THE INFLUENCE OF THE CASIMIR EFFECT ON THE VACUUM STRUCTURE OF 3+1 DIMENTIONAL COMPACT ELECTRODYNAMICS

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XVth Quark Confinement and the Hadron Spectrum
Stavanger, Norway

The emergence of attractive force F_C between two conducting metallic plates in vacuum.

Predicted in 1948 by Casimir.

Indirect experimental evidence in 1958.

The direct experiment in 1997 (Lamoreaux).

$$\frac{F_C}{A} = -\frac{\pi^2}{240} \cdot \frac{\hbar c}{a^4}$$

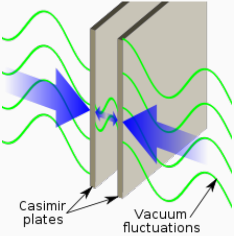

Figure 1: The schematic picture of Casimir effect. Wikipedia

Motivation

- Systems with boundaries
  - MIT bag model
  - Four-fermion theory with the presence of reflective boundaries
  - $\mathbb{C}P^{N-1}$ model with Dirichlet boundary conditions
  - Quenched QCD with Dirichlet boundary conditions
- The effect of boundaries on vacuum structure of the theory

Compact QED on the lattice

Action

$$S_G[\theta] = \beta \sum_{n \in \Lambda} \sum_{\mu < \nu} (1 - \cos \theta_P)$$

$$\theta_{P_{x,\mu\nu}} = \theta_{x,\mu} + \theta_{x+\hat{\mu},\nu} - \theta_{x+\hat{\nu},\mu} - \theta_{x,\nu}$$

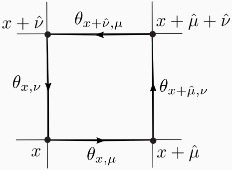

Figure 2: The plaquette angle at site *x*.

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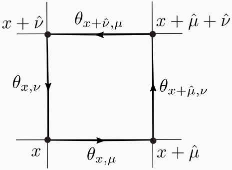
Monopoles

$$\bar{\theta}_P = \theta_P + 2\pi k_P \in [-\pi, \pi)\,, \qquad k_P \in \mathrm{Z}$$

$$j_{x,\mu} = \frac{1}{2\pi} \sum_{P\in\partial C_{x,\mu}} \bar{\theta}_P \in \mathbb{Z}$$

$$\rho = \frac{1}{\mathrm{Vol}_4} \sum_{x,\mu} |j_{x,\mu}|$$



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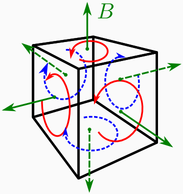

Figure 3: Schematic illustration of monopole charge on the lattice.

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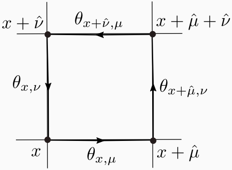
The presence of monopole condensate leads to linear confinement of electric charges.



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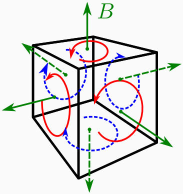

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Casimir boundary conditions on the lattice

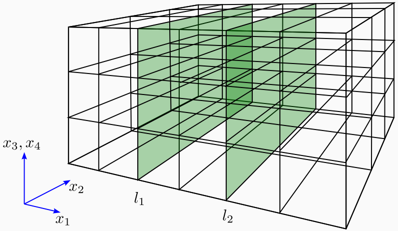


Figure 4: The position of Casimir plates.

$$B_1 \equiv F^{23}(x)\Big|_{x_1=l_a} = 0\,,$$

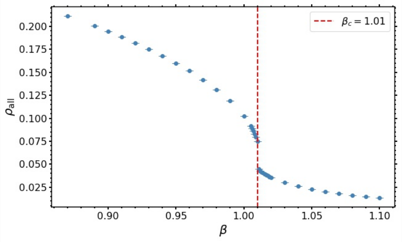
$$E_2 \equiv F^{24}(x)\Big|_{x_1=l_a} = 0\,,$$

$$E_3 \equiv F^{34}(x)\Big|_{x_1=l_a} = 0\,.$$

$$\cos\theta_{x,\mu\nu}\Big|_{x_1=l_a} = 1, \quad (\mu,\nu) = (23, 24, 34)$$

$$S_\varepsilon[\theta] = \sum_P \beta_P(\varepsilon)(1 - \cos\theta_P), \quad \beta_P(\varepsilon) = \beta[1 + (\varepsilon - 1)\delta_{P,\mathcal{V}}]$$

The phase transition in the absence of plates



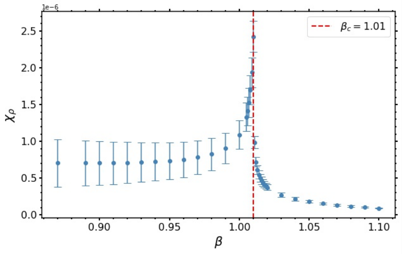


Figure 5: Left: the monopole density ρ vs lattice coupling β; **Right**: its susceptibility. The vertical line marks the position of the phase transition calculated from these observables.

Monopole configurations in the presence of plates

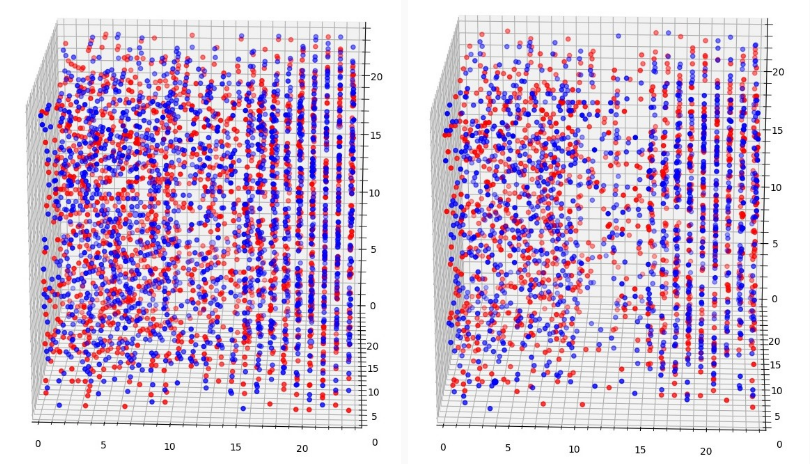


Figure 6: The examples of monopole configurations in the confinement phase(**left**, $\beta = 0.8$) and deconfinement phase (**right**, $\beta = 0.9$) for the plates separated by the distance $R = 3$. Monopoles and antimonopoles are represented by the red and blue dots, respectively. The plates, positioned vertically, are not shown.

The monopole density between plates normalized by monopole density in the absence of plates

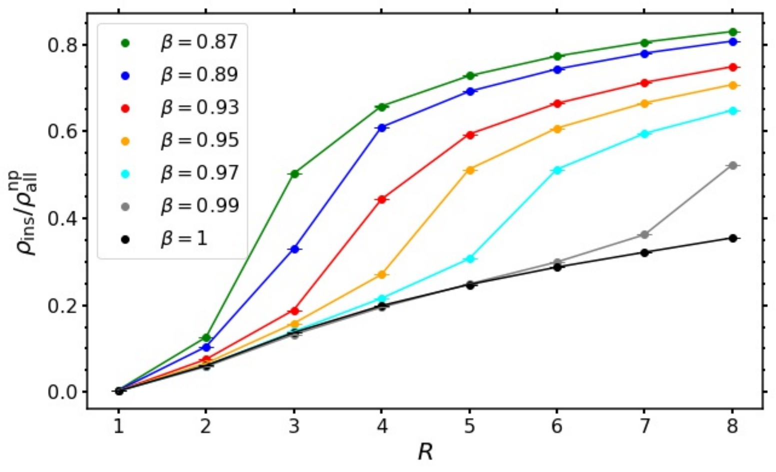


Figure 7: The ratio $\rho_{\rm ins}/\rho_{\rm ins}^{\rm np}$ of the monopole density $\rho_{\rm ins}$ inside the Casimir plates to the monopole density in the absence of the plates, $\rho_{\rm ins}^{\rm np}$ vs interplate separation *R* for a fixed set of lattice coupling β.

The shift of phase transition point

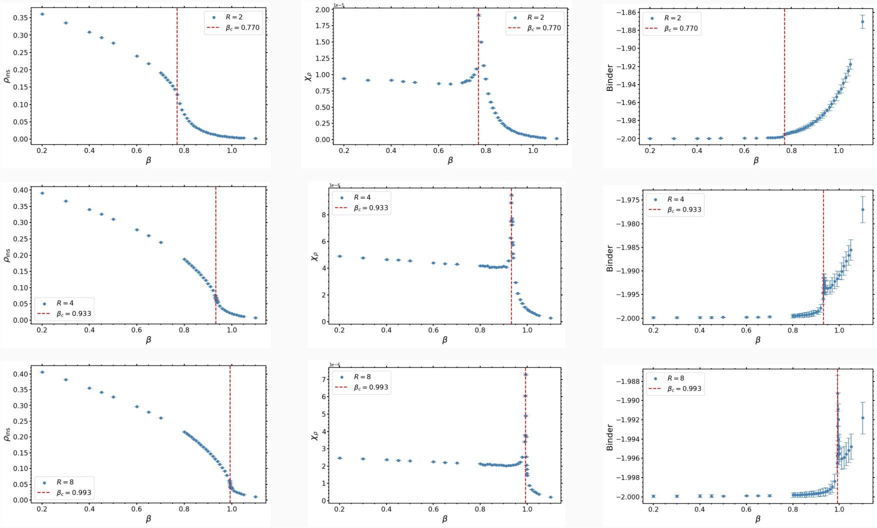


Figure 8: The monopole density (**left**), its susceptibility (**center**) and the Binder cumulant(**right**) for $R = 2, 4, 8$ (from top to bottom).

The phase diagram

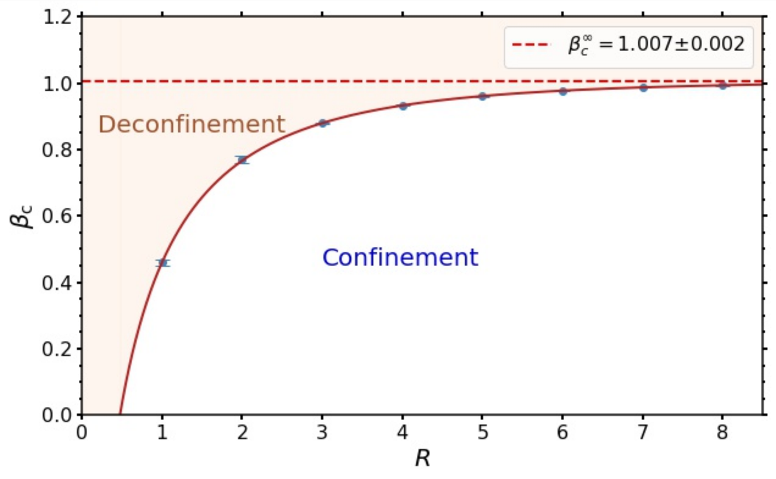


Figure 9: The phase diagram of the vacuum of the compact $U(1)$ gauge theory in between the perfectly metallic plates separated by the distance R. The solid line represents best fit $\beta_c^{\rm fit}(R) = \beta_c^\infty - \alpha \exp[-(R^2/R_0^2)^\nu]$ with $\alpha = 3.7(6)$, $R_0 = 0.28(7)$, $\nu = 0.257(16)$. The limit $R \to \infty$ is shown by the dashed horizontal line.

The Polyakov loop as the deconfinement order parameter

Definitions

$$P_x = \prod_{x_4=0}^{N_T-1} e^{i\theta_{x,x_4;\mu=4}}$$

$$P = \langle P_x \rangle$$

$$|P| = \left| \frac{1}{V_3} \sum_{x \in V_3} P_x \right|$$

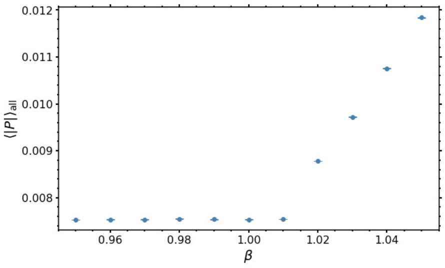


Figure 10: The modulus of Polyakov loop in the absence of plates.

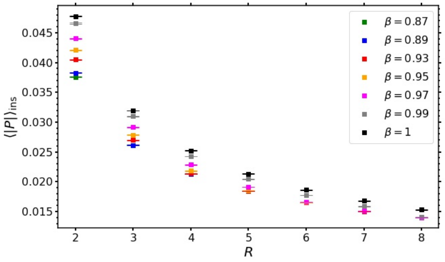


Figure 11: The modulus of the Polyakov loop in the space between the Casimir plates at the separation R at a set of fixed coupling β.

The Polyakov loop inside plates for different R

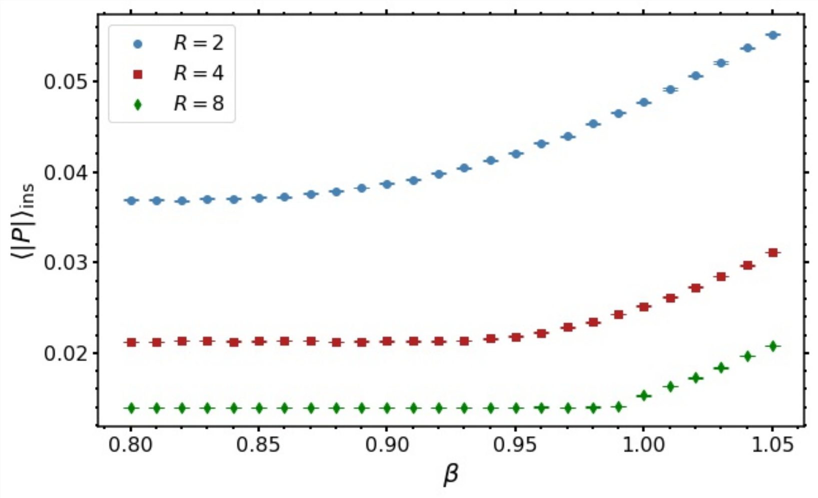


Figure 12: The Polyakov loop inside the plates vs β at fixed R.

Conclusions

- From first-principle numerical simulations we show that *the structure of the vacuum* of the compact U(1) gauge model in $3 + 1$ dimensions *is affected* by closely spaced perfectly conducting parallel plates;
- The non-pertubative Casimir effect *alternates* the dynamics of abelian monopoles, *modifies* the vacuum structure, and *leads* to the Casimir-induced deconfining phase transition in between plates;
- The *phase diagram* in the plane *”lattice coupling constant”-”distance between the plates”* is obtained;
- The presented method now is applied to the non-abelian case – the exiciting results are coming!

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Thank you for attention!