Improving predictions for thermal bubble nucleation

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Cosmological 1^{st}-order phase transitions

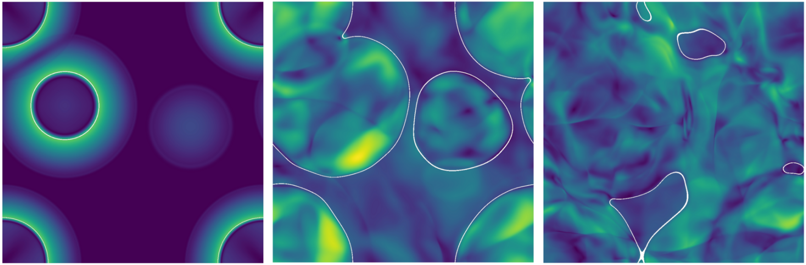


Figure: Cutting et al. arXiv:1906.00480.

- Universe supercools
- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows
- And creates gravitational waves: $\Box h_{ij}^{(TT)} \sim T_{ij}^{(TT)}$

Gravitational wave spectrum

GW signal depends strongly on 4 phase transition quantities,

$$\Omega_{\rm GW} = F(T_*, R_*, \alpha_*, v_{\rm w}),$$

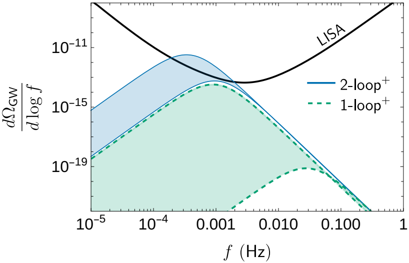
T_* : transition temperature,

R_* : bubble radius,

α_* : transition strength,

$v_{\rm w}$: bubble wall speed.

Each depends on the bubble nucleation rate.



Large uncertainties linked to predictions of nucleation rate.

OG & Tenkanen 2104.04399

Infrared strong coupling

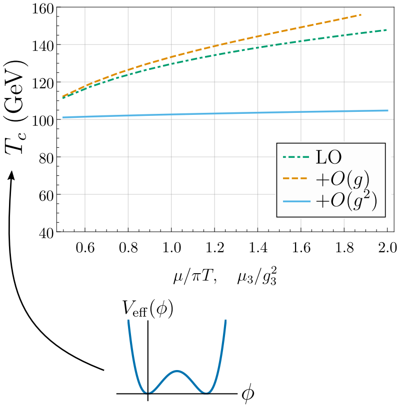
Infrared bosons are highly occupied; the effective expansion parameter α_{eff} grows

$$\alpha_{\mathrm{eff}} \sim g^2 \frac{1}{e^{E/T} - 1} \approx g^2 \frac{T}{E}$$

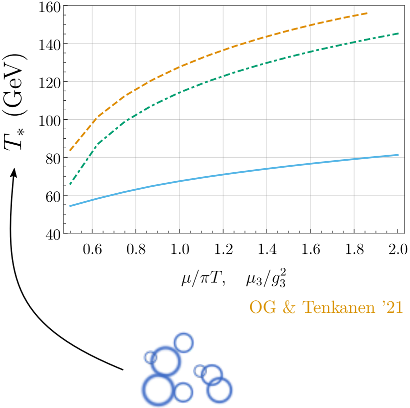
Softer modes are classically occupied and more strongly coupled:

$$\begin{aligned} \text{hard}: \quad & E \sim T \Rightarrow \alpha_{\mathrm{eff}} \sim g^2 \sim 0.03, \\ \text{soft}: \quad & E \sim gT \Rightarrow \alpha_{\mathrm{eff}} \sim g \sim 0.18, \\ \text{ultrasoft}: \quad & E \sim g^2 T \Rightarrow \alpha_{\mathrm{eff}} \sim g^0 \sim 1. \end{aligned}$$

Bubble nucleation uncertainties



critical temperature



percolation temperature

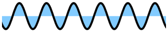
Real scalar model

A simple model,

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \sigma\phi + \frac{m^2}{2}\phi^2 + \frac{\kappa}{3!}\phi^3 + \frac{g^2}{4!}\phi^4 + J_1\phi + J_2\phi^2,$$

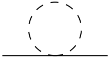
with only two relevant scales:

hard: $E \sim \pi T$ (nonzero Matsubara modes)



$$m_n^2 = m^2 + (n\pi T)^2 \text{ with } n \neq 0$$

soft: $E \sim gT$ (Debye screened)

$m_{\text{eff}}^2 \sim$  $\sim g^2 T^2$

Classicalisation

- Bose enhancement of IR modes

$$n_{\mathrm{B}}(E) = \frac{1}{e^{E/T} - 1}, \approx \frac{T}{E} \gg 1.$$

- Dynamics of QFT at nucleation scale ($\Lambda_{\mathrm{nucl}} \ll T$) expected to be quasi-classical.

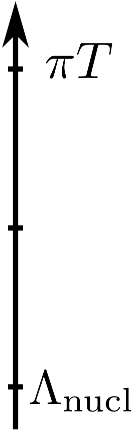


Figure: Nucleation scale much lower than thermal scale.

Classical thermal evolution

Hamilton's equations determine real-time correlation functions,

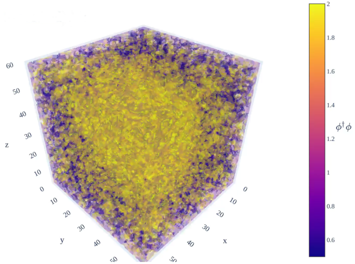
$$\dot{\phi}(t,x) = \{\phi(t,x), H\},$$
$$\dot{\pi}(t,x) = \{\pi(t,x), H\},$$

with thermal initial conditions,

$$\langle \phi(0,x_1)\phi(0,x_2)\rangle_{\mathrm{cl}} \equiv \frac{1}{Z_{\mathrm{cl}}} \int \mathcal{D}\phi\mathcal{D}\pi\,\phi(0,x_1)\phi(0,x_2)e^{-H[\phi,\pi]/T}.$$

Quantum versus classical

- Classical UV catastrophe - the cut-off scale dominates!



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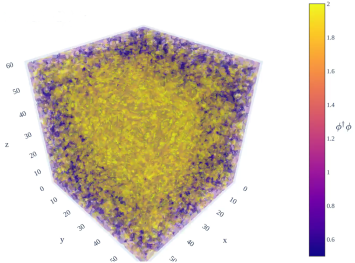
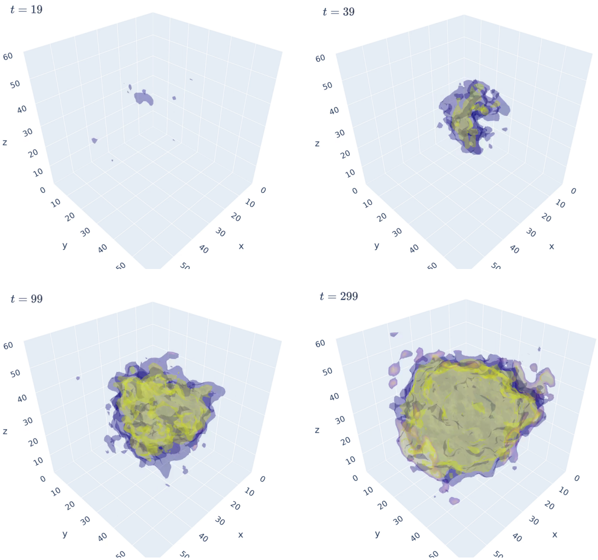


- Using quantum counterterms cancels UV catastrophe, giving finite result. Aarts & Smit '97
- Moreover, the finite classical and quantum remainders agree!

$$\langle \phi(t_1, x_1)\phi(t_2, x_2)\rangle_{\text{cl}} \approx \langle \{\phi(t_1, x_1), \phi(t_2, x_2)\}\rangle_{\text{qm}}$$

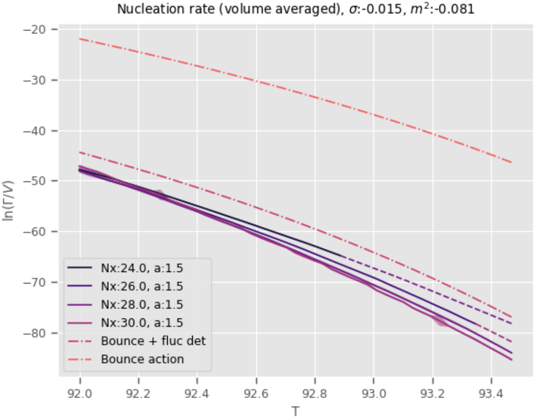
Bödeker '97

Bubble nucleation on the lattice



OG, Güyer & Rummukainen '22

Benchmarking against the lattice



$$\frac{\Gamma}{V} \sim A e^{-B}$$

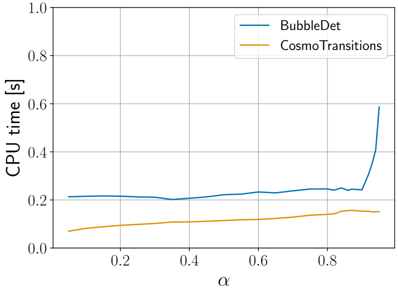
$$H_{\mathrm{eff}} = \int \mathrm{d}^3 x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\partial_i \phi)^2 + \sigma_{\mathrm{eff}}\phi + \frac{1}{2}(m_{\mathrm{eff}}^2 + \delta m_{\mathrm{eff}}^2)\phi^2 + \frac{g_{\mathrm{eff}}^2}{4!}\phi^4 \right]$$

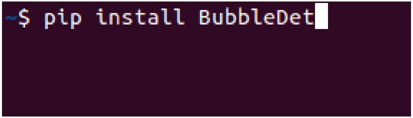
OG, Kormu & Weir (forthcoming), Moore, Rummukainen & Tranberg '01

Perturbation theory to the next order

BubbleDet: a Python library for computing the nucleation prefactor within perturbation theory,

$$\Gamma \sim \underbrace{\sqrt{\left|\frac{\det\left(-\Box + V''(\phi_\mathrm{F})\right)}{\det'\left(-\Box + V''(\phi_\mathrm{b})\right)}\right|}}_{\text{BubbleDet}} \underbrace{e^{-(S[\phi_\mathrm{b}]-S[\phi_\mathrm{F}])}}_{\text{CosmoTransitions}}.$$





Ekstedt, OG, Hirvonen (forthcoming)

Conclusions

- Uncertainties in bubble nucleation infect Ω_{GW} predictions.
- Classical effective theory describes high-T dynamics.
- Lattice simulations of a real scalar theory give sound results.
- Higher order terms in perturbation theory are important.

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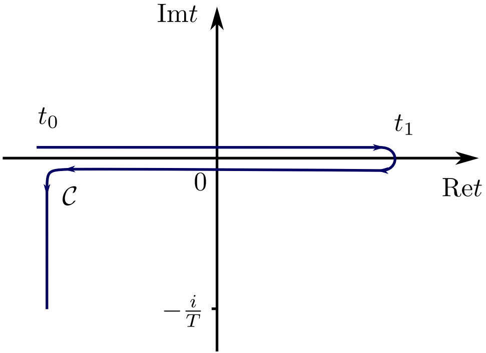
Thanks for listening!

Backup slides

Quantum thermal time evolution



$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle_{\mathsf{qm}} = \frac{1}{Z}\mathrm{Tr}\left[e^{-\hat{H}/T}\left(e^{i\hat{H}t}\mathcal{O}(0)e^{-i\hat{H}t}\right)\mathcal{O}(0)\right]$$
$$= \int_{\mathcal{C}} \mathcal{D}\phi\,\mathcal{O}(t)\mathcal{O}(0)e^{iS[\phi]}$$