

# Primordial Black Hole Review

## LISA CosWG project 11

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# **Primordial Black Hole Review**

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# Primordial Black Hole Review

## Primordial black holes and their gravitational-wave signatures

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# Primordial Black Hole Review

**Abstract.** In the recent years, primordial black holes (PBHs) have emerged as one of the most interesting and hotly debated topics in cosmology. Among other possibilities, PBHs could explain both some of the signals from binary black hole mergers observed in gravitational wave detectors and an important component of the dark matter in the Universe. Significant progress has been achieved both on the theory side and from the point of view of observations, including new models and more accurate calculations of PBH formation, evolution, clustering, merger rates, as well as new astrophysical and cosmological probes. In this work, we review, analyse and combine the latest developments in order to perform end-to-end calculations of the various gravitational wave signatures of PBHs. Different ways to distinguish PBHs from stellar black holes are emphasized. Finally, we discuss their detectability with LISA, the first planned gravitational-wave observatory in space.

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## Contents

### 1 Introduction

- 1.1 A brief history of primordial black holes (PBHs)
- 1.2 Why a(nother) review on PBHs?
- 1.3 Probing PBHs with LISA
- 1.4 Outline of this review

The goal of this paper is to review these most recent developments and discuss how they impact the GW signatures of PBHs. Compared to other recent reviews, we aim at integrating together the most recent and accurate models, e.g., of PBH formation, evolution, clustering, merger rates, in order to compute the GW signatures of PBHs, and to explore their detectability. For this purpose, we have developed in parallel a numerical code that implements all those models and recent scenarios. The code will be released soon in a separate publication, but it has been already used in this paper in order to produce the key figures. The task of including these increasingly complex models has been eased by the specific expertise of many authors, who have actively contributed to these developments, covering a broad range of topics.

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## 1.3 Probing PBHs with LISA

This project is hosted by the cosmology working group of the LISA consortium. Therefore our discussion has been focused on the GW signatures of PBHs in the particular context of the LISA mission.

PBHs cover a very wide range of masses and therefore frequency ranges of GWs. In particular, LISA will be sensitive to a broad band around the millihertz GW frequencies [125, 144–146], which is complementary to the ground-based GW detectors [90, 91] and electromagnetic probes [147]. The search for a SGWB from PBH formation in the radiation era can be indirectly used for their discovery or alternatively to constrain their existence in the  $10^{-12} M_{\odot}$  mass range [125, 144–146, 148]. Moreover, the coalescence of the heavy seeds of supermassive black holes (SMBH) in the late Universe could leave clear signatures of their primordial origin, or even detect individual events at high redshift which could not have arisen from astrophysical mechanisms.

The great sensitivity of LISA at mHz frequencies [149–152] opens the possibility to detect the mergers of  $10^3 - 10^4 M_{\odot}$  PBHs all the way to  $z \simeq 100$ . Furthermore, the isotropic SGWB from the coalescence of PBHs since recombination should have an amplitude and spectral shape that will make it easily detectable by LISA. In this review, we will describe the different features of known PBH scenarios that can be probed with LISA.

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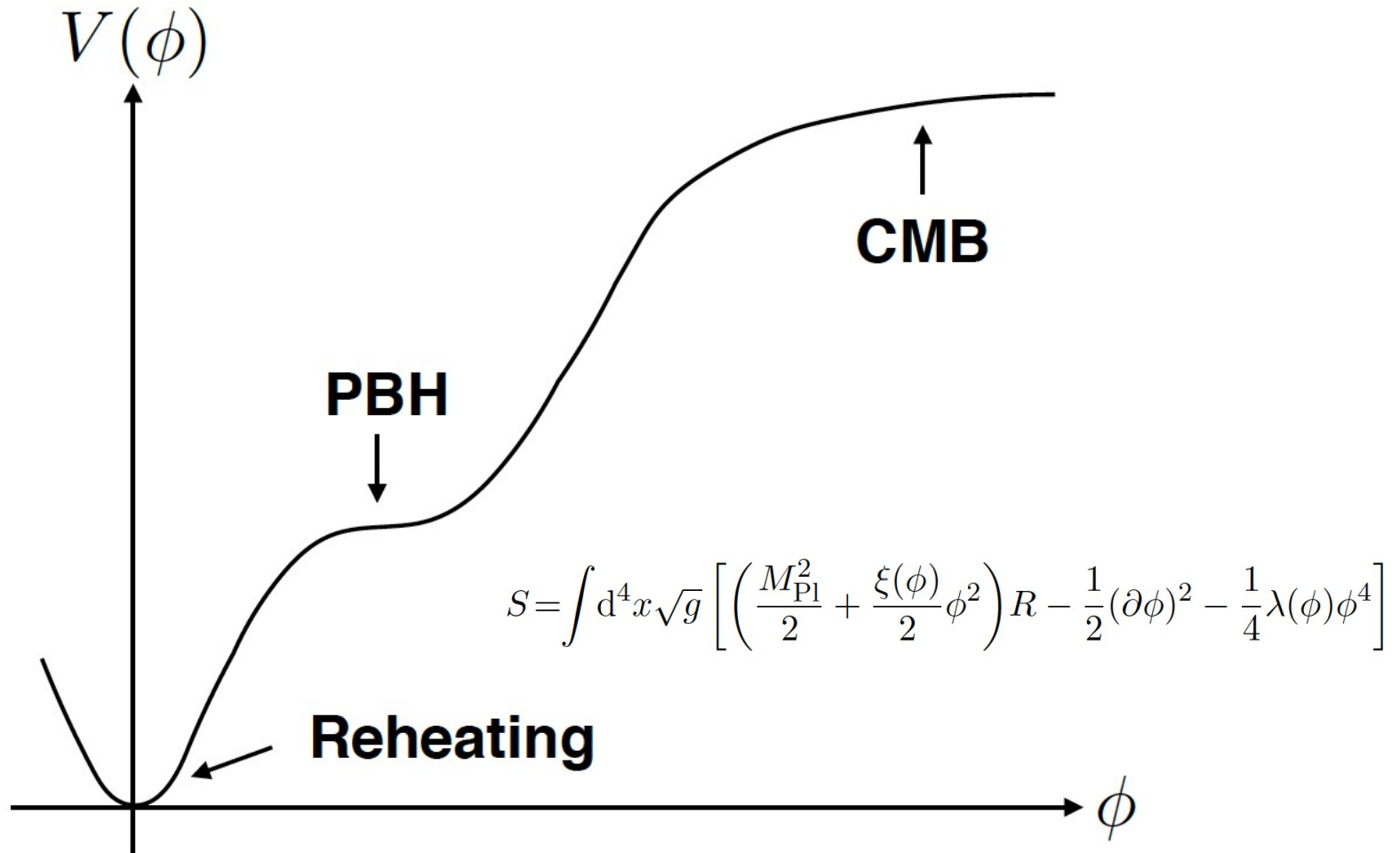
### 2.8 Domain walls and cosmic strings

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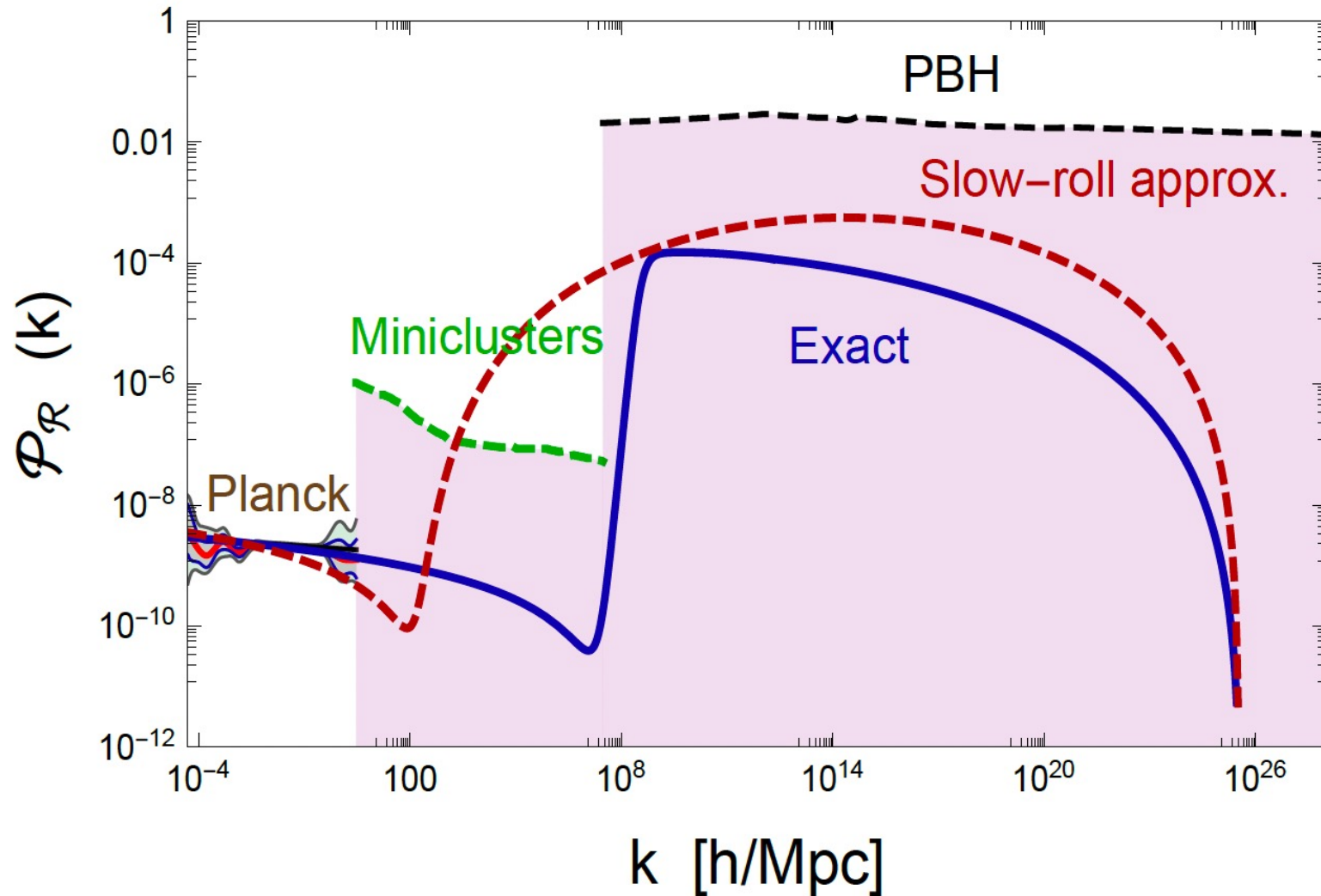
## 2.1 Single-field inflationary models



**Figure 1.** A schematic plot of the inflationary potential with an inflection point.



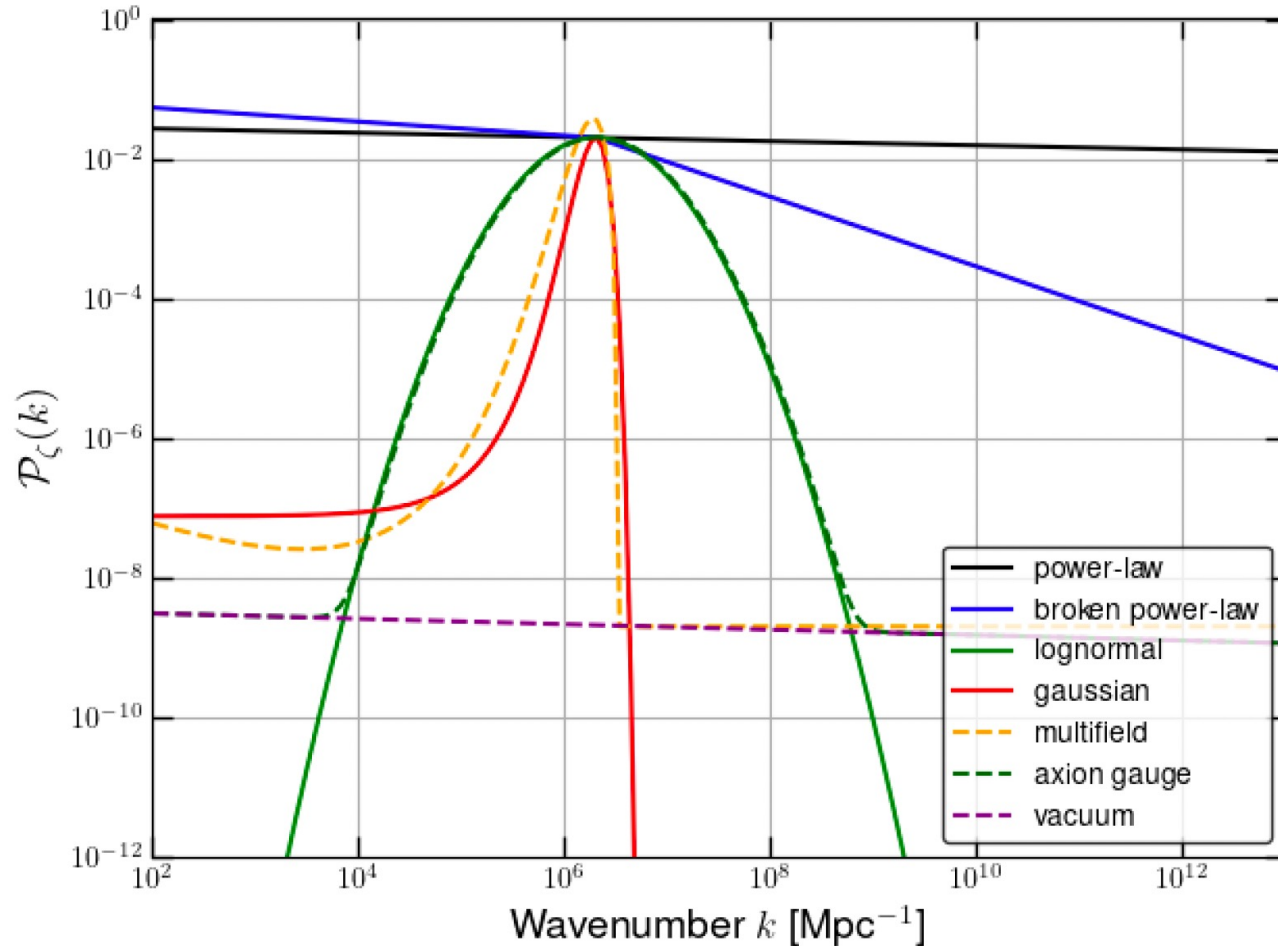
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**Figure 2.** Power spectrum  $\mathcal{P}_k$  for the single field Critical Higgs Inflation model, with an inflection point at  $N \approx 36$  satisfying the Planck 2018 constraints. Adapted from Ref. [154].

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## 2.1.4 Reverse engineering approach

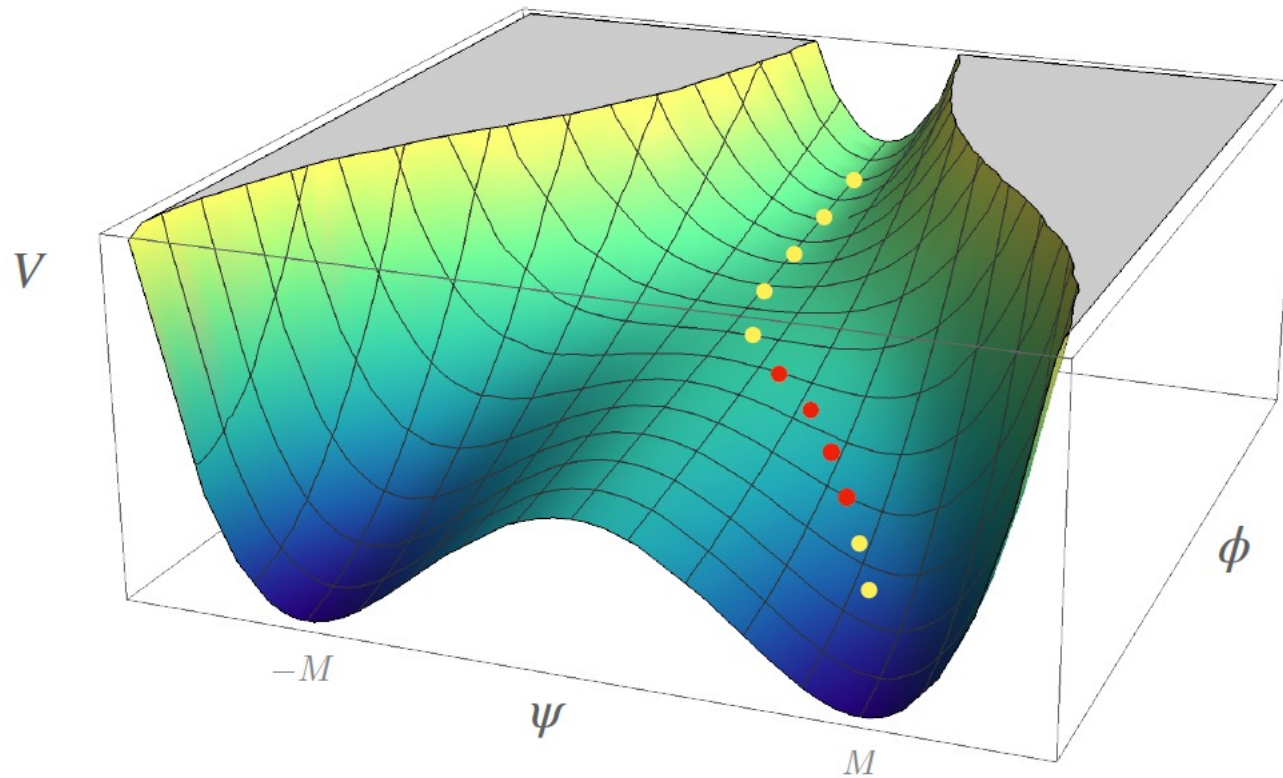


**Figure 3.** Examples of primordial power spectra of curvature fluctuations, leading to (stellar-mass) PBH formation: power-law, broken power-law, Gaussian and log-normal models (solid lines), and particular examples of multifield and axion-gauge models (dashed lines).

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## 2.2 Multi-field inflationary models

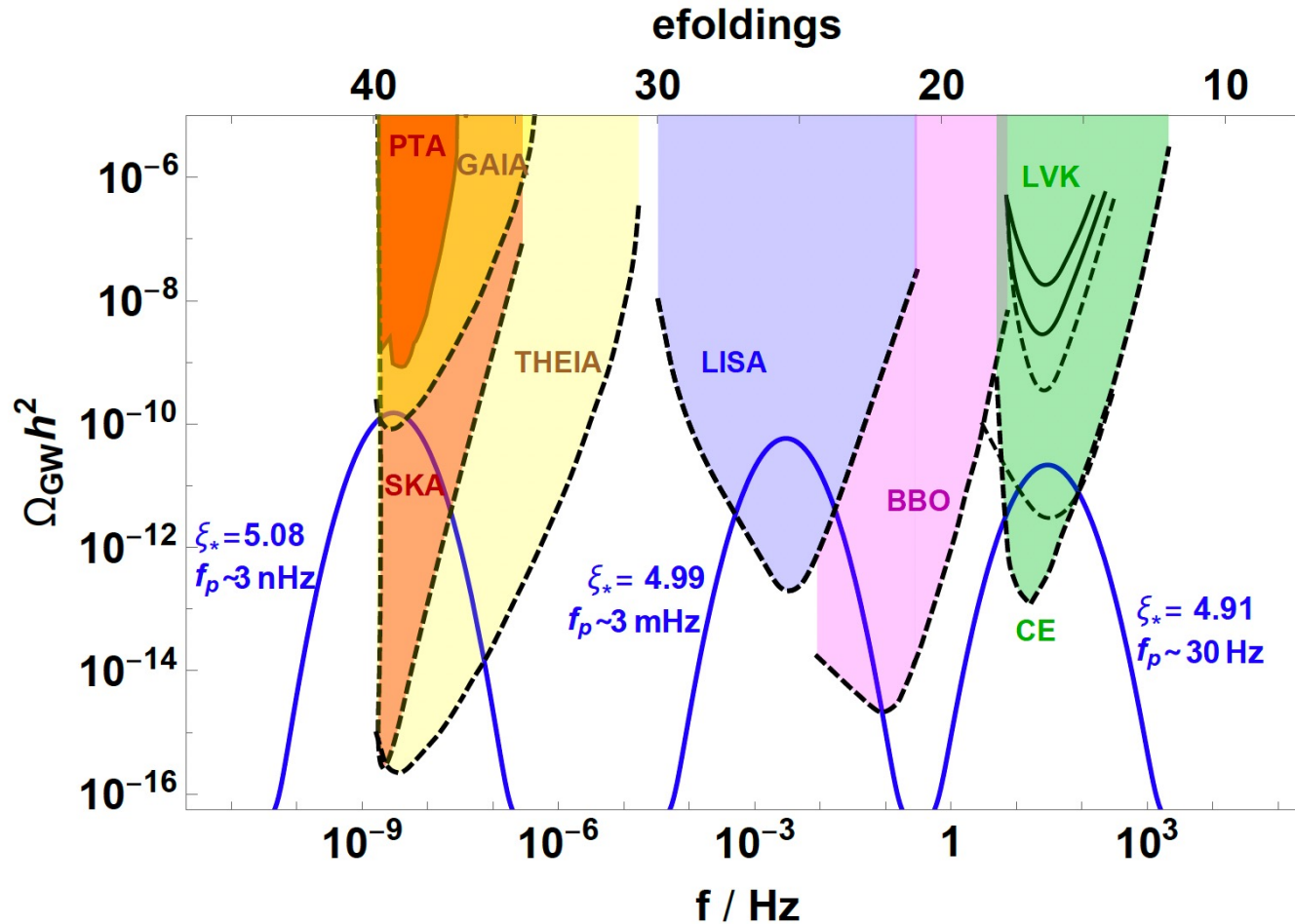
### 2.2.1 Hybrid inflation



**Figure 4.** Representation of a typical hybrid inflation potential,  $V$ , with a possible trajectory in two-field space (dotted line). CMB perturbations are created along the valley at  $\psi = 0$ , during a first phase of inflation. Curvature perturbations suitable for production of PBHs are generated in a second flat part of the potential (red dotted line), when the mass-squared of the field  $\psi$  changes sign.

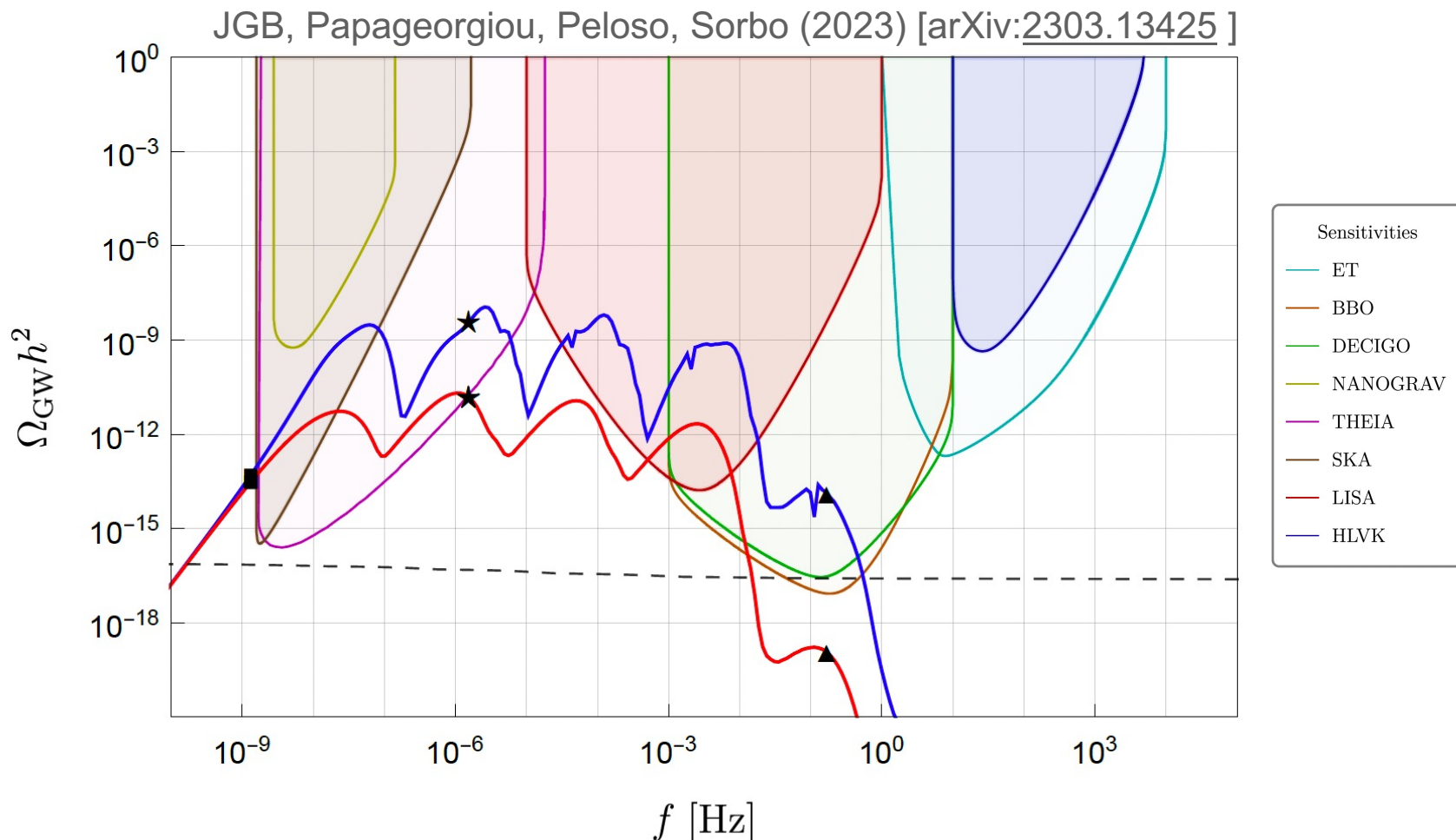
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## 2.2.4 Axion-gauge scenario



**Figure 6.** The stochastic GWs produced by axion inflation at four main scales of interferometers, nHz (PTAs and SKA),  $\mu\text{Hz}$  (Gaia and Theia), mHz (LISA and Taiji) and Hz (LIGO-Virgo-KAGRA (LVK), Einstein Telescope, and Cosmic Explorer). The enhanced density perturbations may produce PBHs which are a significant fraction of dark matter for  $1 - 100 M_\odot$  and the totality in the  $10^{-14} - 10^{-11} M_\odot$

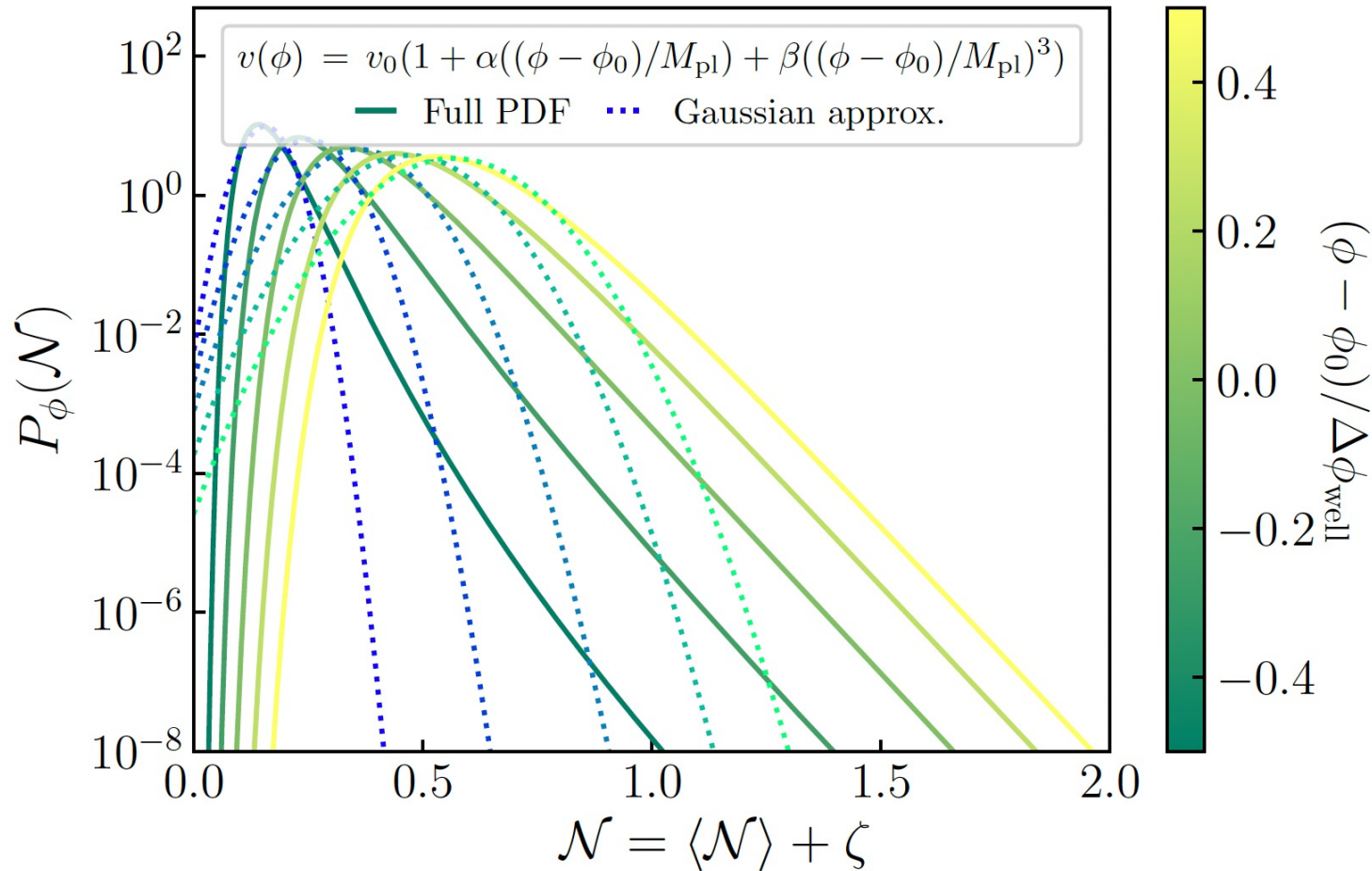
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**Figure 4.** The sourced gravitational wave power spectrum of the *Left* (blue line) and *Right* (red line) polarization as well as the vacuum (back dashed line) superimposed with the power-law-integrated sensitivity curves corresponding to various current and future experiments taken from [39] and [8] in the case of THEIA. The parameters chosen correspond to the coupling strength  $1/f = 57/M_p$ . The squares, stars and triangles are points for which we plot the integrand of the power spectrum defined in (A.16) in Appendix E.

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## 2.3.2 Quantum diffusion



**Figure 8.** Probability density functions of the curvature perturbations generated by an inflection-point inflationary potential (see inset). Solid lines correspond to the full distribution functions computed by mean of the stochastic- $\delta N$  formalism, where different colours correspond to different locations in the potential where the scale under consideration emerges from the Hubble radius.

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3.9 Evolution of the mass function through accretion

3.9.1 Accretion onto isolated PBHs

3.9.2 Accretion onto PBH binaries

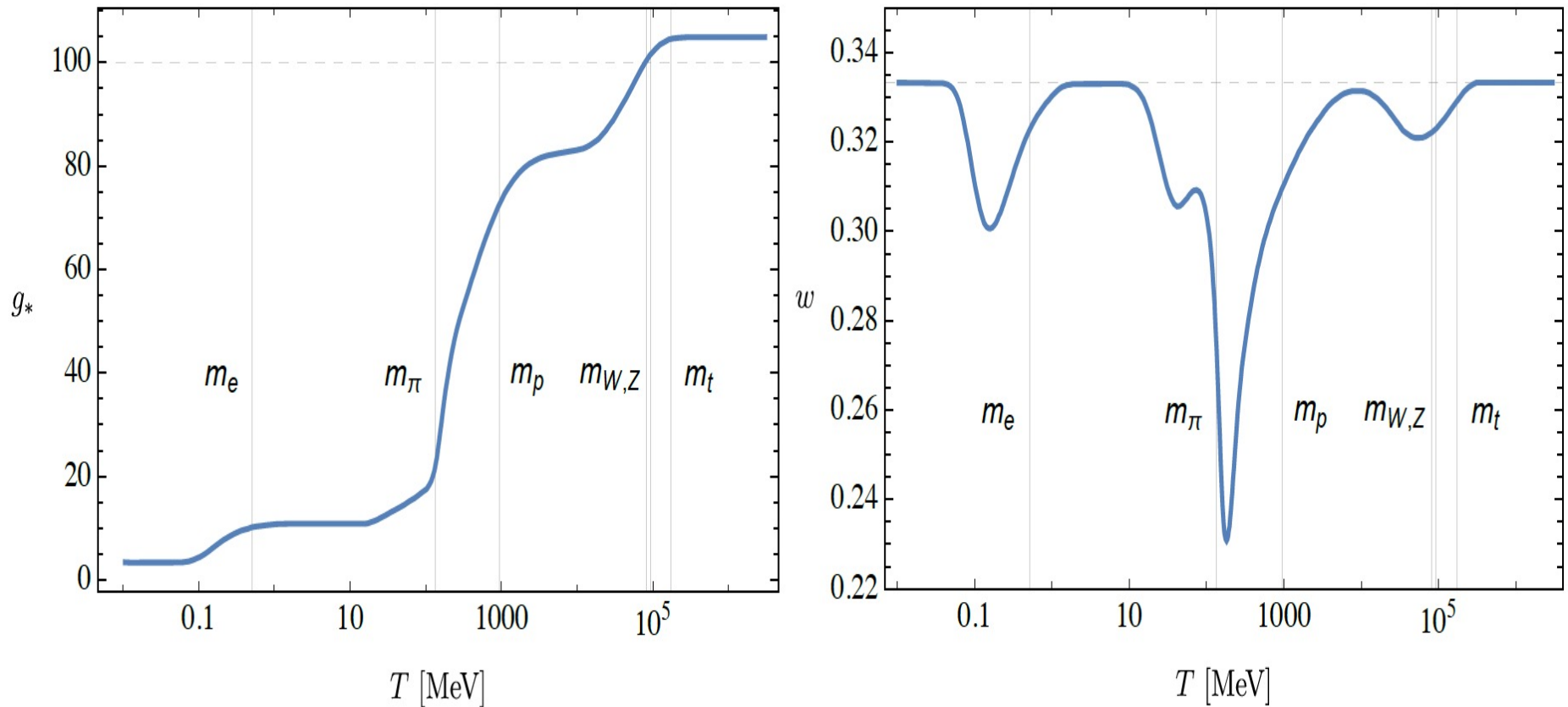
3.10 Spin Distribution

3.11 Comments on the clustering from Poisson initial conditions

3.12 Summary

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## 3.8 Thermal history

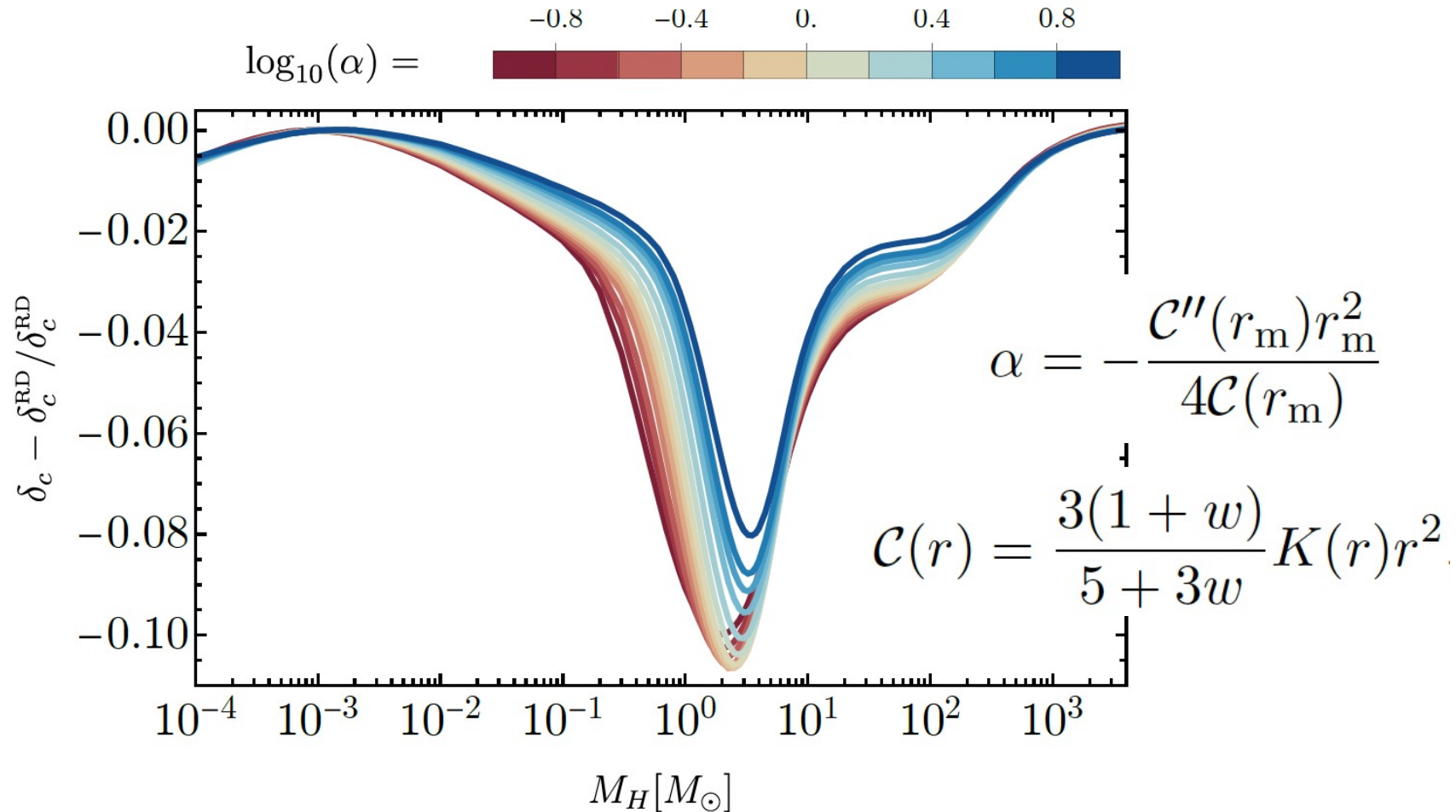


**Figure 11.** Relativistic degrees of freedom  $g_*$  (*Left panel*) and equation-of-state parameter  $w$  (*Right panel*), both as a function of temperature  $T$  (in MeV). The grey vertical lines correspond to the masses of the electron, pion, proton/neutron,  $W$ ,  $Z$  bosons and top quark, respectively. The grey dashed horizontal lines indicate values of  $g_* = 100$  and  $w = 1/3$ , respectively.



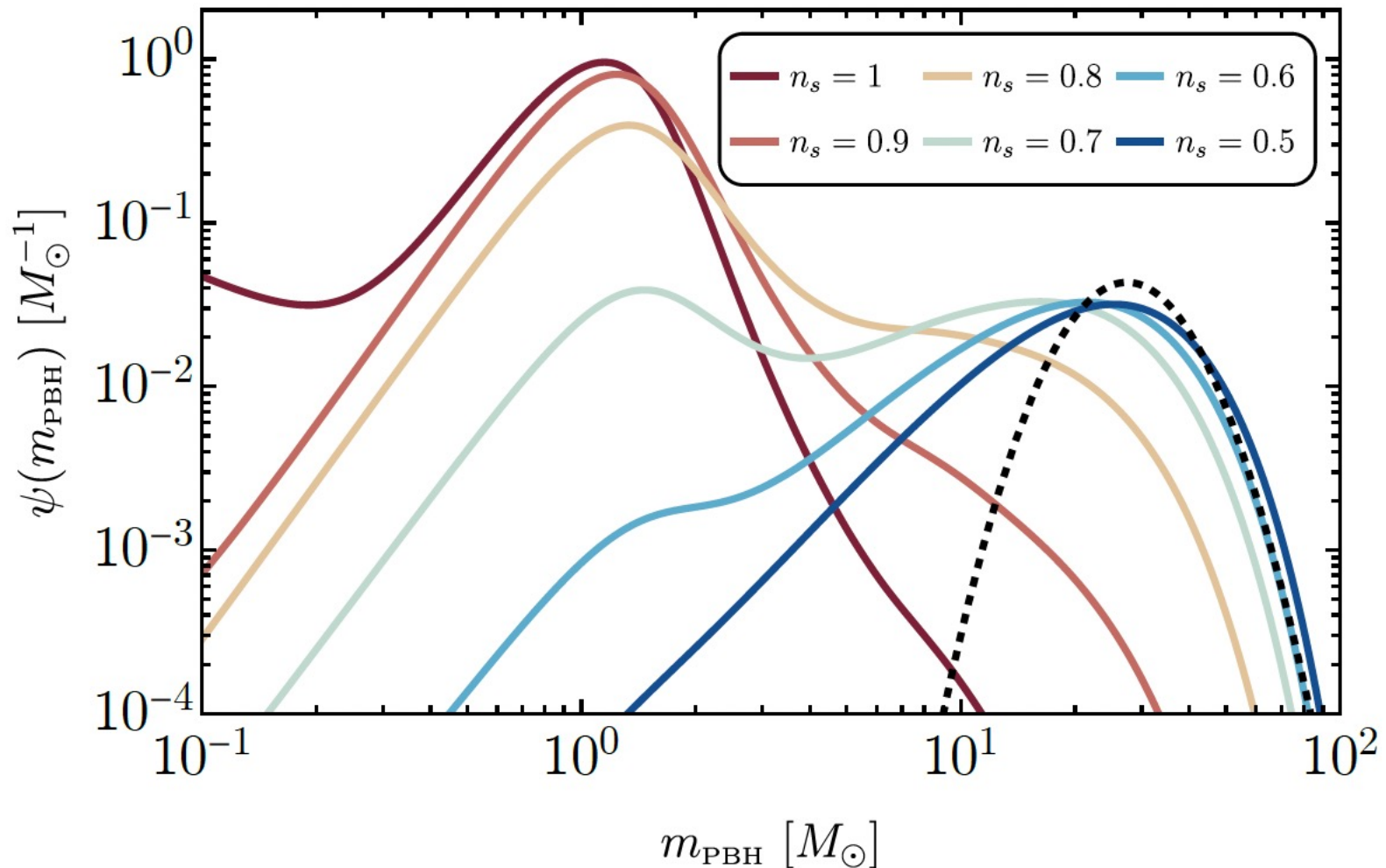
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## 3.3 Threshold for PBH formation



**Figure 13.** Figure readapted from Ref. [335]. Relative variation of the threshold compared to what is obtained assuming perfect radiation as a function of the horizon crossing time (parametrised here with  $M_H$ ) induced by the QCD thermal effects. The color code indicates the different values of  $\log_{10}(\alpha)$  as indicated by the bar on top of the frame.

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**Figure 15.** Plot taken from [84]. Mass function obtained with a few choices of the curvature power spectrum. This plot assumes  $f_{\text{PBH}} = 10^{-3}$ , the minimum horizon mass to be  $\lesssim 10^{-2.5} M_{\odot}$ , the largest mass  $M_H^{\text{max}} = 10^{2.8} M_{\odot}$  and a variable tilt  $n_s$ . The black dashed line reports the lognormal mass distribution found as the best fit in the analysis of Ref. [80].

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4.1.2 Suppression from close PBHs and matter inhomogeneities

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4.1.7 Limitations

### 4.2 Late Binaries

4.2.1 General formula

4.2.2 Link with observations of compact binary coalescences

4.2.3 Limitations

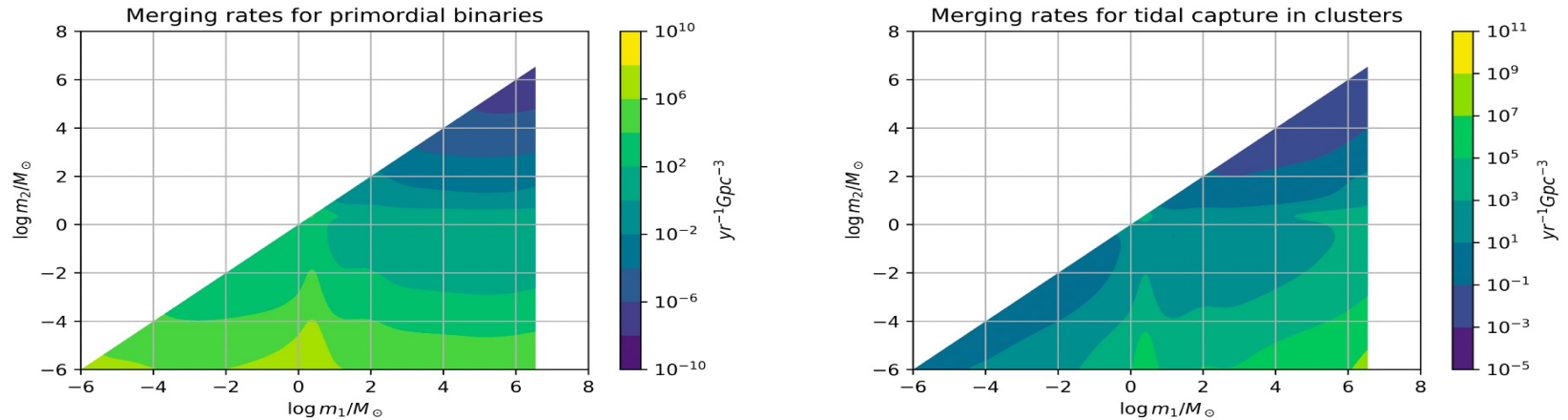
4.3 Hyperbolic encounters

4.4 How to distinguish primordial from astrophysical binaries

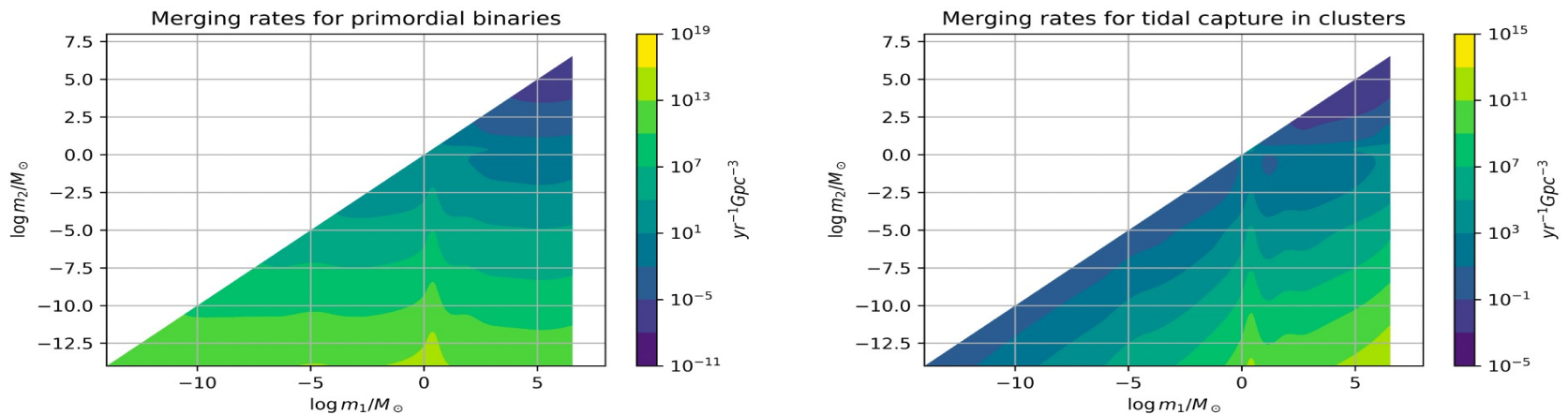
4.5 Summary

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Model 1: Carr 2019,  $n_s = 0.97$

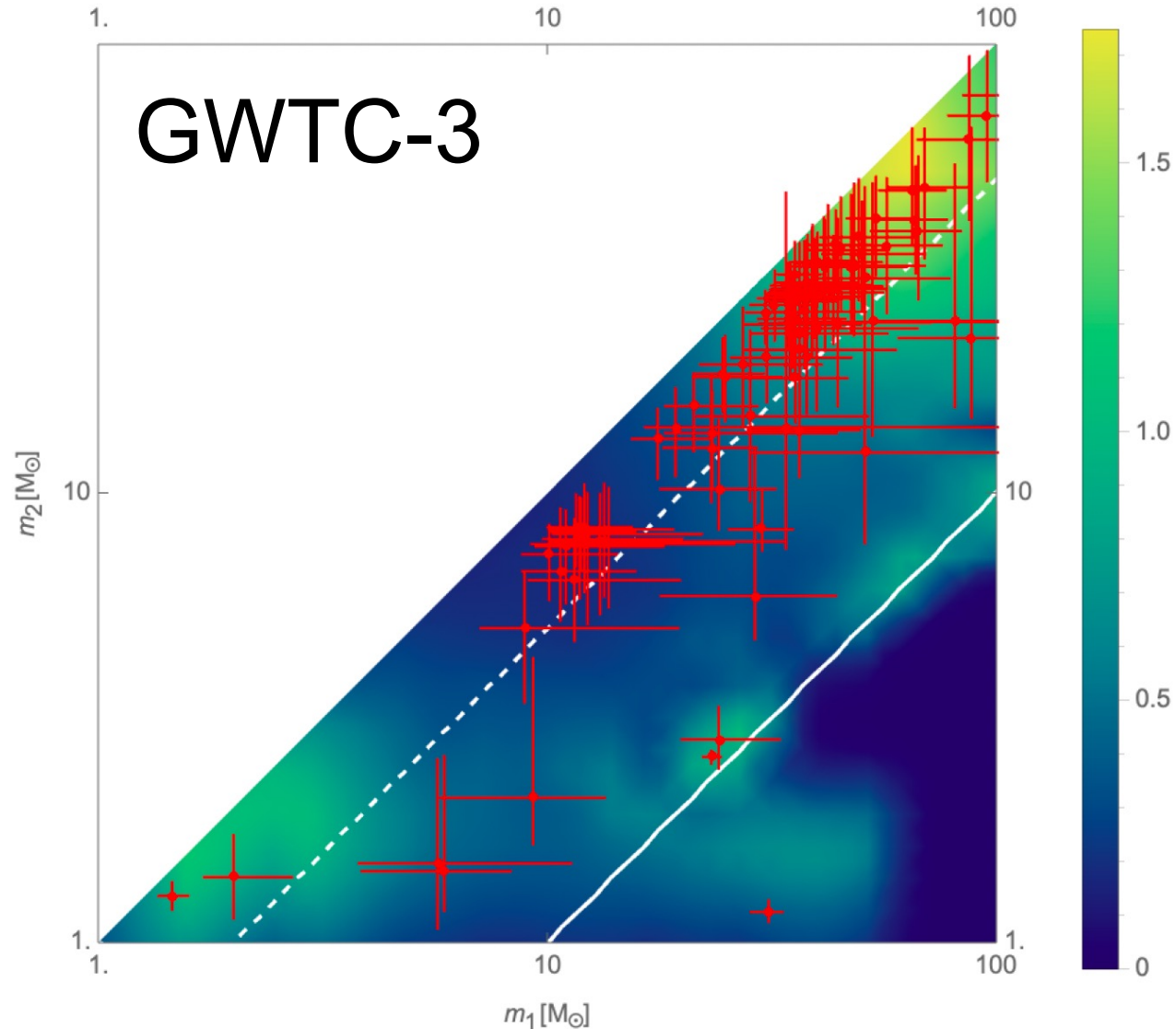


Model 2: De Luca 2020,  $n_s = 1$



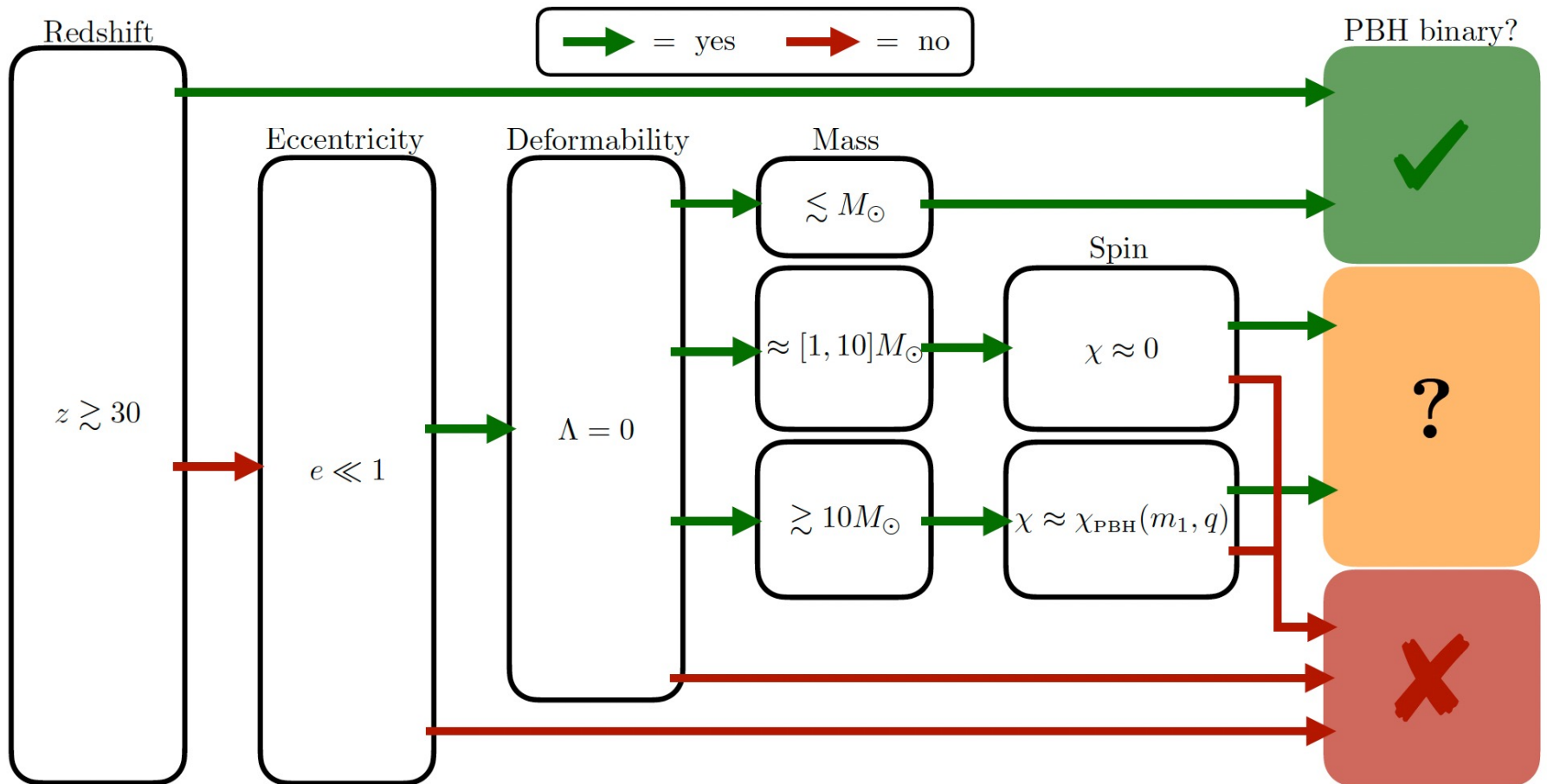
**Figure 21.** Expected merging rates of PBH of masses  $m_1$  and  $m_2$ , for the two mass models represented on Fig. 17 (top panels: Model 1, bottom panels: Model 2), for the two considered binary formation channel: primordial binaries (see Eq. 4.1) on the left panels, and tidal capture in halos (see Eq. 4.11) on the right panels. Figure produced for [359].

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**Figure 29.** Expected probability distribution of PBH mergers with masses  $m_1$  and  $m_2$  for a mass function with  $n_s = 0.97$  and LIGO sensitivity for O3 run. Solid and dashed white lines correspond to mass ratios  $q \equiv m_2/m_1$  of 0.1 and 0.5, respectively. From Ref. [215].

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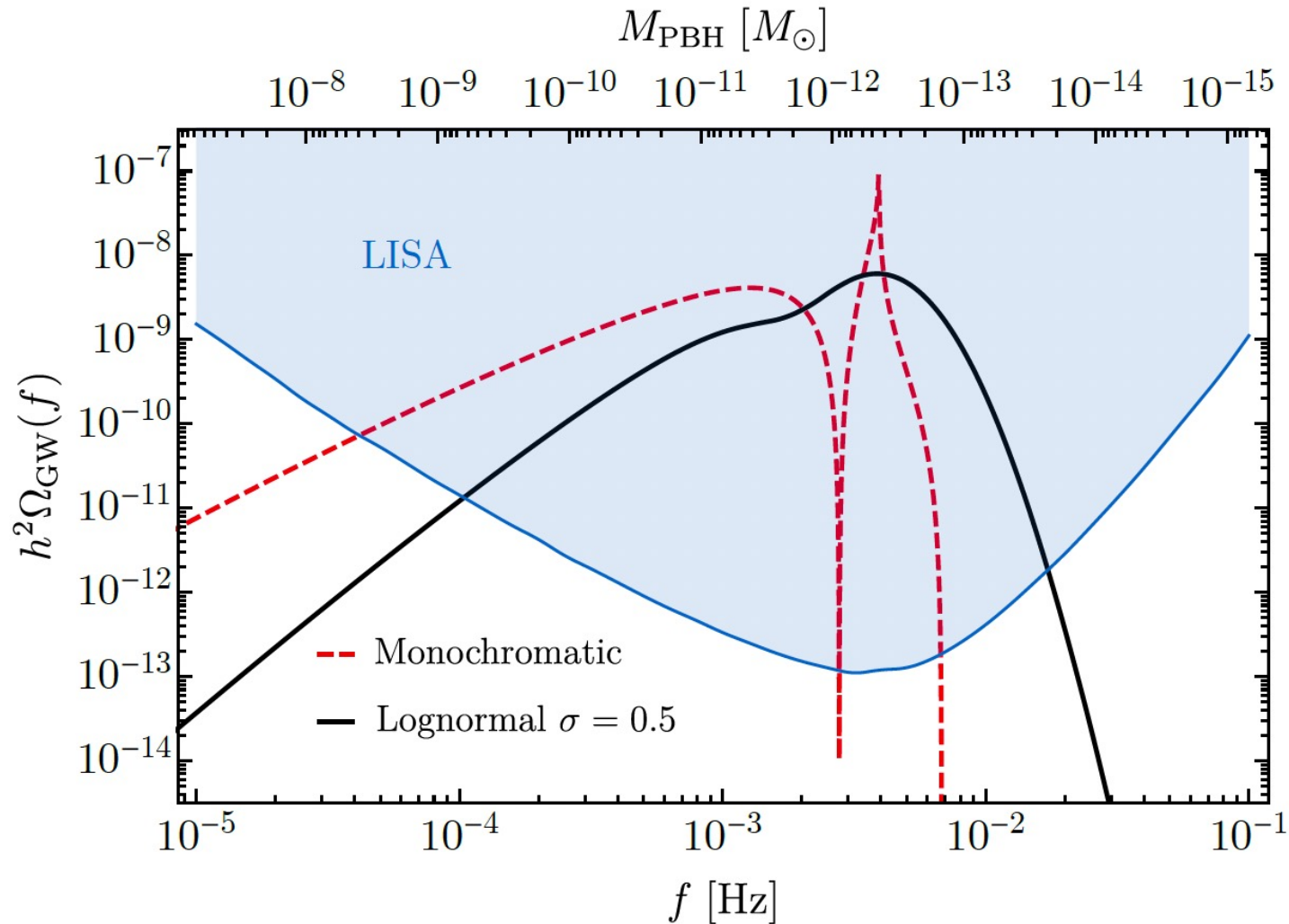
**Figure 23.** Figure taken from Ref. [82]. Schematic flowchart representing how to systematically rule out or potentially assess the primordial origin of a binary merger. These criteria are based on measurements of the redshift  $z$ , eccentricity  $e$ , tidal deformability  $\Lambda$ , component masses  $m$ , and dimensionless spin  $\chi$ . Each arrow indicates if the condition in the box is met (green) or violated (red), while the marks indicate: ✓) likely to be a PBH binary; ✗) cannot to be a PBH binary; ?) may be a PBH binary.

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## 5 Stochastic backgrounds

- 5.1 From second order curvature fluctuations
- 5.2 Gauge invariance of the SGWB spectrum
- 5.3 Impact of Primordial non-Gaussianity on Scalar Induced SGWB
- 5.4 Induced SGWB Anisotropies
- 5.5 From PBH Poisson fluctuations
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  - 5.6.1 Formulation
  - 5.6.2 Early binaries
  - 5.6.3 Late binaries in clusters
- 5.7 SGWB duty cycle
- 5.8 From close encounters
- 5.9 Summary

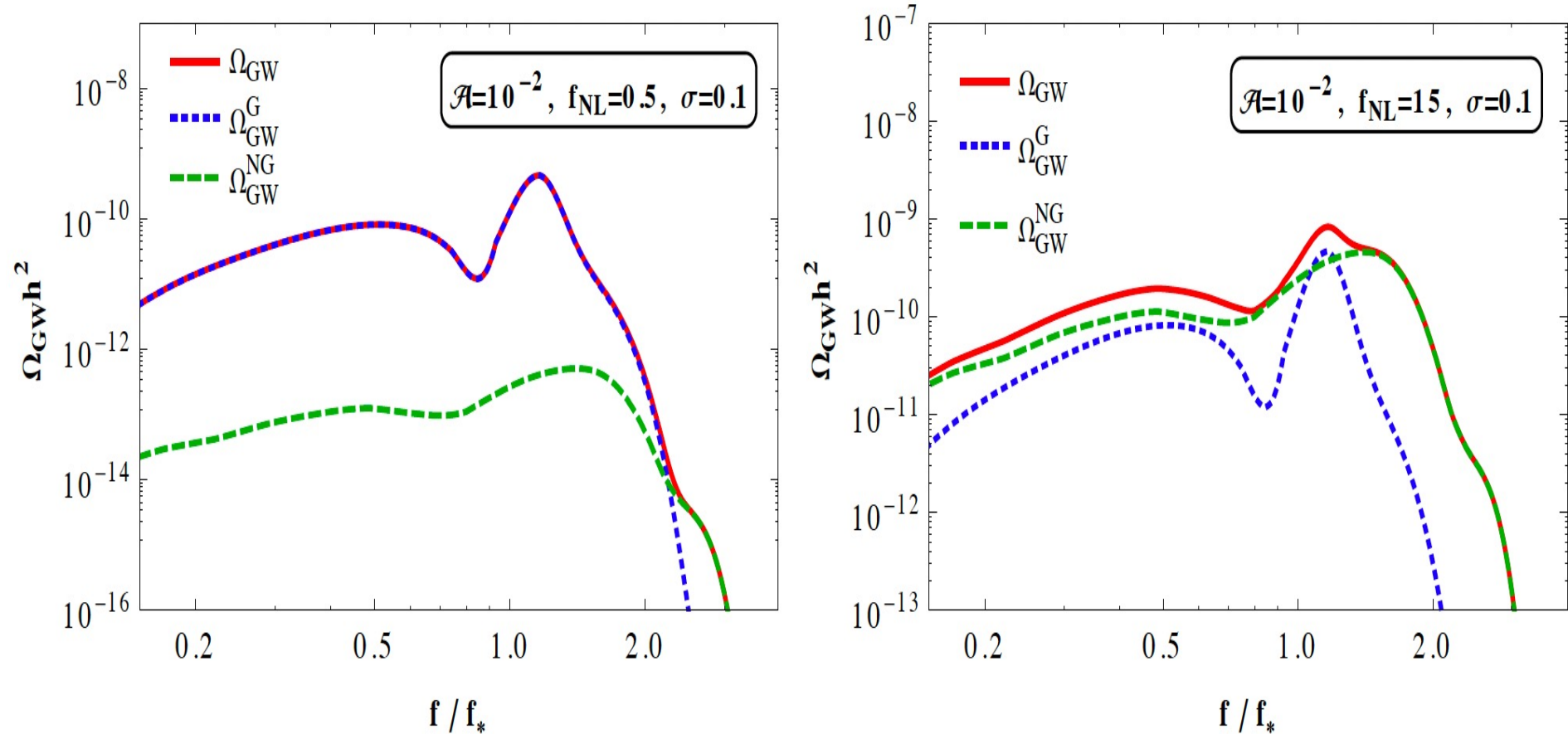
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**Figure 24.** Induced GWs spectra for both examples considered in Eqs. (5.13) and (5.15), for the parameters choice  $A_s = 0.033$ ,  $A_\zeta = 0.044$  and  $\sigma = 0.5$ . For comparison, we also show the estimated sensitivity for LISA [92] (the proposed design (4y, 2.5 Gm of length, 6 links) is expected to yield a sensitivity in between the ones dubbed C1 and C2 in Ref. [526]). The PBH mass corresponding to the characteristic frequency is depicted on the top horizontal axis, according to Eq. 5.1.

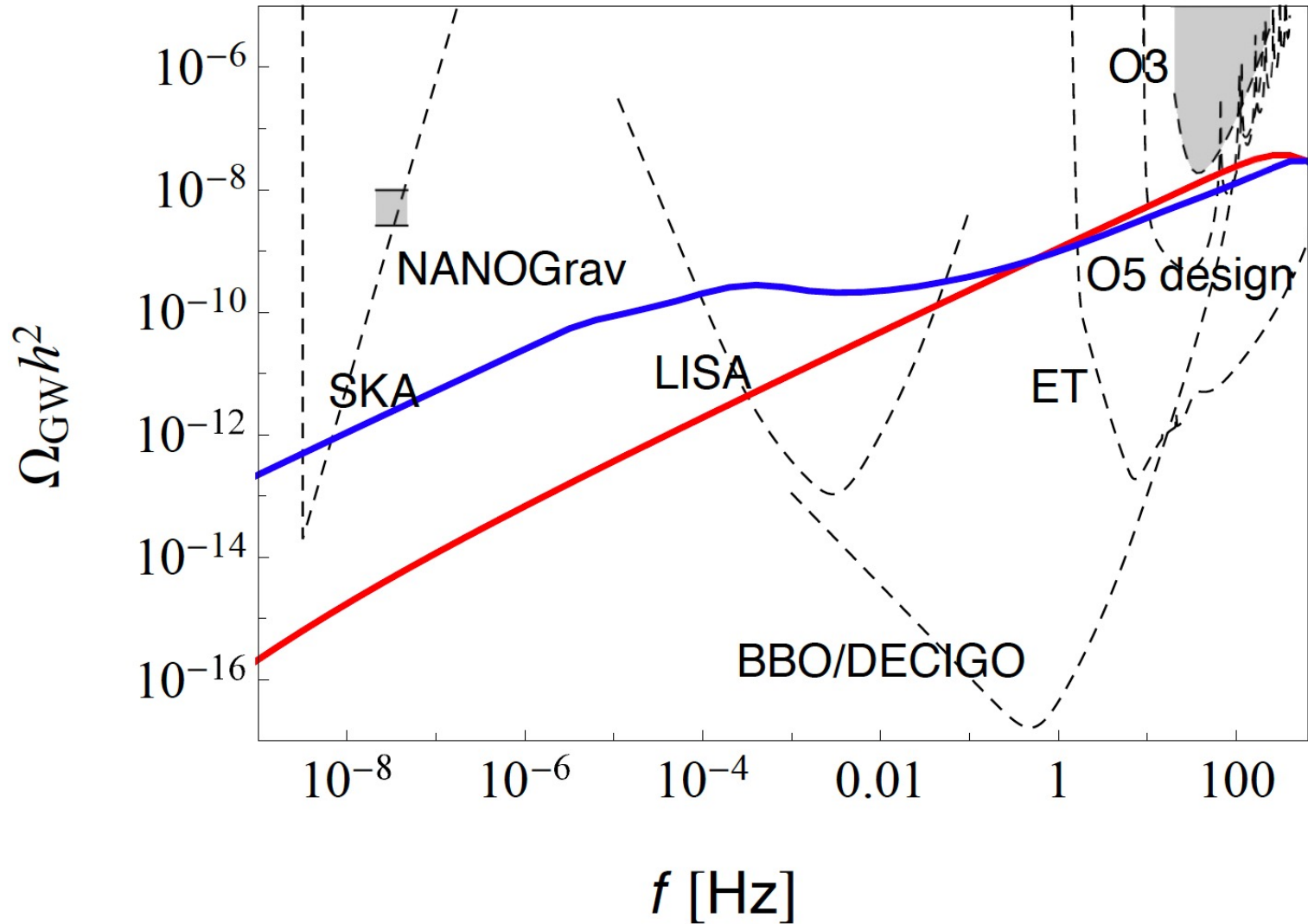


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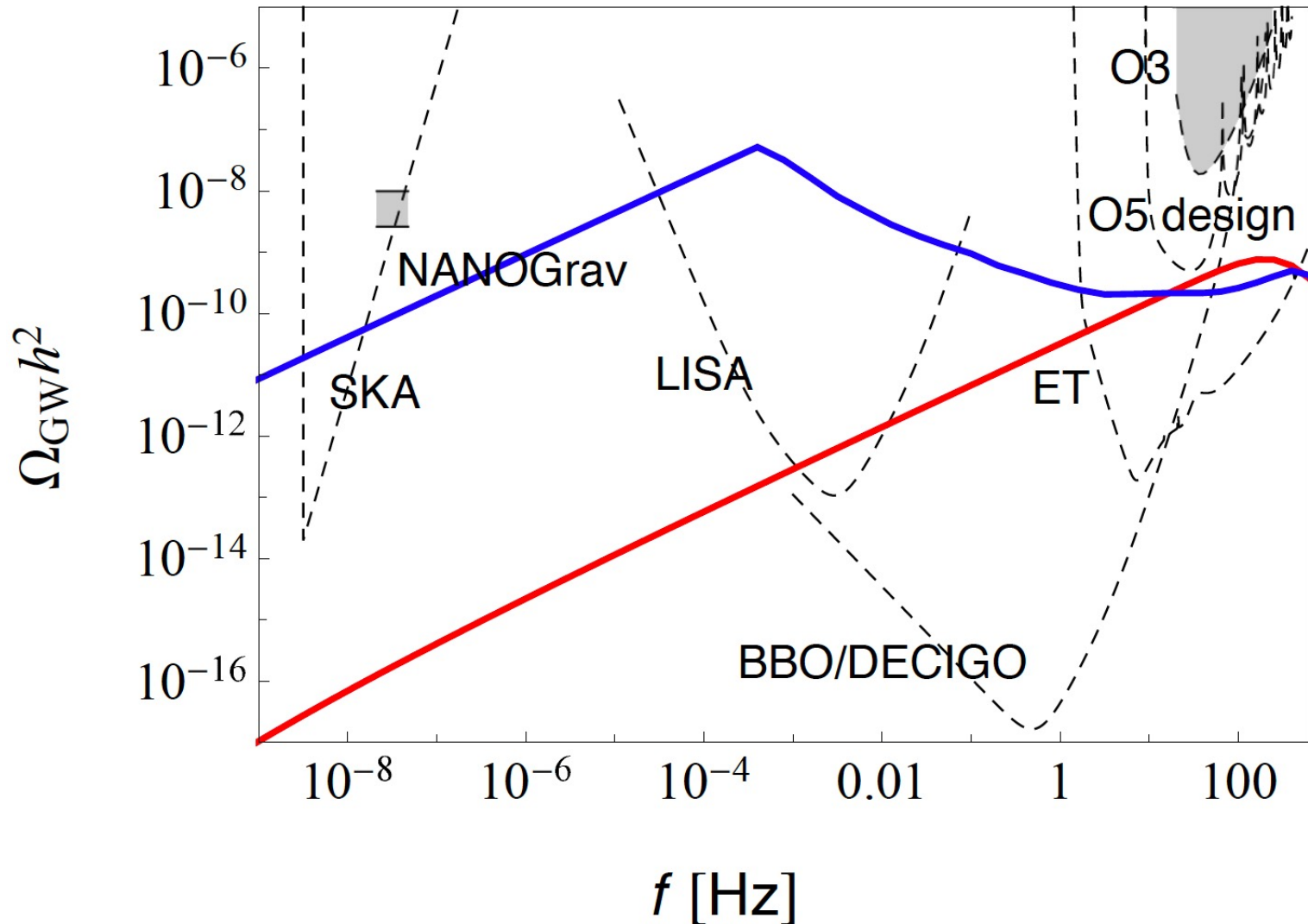
**Figure 27.** Second peak produced by primordial non-Gaussian component of curvature perturbations. Figure taken from [146].

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**Figure 30.** The SGWB spectrum  $\Omega_{\text{GW}} h^2$  for early PBH binaries with a log-normal mass function (in red, with central mass  $\mu = 2.5M_{\odot}$  and width  $\sigma = 1$ ) and a broad mass distribution with scalar spectral index  $n_s = 0.970$  (in blue), and  $f_{\text{PBH}} = 1$ .

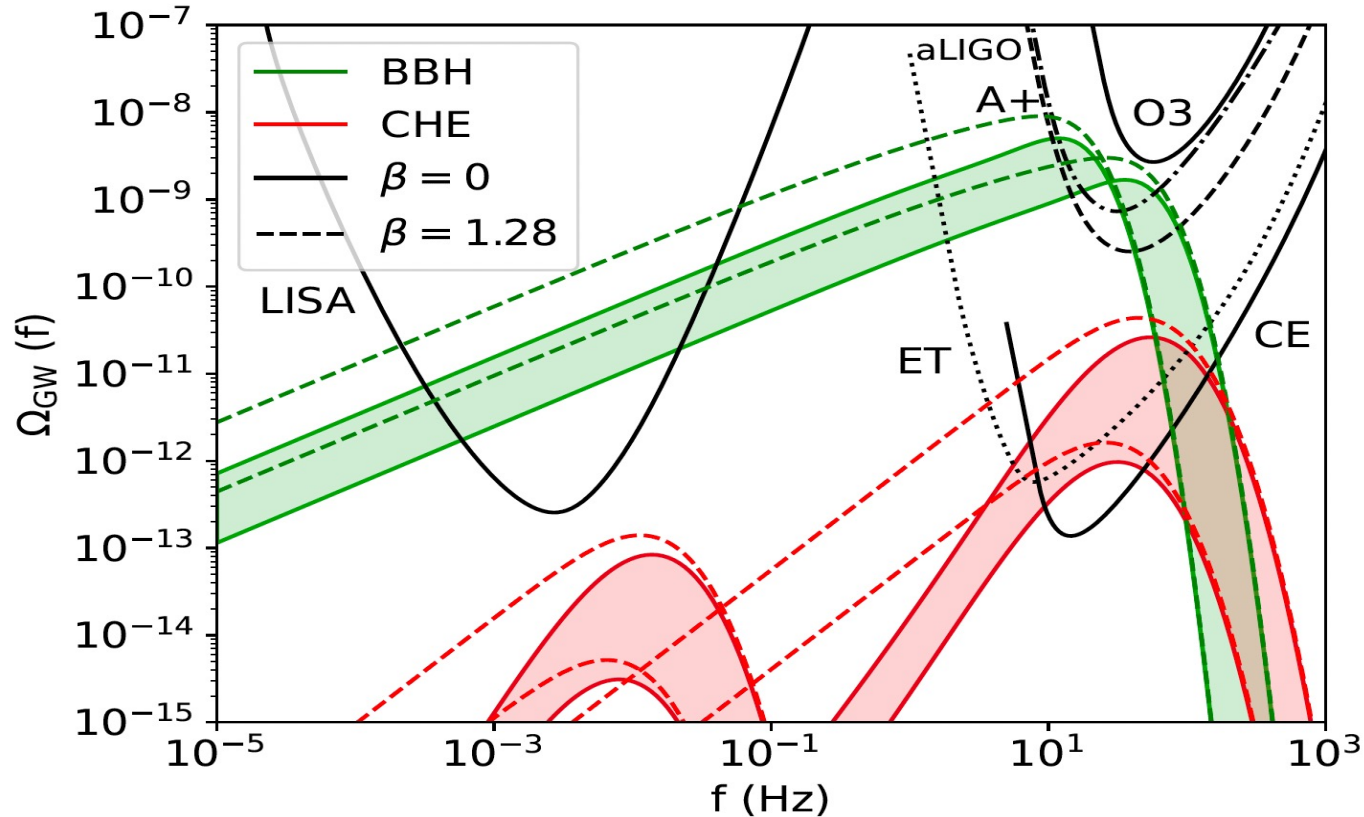
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**Figure 32.** The SGWB spectrum  $\Omega_{\text{GW}} h^2$  for late PBH binaries in clusters with  $f_{\text{PBH}} = 1$  for a log-normal mass function (in red, with the central mass  $\mu = 2.5M_{\odot}$  and the width  $\sigma = 1$ ) and a broad mass distribution (in blue, with  $n_s = 0.97$  and no running).

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## 5.8 From close encounters



**Figure 34.** Comparison of the SGWB spectrum originating from BBHs and CHEs, both for  $\beta = 0$  (solid) and  $\beta = 1.28$  (dashed), where  $\beta$  is a parameter characterizing the redshift dependence of the merger rate as  $\tau^{\text{BBH}} \propto (1+z)^\beta$ . For the BBH curves, we take  $m_1 = m_2 = 100 - 300 M_\odot$  and  $v_0 = 30 \text{ km/s}$ . The CHE curves correspond to the same range of masses with  $a_0 = 5 \text{ AU}$ ,  $y_0 = 2 \times 10^{-3}$  for frequencies around 10 Hz, and  $a_0 = 5 \cdot 10^7 \text{ AU}$ ,  $y_0 = 10^{-5}$  in the mHz range. For all cases, we assume log-normal distributions for  $a_0$ ,  $y_0$  and the PBH mass, of respective widths  $\sigma_a, \sigma_y = 0.1$ ,  $\sigma_m = 0.5$ , as well as  $f_{\text{PBH}} = 1$ . For a smaller fraction of PBHs, the GW spectral amplitude simply scales as  $\Omega_{\text{GW}} \propto f_{\text{PBH}}^2$ .

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## 6 GW and LSS correlations

### 6.1 Introduction

### 6.2 GW-galaxy cross-correlations for resolved events

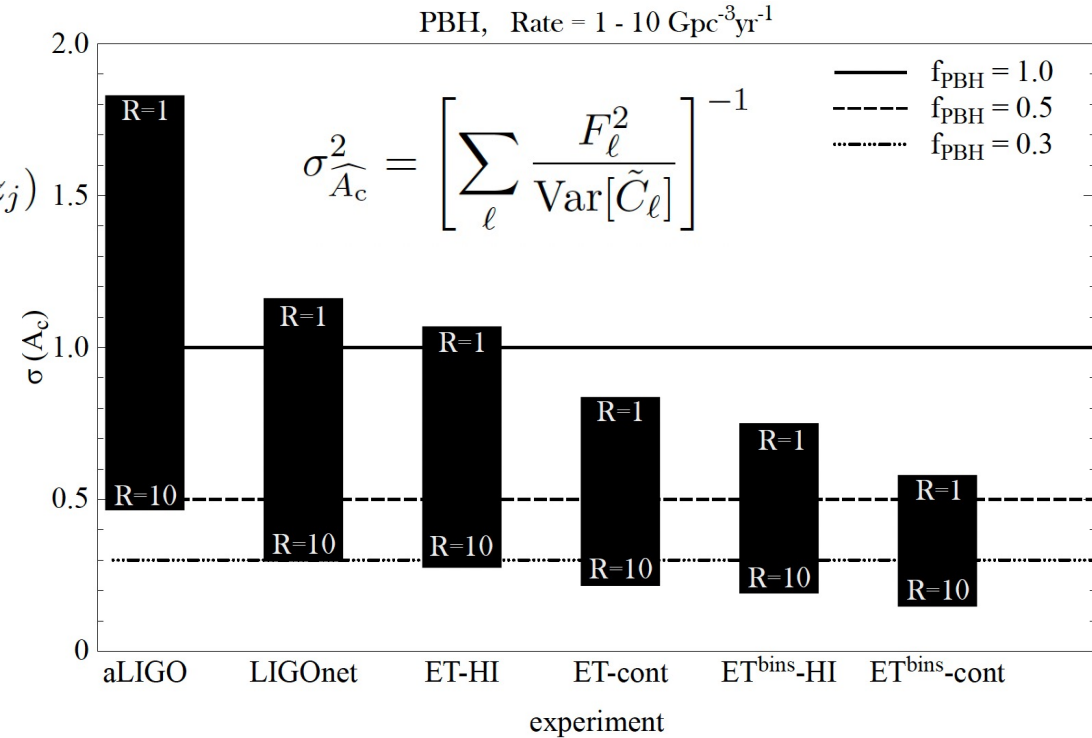
### 6.3 GW-galaxy cross-correlations for the stochastic background

### 6.4 GW $\times$ LSS forecasts

$$\langle a_{\ell m}^X(z_i) a_{\ell' m'}^{Y*}(z_j) \rangle = \delta_{\ell\ell'} \delta_{mm'} \tilde{C}_\ell^{XY}(z_i, z_j)$$

$$\widehat{A}_c = \frac{\sum_\ell \tilde{C}_\ell F_\ell / \text{Var}[\tilde{C}_\ell]}{\sum_\ell F_\ell^2 / \text{Var}[\tilde{C}_\ell]}$$

$$F_\ell \equiv d\tilde{C}_\ell / d\widehat{A}_c$$



**Figure 35.** Forecast errors on the cross-correlation amplitude,  $A_c$ , for different experiment combinations, varying merger rates and years of observations. Each column corresponds to a GW detector experiment, for merger rates from 1 to 10 Gpc<sup>-3</sup>yr<sup>-1</sup>. Horizontal lines show the expected difference in the cross-correlation between (late binary) PBH and stellar binaries, for different values of  $f_{\text{PBH}}$ .

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## 7 Current limits

### 7.1 PBH evaporation

7.1.1 Big-bang nucleosynthesis (BBN)

7.1.2 Gamma-rays and neutrinos from Hawking radiation

7.1.3 Positron annihilation in the Galactic Center

7.1.4  $e^\pm$  observations by Voyager 1

7.1.5 Ultra-faint dwarf galaxies

7.1.6 CMB limits on Dark radiation

7.1.7 Neutrinos

7.1.8 21-cm signal

7.1.9 Limits from DM particle production

7.1.10 Discussion and limitations

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7.2.2 Microlensing from stars in the galactic bulge

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7.2.4 Microlensing from quasars

7.2.5 Microlensing of supernovae

7.2.6 Femtolensing

7.2.7 Discussion and limitations

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## 7.3 Dynamical limits

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7.3.2 Ultra-faint dwarf galaxies

7.3.3 Dark Matter profile of dwarf galaxies (core-cusp problem)

7.3.4 Wide halo binaries

7.3.5 Disruption of stellar streams

7.3.6 PBHs in the Solar System

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7.4.3 21-cm signal

7.4.4 Cosmic Microwave Background (CMB) anisotropies

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7.5.5 Ultra-compact mini haloes

7.5.6 Discussion and limitations

## 7.6 Gravitational waves

7.6.1 LVC GW mergers

7.6.2 LVC sub-solar searches

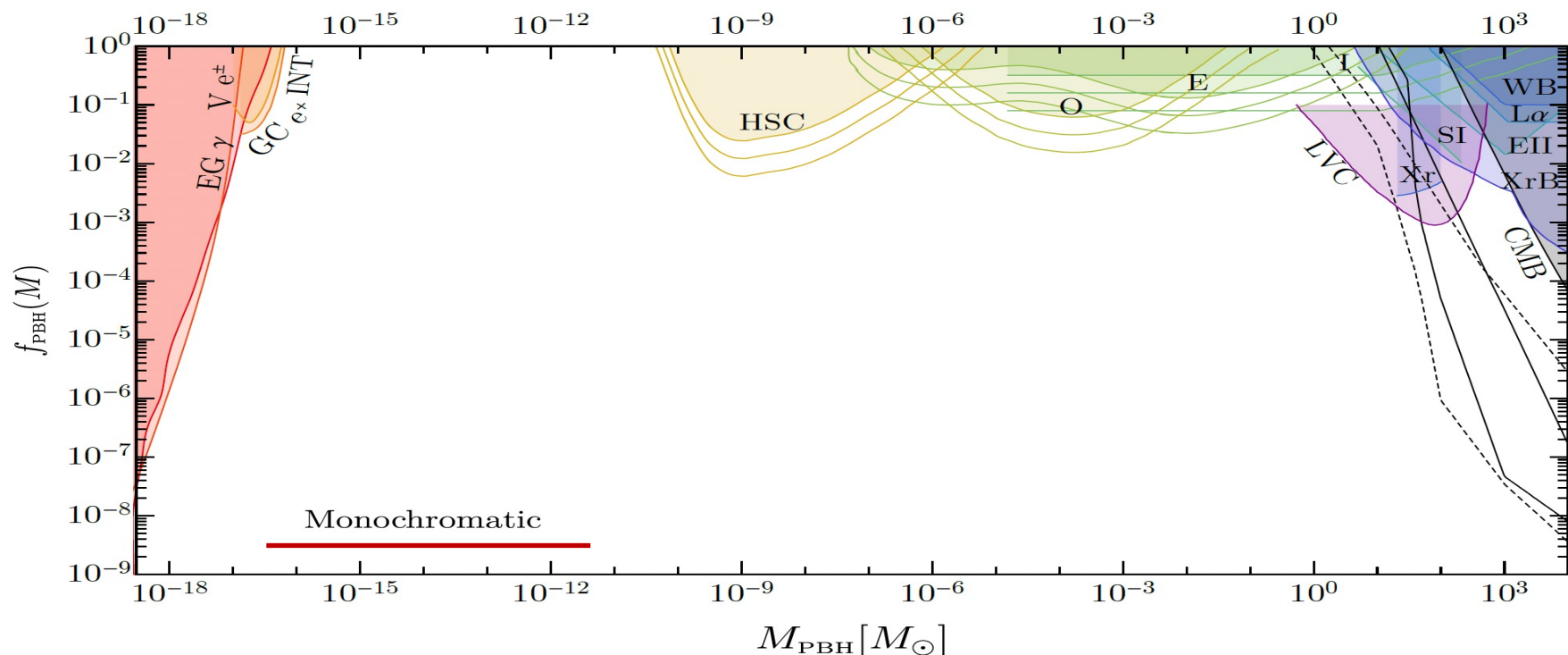
7.6.3 LVC SGWB

7.6.4 PTAs and NANOGrav

7.6.5 Continuous and high-frequency GWs from planetary-mass PBHs

7.6.6 Discussion and limitations

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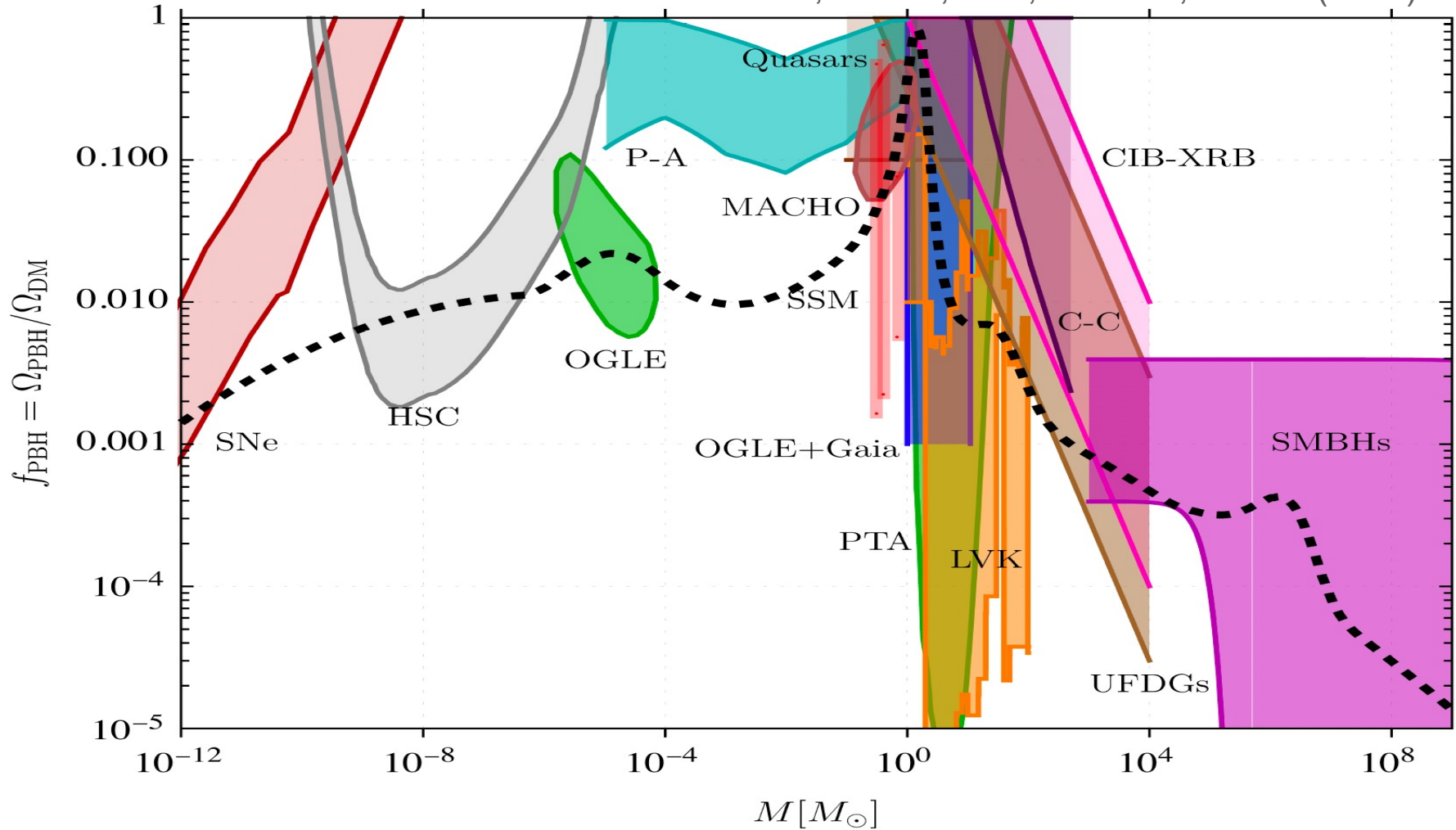


**Figure 39.** We show the most stringent **claimed** constraints in the mass range of phenomenological interest. They come from the Hawking evaporation producing extra-galactic gamma-ray (EG  $\gamma$ ) [627],  $e^\pm$  observations by Voyager 1 (V  $e^\pm$ ) [628], positron annihilations in the Galactic Center (GC  $e^+$ ) [629] and gamma-ray observations by INTEGRAL (INT) [630] (for other constraints in the ultra-light mass range see also [143, 631–635]). We plot microlensing searches by Subaru HSC [110, 113], MACHO/EROS (E) [636, 637], Ogle (O) [114] and Icarus (I) [638]. Other constraints come from CMB distortions. In black dashed, we show the ones assuming disk accretion (Planck D in Ref. [372] and Ref. [639], from left to right) while in black solid the ones assuming spherical accretion (Planck S in Ref. [372] and both photo- and collisional ionization in Ref. [100], from left to right). Only Ref. [372] includes the effect of the secondary dark matter halo in catalysing accretion. Additionally, constraints coming from X-rays (Xr) [640] and X-Ray binaries (XrB) [641] are shown. Dynamical limits coming from the disruption of wide binaries (WB) [642], survival of star clusters in Eridanus II (EII) [106] and Segue I (SI) [643, 644] are also shown. LVC stands for the constraint coming from LIGO/Virgo Collaboration measurements [62, 78, 377, 408]. Constraints from Lyman- $\alpha$  forest observations (L $\alpha$ ) come from Ref. [645]. We neglect the role of accretion which has been shown to affect constraints on masses larger than  $\mathcal{O}(10)M_\odot$  [77, 371] in a redshift dependent manner. See Ref. [65] for a comprehensive review on constraints on the PBH abundance. Notice that there are no stringent bounds in the asteroid mass range [646, 647] where LISA may constrain PBHs through the search of a second order SGWB.



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Carr, Clesse, JGB, Hawkins, Kuhnel (2023)



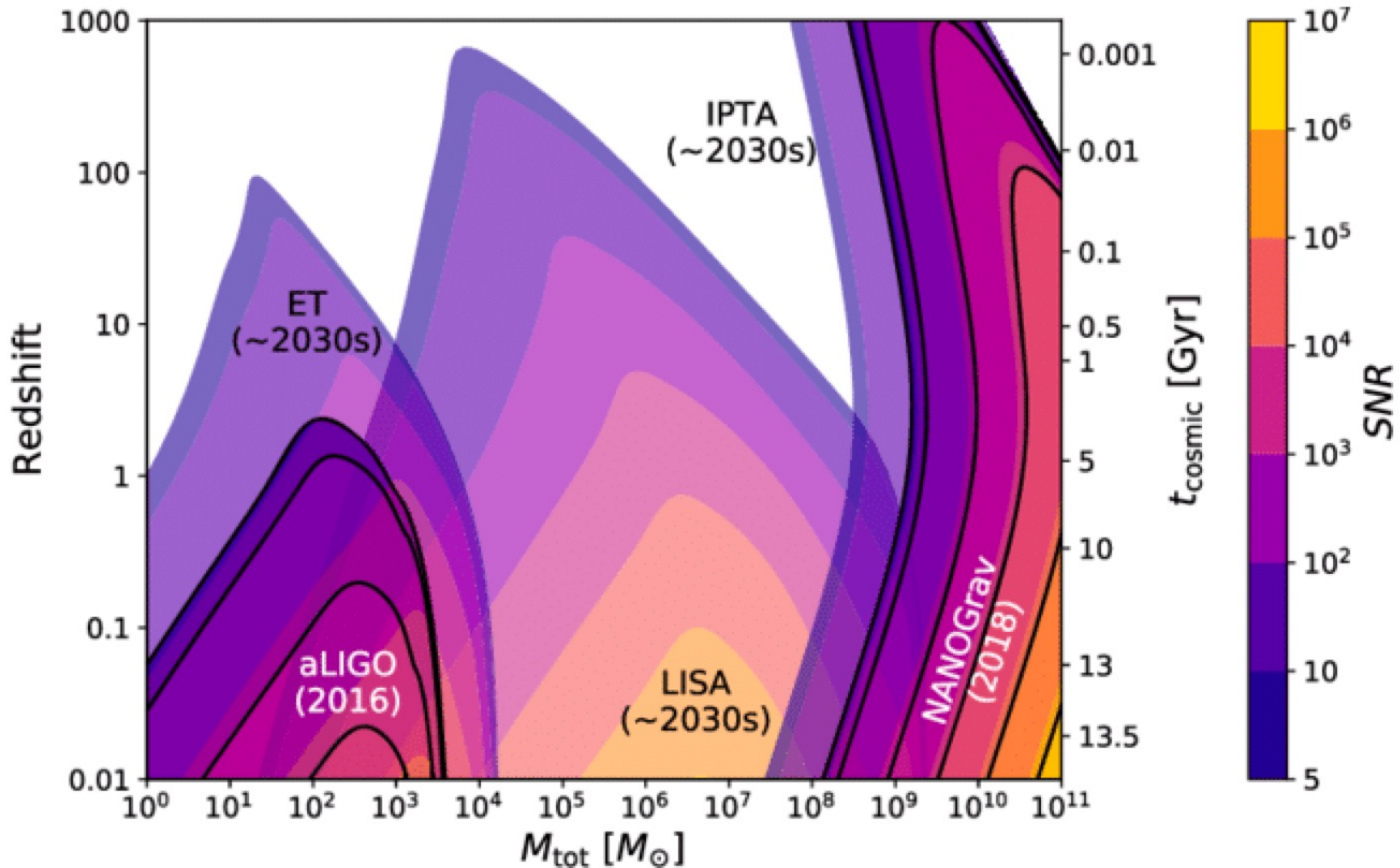
**Figure 1.** Summary of positive evidence for PBHs in terms of  $f_{\text{PBH}}$  values required by or consistent with the claimed detections. These come from PBH-attributed signals from supernovæ (SNe), various microlensing surveys (Gaia, HSC, OGLE, MACHO), pixel-lensing (P-A), gravitational waves (LVK), ultra-faint dwarf galaxies (UFDGs), supermassive black holes (SMBHs), core/cusp (C-C) profiles for inner galactic halos, and correlations of the source-subtracted cosmic infrared and X-ray backgrounds (CIB-XRB).

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## 8 Detectability with LISA

- 8.1 SGWB from second order curvature fluctuations
- 8.2 SGWB from ultralight evaporated PBHs
- 8.3 SGWB from early binaries
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- 8.5 Intermediate-mass binary mergers
- 8.6 Extreme mass ratio inspirals
- 8.7 High-redshift binary mergers
- 8.8 Combination with LVK observations
- 8.9 Quasi-monochromatic continuous waves from subsolar PBHs
- 8.10 Observing near-Earth asteroid-mass primordial black holes
- 8.11 Summary

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**Figure 44.** Redshift range of LISA for equal-mass BBH coalescences as a function of the total system mass and comparison with the range of other detectors and pulsar timing arrays. The color scale represents the expected SNR.

# Primordial Black Hole Review

## 9 Conclusions

One important conclusion of our analysis of the recent literature on PBHs is that strong claims, in one or another direction, are certainly premature and generally rely on hypotheses that are very hard to test. Thus, there is still a huge amount of work to be done before making precise predictions. We therefore encourage others to pursue this direction and hope that our work will provide motivation and tools for researchers from other fields to join the growing PBH community.

### Future

Our work will soon be accompanied by a numerical code to compute all these gravitational-wave observations, for a wide variety of models. This code is currently under development and testing, but some of our key figures were already produced using it. This is a first promising perspective of this work. Then, our review also sheds light on the physical processes that are still subject of large uncertainties, like the role of PBH accretion and clustering, or the PBH binary disruption. We also discuss the fact that all signatures are highly model dependent.