Review of Lattice QCD calculations for Rare Kaon decays

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Motivation

- $K \to \pi \ell \bar{\ell}$, $K \to \pi \nu \bar{\nu}$: Flavour-changing neutral currents
  $\to$ Highly suppressed; sensitive to new physics
- Rare K decays feature long-distance contributions of the form

$$\int d^4x \langle \pi | \mathcal{J}(x) \mathcal{H}_W(0) | K \rangle$$

  $\to$ Long-distance here means distances $\gtrsim 1/m_c$
- Significant or dominant effects for several channels
  $\to$ Non-perturbatively calculable

Background

- Theoretical framework for lattice computations of $K \to \pi \ell \bar{\ell}$ and $K \to \pi \nu \bar{\nu}$ first published by Isidori, Martinelli, and Turchetti (2006)[1]
- Extended for full evaluation by RBC-UKQCD collaborations
  $\to K^+ \to \pi^+ \ell \bar{\ell}$ (2015)[2]
  $\to K^+ \to \pi^+ \nu \bar{\nu}$ (2016)[3]
- Proof-of-concept: RBC-UKQCD Exploratory calculations
  $\to K^+ \to \pi^+ \ell \bar{\ell}$ (2016)[4]
  $\to K^+ \to \pi^+ \nu \bar{\nu}$ (2017)[5] [6], (2019)[7]
- Production runs

[1] Phys. Lett. B 633, 75 (2006) [arXiv:hep-lat/0506026]
[2] Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]
[3] Phys.Rev. D 93 (2016) 114517 [arXiv:1605:04442]
[4] Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]
[5] Phys.Rev.Lett. 118 (2017) 252001 [arXiv:1701.02858]
[6] Phys.Rev. D 98 (2018) 074509 [arXiv:1806.11520]
[7] Phys. Rev. D 100 (2019) 114506 [arXiv:1910.10644]

$$K \to \pi \ell \bar{\ell}$$

$K \to \pi \ell \bar{\ell}$

- Long-distance amplitude:

$$\mathcal{A}_\mu(q^2) = \int d^4x \langle \pi(\mathrm{p}) | T\left[J_\mu(x) H_W(0)\right] | K(\mathrm{k}) \rangle$$

- Re-expressed using EM gauge invariance[1] [2]:

$$\mathcal{A}_\mu(q^2) = -i \frac{G_F}{(4\pi)^2} \left[q^2 (k+p)_\mu - \left(M_K^2 - M_\pi^2\right) q_\mu \right] \underbrace{V(z)}_{non-pert.}$$

$$V(z) = a + bz + V^{\pi\pi}(z) \qquad z = q^2/M_K^2$$

[1] JHEP 08 (1998) 004 [arXiv:hep-ph/9808289]

[2] Rev. Mod. Phys. 84, 399 (2012) [arXiv:1107.6001]

$K \to \pi \ell \bar{\ell}$

- Minkowski and Euclidean spectral representations:

$$\mathcal{A}_\mu(\mathbf{k},\mathbf{p}) = +i\int_0^\infty dE \frac{\rho(E)}{2E}\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E+i\epsilon}$$
$$-i\int_0^\infty dE \frac{\rho_S(E)}{2E}\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})+i\epsilon}$$
$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = -\int_0^\infty dE \frac{\rho(E)}{2E}\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{[E_K(\mathbf{k})-E]T_a}\right)$$
$$+\int_0^\infty dE \frac{\rho_S(E)}{2E}\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- T_a, T_b come from integration of normalised 4pt function:

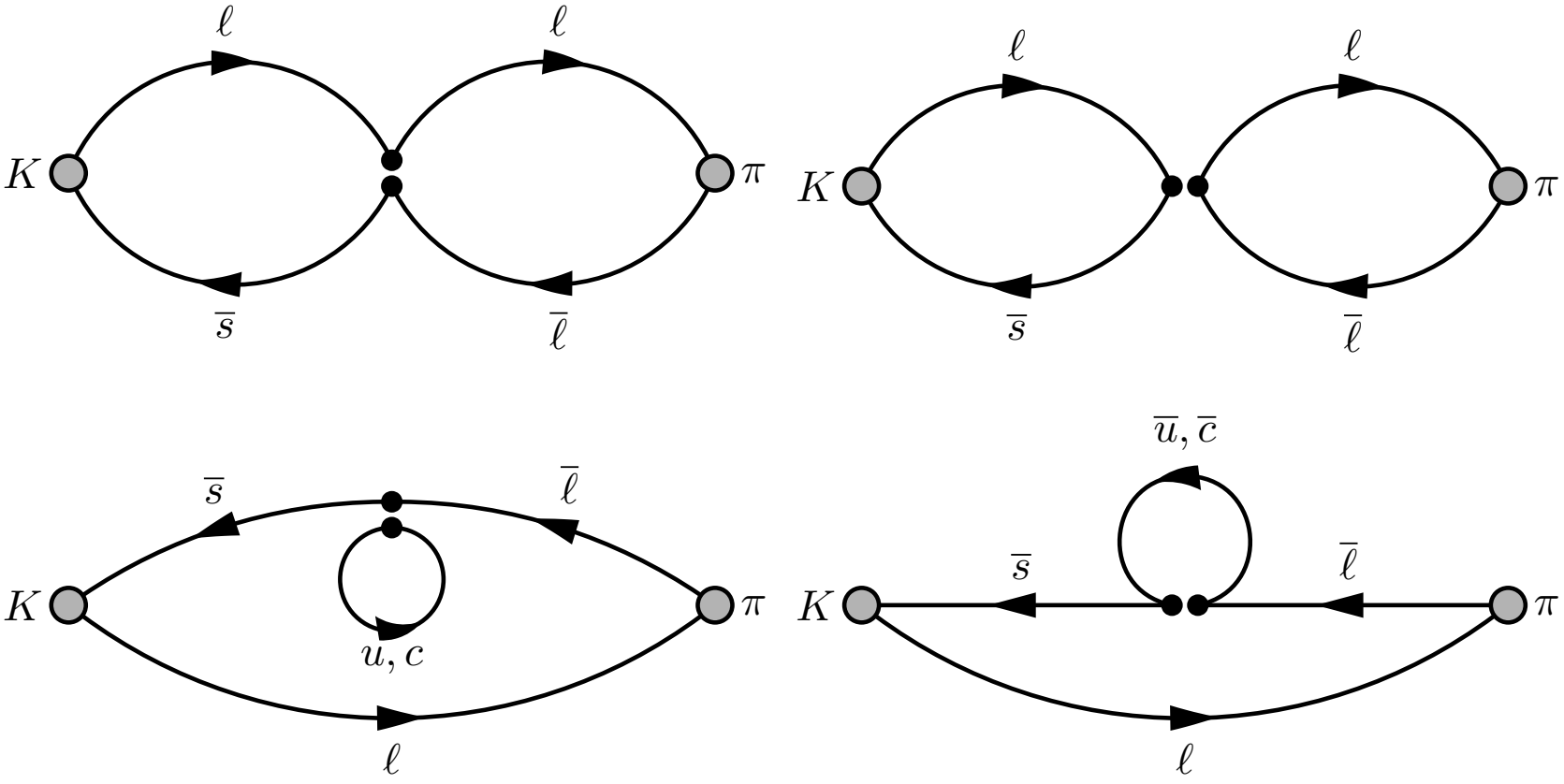
$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = e^{-[E_\pi(\mathbf{p})-E_K(\mathbf{k})]t_J}\int_{t_J-T_a}^{t_J+T_b} dt_H \widetilde{\Gamma}_{\mathrm{4pt}}$$

$K \to \pi \ell \bar{\ell}$

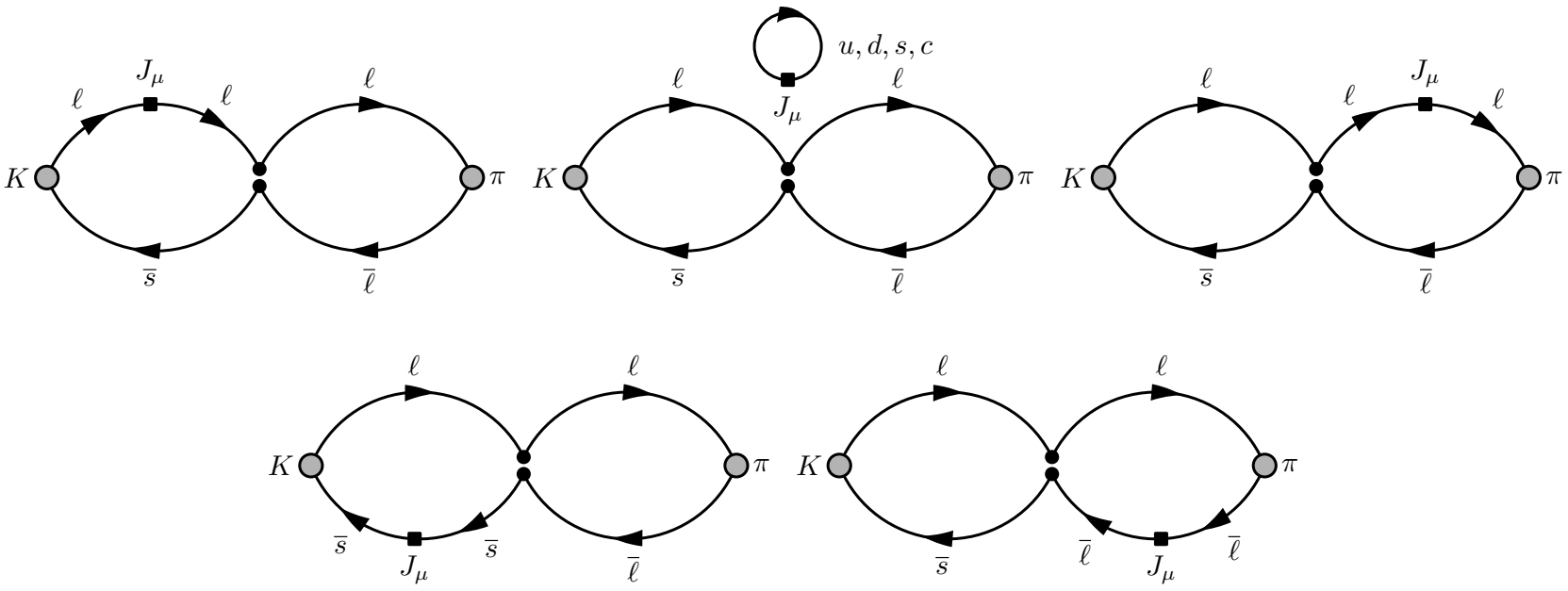
$$I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p}) = -\int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{p})|J_\mu|E, \mathbf{k}\rangle\langle E, \mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E} \left(1 - e^{[E_K(\mathbf{k})-E]T_a}\right)$$
$$+\int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi(\mathbf{p})|H_W|E, \mathbf{p}\rangle\langle E, \mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})} \left(1 - e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- Amplitude corresponds to limit $T_a, T_b \to \infty$
- First line: π, $\pi\pi$, and $\pi\pi\pi$ on-shell intermediate states enter the spectral density (for physical masses)
  $\to E_K > E_\pi$, $E_{\pi\pi}$, $E_{\pi\pi\pi}$: Causes the T_a exponential to diverge!
- Lattice - can't take $T_a, T_b \to \infty$
  $\to$ Must remove exponentially growing terms in T_a due to intermediate states
  $\to$ T_a, T_b must be large enough for exponentials to sufficiently decay

$K \to \pi \ell \bar{\ell}$



$K \to \pi \ell \bar{\ell}$



$K \to \pi \ell \bar{\ell}$

Theoretical proposal:

- Prospects for a lattice computation of rare kaon decay amplitudes: I, $K \to \pi \ell^+ \ell^-$ decays
  RBC-UKQCD (2015)
  Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]

Existing results:

- First exploratory calculation of the long distance contributions to the rare kaon decay $K \to \pi \ell^+ \ell^-$
  RBC-UKQCD (2016)
  Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]
- Simulating rare kaon decays $K \to \pi \ell^+ \ell^-$ using domain wall lattice QCD with physical light quark masses
  RBC-UKQCD (2022)
  [arXiv:2202.08795]

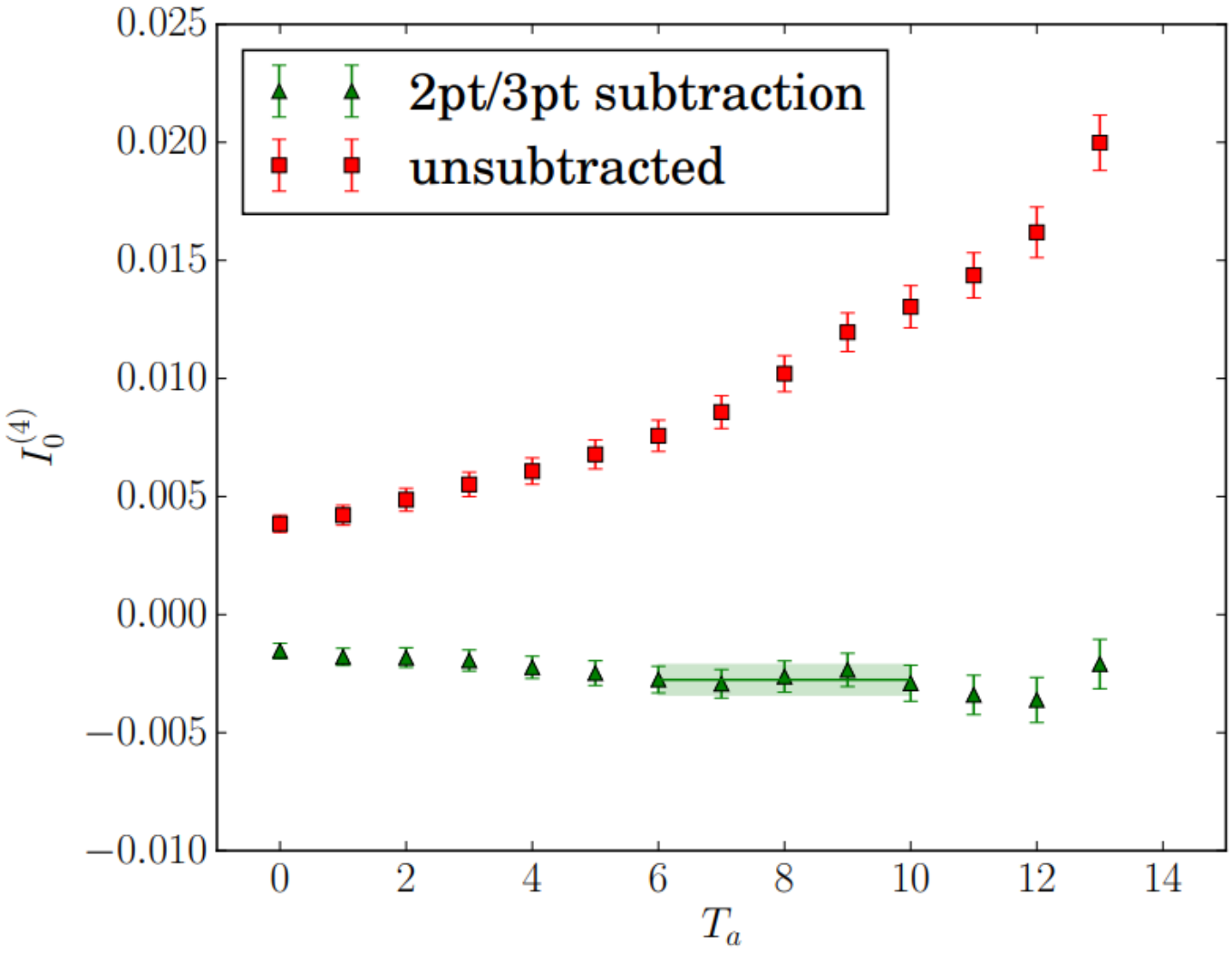
$K \to \pi \ell \bar{\ell}$

RBC-UKQCD Exploratory study (2016)[1]:

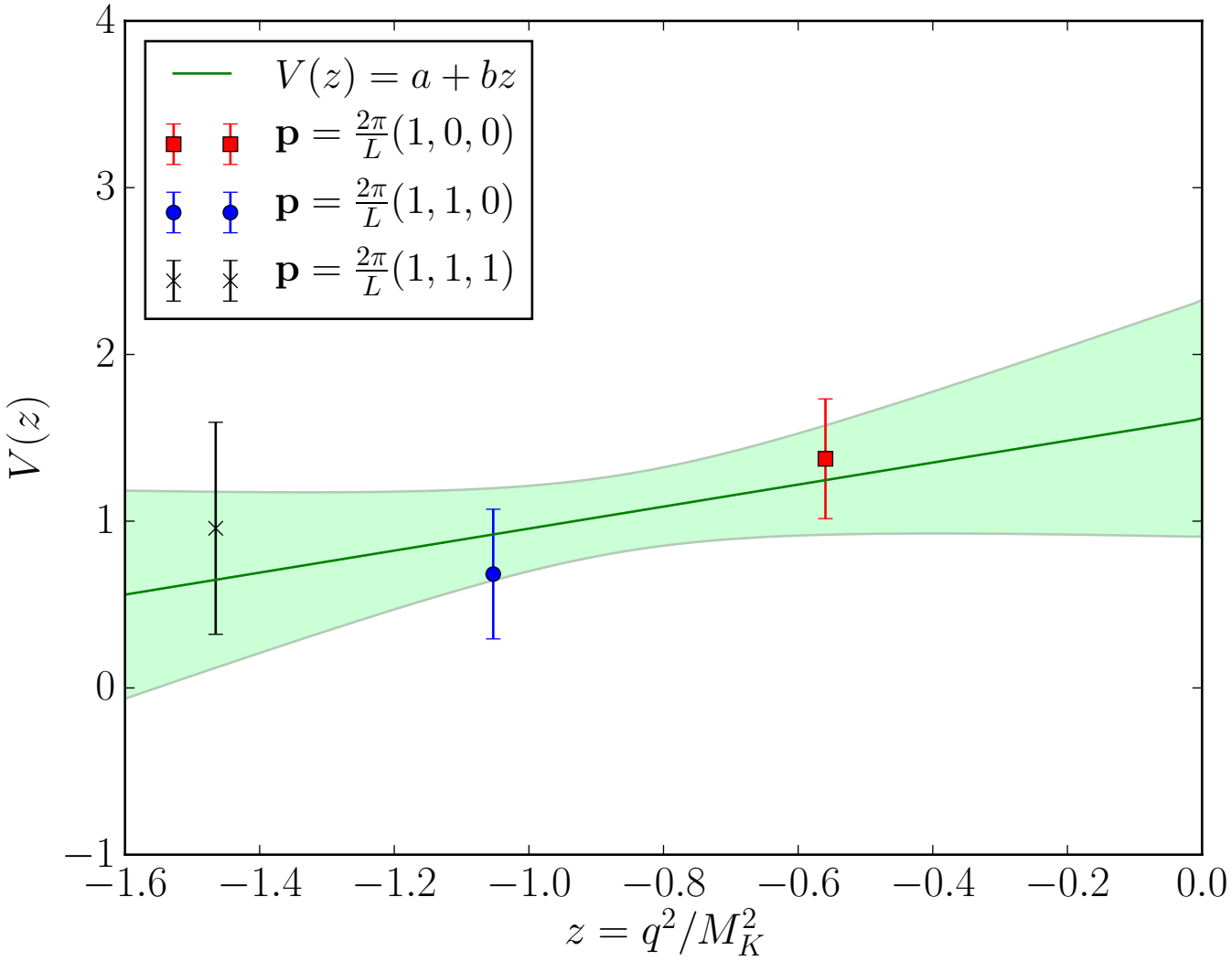
- $2+1$ flavour, $L^3 \times T = 24^3 \times 64$, $a^{-1} = 1.78$ GeV
- $\sim$430 MeV pion, $\sim$625 MeV Kaon
  $\to$ Only single-π intermediate state enters spectral density
- Shamir Domain Wall Fermions: good chiral symmetry
  $\to$ simplified renormalisation

[1] Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]

$K \to \pi \ell \bar{\ell}$



$K \to \pi \ell \bar{\ell}$



$K \to \pi \ell \bar{\ell}$

RBC-UKQCD physical-point calculation (2022)[1]:

- $2+1$ flavour, $L^3 \times T = 48^3 \times 96$, $a^{-1} = 1.73$ GeV
- Physical Pion and Kaon masses
  $\to$ Expensive calculation!
  $\to$ Energy budget allows π, $\pi\pi$, $\pi\pi\pi$ intermediate states
- zMöbius Domain Wall Fermions: **Reduced computational expense** in addition to simplified renormalisation
  $\to$ Requires an All-Modes-Averaging (AMA) style correction to correlators
  $\to$ Allows statistics to be accumulated on a cheaper estimator and then be shifted to the full Möbius action

[1] [arXiv:2202.08795]

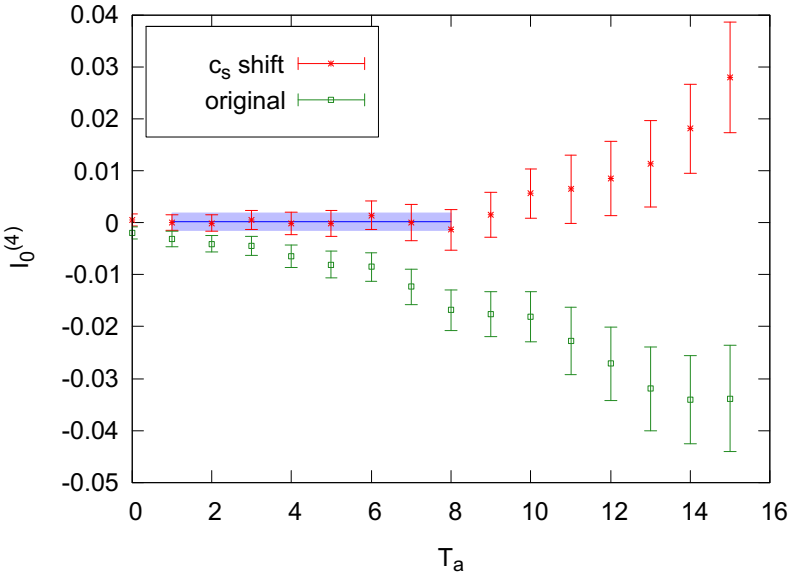
$K \to \pi \ell \bar{\ell}$

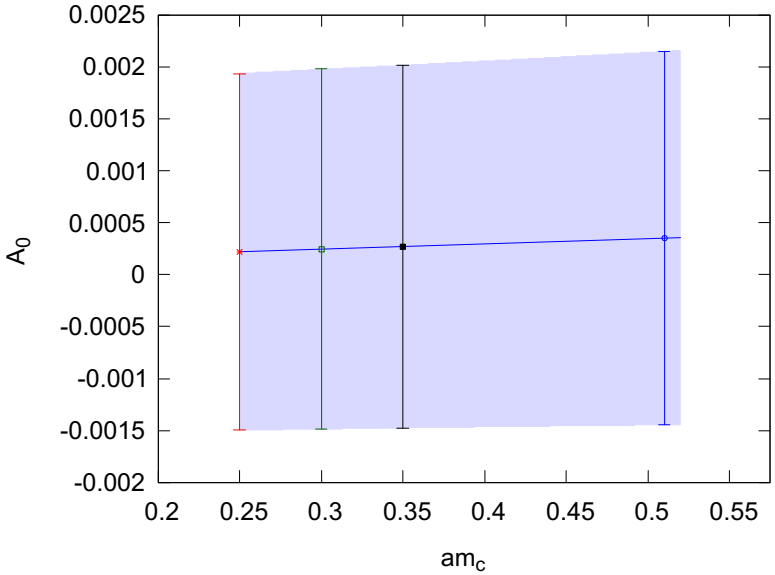
Intermediate states:

- π IS: Significant contribution, can be removed *via* techniques demonstrated in exploratory study
- $\pi\pi$ IS: Introduced by lattice artefacts, at practical values of T_a expected to be percent-level effect[1]
- $\pi\pi\pi$ IS: Compare decay widths of $K_S \to \pi\pi$ to $K_{S,+} \to \pi\pi\pi$: factor $\sim \mathcal{O}(1/500)$ further suppressed, $\pi\pi\pi$ completely negligible for forseeable future with these values of T_a[1]

[1] Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]

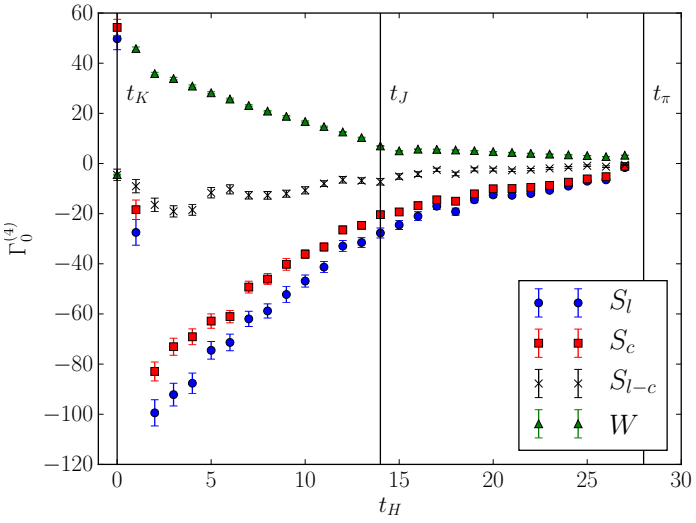
$K \to \pi \ell \bar{\ell}$



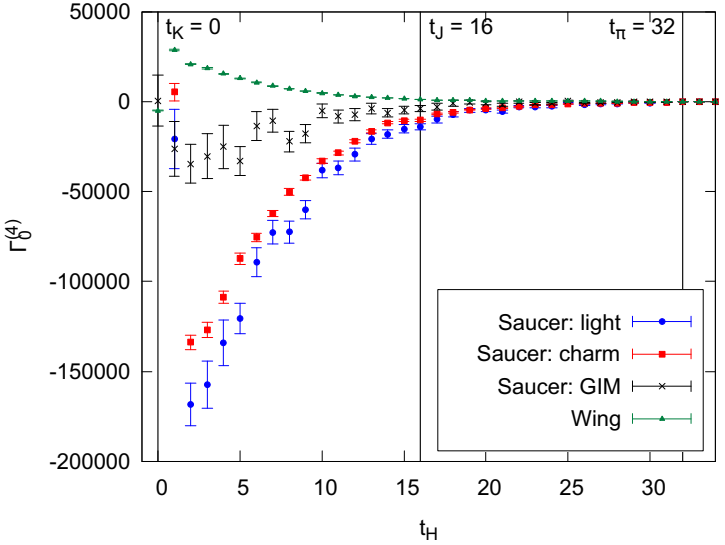


- $A_0 = 0.00035(180)$
- $V(z) = -0.87(4.44)$
- $z = 0.013(2)$
- $V(z) \approx V(0) = a^+$ for our choice of kinematics
- Form factor unfortunately unresolved, but let's investigate why...

$K \to \pi \ell \bar{\ell}$



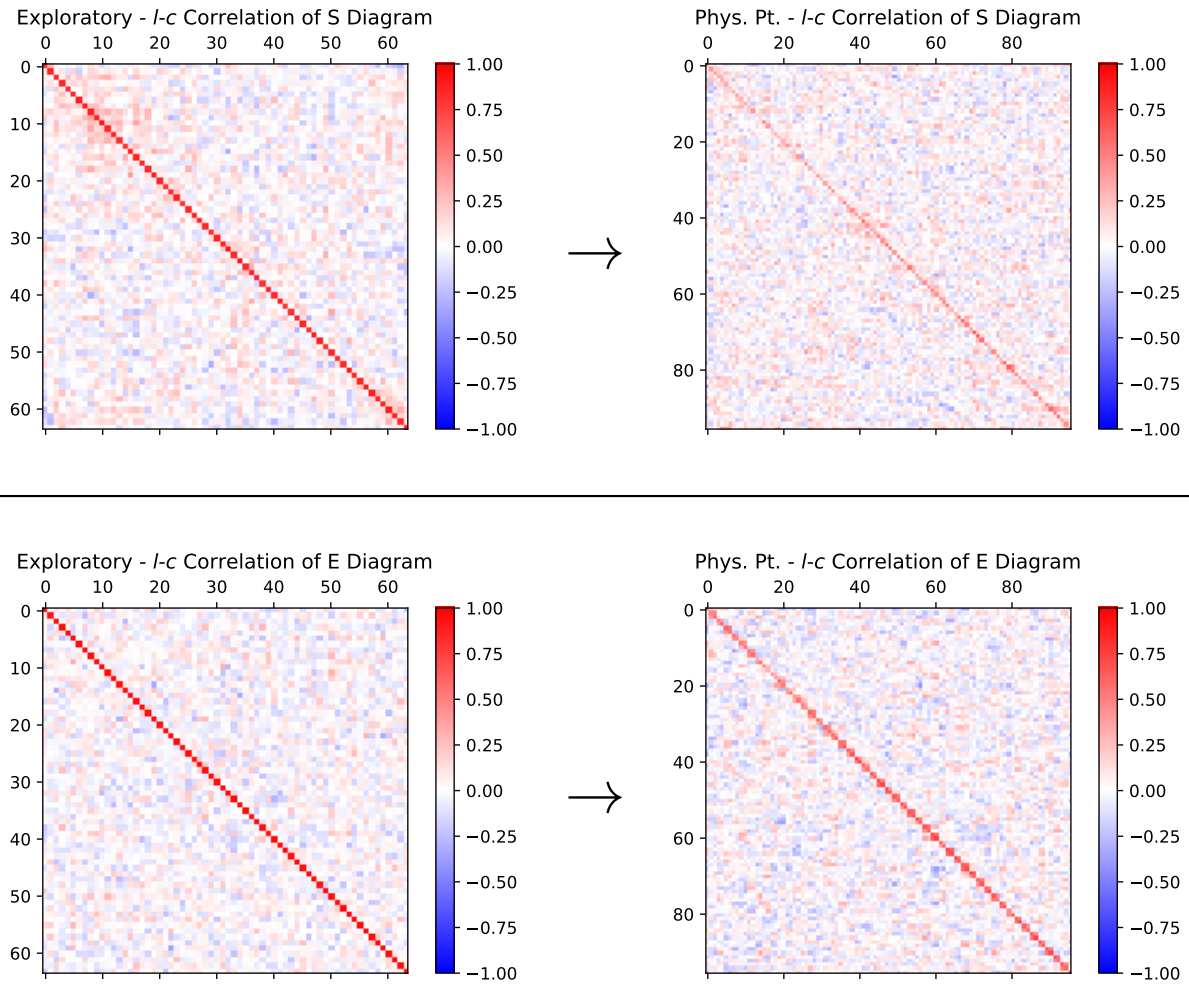
Exploratory Study



Physical-Point

- Plots show the GIM subtraction for the saucer diagram constructed from the l and c quark correlators
- GIM subtraction does not lead to a cancellation of errors with physical light masses

$K \to \pi \ell \bar{\ell}$



- Much reduced correlation between l and c loop quark diagrams at physical point due to large mass difference

$K \to \pi \ell \bar{\ell}$

Statistical error cannot be overcome by square-root scaling of additional statistics alone in near future.
$\to$ Potential ways forward:

- Improvement of estimators for up- and charm-loop propagators
  $\to$ Similar to issues faced in disconnected diagrams
- Forgo explicit charm contribution to GIM loop and handle *via* different renormalisation procedure
  $\to$ Look to $K \to \pi \nu \bar{\nu}$ for lessons learned

$\to$ Combination of algorithmic improvements and next-generation computers makes a competitive lattice result appear feasible in the coming years.

$$K \to \pi \nu \bar{\nu}$$

$K \to \pi\nu\bar{\nu}$

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = \kappa \left(\left[\frac{\mathfrak{Im}(\lambda_t)}{\lambda^5} X_t \left(\frac{m_t^2}{M_W^2} \right) \right]^2 + \left[\frac{\mathfrak{Re}(\lambda_c)}{\lambda} P_c + \frac{\mathfrak{Re}(\lambda_t)}{\lambda^5} X_t \left(\frac{m_t^2}{M_W^2} \right) \right]^2 \right)$$

- $\lambda = |V_{us}|$
- $\lambda_q = V_{qs}^* V_{qd}$
- Top contribution: $\sim$68%
- Charm contribution: $\sim$32%
  $\to$ Short-distance: $\sim$29%
  $\to$ Long-distance: $\sim$3% $\Rightarrow$ Non-negligible contribution

$K \to \pi\nu\bar{\nu}$

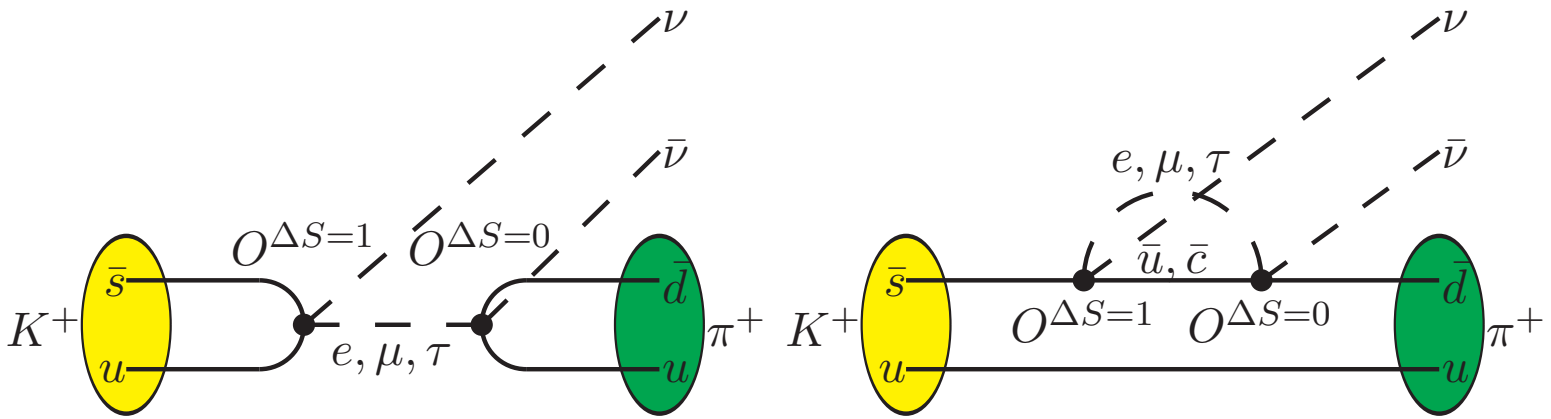
- SM[1]: $Br(K^+ \to \pi^+\nu\bar{\nu}) = (9.11 \pm 0.72) \times 10^{-11}$
- NA62[2]: $Br(K^+ \to \pi^+\nu\bar{\nu}) = \left(10.6^{+4.0}_{-3.4}|_{\text{stat.}} \pm 0.9_{\text{syst.}}\right) \times 10^{-11}$
  $\to$ Based on Run 1 (2016-2018) results
  $\to$ Run 2 (2021-2024) will be of great interest
- Long-distance contributions will comprise a larger portion of SM error as CKM uncertainties continue to fall

[1] JHEP 11 (2015) 033 [arXiv:1503.02693]
[2] JHEP 06 (2021) 093 [arXiv:2103.15389]

$K \to \pi\nu\bar{\nu}$

Diagrams

- Similar to $K \to \pi\ell\bar{\ell}$, but with electromagnetic current $\to$ neutral weak current
- Additionally: W-W box diagrams:



$K \to \pi\nu\bar{\nu}$

Theoretical proposal:

- Prospects for a lattice computation of rare kaon decay amplitudes: II, $K \to \pi\nu\bar{\nu}$ decays
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  Phys.Rev. D 93 (2016) 114517 [arXiv:1605:04442]

Existing results:

- Exploratory lattice QCD study of the rare kaon decay $K^+ \to \pi^+\nu\bar{\nu}$
  RBC-UKQCD (2017)
  Phys.Rev.Lett. 118 (2017) 252001 [arXiv:1701.02858]
- $K^+ \to \pi^+\nu\bar{\nu}$ decay amplitude from lattice QCD
  RBC-UKQCD (2018)
  Phys.Rev. D 98 (2018) 074509 [arXiv:1806.11520]
- Lattice QCD study of the rare kaon decay $K^+ \to \pi^+\nu\bar{\nu}$ at a near-physical pion mass
  RBC-UKQCD (2019)
  Phys. Rev. D 100 (2019) 114506 [arXiv:1910.10644]

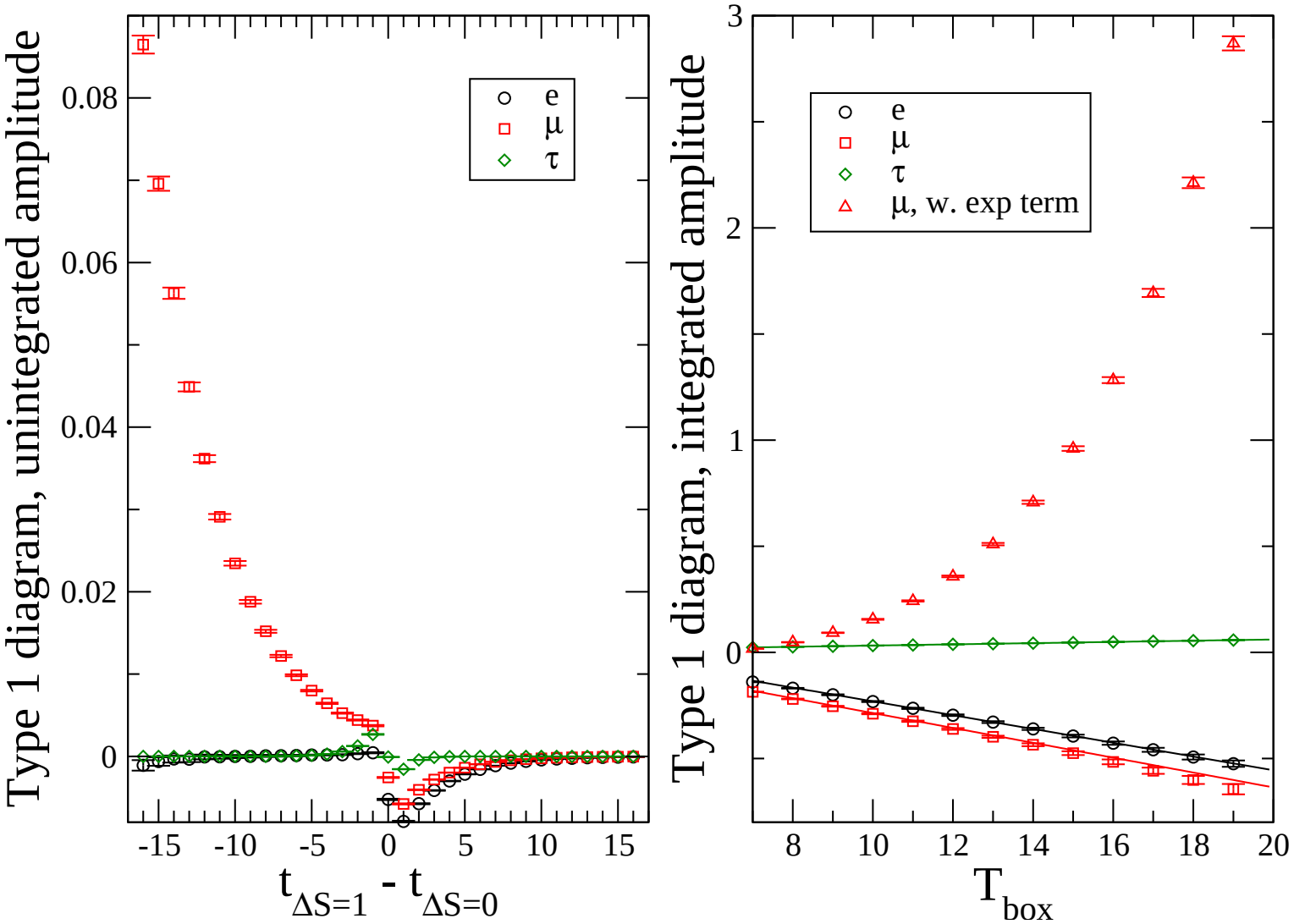
$K \to \pi\nu\bar{\nu}$

RBC-UKQCD Exploratory study (2017)[1] [2], sought to overcome three issues before a full calculation:

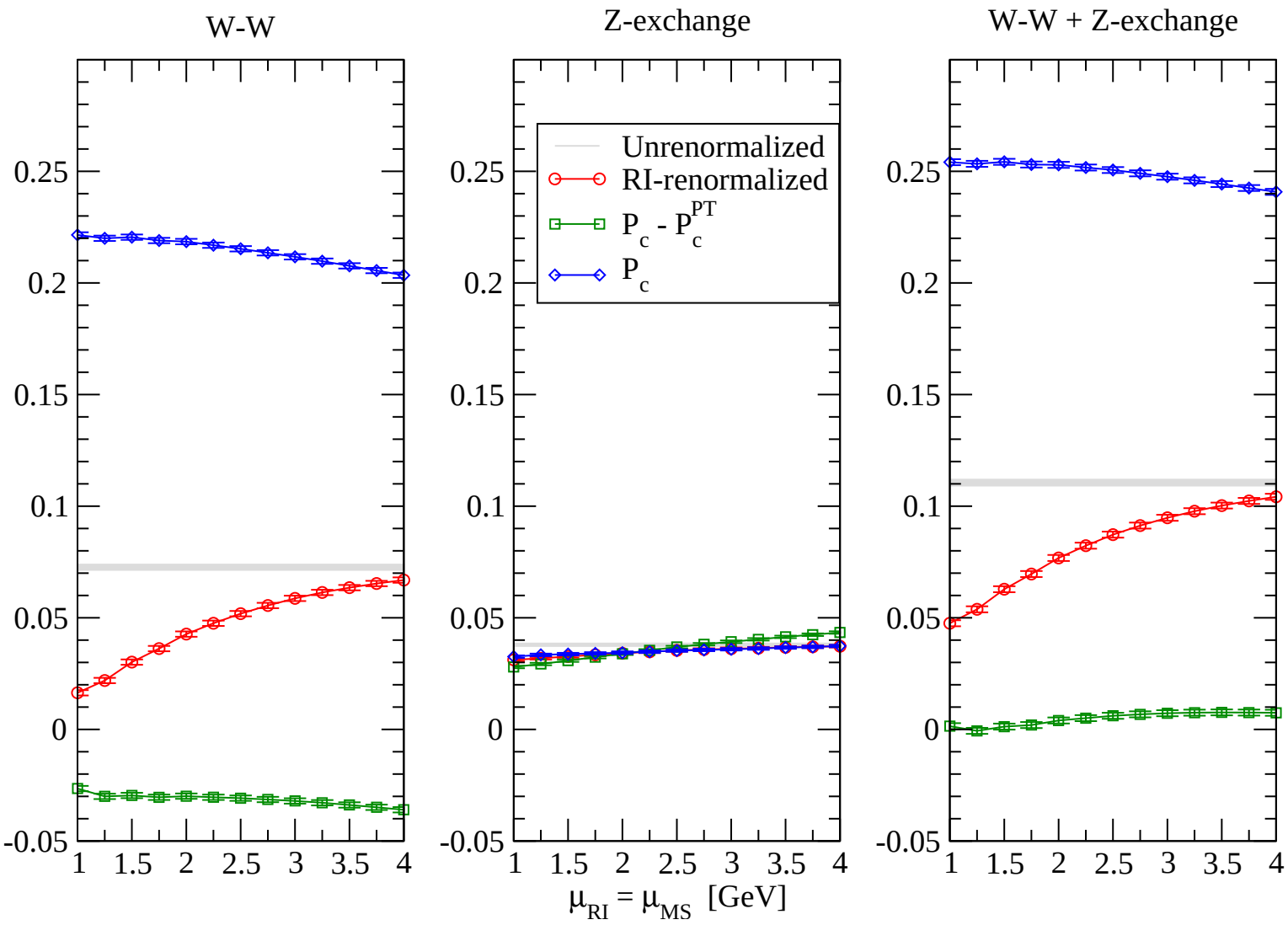
- Contamination from intermediate $|\ell^+\nu\rangle$, $|\pi^0\ell^+\nu\rangle$, $|(\pi^+\pi^0)^{I=2}\rangle$ states
- New UV divergences as neutral current and H_W approach, unlike $K^+ \to \pi^+\ell\bar{\ell}$
  $\to$ Additional counter-terms in renormalisation
- Finite-volume effects introduced by $\pi\pi$ on-shell intermediate states

[1] Phys.Rev.Lett. 118 (2017) 252001 [arXiv:1701.02858]
[2] Phys.Rev. D 98 (2018) 074509 [arXiv:1806.11520]

$K \to \pi\nu\bar{\nu}$



$K \to \pi\nu\bar{\nu}$



$K \to \pi\nu\bar{\nu}$

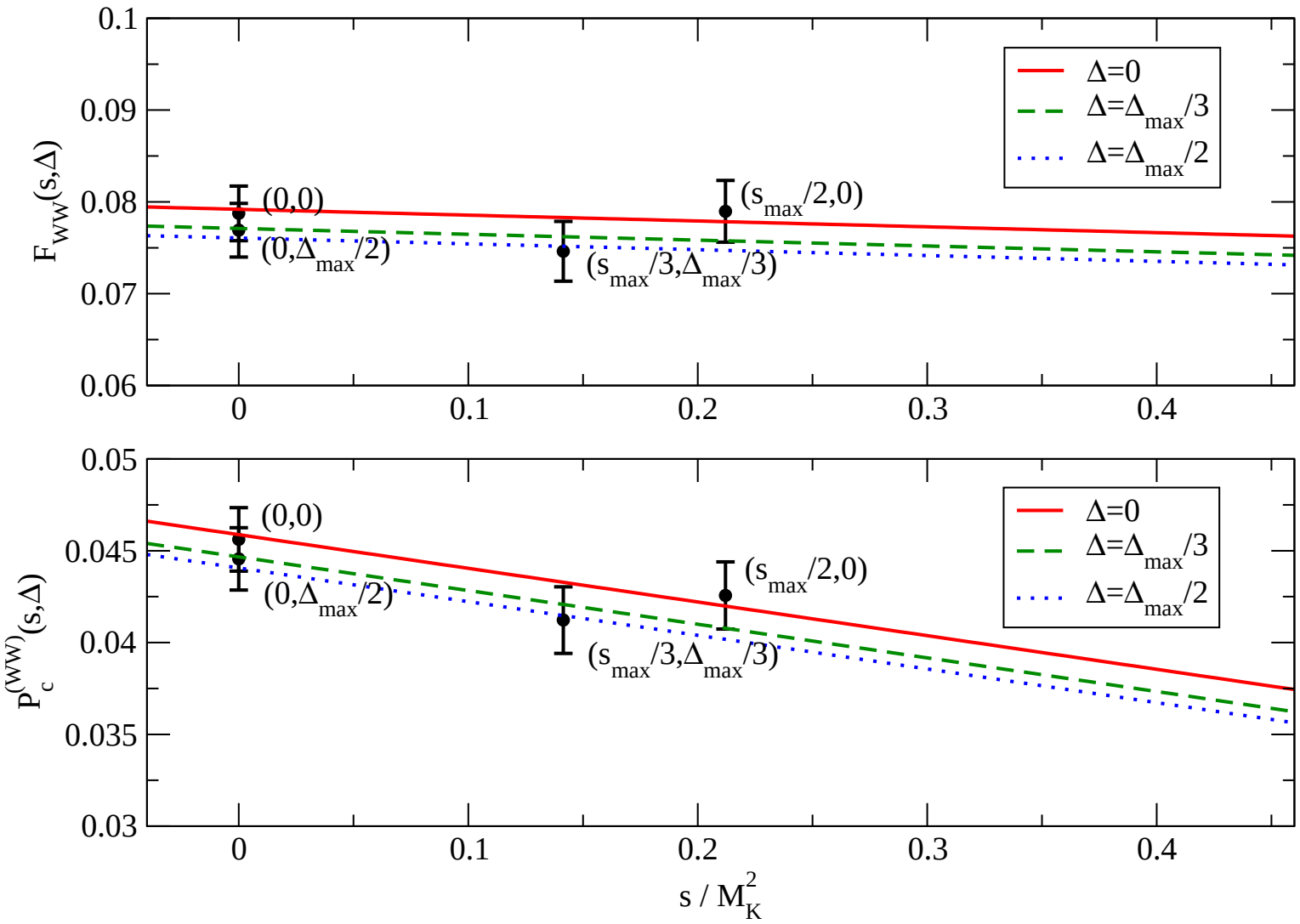
RBC-UKQCD Near-physical-point calculation (2019)[1], main findings:

- Mild momentum-dependence of the amplitude
  $\to$ Lattice QCD is expensive to simulate at different momenta
  $\to$ Implies that a good estimate of long-distance contribution could be obtained at a single kinematic point
- $\pi\pi$ Intermediate state: $< 1\%$ contribution
- Finite-volume effects from $\pi\pi$ intermediate states negligible

Main systematics under control
$\to$ Groundwork now laid out for a full physical-point calculation.

[1] Phys. Rev. D 100 (2019) 114506 [arXiv:1910.10644]

$K \to \pi\nu\bar{\nu}$



Conclusions

Summary

- $K \to \pi \ell \bar{\ell}$
  $\to$ Viability of calculation demonstrated at physical point
  $\to$ Next steps identified: gain control of GIM loop stochastic estimators or sidestep the explicit simulation thereof
- $K \to \pi \nu \bar{\nu}$
  $\to$ Exploratory studies providing important insights for a full physical-point calculation

Outlook

- $K \to \pi \ell \bar{\ell}$: Challenging calculation with physical kinematics achieved. Competitive errors in the next few years?
- $K \to \pi \nu \bar{\nu}$: Next target: calculation with full physical kinematics
- Related efforts: Rare $\Sigma^+ \to p \ell^+ \ell^-$