Preliminary Lattice QCD Study of Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

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Motivation

- Neutrino mass can arise from either Dirac or Majorana mass term
$$\mathcal{L} \supset -m\bar{\nu}_L\nu_R - \frac{M}{2}\bar{\nu}_R\nu_R$$
- ν_R neutral under all charges – Majorana mass term not forbidden
- Majorana mass $\Rightarrow$ ν is own antiparticle

Seesaw Mechanism

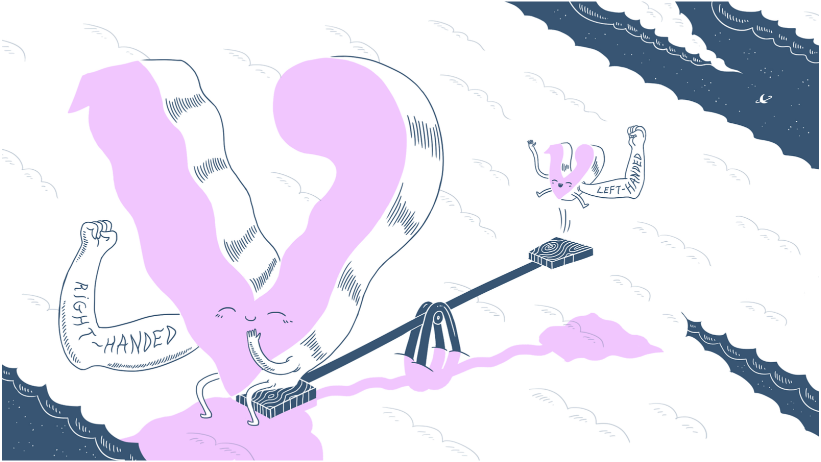


Image credit: Symmetry Magazine, Sandbox Studio, Ana Kova

Leptogenesis

- Universe has more matter than antimatter
- Requires baryon number violation
- Sphalerons: Non-perturbative, high-T SM processes that violate B, L but preserve $B - L$
- Can convert lepton asymmetry into baryon asymmetry
- Majorana neutrinos $\Rightarrow$ lepton number violation

Double-Beta Decay

$$B(Z,A) = \varepsilon_V A - \varepsilon_S A^{2/3} - \varepsilon_C \frac{Z^2}{A^{1/3}} - \varepsilon_{\text{sym}} \frac{(N-Z)^2}{A} + \eta(Z,N)\frac{\Delta}{A^{1/2}}$$

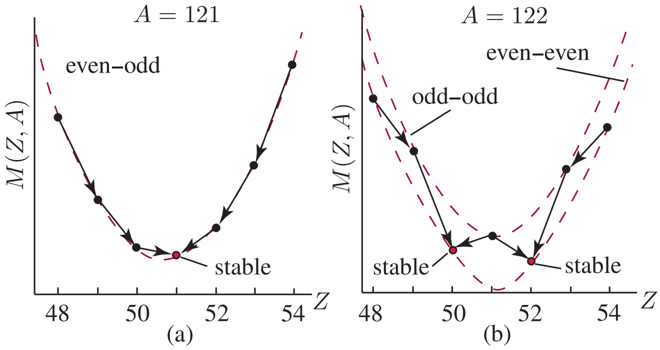
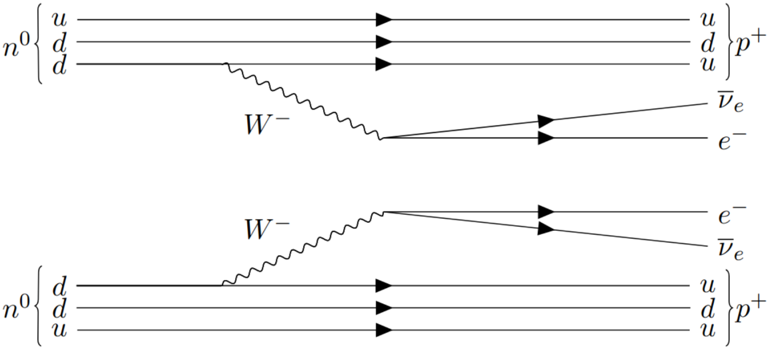
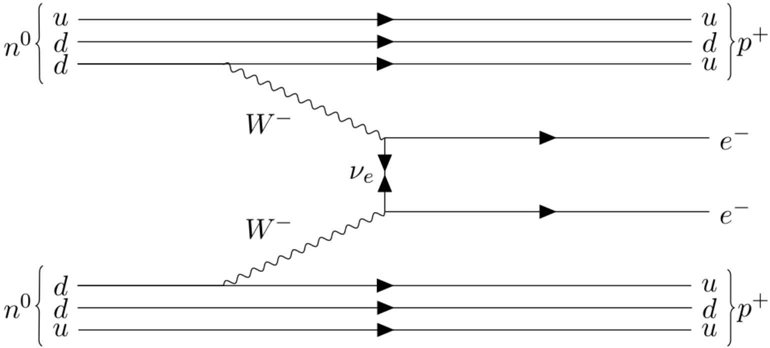


Image credit: Jaffe and Taylor (2018), after J. Lilley (2001)

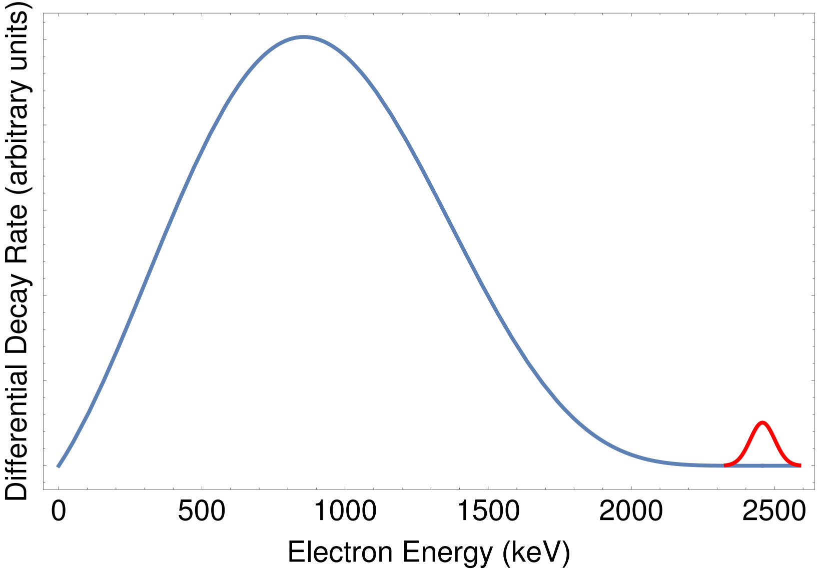
$2\nu\beta\beta$ Diagram



$0\nu\beta\beta$ Diagram



Experimental Signature



Extraction of $m_{\beta\beta}$

$$\left(T_{1/2}^{0\nu}\right)^{-1} \propto |m_{\beta\beta}|^2 G^{0\nu} |M^{0\nu}|^2$$

- $T_{1/2}^{0\nu}$ measured experimentally
- $m_{\beta\beta}$ is effective double-beta neutrino mass
  - $m_{\beta\beta} = |\sum_k U_{ek}^2 m_k|$
- $G^{0\nu}$ is known kinematical factor
- $M^{0\nu}$ is nuclear matrix element
  - Typically estimated with nuclear models (shell models, estimated potentials, etc.)

Nuclear Matrix Element Estimates

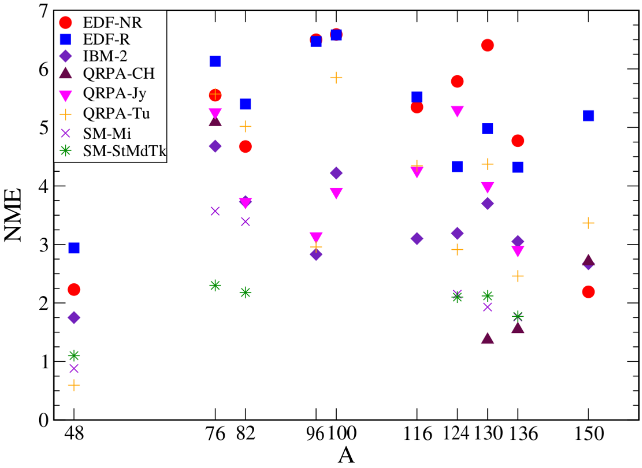


Image credit: Dolinski, Poon, Rodejohann (1902.04097)

KamLAND-Zen Results

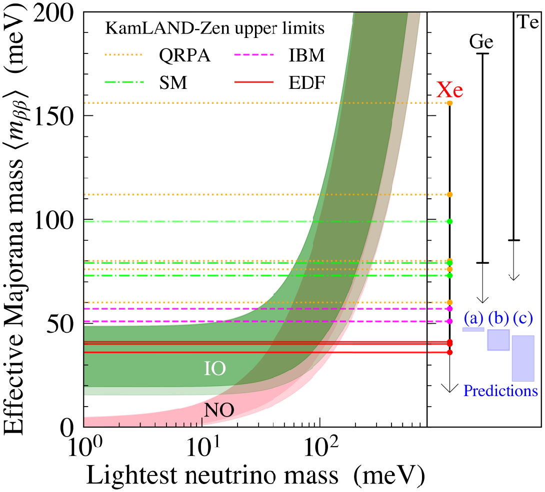


Image credit: KamLAND-Zen (2203.02139)

Lattice QCD

- First-principles, model-independent solution to hadronic physics
- Only input = Lagrangian of QCD ($\{m_q\}, \alpha_s$)
- Systematically controllable errors
- Caveat: Computationally expensive, especially for large systems

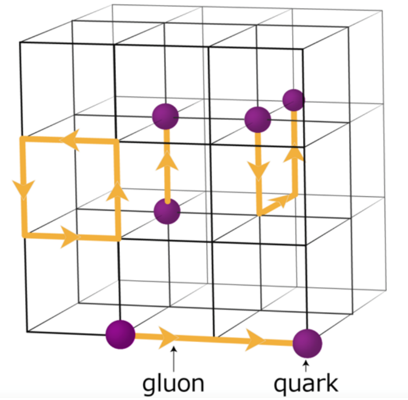


Image credit: JICFuS, Tsukuba

Lattice QCD for $0\nu\beta\beta$

- Goal: Simulate light nuclei to extract parameters for nuclear EFTs
- Previous lattice studies:
  - $2\nu\beta\beta$ for $nn \to pp$: Tiburzi et al. (1702.02929)
  - Short-range piece of $0\nu\beta\beta$ for $\pi^- \to \pi^+$: Nicholson et al. (1805.02634); Detmold et al. (in preparation)
  - $0\nu\beta\beta$ for $\pi^- \to \pi^+$: Tuo, Feng, Jin (1909.13525); Detmold, Murphy (2004.07404)
- This work: Compute matrix elements for $0\nu\beta\beta$ for $nn \to pp$ and $\Sigma^- \to \Sigma^+$
- Note: Dineutron is bound at artificially heavy quark masses ($m_\pi = 800$ MeV)

Effective Field Theory for $0\nu\beta\beta$

- Controlled approximation to nuclear physics based on symmetries of theory
- Degrees of freedom = nucleons (not individual quarks)
- Coupling constants = free parameters – fit to experiment
- Contact interaction specific to $0\nu\beta\beta$ – no data to fit to

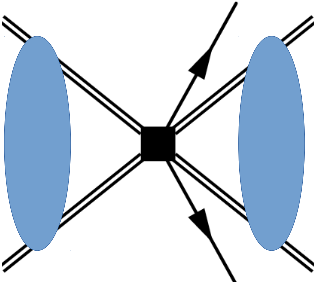
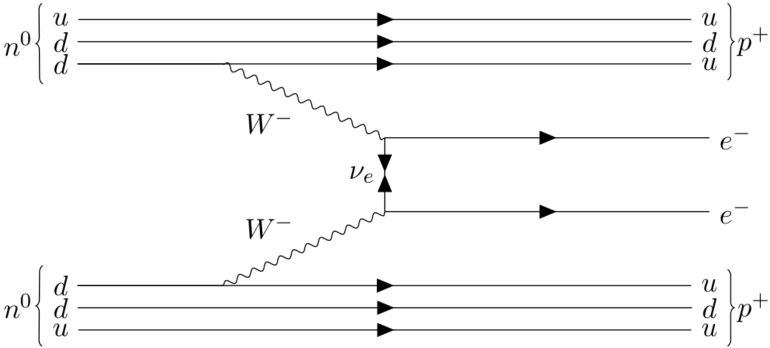


Image credit: Cirigliano et al. (1802.10097)

Long-Distance Quark Contractions



Quark Contractions

- Quarks not confined to specific nucleons within nucleus
- Number of Wick contractions $= N_u! N_d! = 576$
  - Reduced somewhat by symmetries, still $O(100)$ contractions
  - Also $(3!)^4 = 1296$ assignments of color indices
  - Propagators computed as inverses of $\not{D} + m$ (spin-color matrix at each lattice site)
- Algebraically and computationally difficult
  - $O(10^{17})$ 2×2 complex matrix multiplications needed
  - Required manual and automated optimization
  - Current performance $\sim 1000\times$ faster than original expectation

Long-Distance and Short-Distance Contributions

- Long-distance contribution from light Majorana neutrino ν
  - Form of current known:

$$(\bar{d} P_L \gamma_\mu u)(x) S_\nu (x - y)(\bar{d} P_L \gamma^\mu u)(y)$$

- Short-distance contribution from heavy Majorana neutrino N
  - Predicted in some models with Majorana neutrinos
  - Heavy neutrino (and W bosons) integrated out $\rightarrow$ 4-point interaction
  - Form of current model dependent

$$\left(\bar{d} \Gamma u\right) \left(\bar{d} \Gamma u\right)$$

where Γ can be scalar, pseudoscalar, vector, axial, or tensor current

Short-Distance Operators

Five choices of Γ insertion

$$
\begin{aligned}
SS &= (\bar{u}d)\,(\bar{u}d) \\
PP &= (\bar{u}\gamma_5 d)\,(\bar{u}\gamma_5 d) \\
VV &= (\bar{u}\gamma_\mu d)\,(\bar{u}\gamma^\mu d) \\
AA &= (\bar{u}\gamma_\mu\gamma_5 d)\,(\bar{u}\gamma^\mu\gamma_5 d) \\
TT &= \sum_{\mu<\nu} (\bar{u}\gamma_\mu\gamma_\nu d)\,(\bar{u}\gamma^\mu\gamma^\nu d)
\end{aligned}
$$

Can be rearranged into canonical BSM basis

$$
\begin{aligned}
4\,[\mathcal{O}_1]_+ &\equiv VV - AA \\
4\,[\mathcal{O}_2]_+ &\equiv 2\,(SS + PP) \\
4\,[\mathcal{O}_3]_+ &\equiv 2\,(VV + AA) \\
4\,\left[\mathcal{O}_1'\right]_+ &\equiv 2\,(PP - SS) \\
4\,\left[\mathcal{O}_2'\right]_+ &\equiv TT - SS - PP
\end{aligned}
$$

Short-Distance Matrix Elements

- Computed on lattice via ratio of 3-point and 2-point correlation functions:

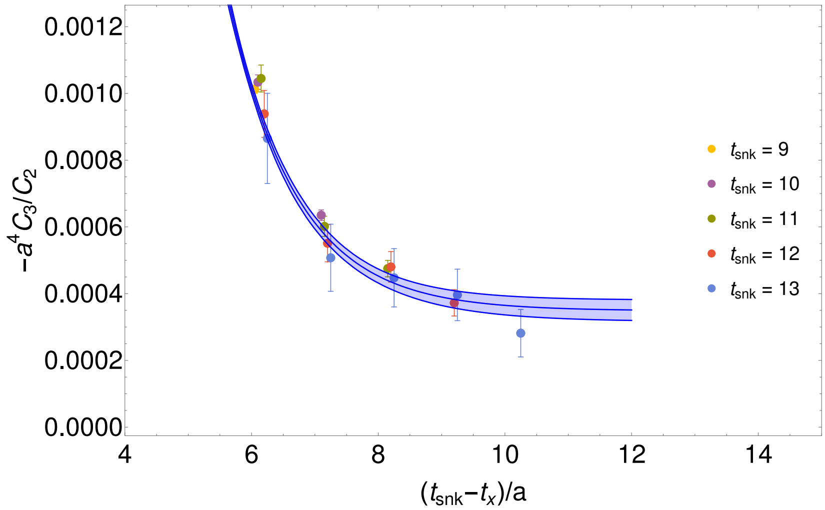
$$R_i(t,\tau) = \frac{C_{3i}(t,\tau)}{C_2(t)} = \frac{\langle \mathcal{O}_{pp}(t)\mathcal{O}_i(\tau)\mathcal{O}^\dagger_{nn}(0)\rangle}{\langle \mathcal{O}_{nn}(t)\mathcal{O}^\dagger_{nn}(0)\rangle}$$

- For $0 \ll \tau \ll t$, extract the desired matrix element

$$R_i(t,\tau) \to 2m\langle pp|\mathcal{O}_i|nn\rangle$$

- For small τ or $t-\tau$, can have significant excited state contamination

$R_{\mathcal{O}_2'}(t,\tau)$



Preliminary Fit Results

$$\langle pp|\mathcal{O}_1|nn\rangle = 0.003(1) \qquad \langle \Sigma^+|\mathcal{O}_1|\Sigma^-\rangle = 0.0088(2)$$
$$\langle pp|\mathcal{O}_2|nn\rangle = -0.014(7) \qquad \langle \Sigma^+|\mathcal{O}_2|\Sigma^-\rangle = 0.0199(3)$$
$$\langle pp|\mathcal{O}_3|nn\rangle = -0.034(3) \qquad \langle \Sigma^+|\mathcal{O}_3|\Sigma^-\rangle = 0.0008(3)$$
$$\langle pp|\mathcal{O}_1'|nn\rangle = -0.012(12) \qquad \langle \Sigma^+|\mathcal{O}_1'|\Sigma^-\rangle = -0.0634(4)$$
$$\langle pp|\mathcal{O}_2'|nn\rangle = -0.006(1) \qquad \langle \Sigma^+|\mathcal{O}_2'|\Sigma^-\rangle = -0.0107(1)$$

(all in units of GeV^4)

- Systematics from excited states not included
- Large lattice spacing, finite volume, unphysical pion mass
- No renormalization

Calculation Procedure

- Computed from 2- and 4-point correlation functions

$$C_2(t_z) = \int d^3\mathbf{z}\, \langle \mathcal{O}_{nn}(z)\mathcal{O}^\dagger_{nn}(0)\rangle$$

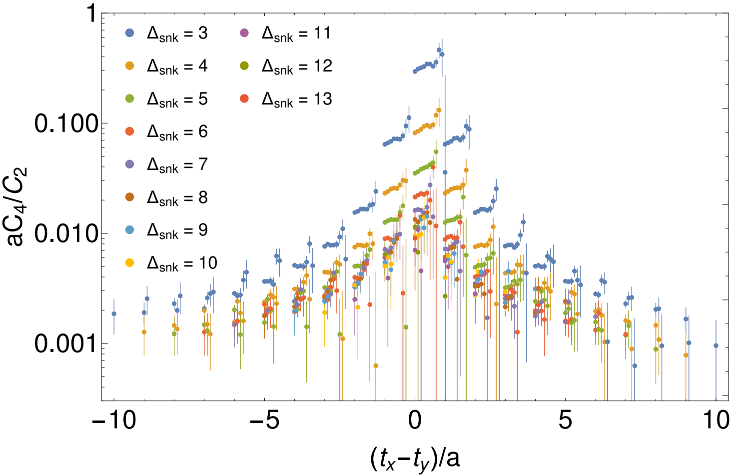
$$C_4(t_z, t_x, t_y) = \int d^3\mathbf{z}\, d^3\mathbf{x}\, d^3\mathbf{y}\, \Gamma^{\text{lept.}}_{\alpha\beta}\, \langle \mathcal{O}_{pp}(z) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(0)\rangle S_\nu(x,y)$$

- Neutrino propagator taken to be massless scalar with UV regulator

$$S_\nu(x,y) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} e^{iq\cdot(x-y)} e^{-q^2/(\pi/a)^2}$$

  - a = lattice spacing (recover full propagator in continuum limit)

Ratio of Correlators



Colored by $\Delta_{snk} = t_z - \max[t_x, t_y]$, ordered by $\Delta_{src} = \min[t_x, t_y]$

Calculation Procedure

- Sum correlator ratio within some window $[\Delta, t_z - \Delta]$

$$\mathcal{C}_\Delta(t_z) = \sum_{t_x=\Delta}^{t_z-\Delta} \sum_{t_y=\Delta}^{t_z-\Delta} \frac{C_4(t_z, t_x, t_y)}{C_2(t_z)}$$

Calculation Procedure

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$$= \sum_{k=0}^{\infty} \sum_{\mathbf{x},\mathbf{y}} \int \frac{d^3q}{(2\pi)^3} \frac{\Gamma^{\text{lept.}}_{\alpha\beta} \langle pp | j_\alpha(\mathbf{x}) | k \rangle \langle k | j_\beta(\mathbf{y}) | nn \rangle}{2E_k |\mathbf{q}| \left(|\mathbf{q}| + E_k - m_{nn} \right)} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}$$

$$\times \frac{1}{2m_{nn}} \left((t_z - 2\Delta) + \frac{e^{-(|\mathbf{q}|+E_k-m_{nn})(t_z-2\Delta)} - 1}{|\mathbf{q}| + E_k - m_{nn}} \right)$$

Calculation Procedure

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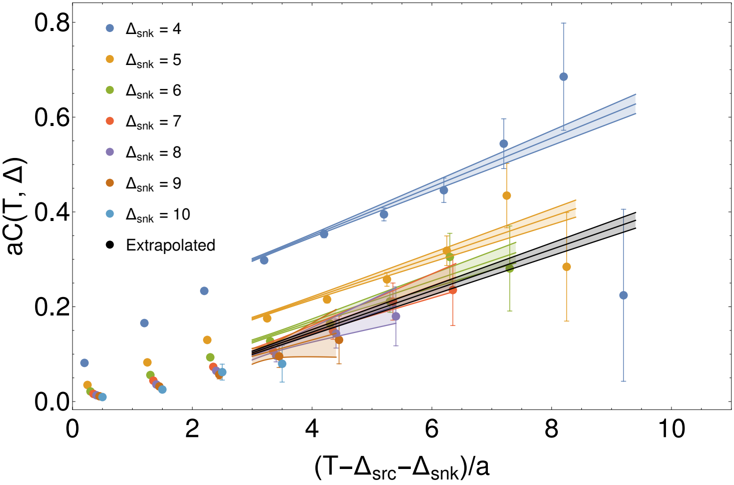
$$= \sum_{k=0}^{\infty} \sum_{\mathbf{x},\mathbf{y}} \int \frac{d^3q}{(2\pi)^3} \frac{\Gamma^{\text{lept.}}_{\alpha\beta} \langle pp|j_\alpha(\mathbf{x})|k\rangle\langle k|j_\beta(\mathbf{y})|nn\rangle}{2E_k|\mathbf{q}|\left(|\mathbf{q}| + E_k - m_{nn}\right)} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}$$

$$\times \frac{1}{2m_{nn}} \left((t_z - 2\Delta) + \frac{e^{-(|\mathbf{q}|+E_k-m_{nn})(t_z-2\Delta)} - 1}{|\mathbf{q}| + E_k - m_{nn}} \right)$$

- Allows extraction of matrix element

$$M^{0\nu} = \sum_{k=0}^{\infty} \sum_{\mathbf{x},\mathbf{y}} \int \frac{d^3q}{(2\pi)^3} \frac{\Gamma^{\text{lept.}}_{\alpha\beta} \langle pp|j_\alpha(\mathbf{x})|k\rangle\langle k|j_\beta(\mathbf{y})|nn\rangle}{2E_k|\mathbf{q}|\left(|\mathbf{q}| + E_k - m_{nn}\right)} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}$$

Preliminary Extraction of $M^{0\nu}$



Sum over insertion times and then take slope in $t_z \to \infty$ limit

Preliminary Fit Results for $M^{0\nu}$

- $nn \to pp$: $0.386(26)$ GeV^2
- $\Sigma^- \to \Sigma^+$: $0.0162(3)$ GeV^2
- As before, many systematics missing from preliminary results
- Light intermediate states (Λ/Σ^0 or d) must be analyzed separately

Future Plans

- Cannot simulate large nuclei on lattice
  - Costs of simulation scale factorially in number of quarks
  - Very challenging to get beyond ^{4}He on lattice
- Goal: Fit EFT parameters to lattice results
- Parameters can be used as input to current nuclear models: R. Weiss et al. (2112.08146)
- Direct *ab initio* nuclear calculations computationally tractable at least up to ^{82}Se: A. Belley et al. (2008.06588)
  - Require some approximations but systematically controllable

Lattice QCD Details

- Dirac operator $\not{D} + m$ implemented as matrix coupling adjacent vertices of lattice
- Quark propagators computed by inverting Dirac matrix (expensive!)
- Hadronic states created and annihilated by interpolating operators, e.g.

$$\mathcal{O}_n = \varepsilon^{abc} P_+ d_a \left(d_b^T P_+ C \gamma_5 u_c \right)$$

- Correlation functions built from interpolating operators and current insertions

$$C_2(t_z) = \int d^3\mathbf{z} \, \langle \mathcal{O}_{nn}(z) \mathcal{O}^\dagger_{nn}(0) \rangle$$

$$C_4(t_z, t_x, t_y) = \int d^3\mathbf{z} \, d^3\mathbf{x} \, d^3\mathbf{y} \, \Gamma^{\text{lept.}}_{\alpha\beta} \langle \mathcal{O}_{pp}(z) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(0) \rangle S_\nu(x, y)$$

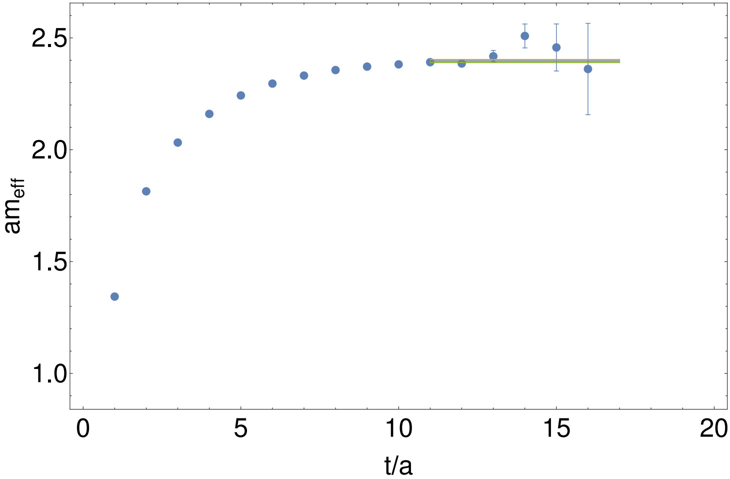
Interpretation of Correlation Functions

- 2-point correlation functions related to energies

$$
\begin{aligned}
\langle \mathcal{O}_H(t)\mathcal{O}_H^\dagger(0)\rangle &= \sum_n \frac{1}{2E_n}\langle 0|\mathcal{O}_H(t)|n\rangle\langle n|\mathcal{O}_H^\dagger(0)|0\rangle \\
&= \sum_n \frac{1}{2E_n}\langle 0|\mathcal{O}_H(0)|n\rangle e^{-E_n t}\langle n|\mathcal{O}_H^\dagger(0)|0\rangle \\
&= \sum_n \frac{1}{2E_n} e^{-E_n t}|\langle 0|\mathcal{O}_H|n\rangle|^2 \\
&\to \frac{1}{2E_0} e^{-E_0 t}|\langle 0|\mathcal{O}_H|H\rangle|^2
\end{aligned}
$$

- In Euclidean time, correlation functions exponentially decay instead of oscillating
- At small time, signal has not just ground state H but tower of unwanted excited states with energies $> E_0$

Effective Mass



$$m_{\text{eff}}(t) \equiv \log \frac{C(t)}{C(t+1)} \to E_0$$

$R_{\mathcal{O}_2'}(t,\tau)$