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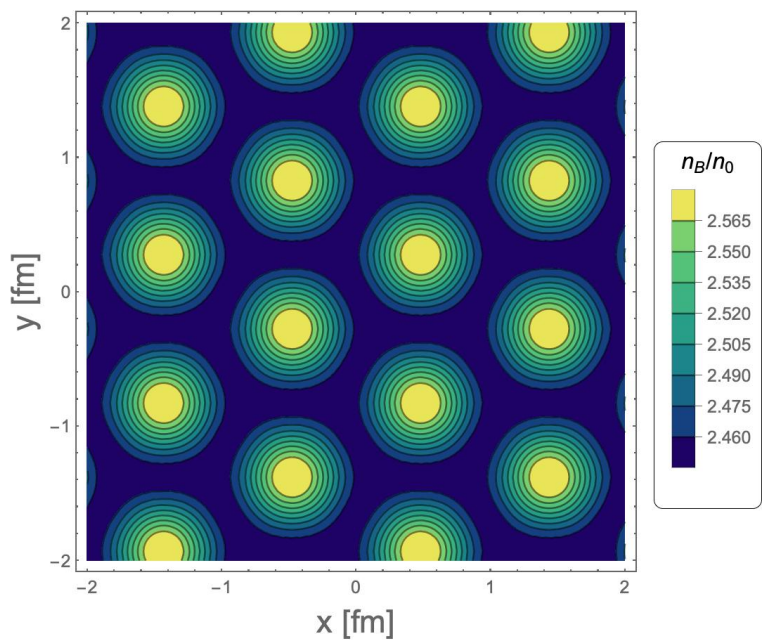
Chiral anomaly induces superconducting baryon crystal

G. W. Evans and A. Schmitt, arXiv:2206.01227 [hep-th]

- QCD phase structure in the $\mu - B$ plane
- chiral soliton lattice and its instability
- charged pion condensation (type-II superconductor)



Chiral perturbation theory with chiral anomaly

- $N_f = 2$ chiral perturbation theory +electromagnetism+ chiral anomaly
  D.T. Son, M.A. Stephanov, PRD 77, 014021 (2008)
  T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}[\Sigma + \Sigma^\dagger] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(A_\mu^B - \frac{e}{2}A_\mu\right) j_B^\mu$$

with $\nabla^\mu \Sigma = \partial^\mu \Sigma - i[\mathcal{A}^\mu, \Sigma]$ and anomalous baryon current
J. Wess, B. Zumino, PLB 37, 95 (1971); E. Witten, NPB 223, 422 (1983)
J. Goldstone, F. Wilczek, PRL 47, 986 (1981)

$$j_B^\mu = -\frac{\epsilon^{\mu\nu\rho\lambda}}{24\pi^2}\mathrm{Tr}\left[(\Sigma\nabla_\nu\Sigma^\dagger)(\Sigma\nabla_\rho\Sigma^\dagger)(\Sigma\nabla_\lambda\Sigma^\dagger) + \frac{3ie}{4}F_{\nu\rho}\tau_3\left(\Sigma\nabla_\lambda\Sigma^\dagger + \nabla_\lambda\Sigma^\dagger\Sigma\right)\right]$$

- parametrize $\pi^0, \pi^\pm \to \alpha \in \mathbb{R}, \varphi \in \mathbb{C}$

- anomalous current receives contributions from magnetic field and vorticity

$$j_B^\mu = -\frac{\epsilon^{\mu\nu\rho\lambda}}{4\pi^2}\partial_\nu\alpha\left(\frac{e}{2}F_{\rho\lambda} + \frac{\partial_\rho j_\lambda}{e f_\pi^2}\right)$$

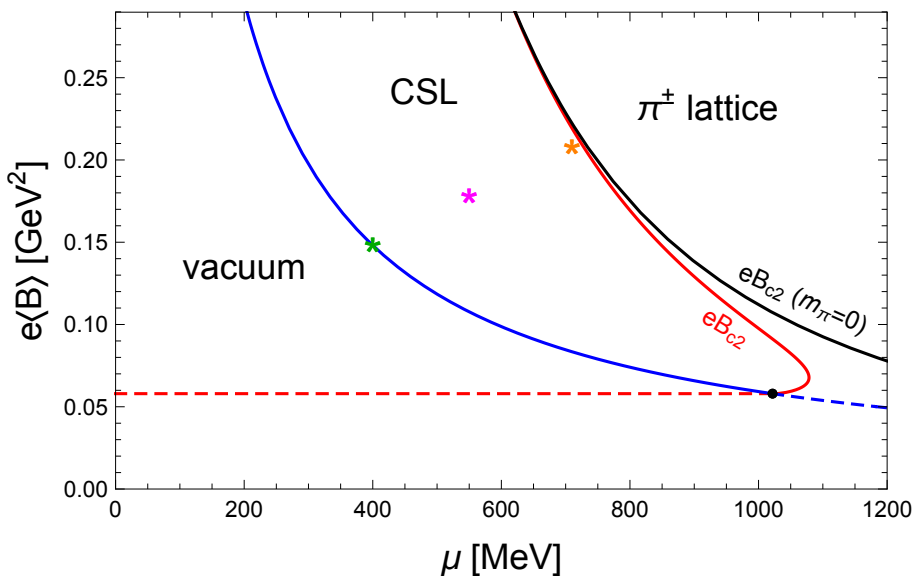
Chiral soliton lattice (CSL)

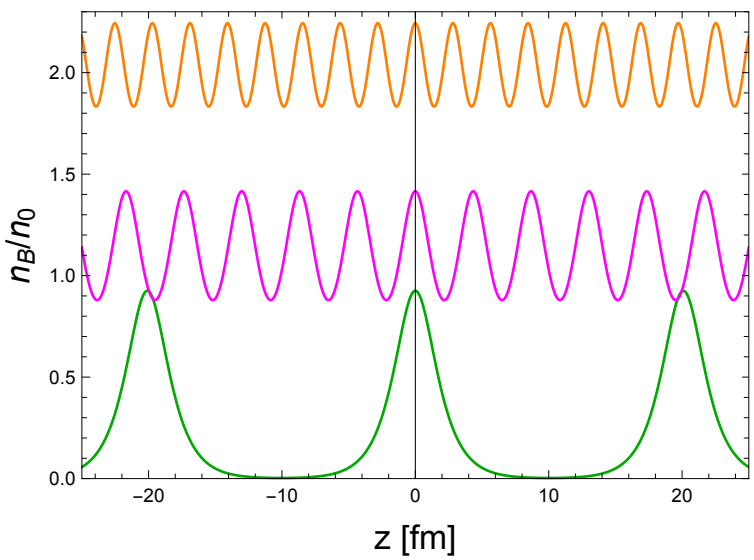
T. Brauner, N. Yamamoto, JHEP 04, 132 (2017)

- anomaly couples neutral pions to magnetic field via $\mu \nabla \pi^0 \cdot \vec{B}$
  D. T. Son and A. R. Zhitnitsky, PRD 70, 074018 (2004)

- "stack of domain walls" for

$$eB > \frac{16\pi m_\pi f_\pi^2}{\mu}$$





- $\nabla \pi^0 \propto n_B$ oscillates in the direction of $\vec{B}$
- chiral limit: $\nabla \pi^0 = \text{const}$

Instability towards pion condensation

- charged pion fluctuations in CSL (chiral limit)

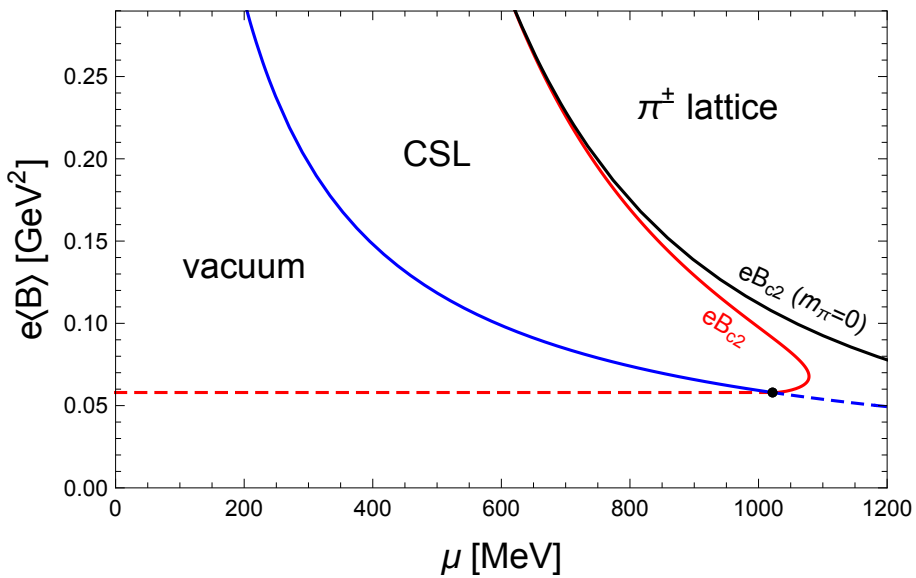
$$\omega_n = \sqrt{(2n+1)eB + m_*^2 + k_z^2} - \mu_*$$

with

$$m_*^2 = \mu_*^2 \left[1 - 4 \left(\frac{4\pi^2 f_\pi^2}{eB} \right)^2 \right], \qquad \mu_* = \frac{\mu}{2} \left(\frac{eB}{4\pi^2 f_\pi^2} \right)^2$$

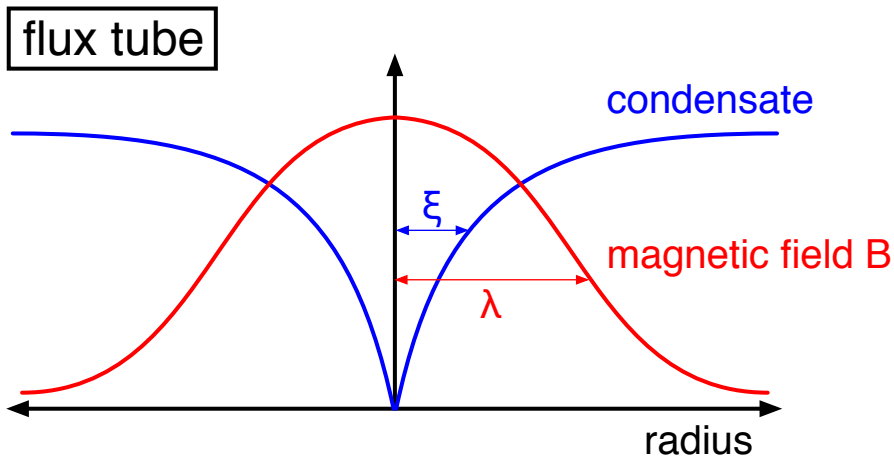
$\Rightarrow$ instability of $n = k_z = 0$ mode for

$$\boxed{B > B_{c2} = \frac{16\pi^4 f_\pi^4}{e\mu^2}}$$



- analogous to ordinary type-II superconductor, where m_*, μ_* = const and instability for $B < B_{c2}$

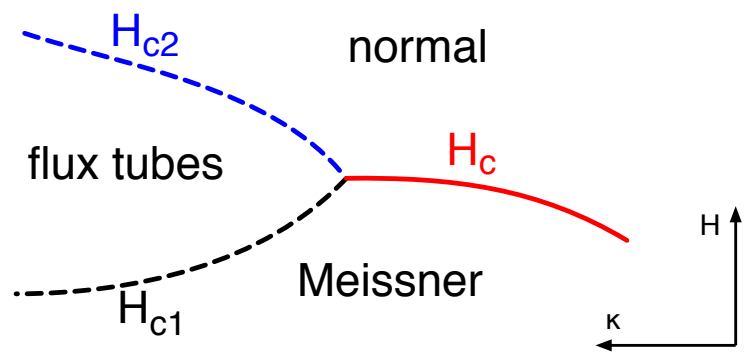
Recall type-II superconductivity



- Ginzburg-Landau parameter

$$\kappa = \frac{\lambda}{\xi}$$

- type-II superconductivity
  for $\kappa > 1/\sqrt{2}$: flux tube lattice
  for $H_{c1} < H < H_{c2}$



Expansion close to B_{c2} (page 1/2)

method based on A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)
W.H. Kleiner, L.M. Roth, S.H. Autler, Phys. Rev. 133, A1226 (1964)

- work in chiral limit $m_\pi = 0$ for simplicity
- expand α, φ, $\vec{A}$ about CSL solution for small $\sqrt{B - B_{c2}}$
- solve equations of motion to obtain charged pion condensate

$$\varphi_0(x,y) = \sum_{n=-\infty}^{\infty} C_n e^{inqy} e^{-\frac{(x-nq\xi^2)^2}{2\xi^2}}$$

- lattice structure encoded in

$$\beta \equiv \frac{\langle|\varphi_0|^4\rangle}{\langle|\varphi_0|^2\rangle^2} = \sqrt{\frac{a}{2}}\Big\{\left[\vartheta_3(0,e^{-2\pi a})\right]^2 + 2e^{-\frac{\pi a}{2}}\vartheta_3(0,e^{-2\pi a})\vartheta_3(i\pi a,e^{-2\pi a})$$

$$-e^{-\pi a}\left[\vartheta_3(i\pi a,e^{-2\pi a})\right]^2\Big\}$$

for $C_{\text{even } n} = \pm i C_{\text{odd } n}$ and $a \equiv q^2\xi^2/\pi$

Expansion close to B_{c2} (page 2/2)

- free energy density for fixed μ, $\langle B \rangle$

$$\mathcal{F} = \mathcal{F}_{\mathrm{CSL}} - \frac{1}{2}\frac{\left(\langle B \rangle - B_{c2}\right)^2}{\left(2\kappa^2 - 1\right)\beta + 1}$$

- effective Ginzburg-Landau parameter

$$\kappa \equiv \frac{\sqrt{eB_{c2}}}{\sqrt{2}\, e f_\pi} \gg 1 \qquad (\rightarrow \text{type II})$$

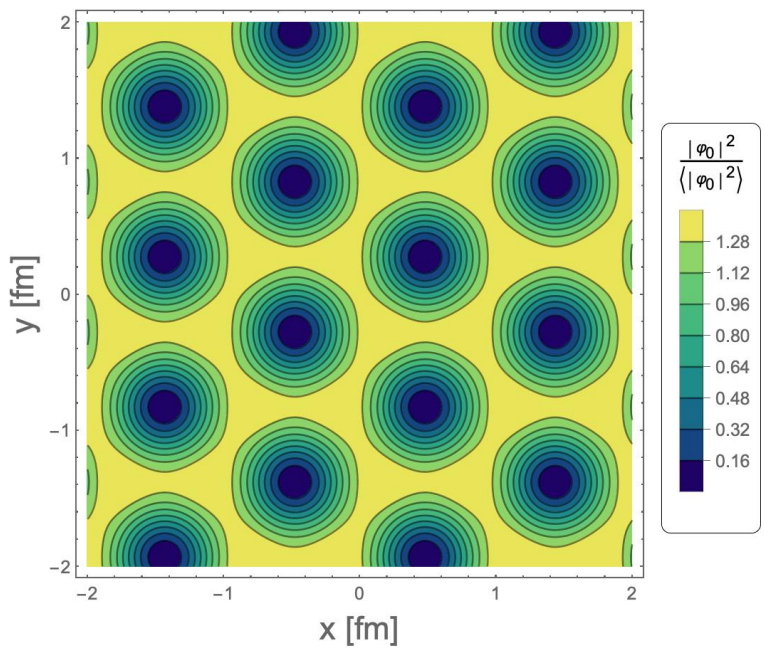
- minimization of $\beta \rightarrow$ hexagonal lattice

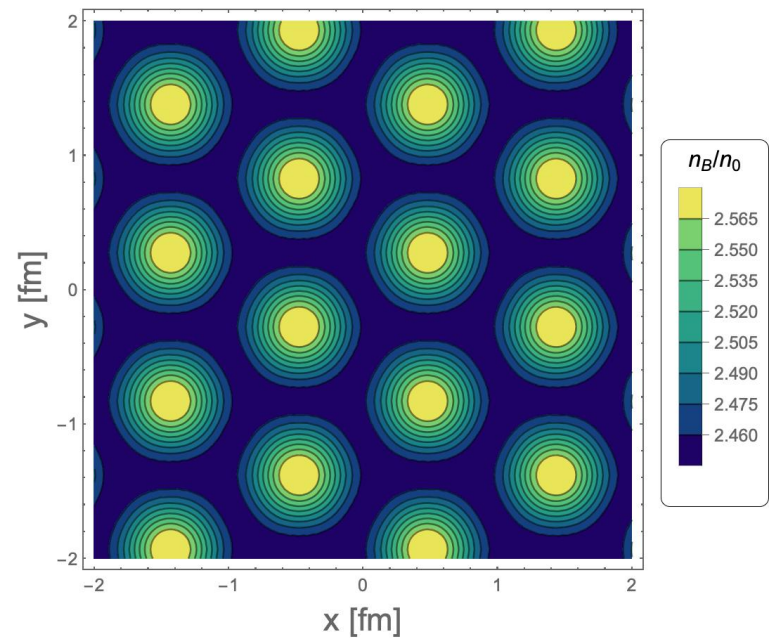
$\pi^\pm$ lattice favored over CSL for $\langle B \rangle > B_{c2}$

Superconducting baryon crystal

- baryon number enhanced in magnetic flux tubes
  – main contribution from vorticity $\nabla \times \vec{j} \simeq e\Delta|\varphi_0|^2\hat{\vec{e}}_z$

- lattice for $\mu = 700\,\mathrm{MeV}$, $\langle B \rangle = 1.01\, B_{c2}$





- 2D lattice in plane perpendicular to $\vec{B}$ ("baryon tubes")
  → 3D lattice expected for nonzero m_π

Summary

- QCD at nonzero B and μ exhibits a chiral soliton lattice, which becomes unstable at sufficiently large B and/or μ

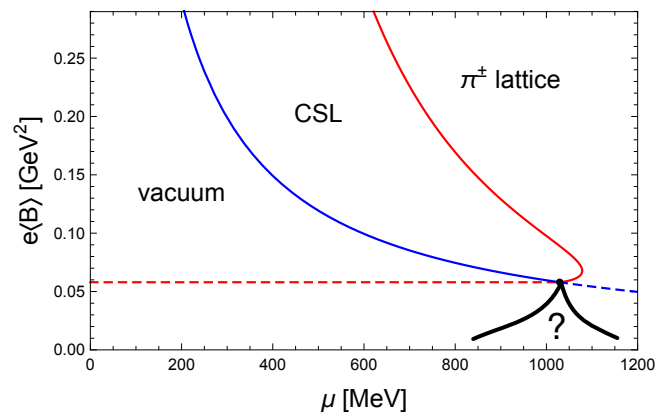
- we have constructed the resulting phase in the chiral limit: a superconducting charged pion lattice with magnetic and baryonic tubes

Outlook

- construct 3D structure with physical m_π
  G. Evans, A. Schmitt, work in progress

- 1st order transition $\pi^\pm$/vacuum?

- include neutrons and protons
  F. Preis, A. Rebhan, A. Schmitt,
  JPG 39, 054006 (2012)



- go beyond chiral perturbation theory for rigorous conclusions

- include isospin chemical potential
  M.S. Grønli, T. Brauner, EPJ C 82, 354 (2022)

- include nonzero temperatures
  T. Brauner, H. Kolešová, N. Yamamoto, PLB 823, 136737 (2021)

- relevance for compact stars?