QCD Topology and Axions

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STRONG CP VIOLATION

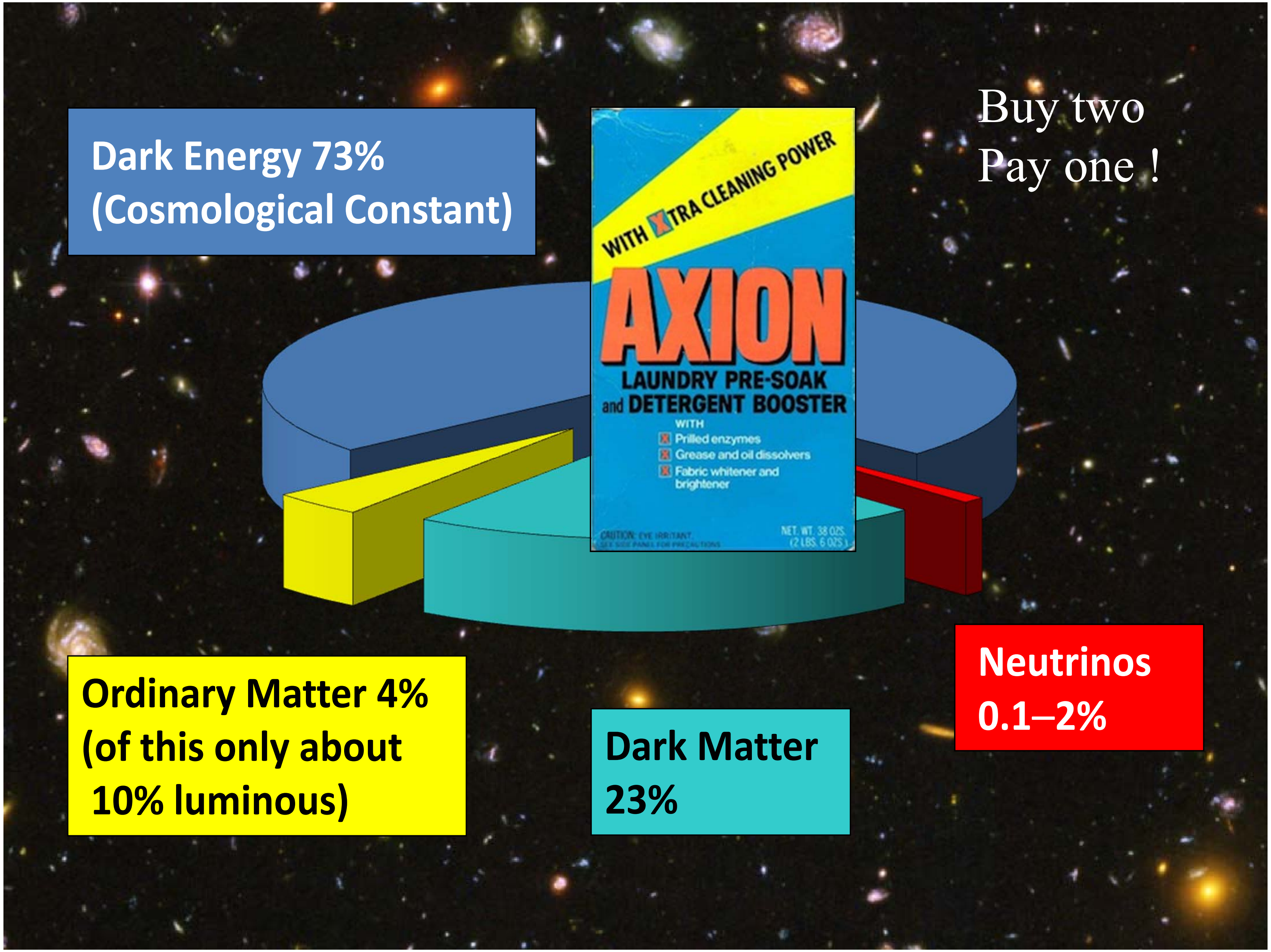
$$\mathcal{L}_\theta = \theta\, G^{\mu\nu a}\, \tilde{G}^a_{\mu\nu} \qquad\qquad \tilde{G}^a_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}\, G^a_{\rho\sigma}$$

$$\mathcal{L}_\theta \sim \theta\; E^a \cdot B^a$$

This term violates CP and gives a contribution to the electric dipole moment of the neutron

$$e_n < 3\;10^{-26}\; e\,\mathrm{cm}$$

$\theta < 10^{-10}$ which is quite unnatural !!


Dark Energy 73%
(Cosmological Constant)
WITH XTRA CLEANING POWER
AXION
LAUNDRY PRE-SOAK
and DETERGENT BOOSTER
WITH
Prilled enzymes
Grease and oil dissolvers
Fabric whitener and brightener
Buy two
Pay one !
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%

Minimizing the Axion Potential

$$e^{-S_{\text{eff}}(\partial_\mu \phi, \phi)} = \int \mathcal{D}\left[\psi, \bar{\psi}, G_\mu^A \dots\right] e^{-S\left(\psi, \bar{\psi}, G_\mu^A, \dots, \partial_\mu \phi, \phi\right)}$$

The minimum of the effective potential

$$\frac{\partial V_{\text{eff}}}{\partial \phi} = -\frac{1}{f_\phi} \frac{g^2}{32\pi^2} \langle G_{\mu\nu}^A \tilde{G}_A^{\mu\nu}(x) \rangle = 0$$

corresponds to $\varphi / f_\phi = -\theta$ and solve the strong CP problem; from the second derivative of the effective potential we may compute the axion mass

$$m_\phi^2 = \frac{1}{f_\phi^2} \chi_t = \frac{1}{f_\phi^2} \int d^4x \, \langle Q(x) Q(0) \rangle$$

1) Necessity

Necessity means that we need to compute the axion potential as a function of the temperature or the scattering amplitudes *for axion + pion* or *axion+ nucleon* scattering as a function of the energy (T)

2) Contingency

Contingency means that the present distribution of axions in the Universe depends on both the story of the evolution of the Universe as well as on the mechanism of axion production

3) Heresy

a) Towards a dynamical solution of the strong CP problem Schierolz& Nakamura

b) Consequences of the order of the limit of infinite spacetime volume and the sum over topological sectors for *CP* violation in the strong interactions Wen-YuanAia, Cruzb, Garbrechtb, Tamaritb

$\chi(T)$ needed at high T

Very high temperature: semi-classical approximation reliable

Compare lattice with semi-classical result, find temperature where they agree to some prescribed accuracy

Lattice calculation very challenging for high T

Main difficulty for lattice: continuum limit

Typically, with staggered fermions, $\chi(T)$ over-estimated at finite N_t, not suppressed enough because of non-zero would-be zero modes

Semi-classical calculation

You would think correctly done a long time ago

You would be mostly right

But not completely

$\rightarrow$ parallel E, 4:10 PM

Lattice work $\rightarrow$ parallel E, 4:30 PM, Claudio Bonati

The θ dependence of the free energy

BONATI

$$F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2\left[1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right] ,$$

where

$$\chi = \frac{1}{V_4}\langle Q^2\rangle_0 \quad , \quad b_2 = -\frac{\langle Q^4\rangle_0 - 3\langle Q^2\rangle_0^2}{12\langle Q^2\rangle_0}$$

and $\langle \quad \rangle_0$ denotes the average at $\theta = 0$.

For $T \ll \Lambda_{QCD}$ known from ChPT, for $T \gg \Lambda_{QCD}$

$$\mathrm{DIGA:} \quad F(\theta, T) - F(0, T) \simeq \chi(T)(1 - \cos\theta)$$

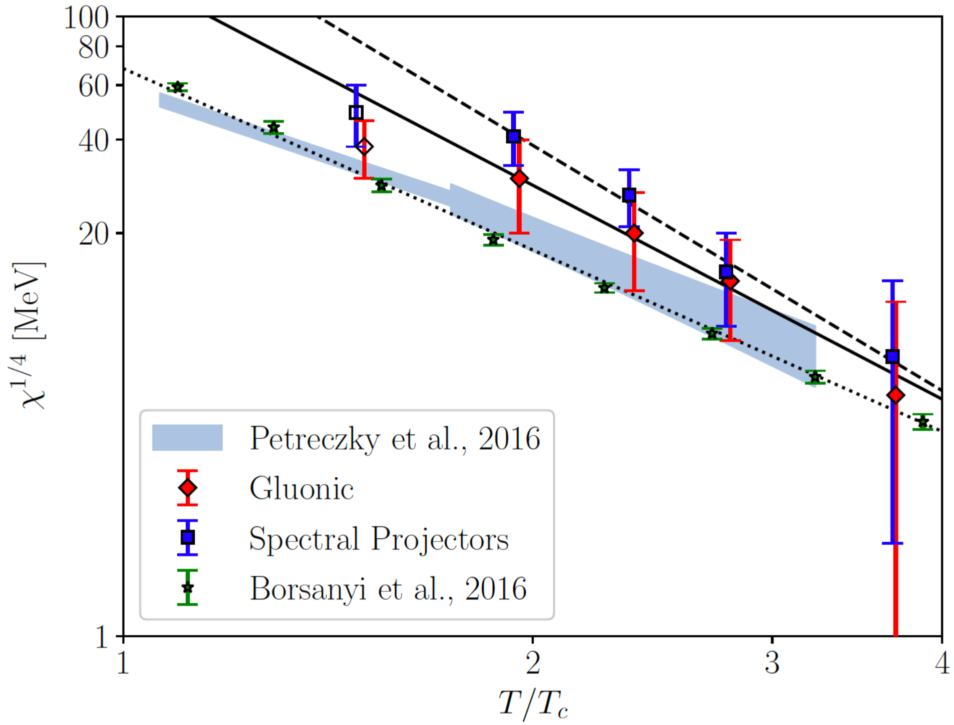
$$\mathrm{PT:} \quad \chi(T) \propto m^{N_f} T^{4-\frac{11}{3}N-\frac{1}{3}N_f}$$

In YM (no quarks) θ dependence well described by $1 - \cos\theta$ and $\chi(T) \propto T^{-7}$ starting practically from T_c (1^{st} order transition).

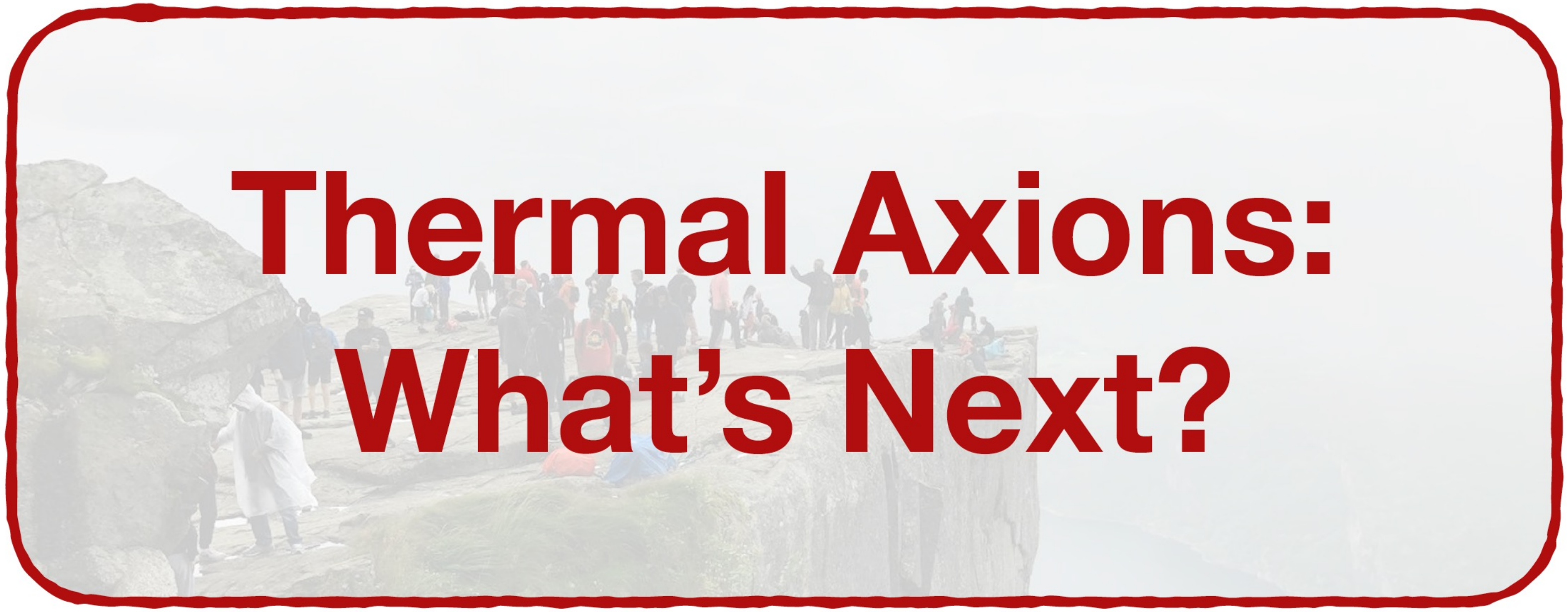
Why the QCD case is extremely challenging

1. QCD with physical quark masses is computationally much more difficult than YM theory
2. topological observables are extremely sensitive to the explicit chiral symmetry breaking of the lattice fermion discretization
3. at high T $\chi(T) \to 0$: the probability $P(Q)$ of observing a configuration with charge Q gets strongly peaked at $Q = 0$ (remember that $\langle Q^2 \rangle = \chi V_4$)
4. autocorrelation times grows exponentially fast with the inverse lattice spacing: simulations get stuck in a fixed topological sector (no estimate of χ but also no ergodicity!)

A summary of results and problems



- Tensions between different determinations. Biases?
- Very different error estimates. Biases?
- Algorithmic development needed to solve the problem in a stochastically exact way.
- Mass dependence of $\chi(T)$?



Thermal Axions: What's Next?

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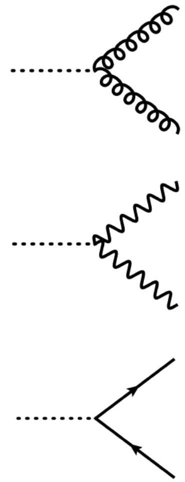
Round Table: QCD Topology and Axions

XVth Quark Confinement and the Hadron Spectrum Conference, University of Stavanger — 5 August 2022

Unavoidable Production Source

Scatterings and/or decays involving primordial thermal bath particles (axion energy ≫ m_a, i.e. "hot")

$$B_i \, B_j \;\rightarrow\; B_k \, a$$



$$\frac{\alpha_s}{8\pi}\frac{a}{f_a} G^{\mu\nu}\widetilde{G}_{\mu\nu}$$

$$c_{\gamma\gamma}\frac{\alpha_{\rm em}}{8\pi}\frac{a}{f_a} F^{\mu\nu}\widetilde{F}_{\mu\nu}$$

$$c_\psi \frac{\partial_\mu a}{f_a}\overline{\psi}\gamma^\mu\gamma^5\psi$$

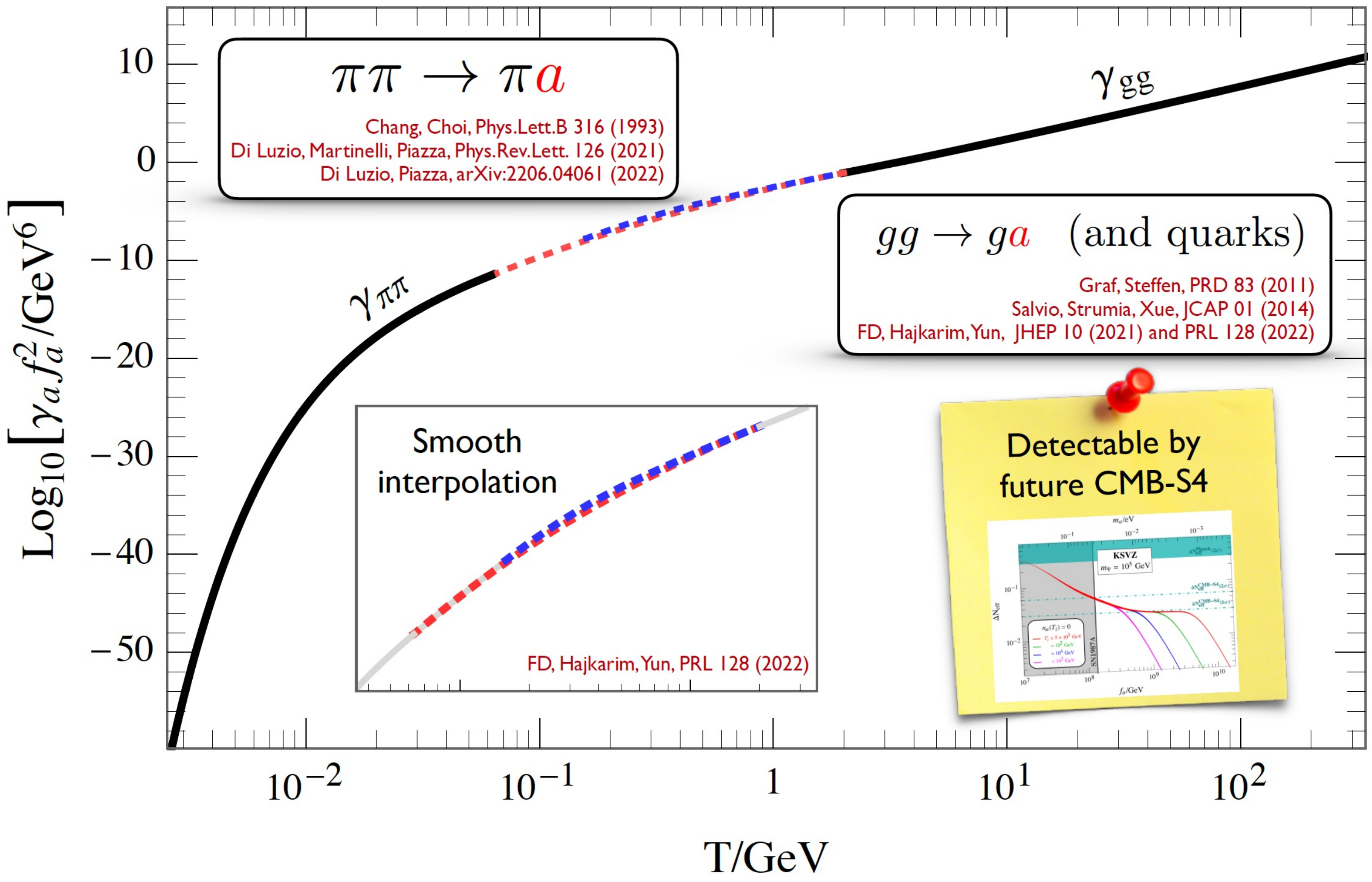
GOALS:

- Compute how many axions are produced in the early universe
- Quantify the resulting effect on cosmological observables (BBN, CMB, LSS)

$$\boxed{\frac{dn_a}{dt} + 3Hn_a = \gamma_a}$$

Turner, Phys.Rev Lett. 59 (1987)
Brust, Kaplan, Walters, JHEP 12 (2013)
Baumann, Green, Wallisch, Phys. Rev. Lett. 117 (2016)

Where We Stand



What's Next?

Axion production rate

across the confinement scale still unknown

$$\gamma_a = n_i n_j \times \langle \sigma_{ij \to ja} v_{\rm rel} \rangle$$

Thermal bath Particle Physics

1. Pion scattering above 100 MeV?
2. Production via other hadrons?
3. How to describe the thermal bath?
4. Boltzmann equation evolution and cosmological observables?

HERESY:

Wen-YuanAia, Cruzb, Garbrechtb, Tamaritb

For the resulting correlation functions, we therefore take the in infinite-volume limit before summing over topologicalsectors. In contrast to the opposite order of limits, the chiral phases from the mass terms and from the instantoneffects then are aligned so that, in absence of additional phases,

these do not give rise to observables violating charge-parity symmetry.

This result is confirmed when constraining the correlations at coincident points byusing the index theorem instead of instanton calculus.