0.00}_{-0.00}$	$0.07^{+0.02}_{-0.01}$

- Fits are not well fixed –large errorbars
- Coupling to $\psi(3770)J/\psi$ are much smaller than to $\chi_{c0,1}\chi_{c0,1}$ –much less important role of the $\psi(3770)J/\psi$ channel

Perturbative treatment of $\psi(3770)J/\psi$

$J/\psi J/\psi$ (1), $\chi_{c0}\chi_{c0}$ (2), $\chi_{c1}\chi_{c1}$ (3) and $\psi(3770)J/\psi$ (4)

$b_{14} = b_{44} = d_4 = 0$ Perturbative Treatment

Only one more free parameter $b_{24} = b_{34}\sqrt{3}$

Fits I and II are stable

Saturating X and Γ

Taking X from the fits

$$X = X_1 + X_2 + X_3$$

$$= |\gamma_1|^2 \left| \frac{dG_1^{\rm II}(s_R)}{ds} \right| + |\gamma_2|^2 \left| \frac{dG_2^{\rm II}(s_R)}{ds} \right| + \frac{|\gamma_2|^2}{3} \left| \frac{dG_3(s_R)}{ds} \right| ,$$

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$$

$$= |\gamma_1|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |\gamma_2|^2 \int_{m_{\rm th,2}}^{M_R+2\Gamma_R} dw \, \frac{q_2(w^2)}{16\pi^2 \, w^2} \frac{\Gamma_R}{(M_R - w)^2 + \Gamma_R^2/4}$$

$$+ \frac{|\gamma_2|^2}{3} \int_{m_{\rm th,3}}^{M_R+2\Gamma_R} dw \, \frac{q_3(w^2)}{16\pi^2 \, w^2} \frac{\Gamma_R}{(M_R - w)^2 + \Gamma_R^2/4} ,$$

recall $|\gamma_3| \approx |\gamma_2|/\sqrt{3}$

Fit I $X = 0.17$

$$|\gamma_1| = 6.2 \text{ GeV} , \quad |\gamma_2| = 9.5 \text{ GeV}$$

$$\Gamma_1 = 49.7 \text{ MeV} , \quad \Gamma_2 = 30.1 \text{ MeV} , \quad \Gamma_3 = 0.2 \text{ MeV} ,$$

$$X_1 = 0.018 , \quad X_2 = 0.126 , \quad X_3 = 0.026 ,$$

Fit II $X = 0.13$

$$|\gamma_1| = 11.1 \text{ GeV}\,, \quad |\gamma_2| = 6.7 \text{ GeV}$$
$$\Gamma_1 = 154.7 \text{ MeV}\,, \quad \Gamma_2 = 12.8 \text{ MeV}\,, \quad \Gamma_3 = 0.5 \text{ MeV}\,,$$
$$X_1 = 0.06\,, \quad X_2 = 0.06\,, \quad X_3 = 0.01\,,$$

The $|\gamma_i|$ are in good agreement with the fit values

Decay partial widths and partial compositeness coefficients are provided

7.- $P_{cs}(4459)$. X-Γ studies

Du,Guo,JAO,PRD104,114034(2021)

Charmonium pentaquark resonance with strangeness $P_{cs}(4459)$ by the LHCb Sci.Bull.66,1278(2021)

$$M_R = 4458.8 \pm 2.9^{+4.7}_{-1.1} \text{ MeV}, \quad \Gamma_R = 17.3 \pm 6.5^{+8-0}_{-5.7} \text{ MeV}$$

$J/\psi\Lambda$ event distributions –one or two resonances , $J = 1/2$ or $3/2$

Theoretical predictions Wu,Molina,Oset,Zou,PRL(2010) and others

Our three methods → Molecular nature of the $P_{cs}(4459)$

Elastic-ERE Study

Resonance	Mass (MeV)	Width (MeV)	Threshold (MeV)	a (fm)	r (fm)	X
P_{cs}	4458.8 ± 5.5	17.3 ± 10.3	$\Xi'_c D$ (4446.0)	-0.63 ± 0.38	-3.68 ± 2.11	0.31 ± 0.19
P_{cs}	4458.8 ± 5.5	17.3 ± 10.3	$\Xi_c D^*$ (4478.0)	-1.79 ± 0.23	-0.94 ± 0.13	$\cdots$

Saturating X and Γ

Main difference: To take into account the partial-decay width to $J/\psi\Lambda \to$ reproducing with the elastic ERE not the total width but partial-decay widths

Phase-space suppression:
Distance from M_R to threshold $\sim \Gamma_R$

$$X = X_1 + X_2 = |g_1|^2 \left| \frac{dG_1^{\rm II}(s_R)}{ds} \right| + |g_2|^2 \left| \frac{dG_2(s_R)}{ds} \right|$$

$$\Gamma = \Gamma_1 + \Gamma_2 = |g_1|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |g_2|^2 \int_{m_{\rm th,2}}^{M_R+n\Gamma_R} dw \, \frac{q_2(w^2)}{16\pi^2 \, w^2} \frac{\Gamma_R}{(M_R - w)^2 + \Gamma_R^2/4}$$

(1) $J/\psi\Lambda$, (2) $\Xi_c\bar{D}^*$

	$\lvert g_1\rvert$ (GeV)	$\lvert g_2\rvert$ (GeV)	Γ_1 (MeV)	Γ_2 (MeV)	X_1	X_2
$X = 0.1$	$3.5^{+1.0}_{-0.8}$	$4.3^{+0.2}_{-0.4}$	$16.4^{+9.5}_{-6.7}$	$0.9^{+0.5}_{-0.5}$	$0.02^{+0.01}_{-0.01}$	$0.08^{+0.01}_{-0.01}$
$X = 0.5$	$3.1^{+0.7}_{-0.7}$	$10.4^{+0.6}_{-0.8}$	$12.3^{+5.9}_{-4.9}$	$5.0^{+4.7}_{-2.9}$	$0.01^{+0.01}_{-0.01}$	$0.49^{+0.01}_{-0.01}$
$X = 1.0$	$2.3^{+0.4}_{-0.4}$	$14.8^{+1.0}_{-1.0}$	$7.1^{+1.7}_{-2.3}$	$10.2^{+9.5}_{-5.5}$	$0.0^{+0.0}_{-0.0}$	$1.0^{+0.0}_{-0.0}$

(1) $J/\psi\Lambda$, (2) $\Xi_c'\bar{D}$

	$\lvert g_1\rvert$ (GeV)	$\lvert g_2\rvert$ (GeV)	Γ_1 (MeV)	Γ_2 (MeV)	X_1	X_2
$X = 0.1$	$3.2^{+1.2}_{-1.0}$	$3.8^{+0.2}_{-0.4}$	$13.0^{+12.2}_{-6.4}$	$4.3^{+1.8}_{-1.4}$	$0.01^{+0.02}_{-0.00}$	$0.09^{+0.00}_{-0.02}$
$X = 0.3$	$1.4^{+2.0}_{-0.0}$	$7.0^{+0.4}_{-0.8}$	$2.5^{+12.2}_{-0.0}$	$14.8^{+4.3}_{-6.1}$	$0.00^{+0.02}_{-0.00}$	$0.30^{+0.00}_{-0.02}$

No solution for $X \gtrsim 0.3 \rightarrow$ If P_{cs} is of molecular type then it must be made up by $\Xi_c\bar{D}^*$

Still one has to provide as input X...

Fits to data: $P_{cs}(4459)$

Two-channel case

Channel	Threshold [MeV]
(1) $J/\psi\Lambda$	4212.6
(2) $\Xi_c\bar{D}^*$	4478.0

S-wave scattering

- $P_{cs}(4459)$ is very close to the thresholds of (2) and (3)
- HQSS. The $J/\psi\Lambda$ ($\eta_c\Lambda$) cannot couple to (2)-(3) in D and higher partial waves

$$\mathcal{T}_J(s) = [\mathbb{I} - \mathcal{V}_J \cdot G(s)]^{-1} \cdot \mathcal{V}_J(s)\,.$$

$$\mathcal{V}_{\frac{1}{2}} = \begin{pmatrix} 0 & g \\ g & C_{\frac{1}{2}} \end{pmatrix}, \quad \mathcal{V}_{\frac{3}{2}} = \begin{pmatrix} 0 & g \\ g & C_{\frac{3}{2}} \end{pmatrix}.$$

Direct $J/\psi\Lambda$ and $\eta_c\Lambda$ scattering is OZI suppressed. LQCD Skerbis,Prelovsek PRD99(2019),...

HQSS: $C_{\frac{1}{2}} = C_{\frac{1}{3}}$

We let them float as a check of completeness of the model

J/ψ production amplitude and event distribution

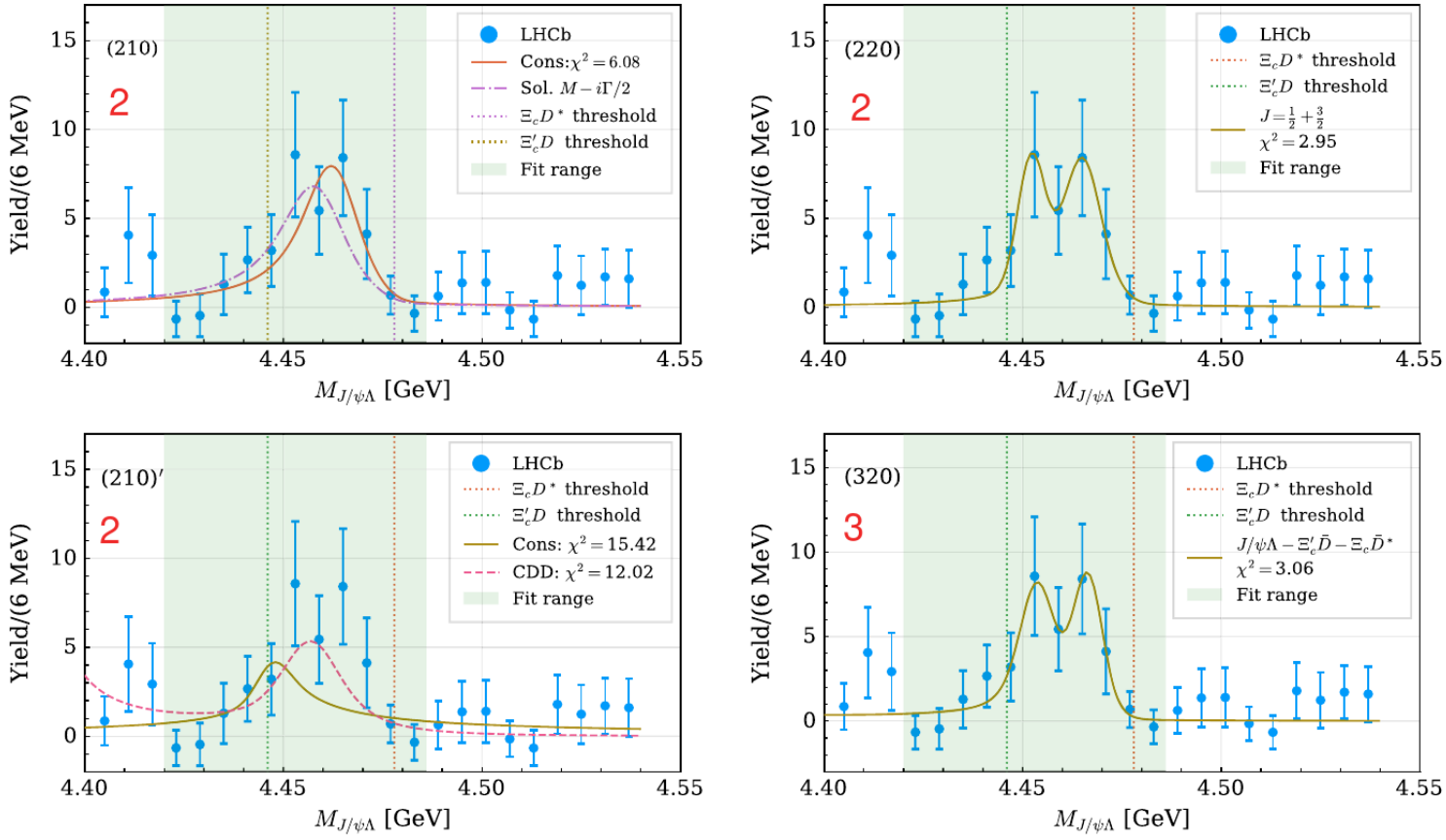
$$F_J(s) = \frac{d_J}{\Delta_J(s)} = \frac{d_J}{1-(C_J + G_1(s)g^2)G_3(s)}$$

$$\Delta_J(s) = \det\left[\mathbb{I} - \mathcal{V}_J \cdot G(s)\right]$$

$$\frac{dN(s)}{d\sqrt{s}} = \frac{1}{128\pi^3 M_{\Xi_b}^3} \frac{\sqrt{\lambda(M_{\Xi_b}^2, s, M_K^2)\lambda(s, M_{J/\psi}^2, M_\Lambda^2)}}{\sqrt{s}} \sum_J |F_J|^2 \ .$$

$\times$convolution to take into account energy resolution

$J/\psi\Lambda$, $\Xi_c\bar{D}^*$ Fit Results



(210), (220) $J/\psi\Lambda$,$\Xi_c\bar{D}^*$; (210)' $J/\psi\Lambda$, $\Xi_c'\bar{D}$
(320) $J/\psi\Lambda$, $\Xi_c\bar{D}^*$, $\Xi_c'\bar{D}$

(320) $C_{\frac{1}{2}} = C_{\frac{3}{2}}$ Lack of $\Xi_c'\bar{D}$ was the reason for HQSS breaking in (220) with $C_{\frac{1}{2}} \neq C_{\frac{3}{2}}$

Fit	χ^2	g	$C_{\frac{1}{2}}$	$d_{\frac{1}{2}}$	$C_{\frac{3}{2}}$	$d_{\frac{3}{2}}$
(210)	6.08	$316.0^{+86.2}_{-69.3}$	$1126.6^{+327.8}_{-214.9}$	$290.5^{+71.1}_{-77.2}$	$\times$	$\times$
(220)	2.95	$217.8^{+80.6}_{-79.3}$	$1125.5^{+190.6}_{-185.9}$	$174.7^{+86.5}_{-77.3}$	$3862.7^{+1466.1}_{-1003.3}$	$97.6^{+37.9}_{-35.5}$
(320)	3.06	$124.5^{+130.4}_{-164.7}$	$1105.8^{+191.9}_{-132.5}$	$250.8^{+62.0}_{-40.2}$	$C_{\frac{3}{2}} = C_{\frac{1}{2}}$	$82.1^{+292.1}_{-128.0}$
$(320)_1$	3.00	$145.8^{+122.9}_{-414.5}$	$1108.7^{+192.3}_{-142.9}$	$237.8^{+84.6}_{-72.5}$	$C_{\frac{3}{2}} = C_{\frac{1}{2}}$	$100.1^{+125.1}_{-144.2}$
$(320)_2$	2.81	$0.06^{+289.5}_{-288.6}$	$1098.0^{+175.7}_{-137.5}$	$197.9^{+177.8}_{-78.3}$	$C_{\frac{3}{2}} = C_{\frac{1}{2}}$	$19.1^{+90.2}_{-90.2}$

CDD poles: If including linear s dependence

$C_{\frac{1}{2}}(\frac{s}{M^2_{\rm CDD}-1}-1) \to$ Resonance poles in the 1st RS!

$$V_{\frac{1}{2}} = \begin{pmatrix} 0 & g' & g \\ g' & C'_{\frac{1}{2}} & C_{\rm mx} \\ g & C_{\rm mx} & C_{\frac{1}{2}} \end{pmatrix} \ , \quad P_{\frac{1}{2}} = \begin{pmatrix} 0 \\ d'_{\frac{1}{2}} \\ d_{\frac{1}{2}} \end{pmatrix}$$

(320) Perturbative treatment of $\Xi'_c \bar{D}$

$$g' = C'_{\frac{1}{2}} = d'_{\frac{1}{2}} = 0$$

Fit	χ^2	$C_{\rm mx}$	g'	$C'_{\frac{1}{2}}$	$d'_{\frac{1}{2}}$
(320)	3.06	$851.1^{+341.0}_{-148.1}$	0	0	0
$(320)_1$	3.00	$885.6^{+353.4}_{-204.9}$	$59.6^{+160.7}_{-249.2}$	0	0
$(320)_2$	2.81	$579.8^{+1014.2}_{-2201.5}$	0	$187.6^{+174.8}_{-831.8}$	0

Poles

RSII $(-,+,+)$, RSIII $(-,-,+)$, RSIV $(-,-,-)$

Type	J	RS	$\sqrt{s_R}$ (MeV)	$\lvert g_1\rvert$ (MeV)	$\lvert g_2\rvert$ (MeV)	$\lvert g_3\rvert$ (MeV)
(320)	3/2 $(-+)$	RSII	$4466.6^{+1.9}_{-2.7} - i\ 1.3^{+1.3}_{-3.7}$	$1.4^{+1.4}_{-1.4}$	$\times$	$12.6^{+0.8}_{-0.6}$
(320)	1/2 $(--+)$	RSIII	$4453.8^{+2.4}_{-3.3} - i2.8^{+0.9}_{-0.8}$	$0.6^{+0.6}_{-0.6}$	$4.2^{+0.2}_{-0.4}$ <<	$15.0^{+0.5}_{-0.3}$

$$J = 1/2 : \Gamma_1 = 0.5^{+1.9}_{-0.5}\ \mathrm{MeV}\,,\ \Gamma_2 = 4.3^{+1.2}_{-1.4}\ \mathrm{MeV}\,,\ \Gamma_3 = 0.9^{+1.2}_{-0.6}\ \mathrm{MeV}$$
$$X_1 = 0.0 \pm 0.0\,,\ X_2 = 0.15 \pm 0.05\,.$$

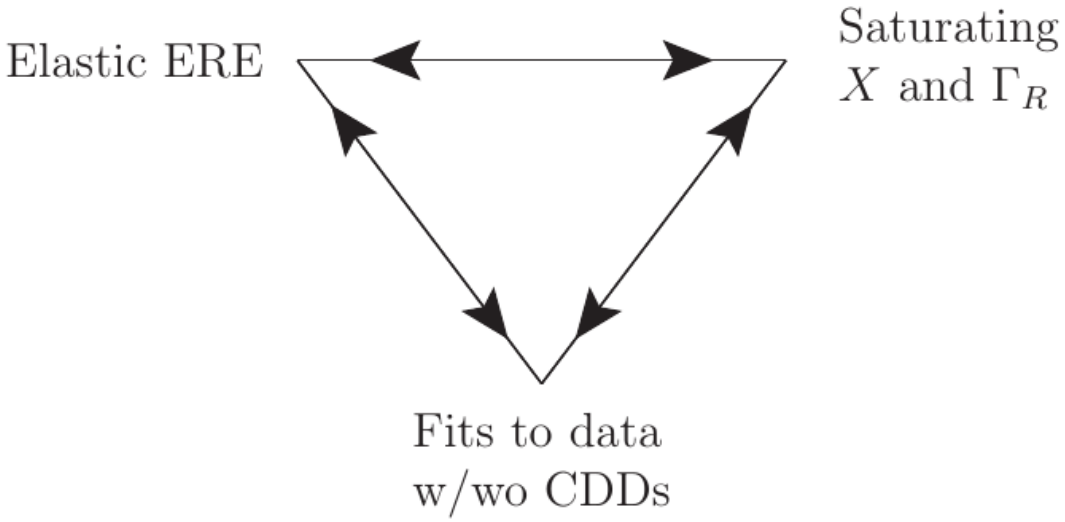
$$J = 3/2 : \Gamma_1 = 2.6^{+8.2}_{-2.6}\ \mathrm{MeV}\,,\ \Gamma_3 = 0.4^{+2.5}_{-0.4}\ \mathrm{MeV}$$
$$X_1 = 0.0 \pm 0.0\,,\ X_3 = 1.0^{+0.2}_{-0.2}$$

Composite resonances, like $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$ Du,*et. al.* PRL124,072001(2020)

$J/\psi\Lambda$, $\Xi_c\bar{D}^*$ only

Fit	RS	$\sqrt{s_R}$ (MeV)	$\lvert g_1\rvert$ (MeV)	$\lvert g_2\rvert$ (MeV)
(210)	$(-+)$	$4463.2^{+2.8}_{-4.4} - i7.1^{+2.5}_{-2.8}$	$3.29^{+0.64}_{-0.68}$	$13.81^{+0.87}_{-0.68}$
(220)	$(-+)$	$4465.5^{+2.3}_{-2.3} - i3.8^{+2.3}_{-3.4}$	$1.20^{+0.46}_{-0.44}$	$13.01^{+0.65}_{-0.62}$
(220)	$(-+)$	$4452.1^{+2.4}_{-2.0} - i0.5^{+0.3}_{-0.7}$	$0.88^{+0.46}_{-0.33}$	$15.73^{+0.33}_{-0.41}$

8.- Conclusions



1. Classic formalism for elementariness and compositeness
2. New formalism based on the use of the number operators of free particles JAO,ANP(2018)
3. $Z_b(10610)$, $Z_b(10650)$ Admixture
4. $Z_{cs}(3985)$, $Z_c(3900)$, $X(4020)$ Related dynamics
5. $X(6900)$ Elementary. $X(6825)$ Composite virtual state.
6. $P_{cs}(4459)$ Composite

Contact interactions: $|k_{n,B}| \ll \Lambda$

Guo,JAO,PRD103,054021(2021)

$$X = \frac{2m^2}{\pi^2}\int_0^\infty k^2 \frac{g(k^2)^2}{(k^2-k_B^2)^2}dk \ , \ k_B^2 = 2mE_B$$

- Expansion of $g(k^2)^2$ in powers of $k^2 - k_B^2$

$$g(k^2) = g(k_B^2) + c_1(k^2-k_B^2) + c_2(k^2-k_B^2)^2 + \ldots$$

- Dimensional regularization$\rightarrow$ power-like divergences vanish

$$X = -g(k_B^2)^2 \left.\frac{\partial G(E)}{\partial E}\right|_{E_B} - \frac{m^2|k_B|}{\pi}\left.\frac{\partial g(k^2)^2}{\partial k^2}\right|_{E_B}$$
$$= -g(k_B^2)^2\frac{i\mu^2}{2\pi k_B} + \mathcal{O}\left(\frac{k_B^2}{\Lambda^2}\right)$$

If k_B^2 dependence of $g(k_B^2)$ is neglected$\rightarrow$ Weinberg's formula for $1-Z$ for a shallow bound state

Equality of the wave functions of the Gamow state and its dual

$$|\psi_\alpha^+\rangle = |\varphi_\alpha\rangle + \int d\gamma \frac{T_{\gamma\alpha}(E+i\varepsilon)}{E+i\varepsilon-E_\gamma}|\varphi_\gamma\rangle + \sum_n \frac{T_{n\alpha}(E)}{E-E_n}|\varphi_n\rangle$$

$$\langle\psi_\alpha^-| = \langle\varphi_\alpha| + \int d\gamma \frac{T_{\gamma\alpha}(E+i\varepsilon)}{E+i\varepsilon-E_\gamma}\langle\varphi_\gamma| + \sum_n \frac{T_{n\alpha}(E)}{E+i\varepsilon-E_n}\langle\varphi_n|$$

Therefore,

$$\langle\psi_\alpha^-|\varphi_\gamma\rangle = \langle\varphi_\gamma|\psi_\alpha^+\rangle = \frac{T_{\gamma\alpha}(E+i\varepsilon)}{E+i\varepsilon-E_\gamma} \to \frac{g_\gamma(k_n)^2}{(E_n-E_\gamma)^2}$$

Instead of $|g_\gamma(k_n)|^2$. Wave-function squared

Hernández,Mondragón (1984)

$$u_{nl}(q;k_n) = \widetilde{u}_{nl}(q;k_n)$$

$$\left(\frac{1}{T_2+T_4+GT_2^2}+G\right)^{-1}$$

$$\frac{1}{T_2+T_4+GT_2^2}+G=\frac{1}{T_2}-\frac{T_4}{T_2^2}+\ldots$$

$$T_{IAM}=\left(\frac{1}{T_2}-\frac{T_4}{T_2^2}\right)^{-1}=\frac{T_2^2}{T_2-T_4}$$