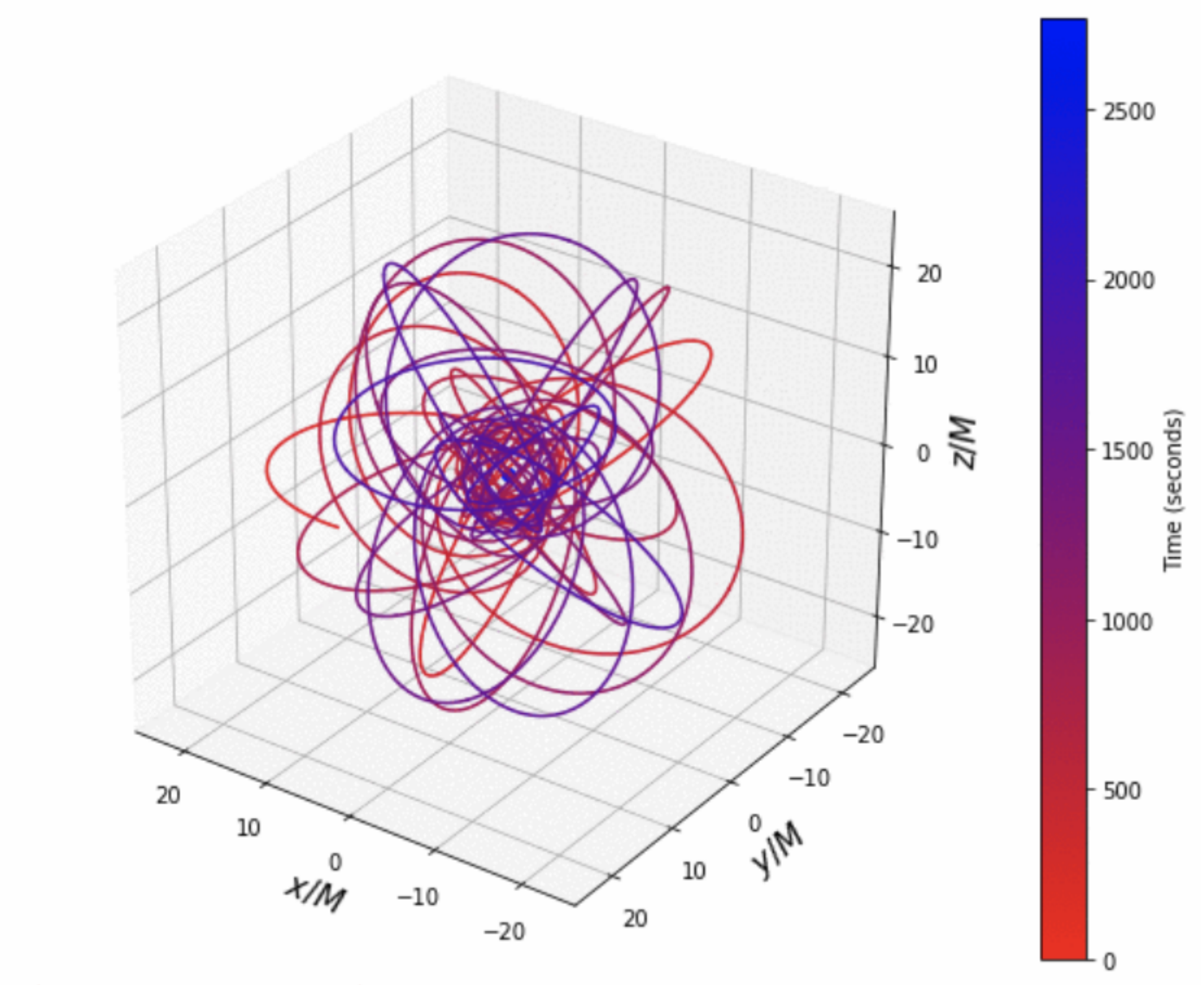
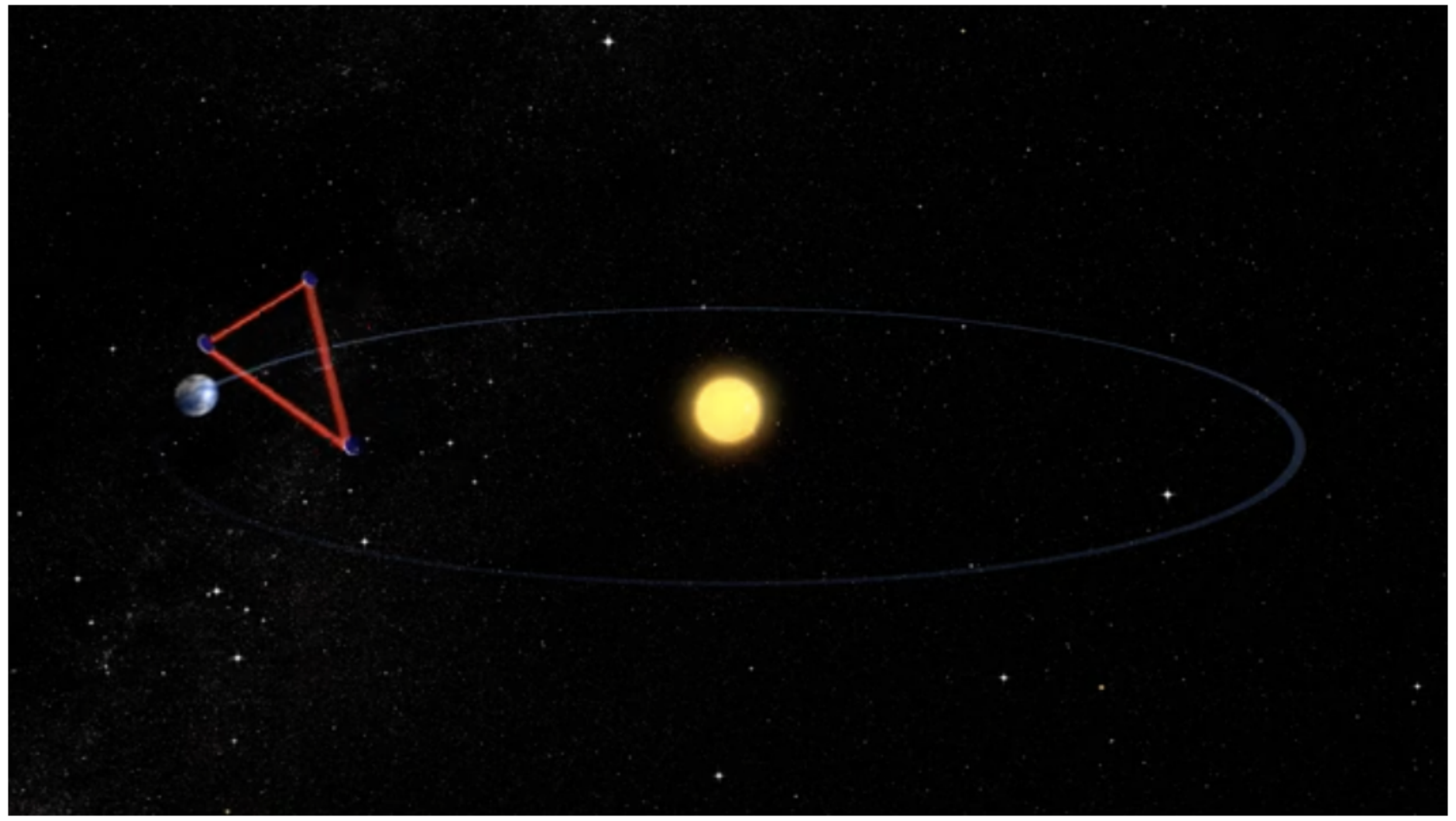
Constraining modified GW propagation with extreme mass-ratio inspirals



Credit: Lorenzo Speri

Chang Liu



In collaboration with D. Laghi, N. Tamanini et al.
Laboratoire des 2 Infinis, Toulouse & Peking University

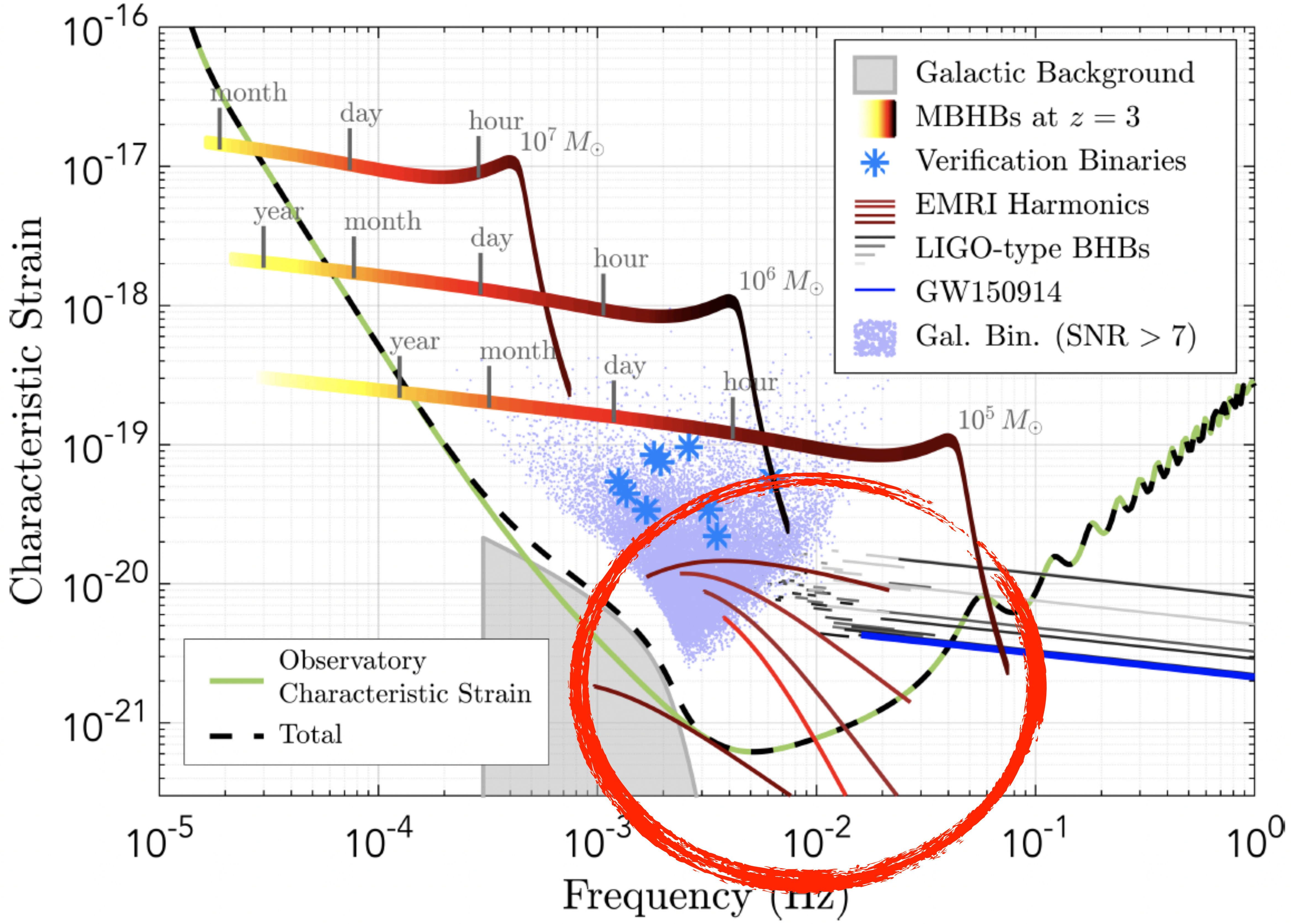
10th LISA Cosmology Working Group Workshop 07/06/2023

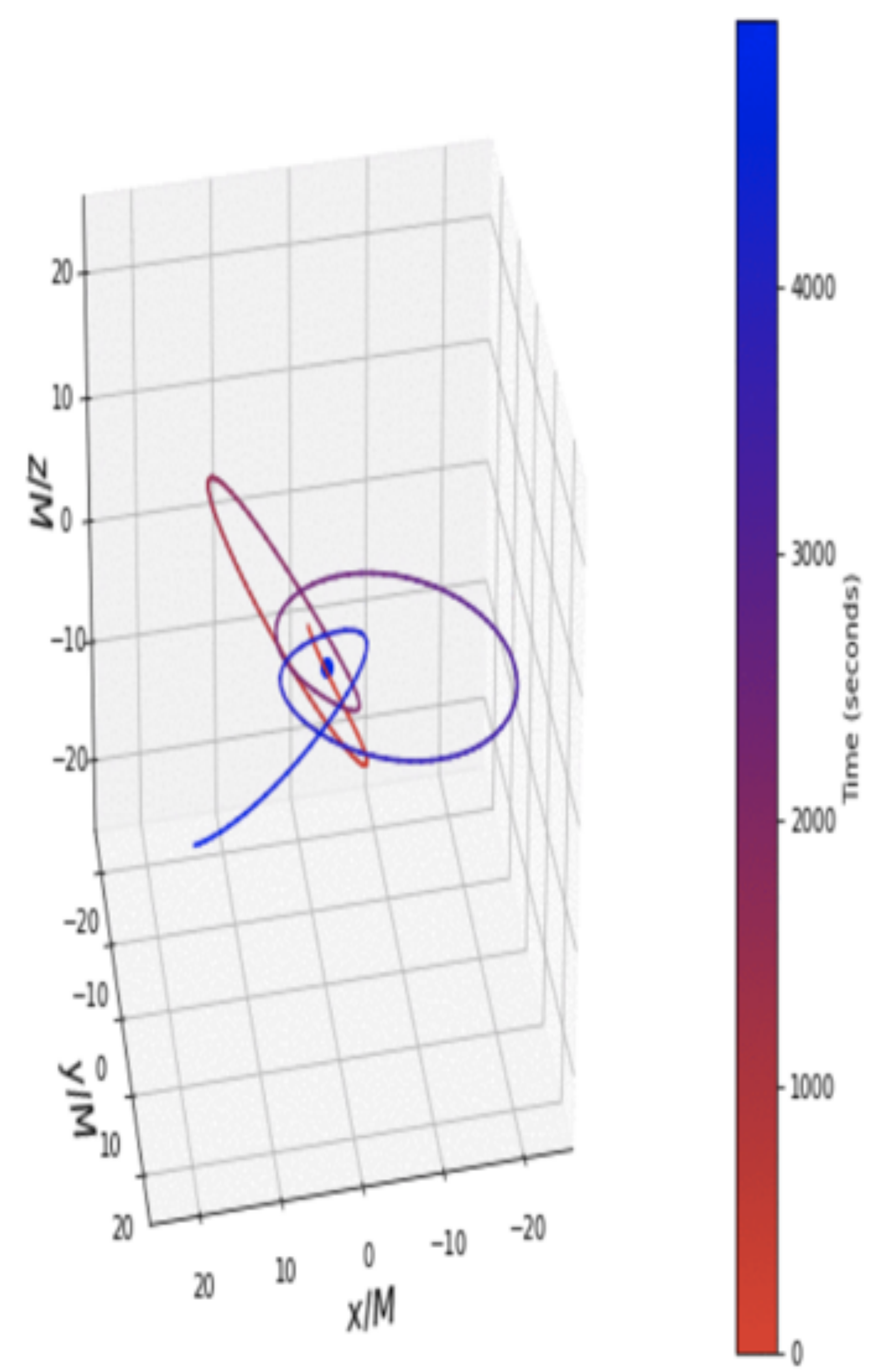
Outline

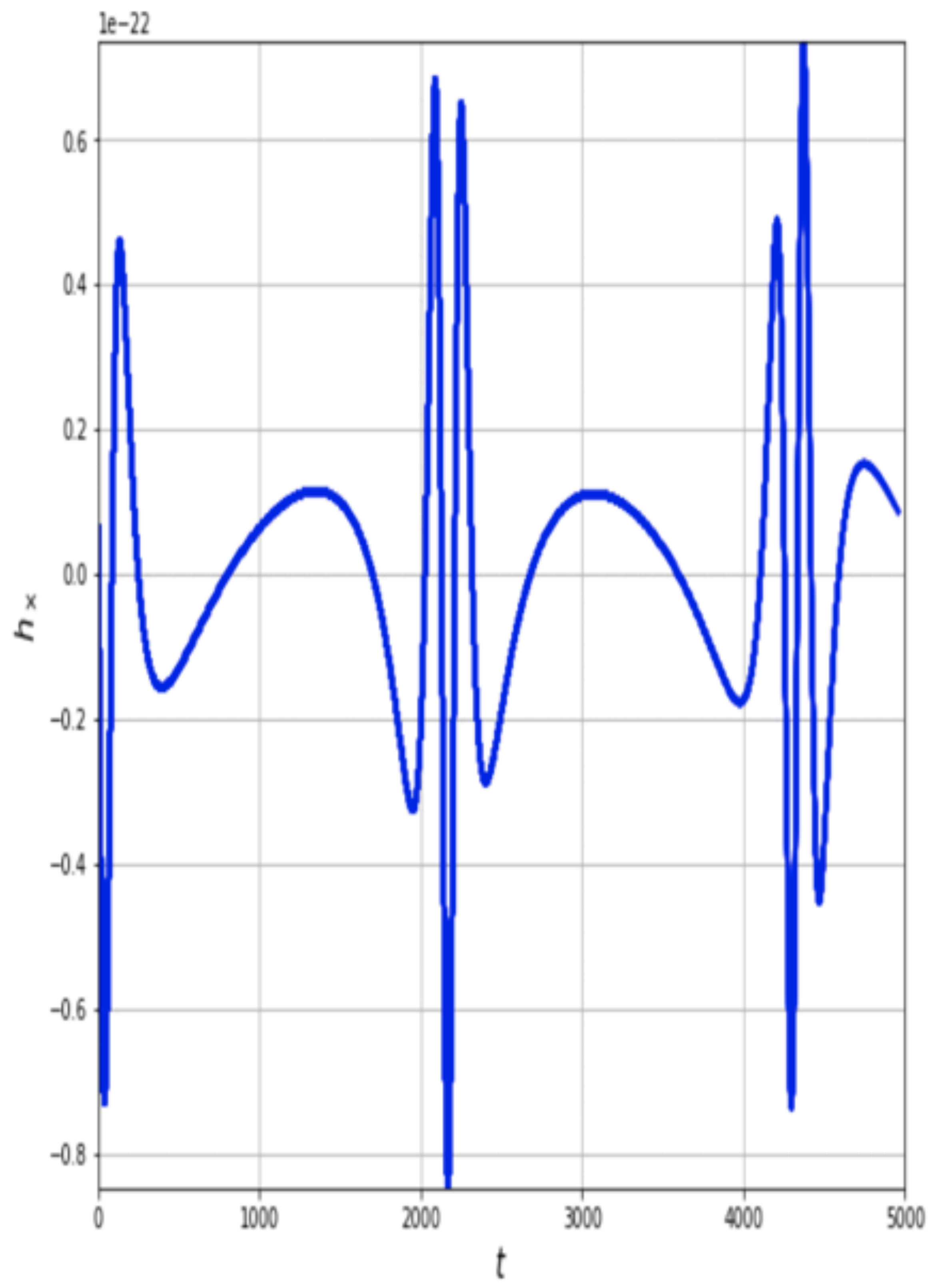
- LISA and extreme mass-ratio inspirals (EMRIs)
- Cosmology and modified GW propagation
- Inference with LISA dark sirens
- Preliminary results
- Conclusion and future prospects

Extreme Mass-Ratio Inspirals (EMRIs)

Credit: Lorenzo Speri, Ollie Burke







Mass of the Kerr Black Hole
$M \sim 10^5 - 10^7 M_\odot$

Mass Compact Object
$\mu \approx 10\, M_\odot$

Mass Ratio
$\eta \approx 10^{-6} - 10^{-4}$

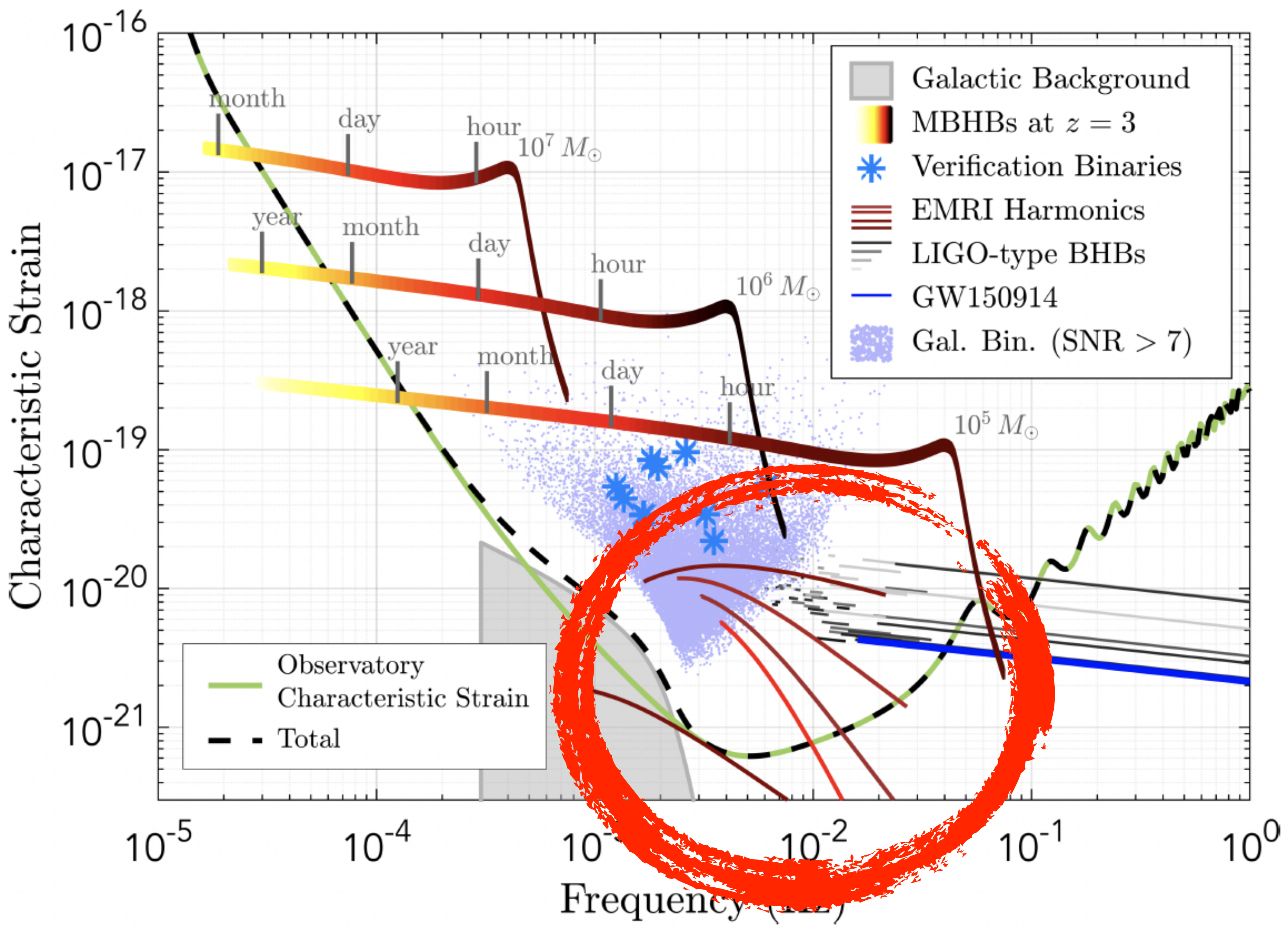
Evolution Scale
$T_{\rm ev} \sim \frac{1}{\eta}$

GW Frequency
$f \approx 10^{-4} - 10^{-2}$ Hz

Observation of EMRIs with LISA

[Babak et al. 2017]

- EMRI waveform: AKK (Analytical Kludge Kerr, optimistic)
- Sensitivity curve: 2.5 Gm LISA configuration, 2017
- Observation time: 10 yrs
- Parameter estimation: based on EMRI catalog M1 by Fisher Matrix analysis (SNR>20)



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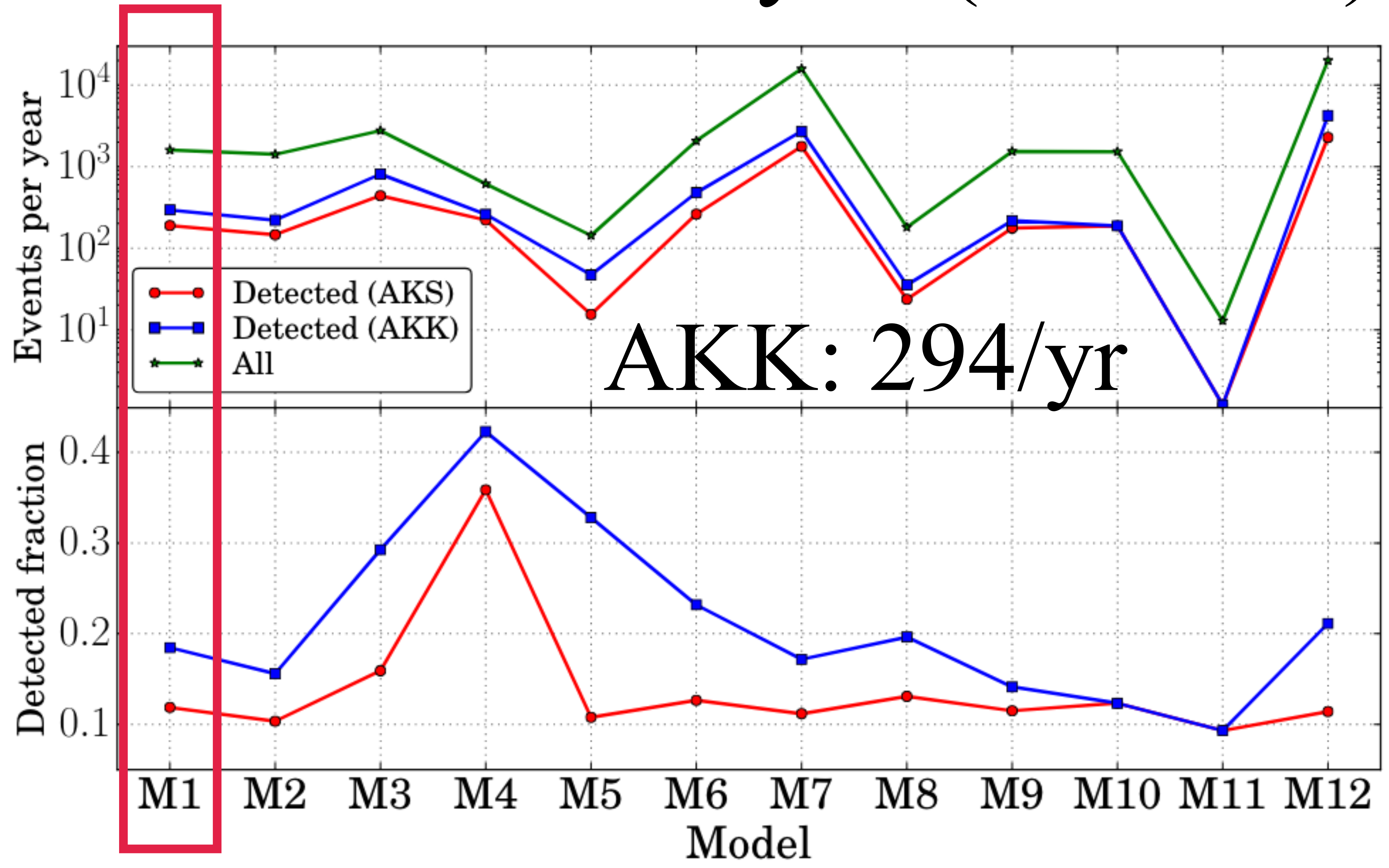
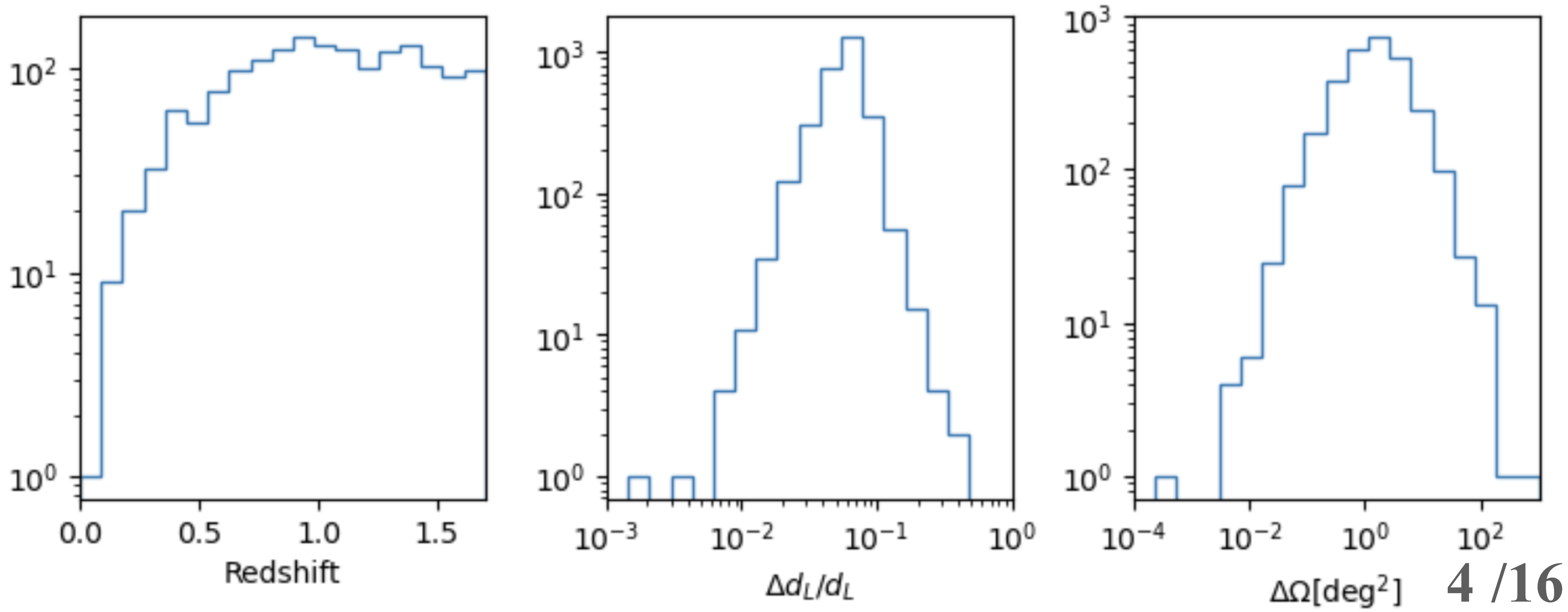


M1 Parameter distribution of the detected sources:



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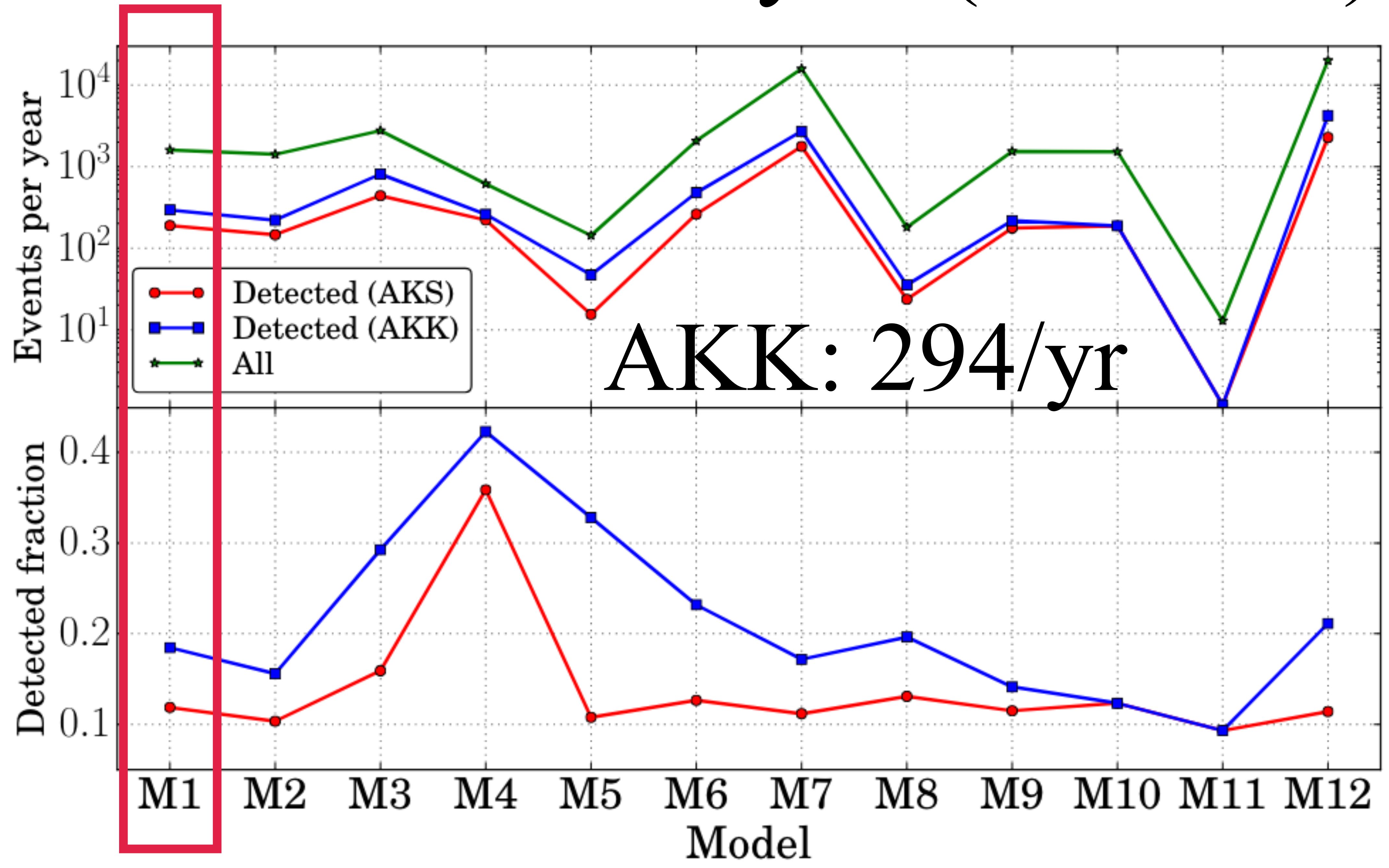

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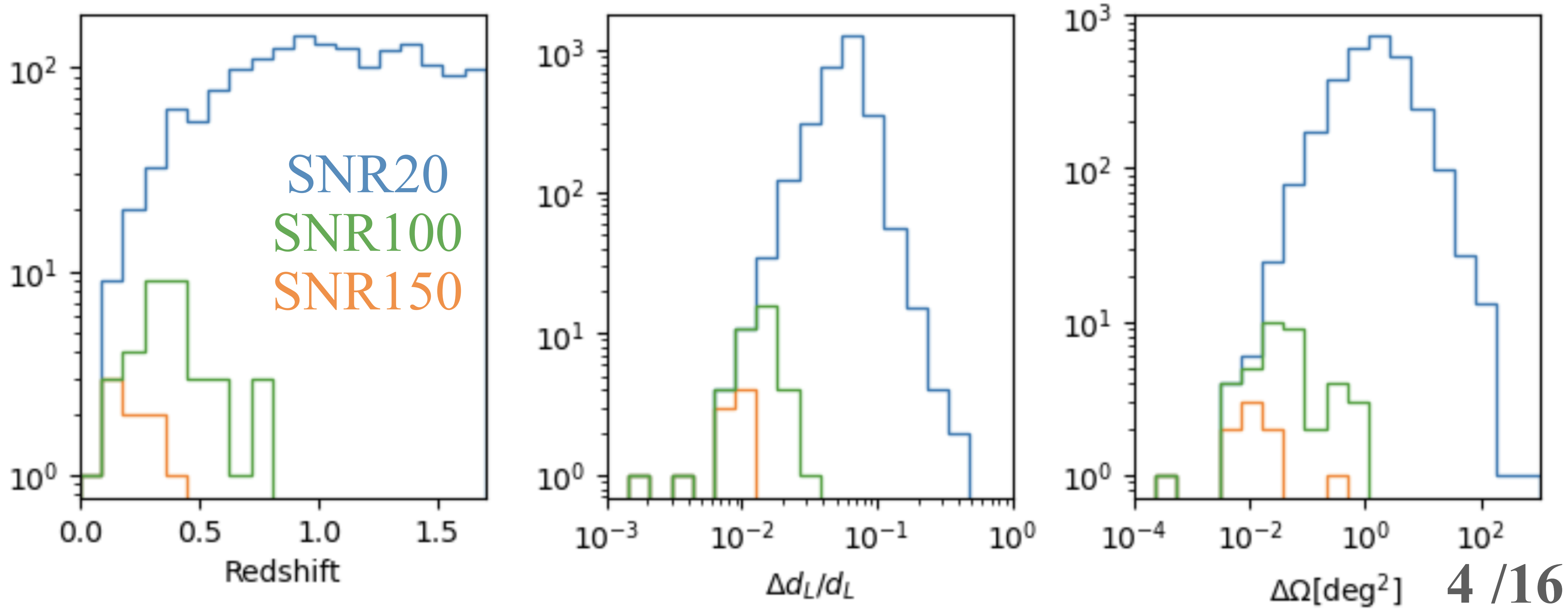


Cosmology with GWs

[Schutz 1986]

From GW:

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

From EM: z { Counterpart: Bright sirens / No counterpart: Dark sirens

$$d_L(z) = \frac{1+z}{H_0}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\,,$$

$\xrightarrow{\Lambda\mathrm{CDM}}$ $H_0, \Omega_m, \Omega_\Lambda$

$$E(z) = \sqrt{\Omega_M(1+z)^3 + \rho_{\mathrm{DE}}(z)/\rho_0}\,,$$

Modified GW propagation

Considering how GWs propagate across cosmological distances, the free propagation of tensor perturbations over FRW is governed by the equation:

$$\tilde{h}''_A + 2\mathcal{H}\tilde{h}'_A + c^2k^2\tilde{h}_A = 0$$

$\tilde{h}'_A$: derivative with respect to cosmic time η

Friction term

- Affect amplitude

$$\tilde{h}''_A + 2\mathcal{H}[1-\delta(\eta)]\tilde{h}'_A + c^2k^2\tilde{h}_A = 0$$

$$d_L^{\rm gw}(z) = d_L^{\rm em}(z)\exp\left\{-\int_0^z \frac{dz'}{1+z'}\,\delta(z')\right\}$$

GW speed

- Affect speed

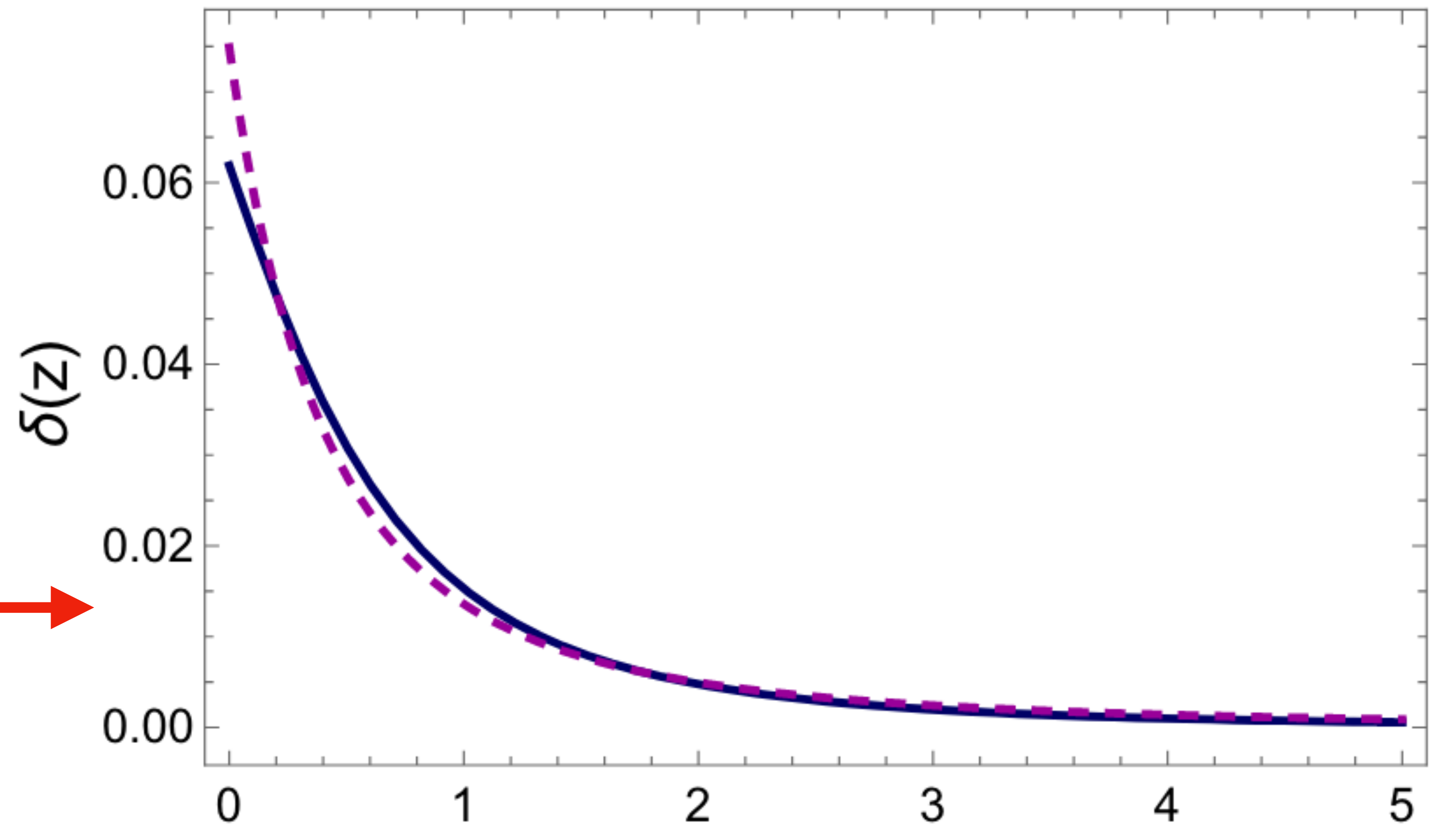
From GW170817

$$|c_{\rm gw} - c|/c < O(10^{-15})$$

Modified gravity: an example

[Belgacem et al., 2018]

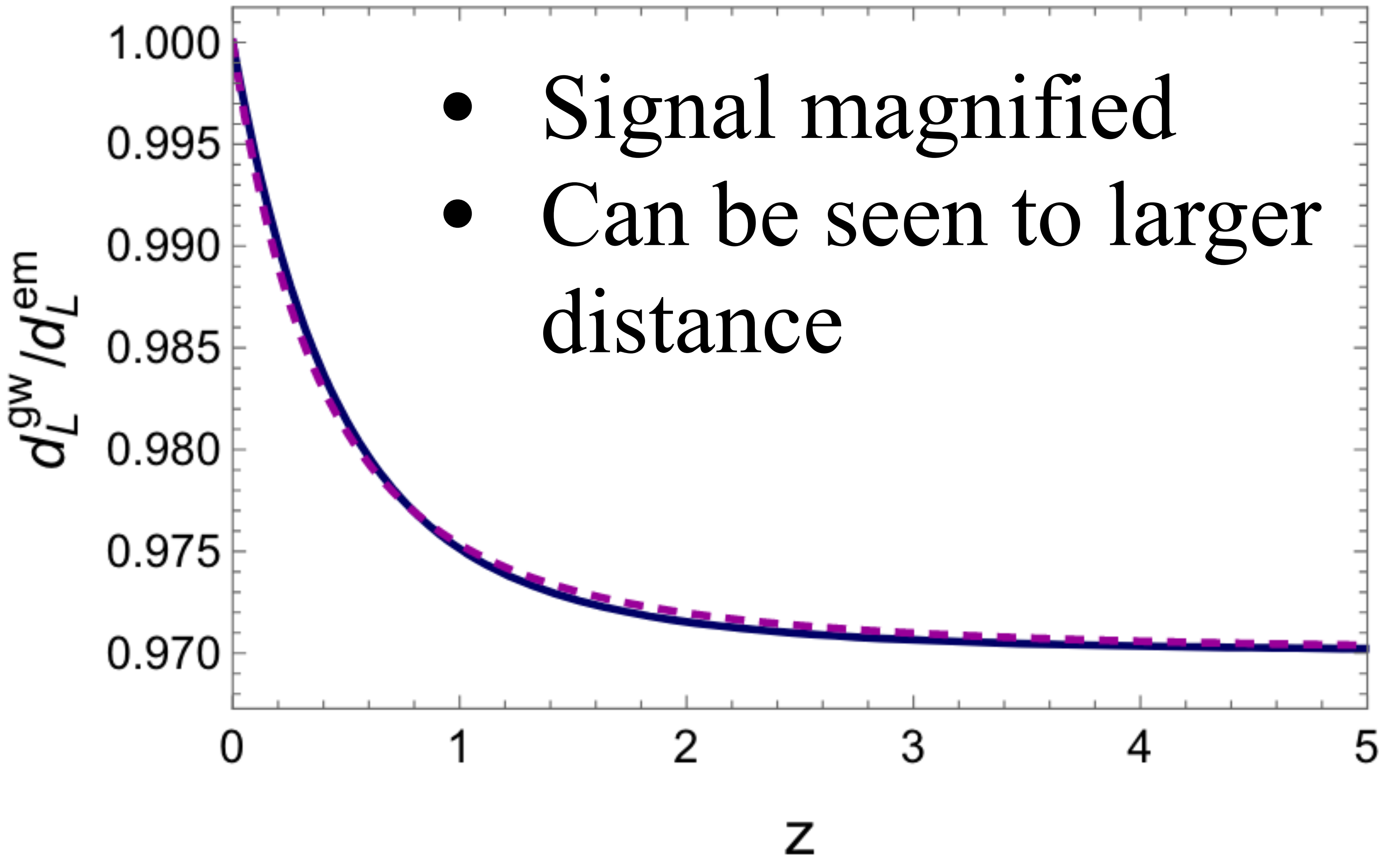
The function δ(η) is predicted explicitly by the RR model:



$$d_L^{\mathrm{gw}}(z) = d_L^{\mathrm{em}}(z) \exp\left\{ -\int_0^z \frac{dz'}{1+z'}\, \delta(z') \right\}$$

Parametrization

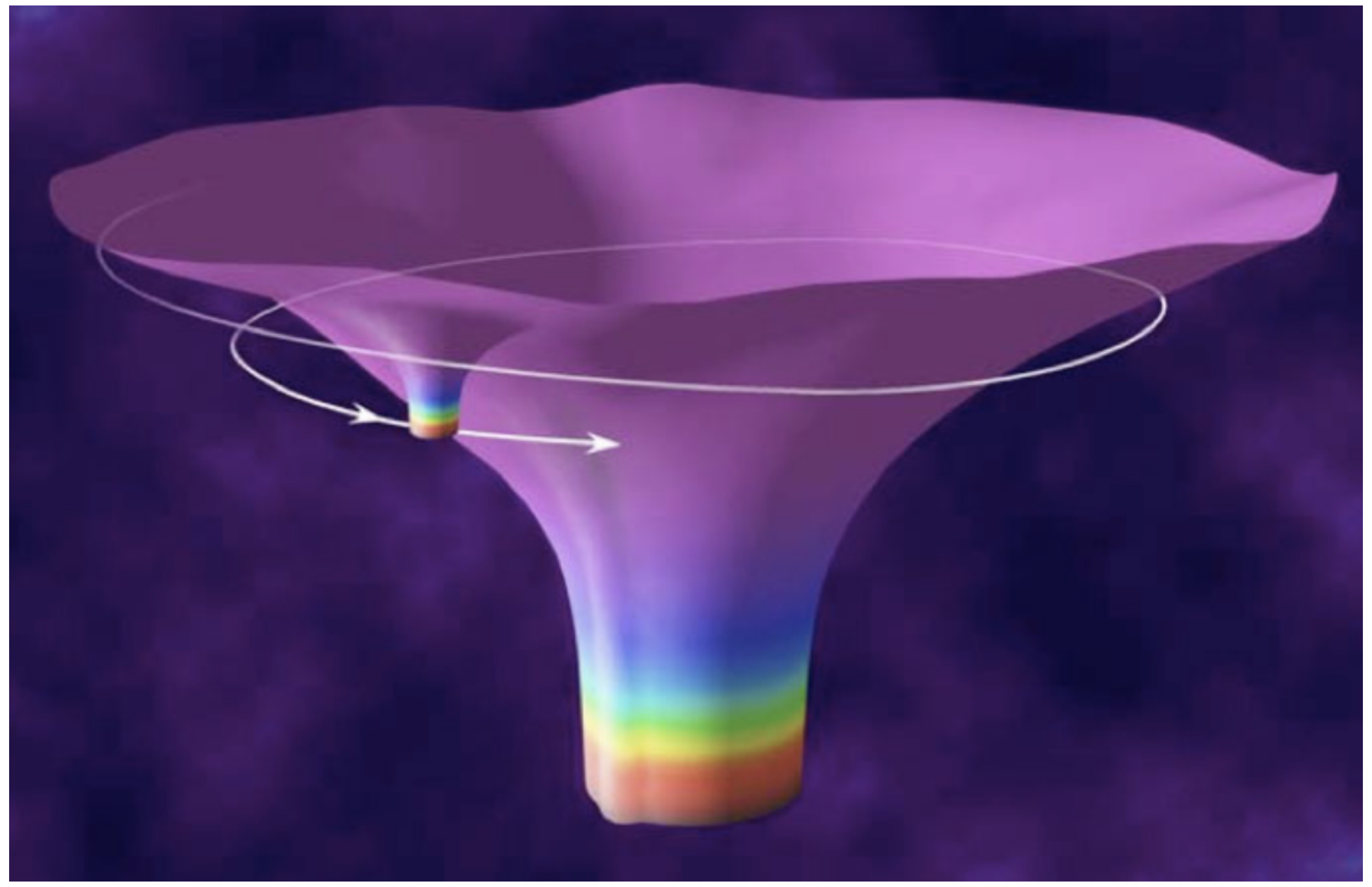
$$\frac{d_L^{\mathrm{gw}}(z)}{d_L^{\mathrm{em}}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$

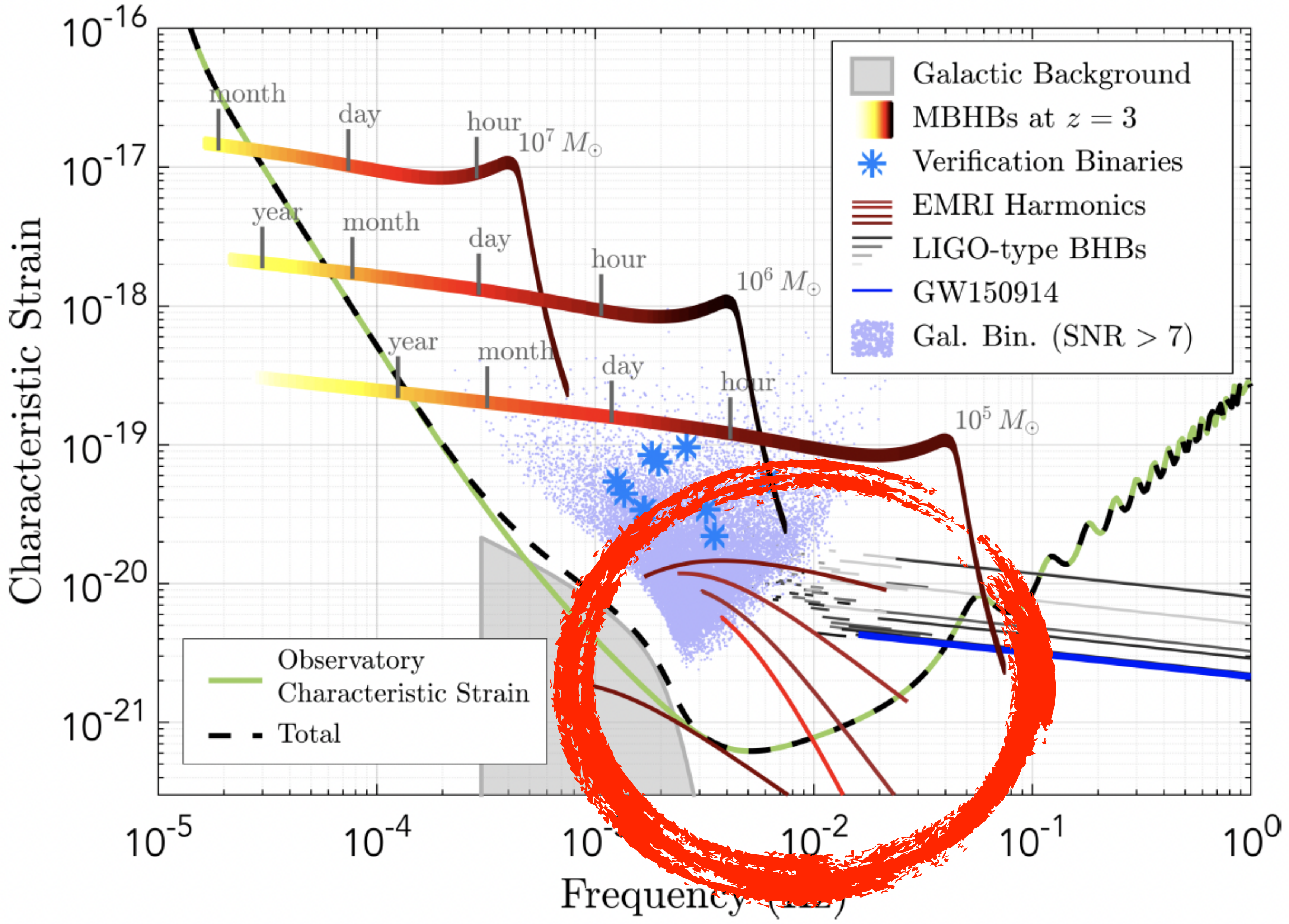


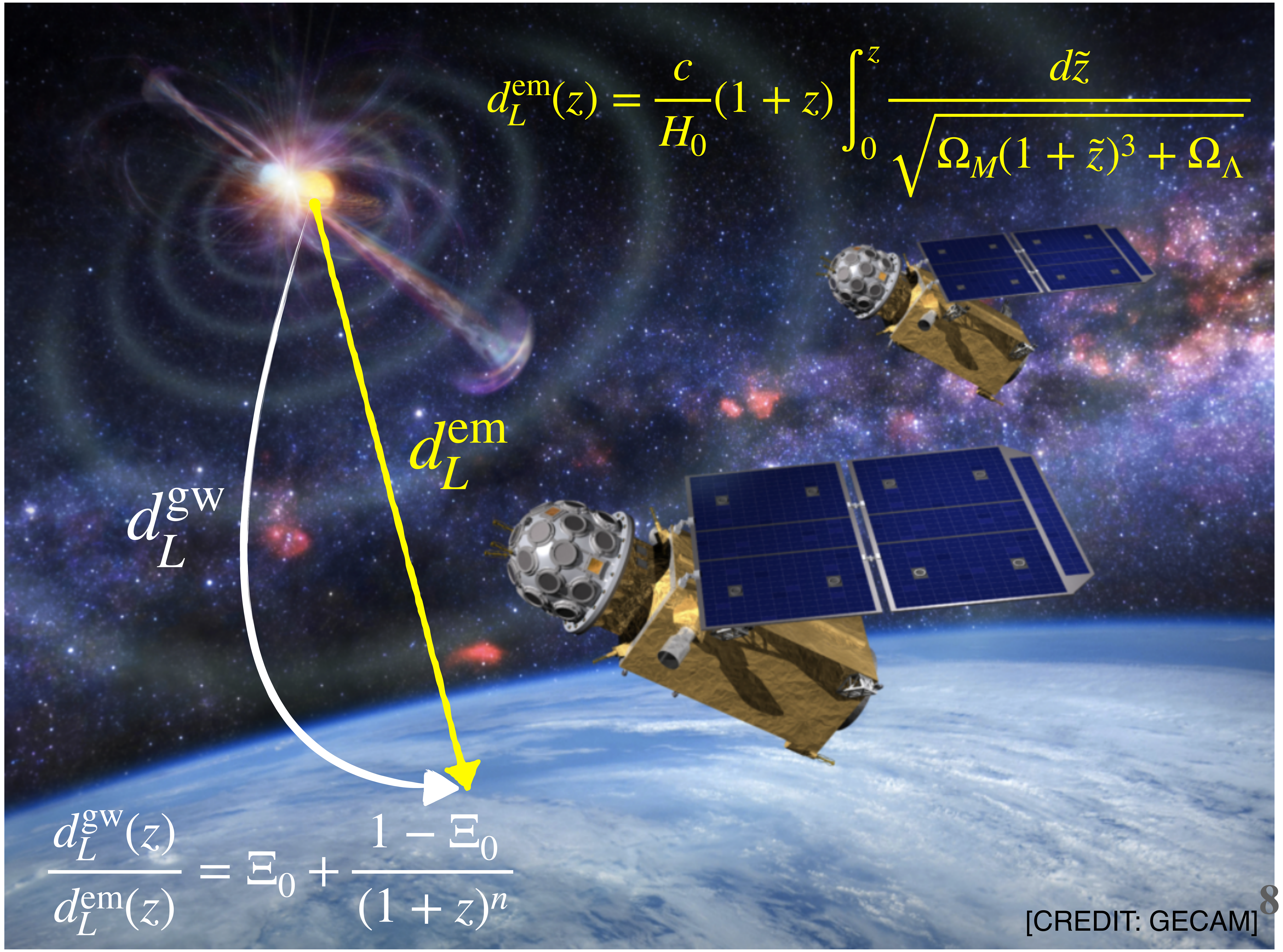
The model predicts $\delta(z=0) = 0.062$ [n=5/2 and Ξ_0=0.970]

- $d_L^{\mathrm{GW}}(z=0)/d_L^{\mathrm{EM}}(z=0) = 1$
- $d_L^{\mathrm{GW}}(z)/d_L^{\mathrm{EM}}(z)$ saturates to a constant Ξ_0
- Ξ_0: crucial parameter, fix the asymptotic value of $d_L^{\mathrm{GW}}(z)/d_L^{\mathrm{EM}}(z)$ at large z
- n only determines the precise shape of the function that interpolates from z = 0 and large z

Constraining **modified GW propagation** with **extreme mass-ratio inspirals**







Bayesian theorem in cosmology

$D \equiv \{D_1, \ldots, D_N\}$ i_{th}GW data → $D_i = \left\{\hat{d}_L, \hat{\theta}_{gw}, \hat{\phi}_{gw}\right\}_i$ + errors

$\Omega \equiv \{H_0, \Omega_m, \Xi_0, n\}$

Likelihood

Prior

Posterior

$$p(\Omega \,|\, D\, \mathcal{H}\, I) = p(\Omega \,|\, \mathcal{H}\, I) \frac{p(D \,|\, \Omega\, \mathcal{H}\, I)}{p(D \,|\, \mathcal{H}\, I)}$$

Calculated based on cosmological model and the EM information

Cosmological model:
Defines the relation between
$d_L^{\rm gw}$, z, Ω

$\Xi_0 = [0.3\ ,\ 2.0]$
$n = [0.5\ ,\ 3.0]$
$H_0 = [60,\ \ 86\ ]$
$\Omega_M = [0.04, 0.5]$

1. EMRI Catalog M1

2. Parameter estimation $\rightarrow d_L, \Delta d_L, \Delta\theta, \Delta\phi$

3. Apply modified GW propagation relation $\rightarrow z$

From GW:

$d_L^{gw}, \Delta d_L^{gw}, \Delta\theta_{gw}, \Delta\phi_{gw}$

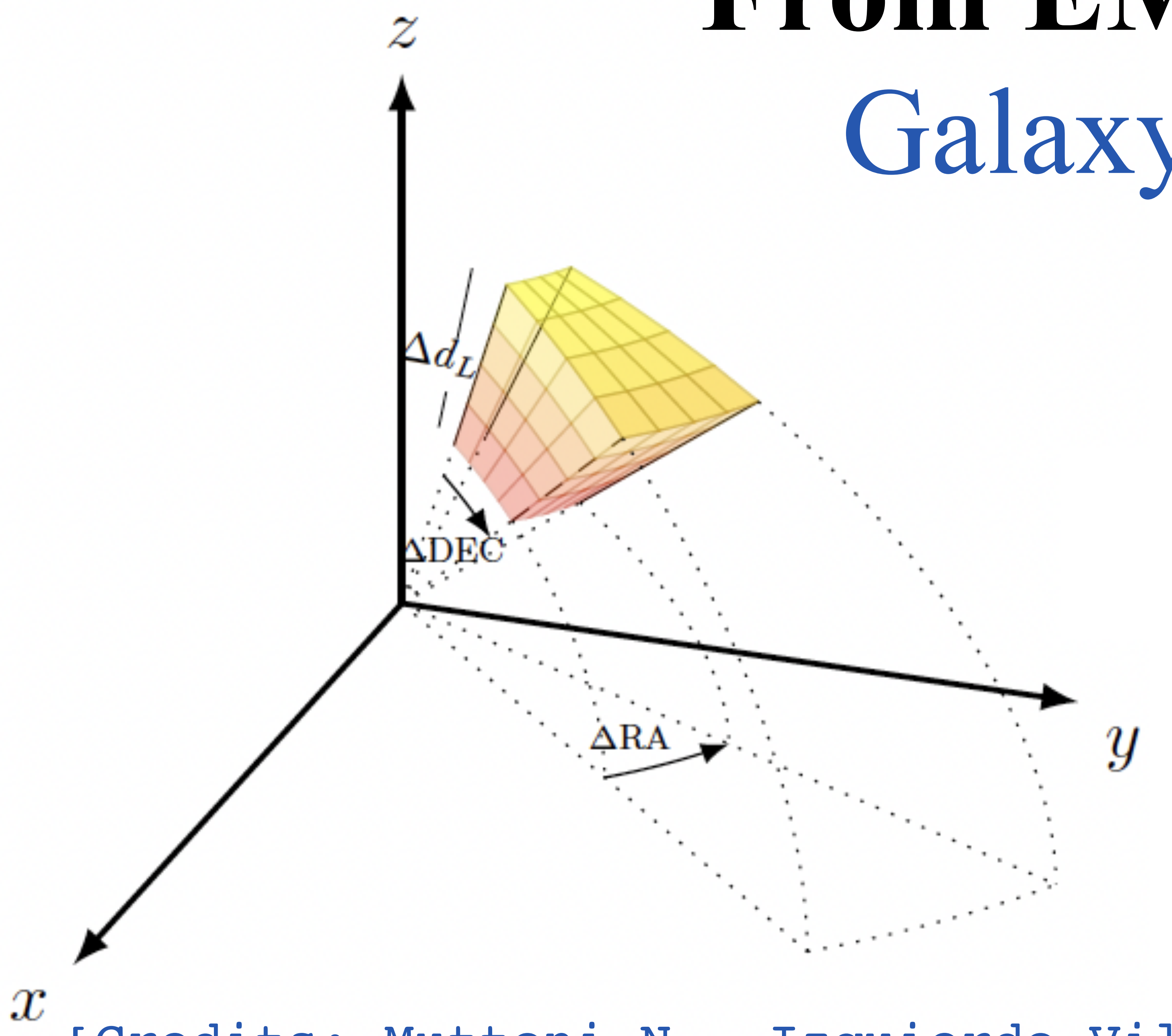
From EM:

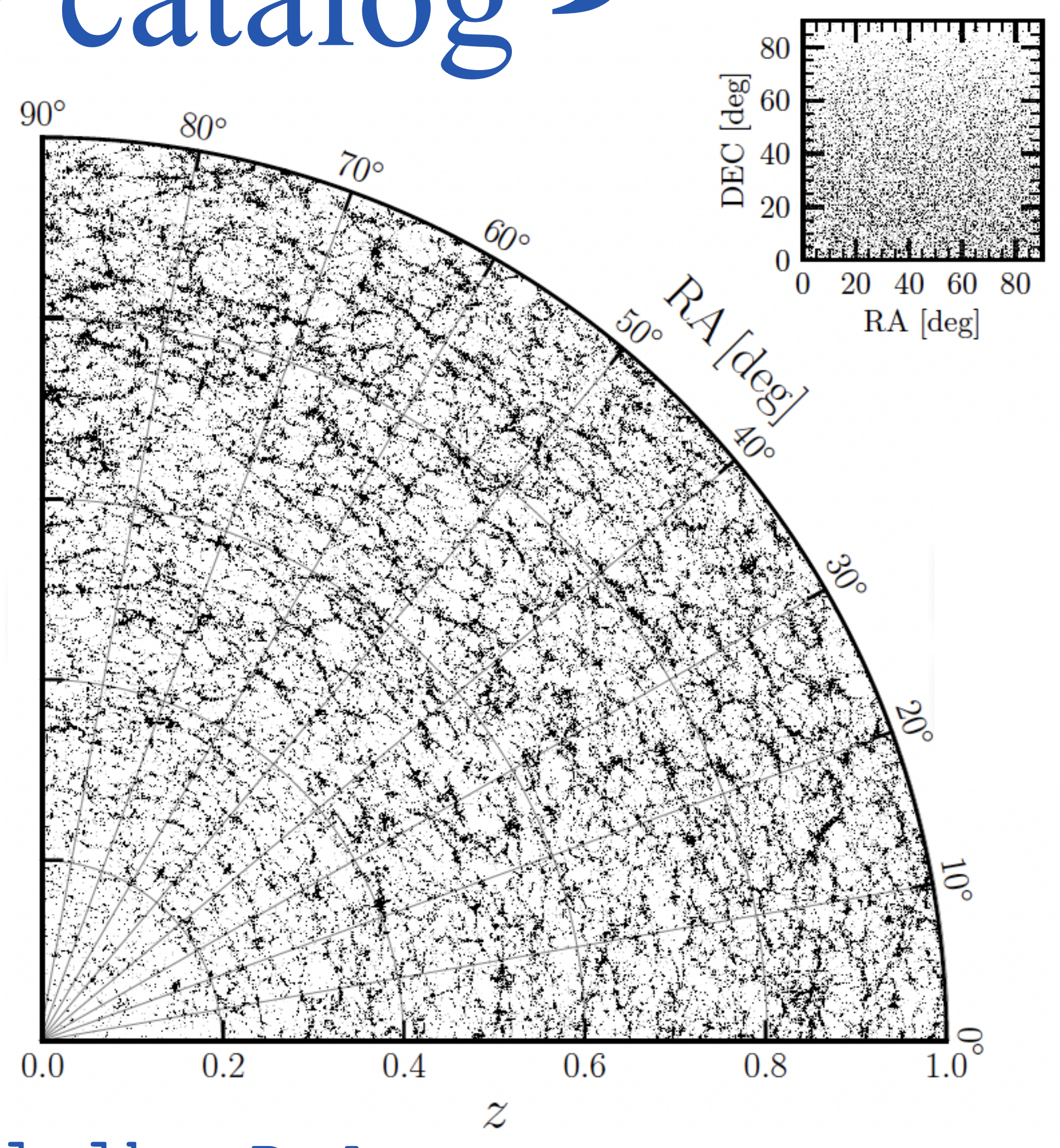
Galaxy catalog

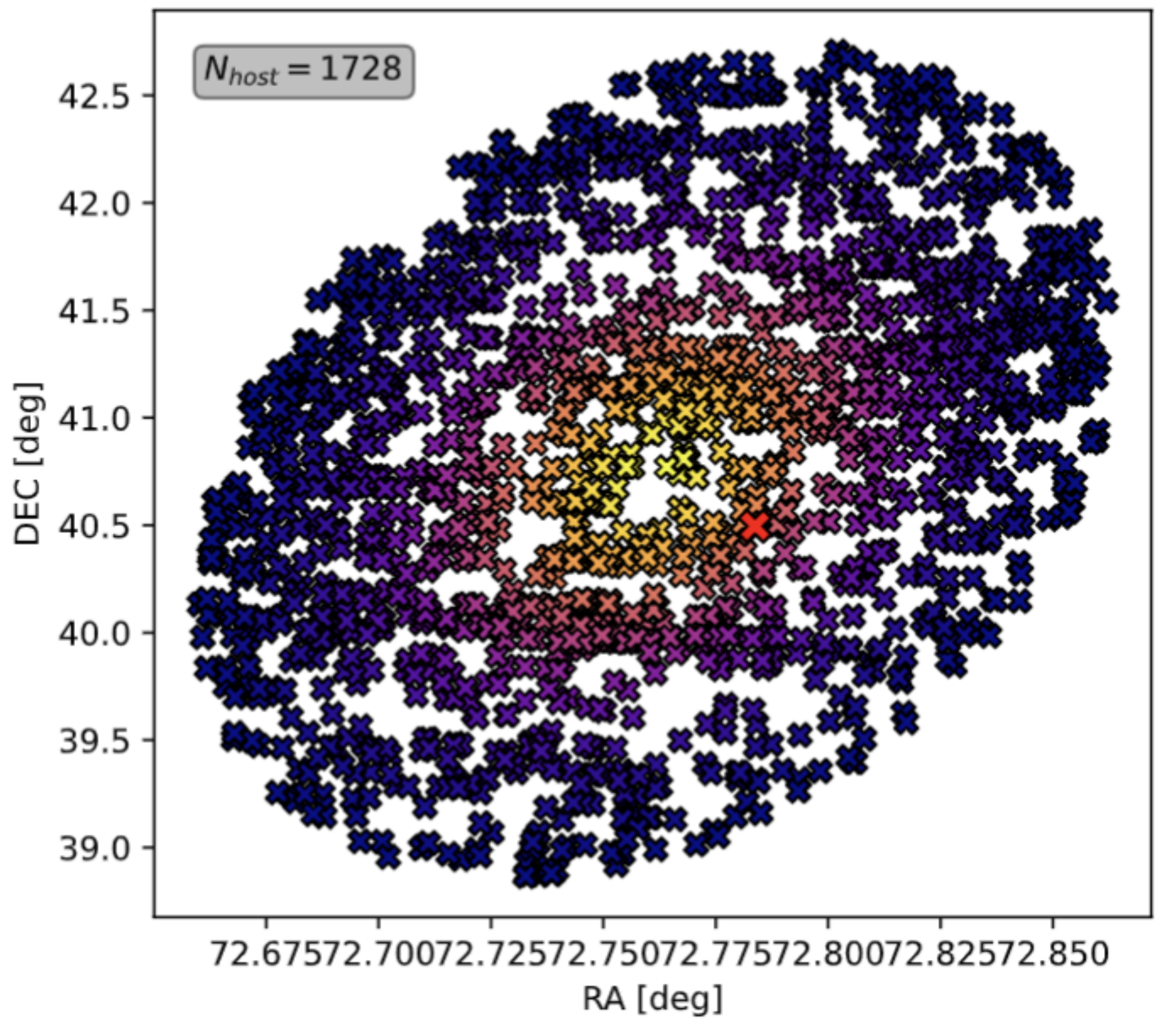
} + prior:

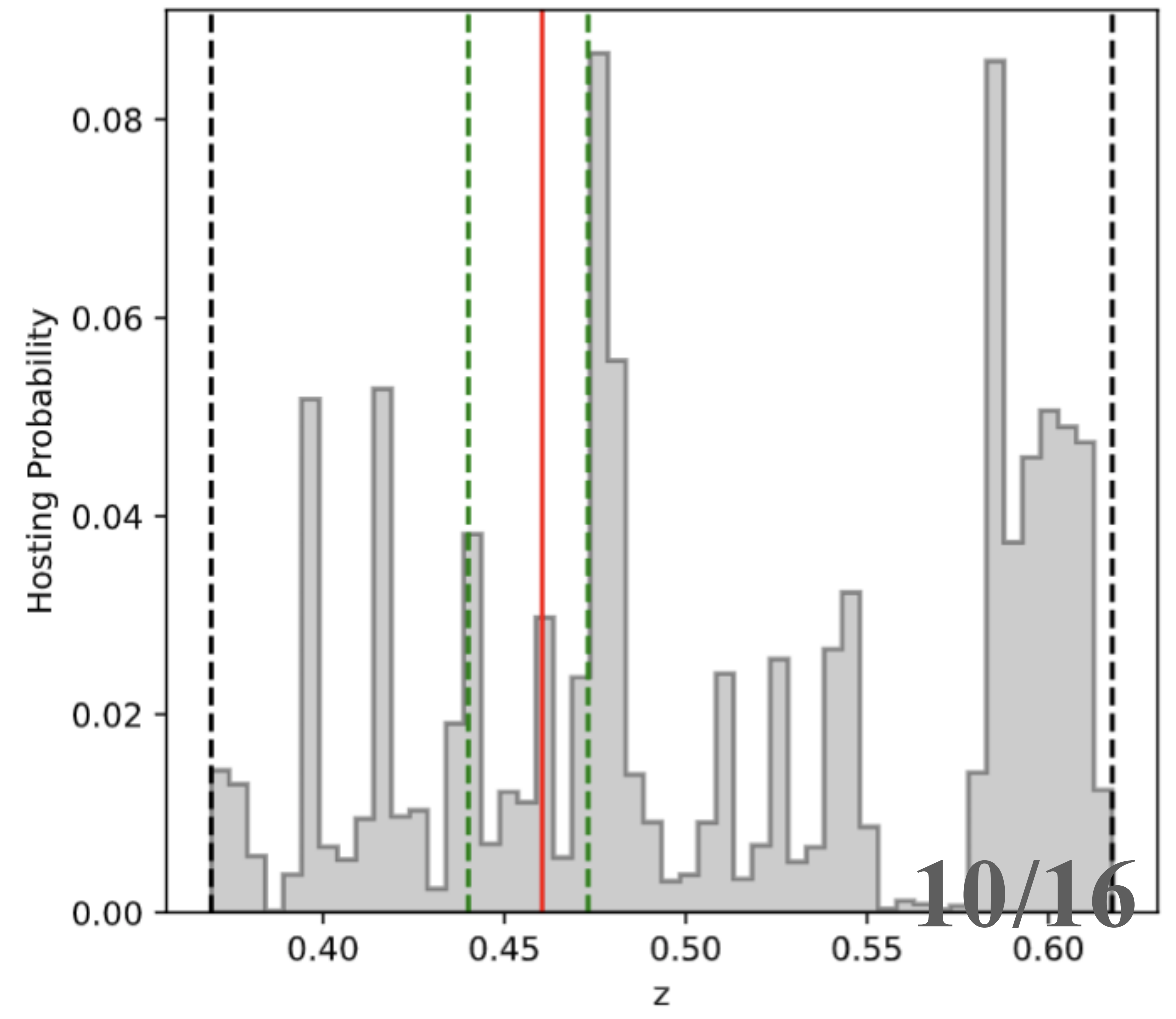
4. 3σ Error box construction:

Statistical information on: z, σ_z









[Credits: Muttoni N., Izquierdo Villalba D.]

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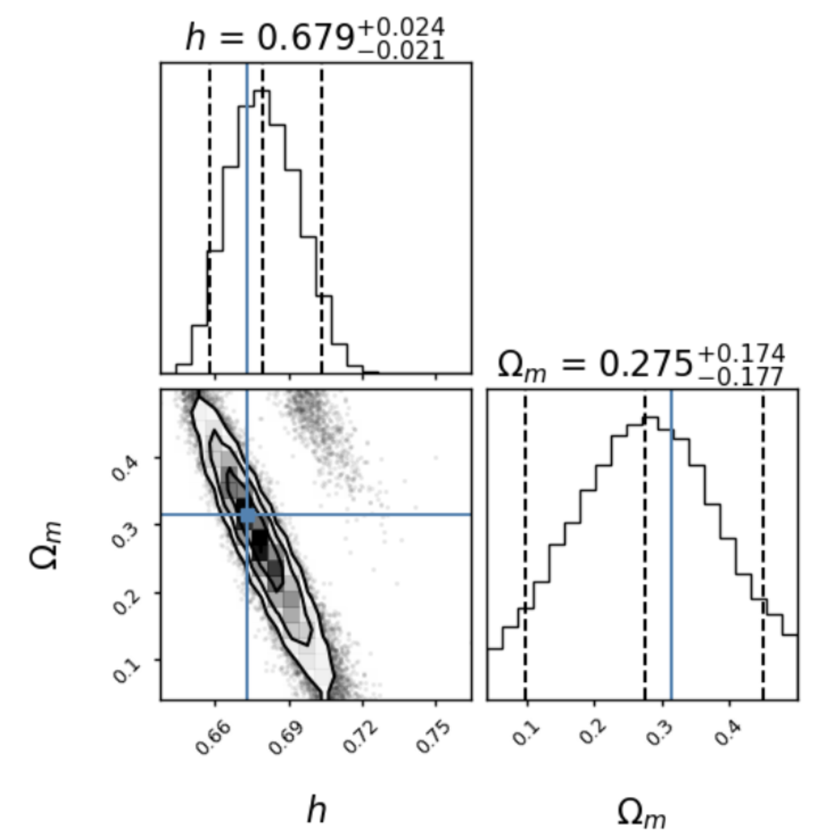
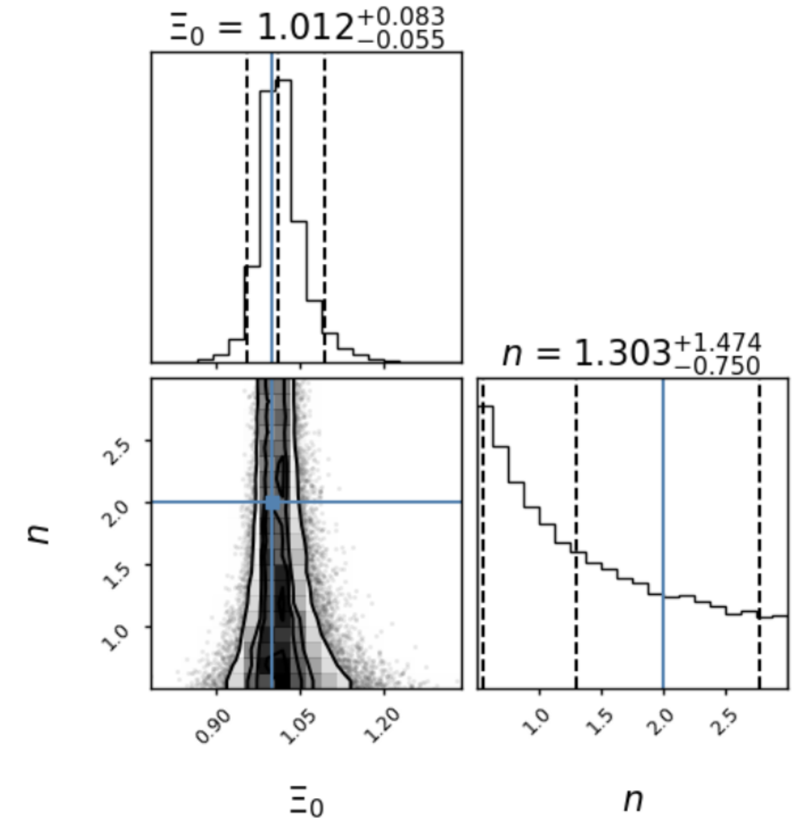




6. Constrain on cosmological parameters

From GW:

$d_L^{gw}, \Delta d_L^{gw}, \Delta\theta_{gw}, \Delta\phi_{gw}$

5. Bayesian theorem [nested sampling by cosmolisa]

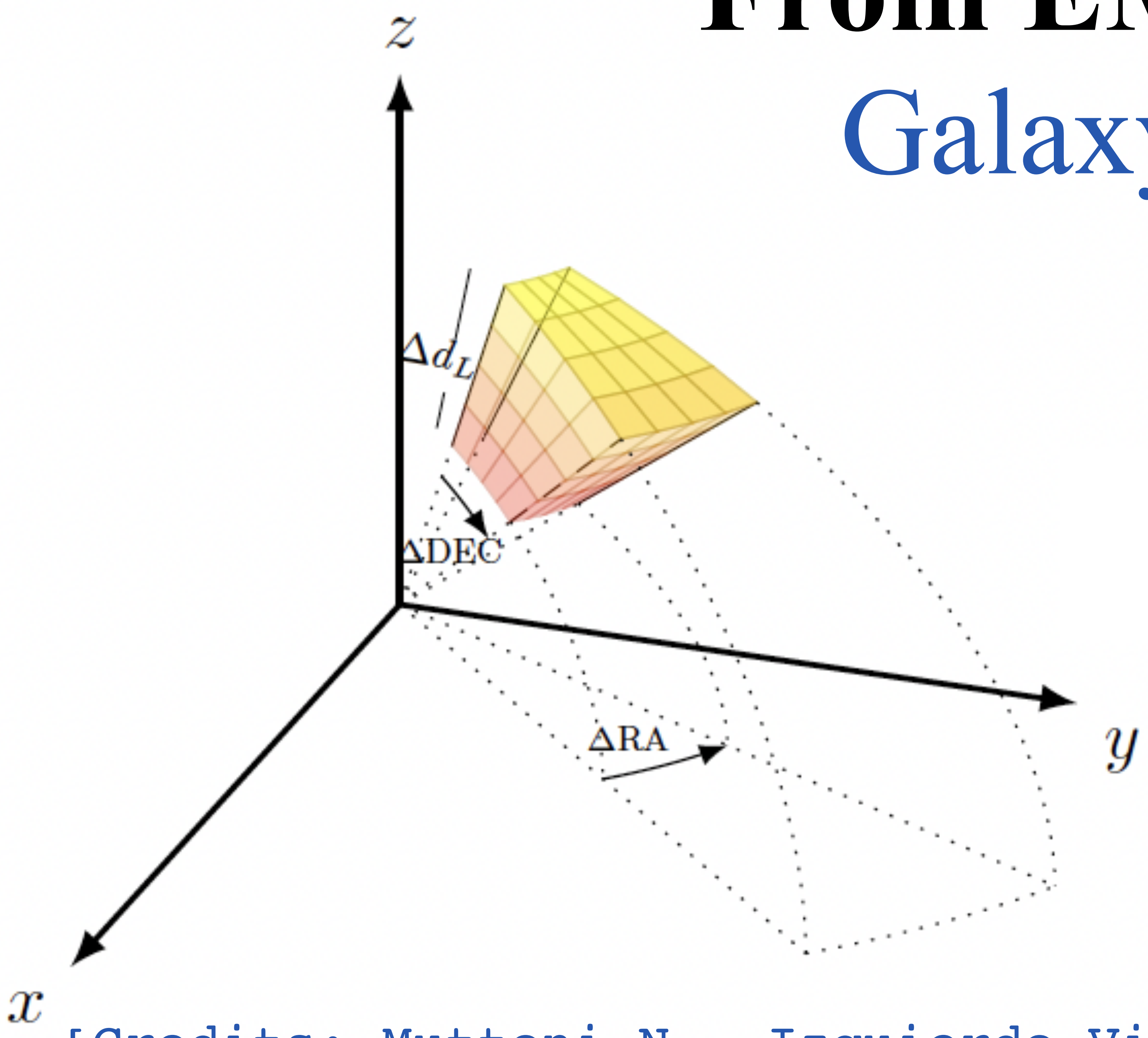
From EM:

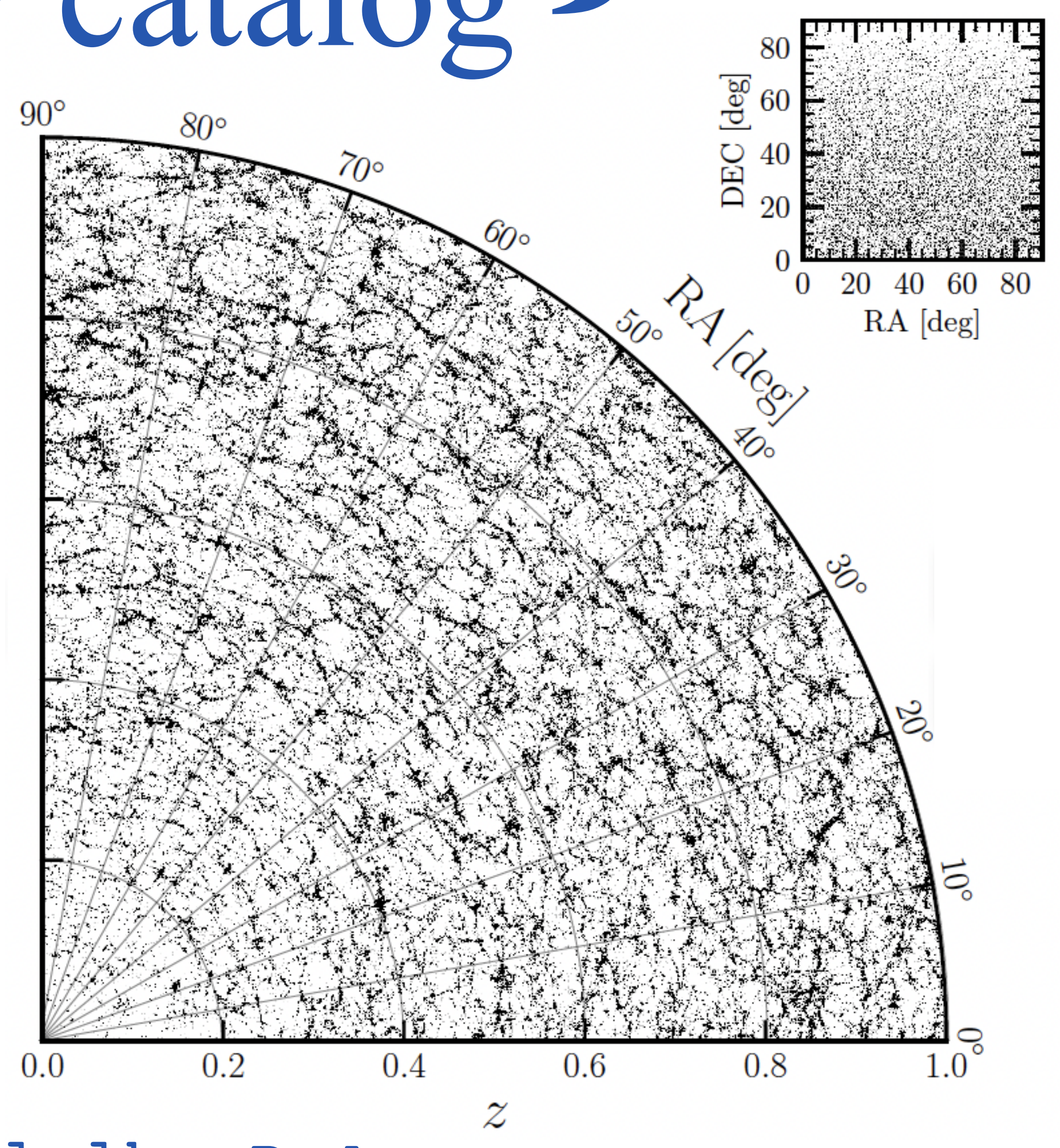
Galaxy catalog

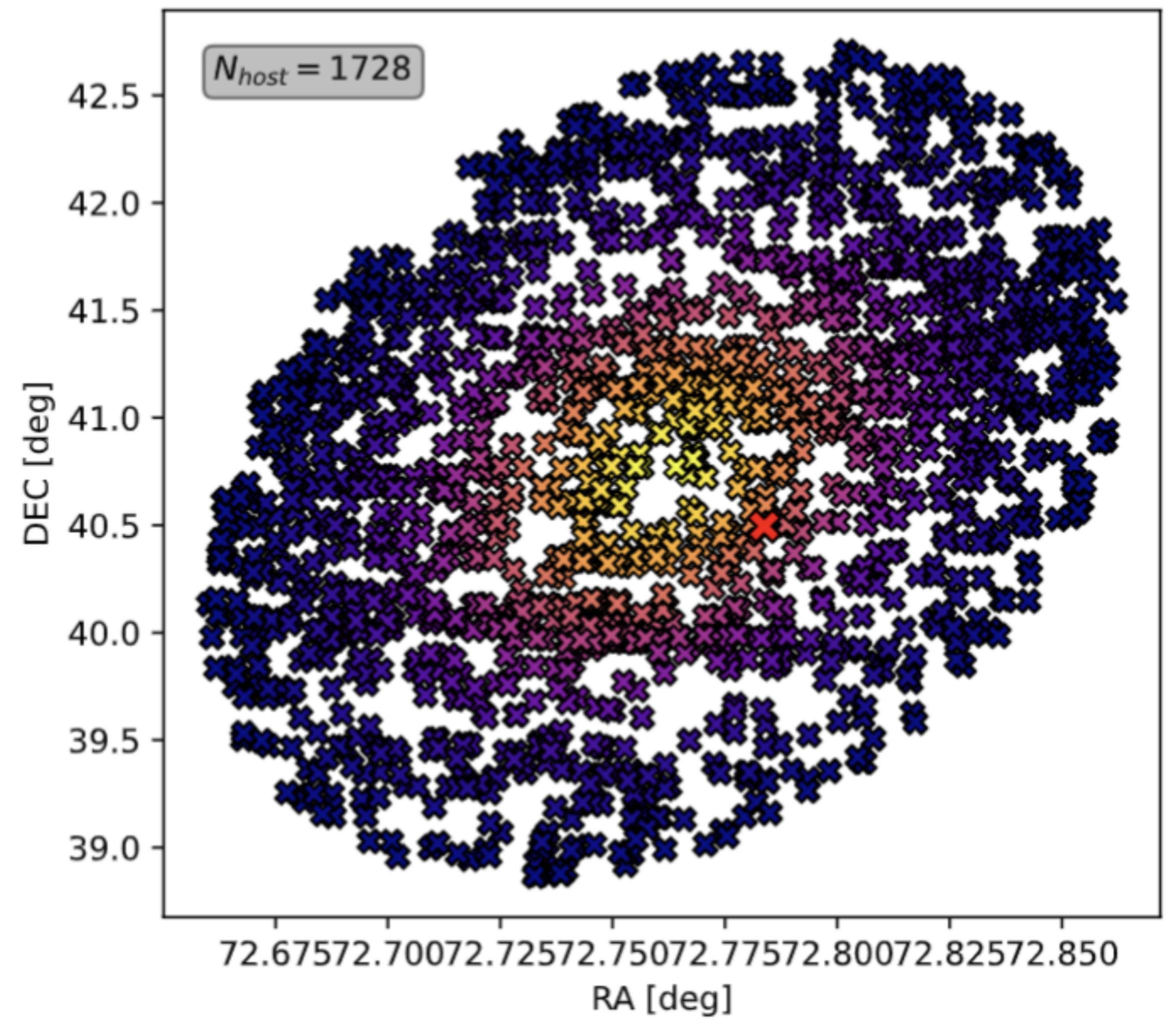
+ prior:

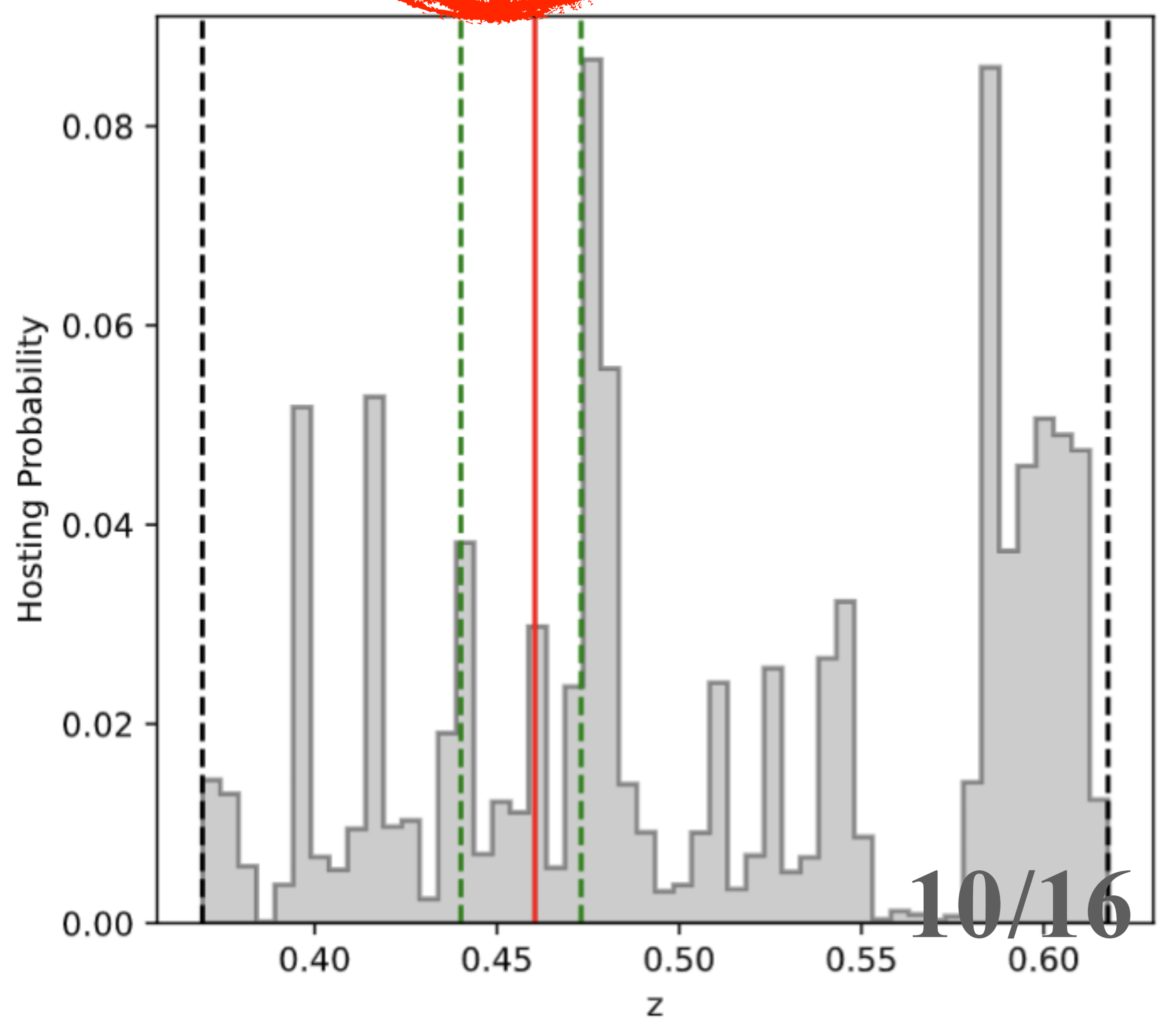
4. 3σ Error box construction:

Statistical information on: z, σ_z





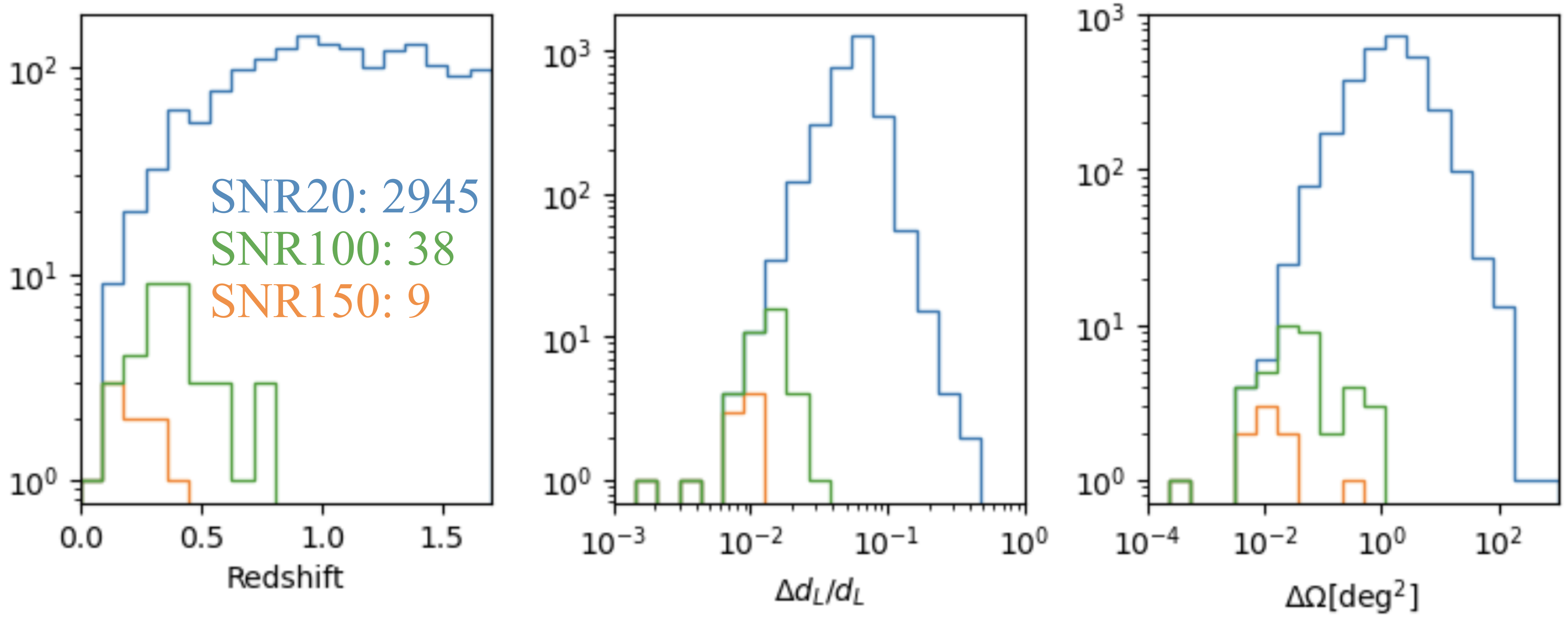




[Credits: Muttoni N., Izquierdo Villalba D.]

Preliminary results: number of detected EMRIs

- 10 yrs observation
- low SNR events tend to produce a bias in the estimation



Number of events used in the analysis ($z < 1$)

$\Xi_0 = [0.3 , 2.0]$
$n = [0.5 , 3.0]$
$H_0 = [60, 86]$
$\Omega_M = [0.04,0.5]$

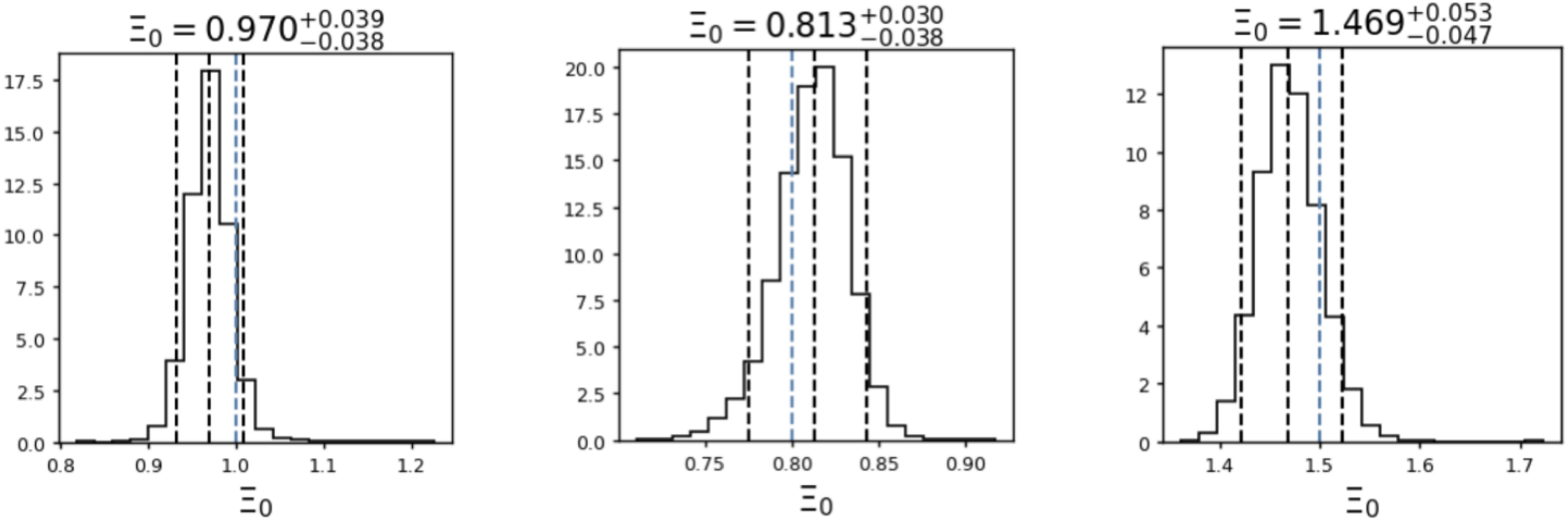
M1	$H_0+\Omega_M+\Xi_0+n$	$H_0+\Omega_M+\Xi_0$	Ξ_0+n	Ξ_0	$H_0+\Omega_M$
SNR >= 150	8	8	9	9	9

Preliminary results: Ξ_0

Injected value and [prior]:
$\Xi_0 =$ **1.0/0.8/1.5** [0.3 , 2.0]
$n = 2$
$H_0 = 67.3$
$\Omega_M = 0.315$

- 90% CI
- Median of 5 realizations

$\Delta\Xi_0/\Xi_0 \sim 4\,\%$



Liu et al. in prep

Preliminary results: $\Xi_0 + n$

Injected value and [prior]:
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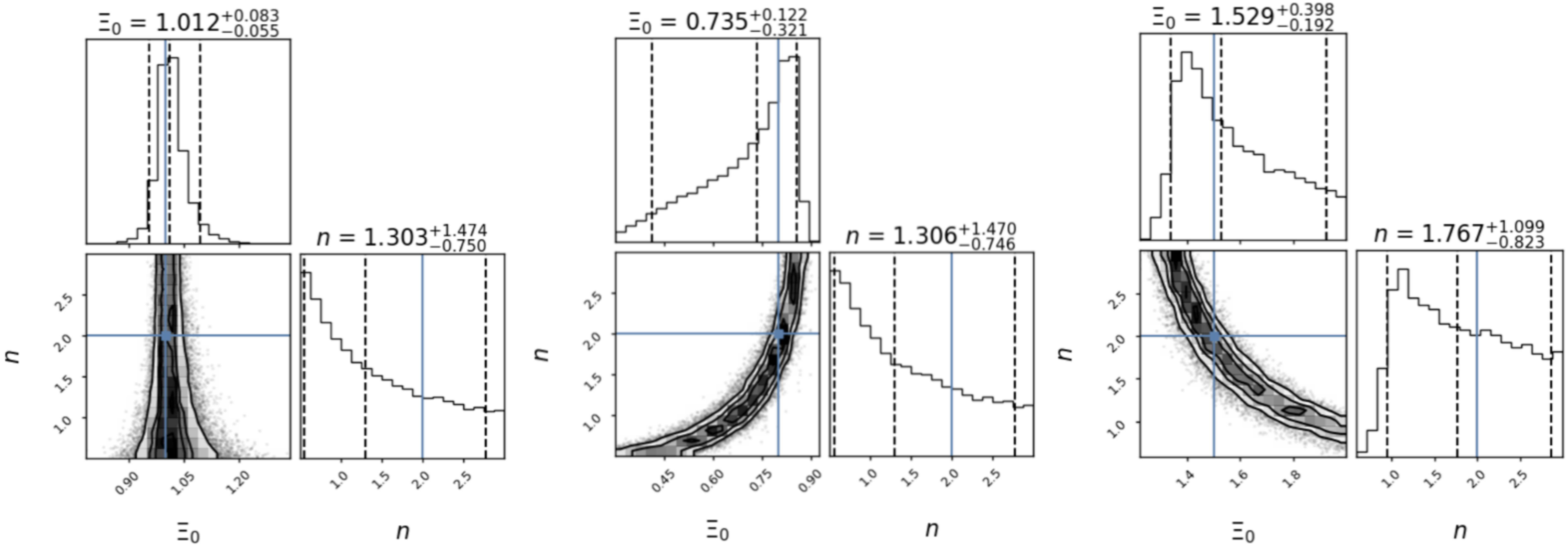
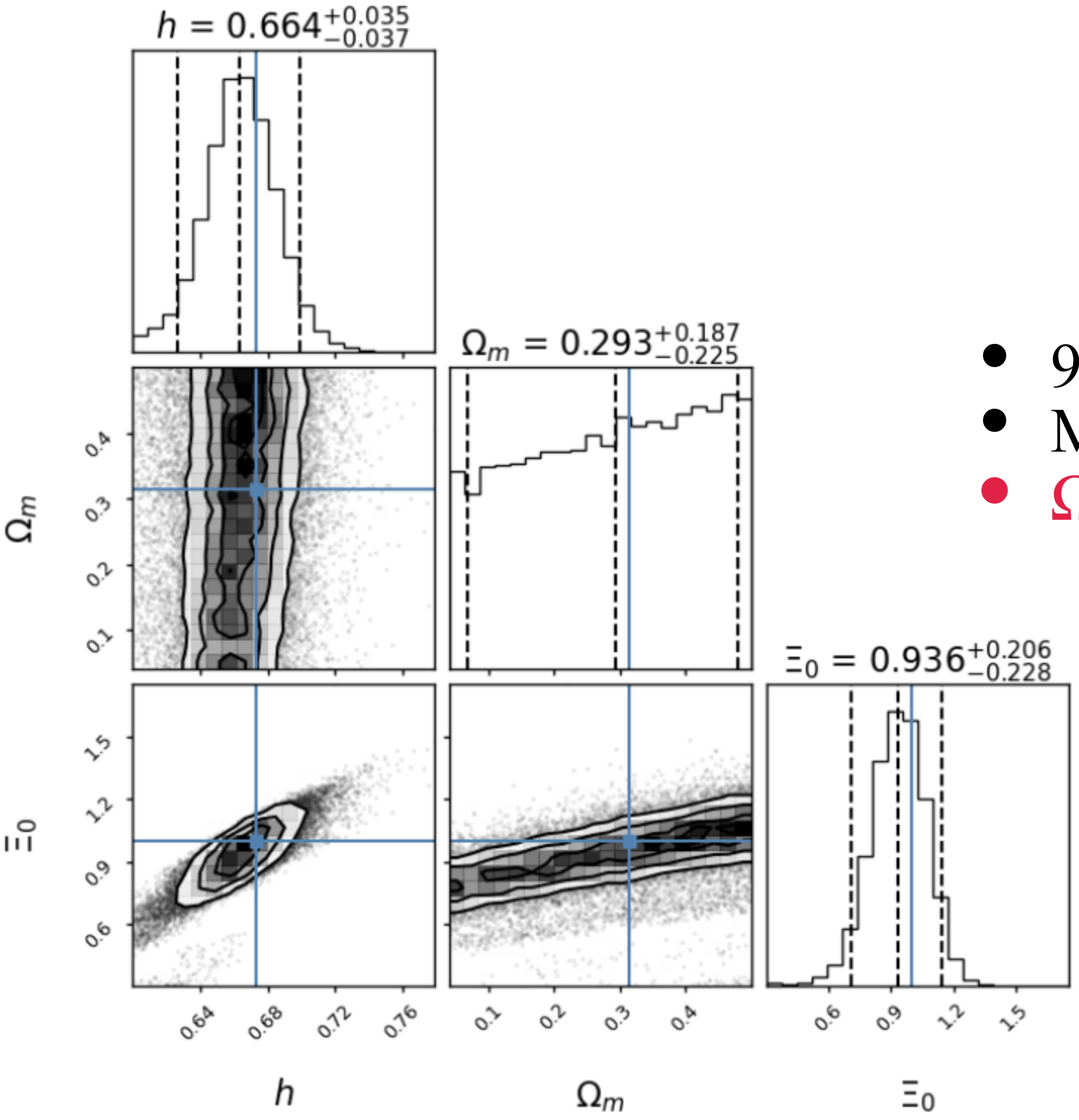
$H_0 =$ 67.3
$\Omega_M = 0.315$

- 90% CI
- Median of 5 realizations
- **Strongly correlated $\Xi_0 + n$**



Liu et al. in prep

Preliminary results: $H_0 + \Omega_M + \Xi_0$



Injected value and [prior]:
$\Xi_0 =$ **1.0** [0.3 , 2.0]
$H_0 =$ 67.3 [60, 86]
$\Omega_M =$ 0.315[0.04,0.5]

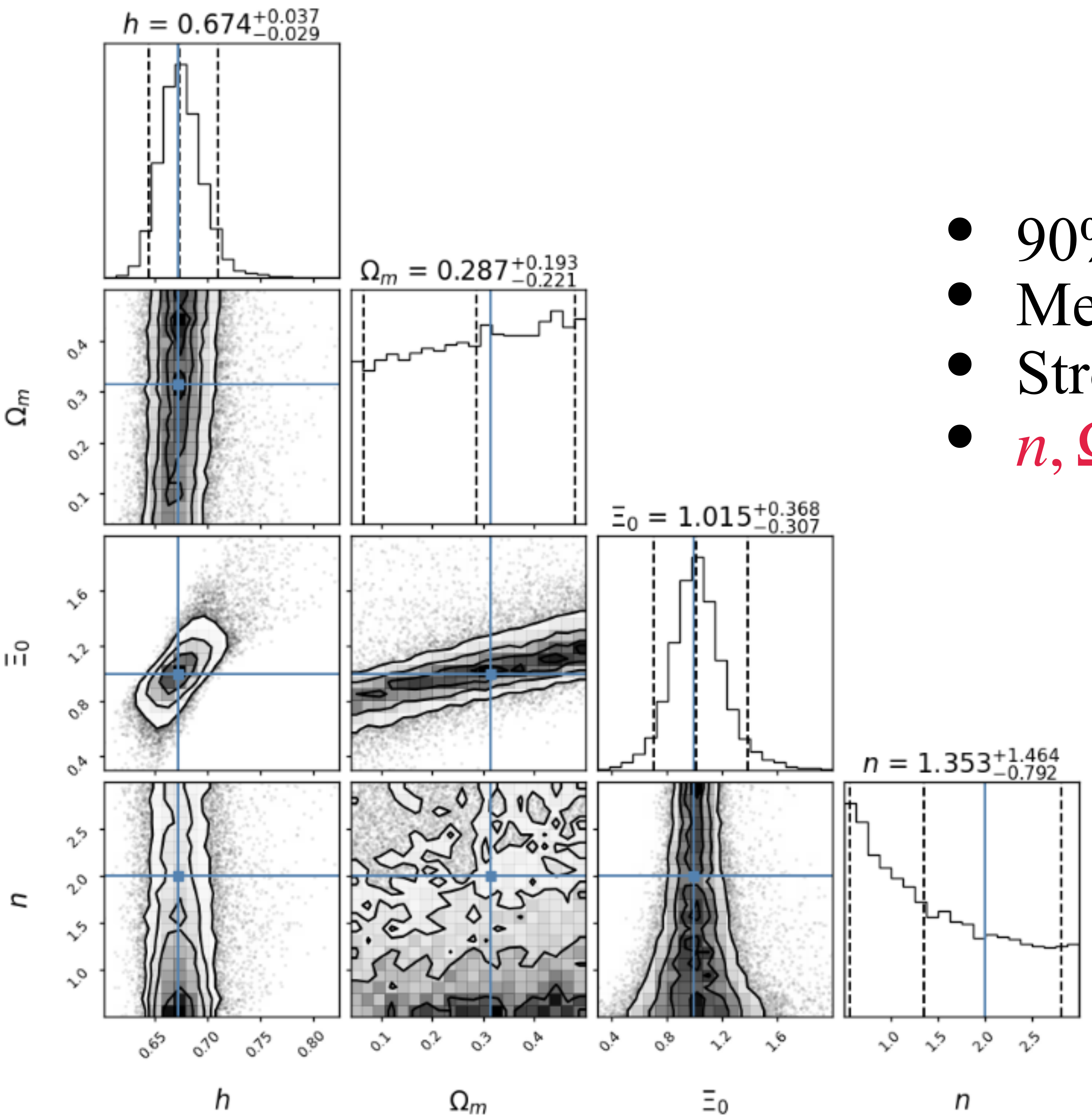
- 90% CI
- Median of 5 realizations
- **Ω_m can not be measured**

$$\Delta \Xi_0 / \Xi_0 \sim 21\,\%$$
$$\Delta H_0 / H_0 \sim 5\,\%$$

Liu et al. in prep

Preliminary results: $H_0 + \Omega_M + \Xi_0 + n$

Injected value and [prior]:
$\Xi_0 =$ **1.0** [0.3 , 2.0]
$n =$ [0.5 , 3.0]
$H_0 =$ 67.3 [60, 86]
$\Omega_M =$ 0.315[0.04,0.5]



- 90% CI
- Median of 5 realizations
- Strongly correlated $\Xi_0 + n$
- n, Ω_m can not be measured

$$\Delta\Xi_0/\Xi_0 \sim 33\,\%$$
$$\Delta H_0/H_0 \sim 5\,\%$$

Liu et al. in prep

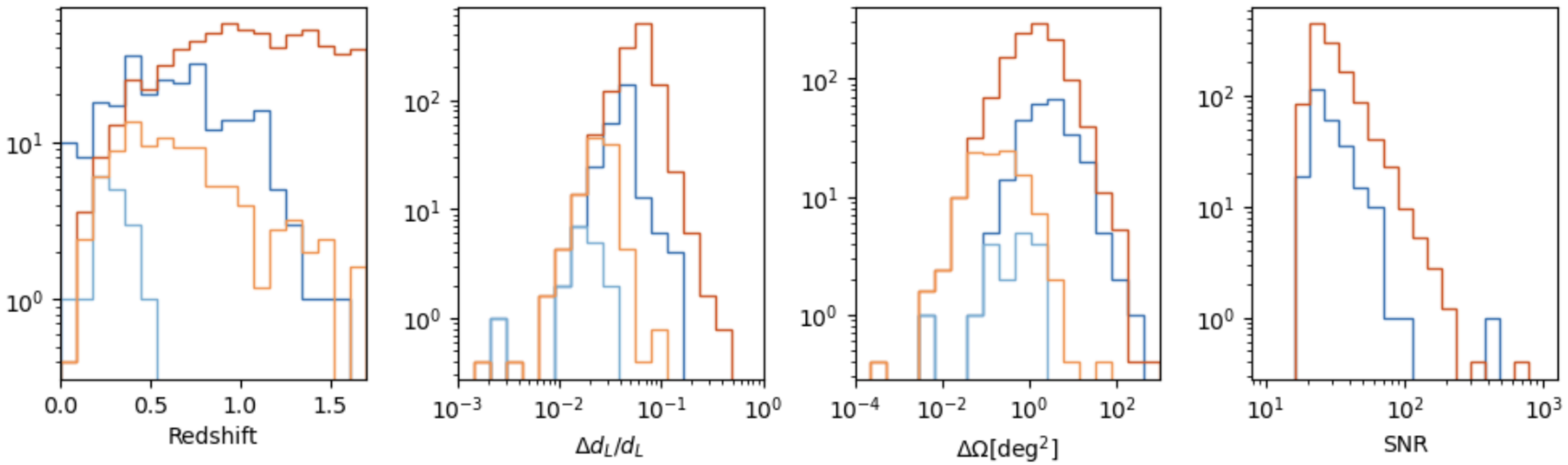
Conclusion & future prospects

- Conclusion:
- Ξ_0 alone $\sim 4\,\%$
- Ξ_0 and n: Strongly correlated
  - Different trend when $\Xi_0 > 1, = 1, < 1$
- When also considering other parameters, $\Delta\Xi_0/\Xi_0 > \sim 20\,\%$
- $H_0 \sim$ few percent

- Future prospects:
- New waveform model:
  - Augmented Analytic Kludge with 5PN trajectories
- New sensitivity curve + full response TDI
- Other modified GR model

Backups

M1 4yr detected sources properties: AKK vs AAK5pn



Numbers of detection:

- **AKK SNR 20: 1178**
- **AKK SNR 50: 111**
- **AAK5pn SNR 20: 257**
- **AAK5pn SNR 50: 17**

Modified gravity theory

RR model:

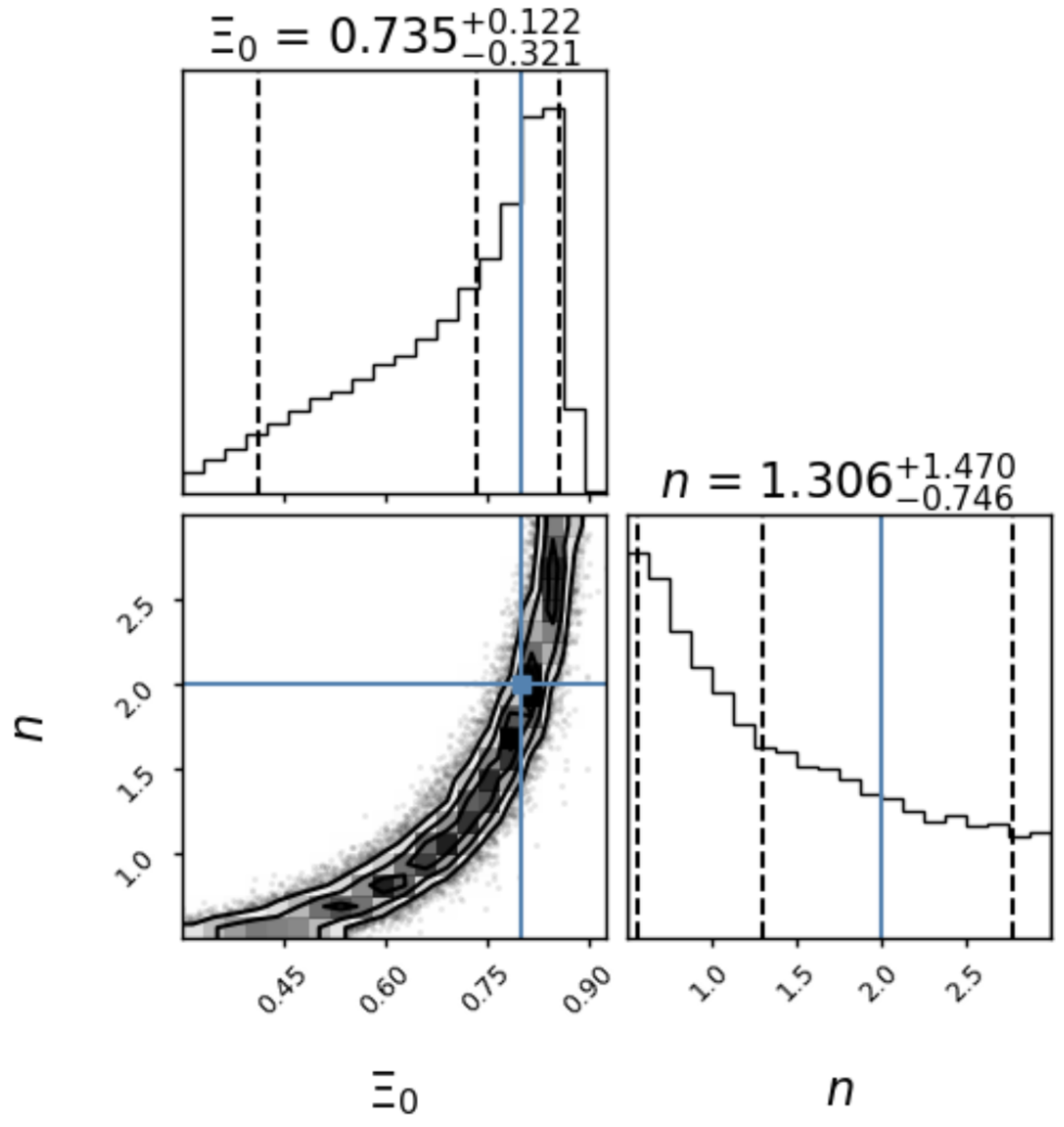
Gravity is modified by the addition of a nonlocal term

$$\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{6}m^2 R\frac{1}{\Box^2}R\right]$$

Other applicable theories: Horndeski, DHOST theories, RT non-local gravity model ($\Xi_0^{\rm max} = 1.8$, $n = 1.91$)

[Finke et al. 2021]

Explanation of the correlation



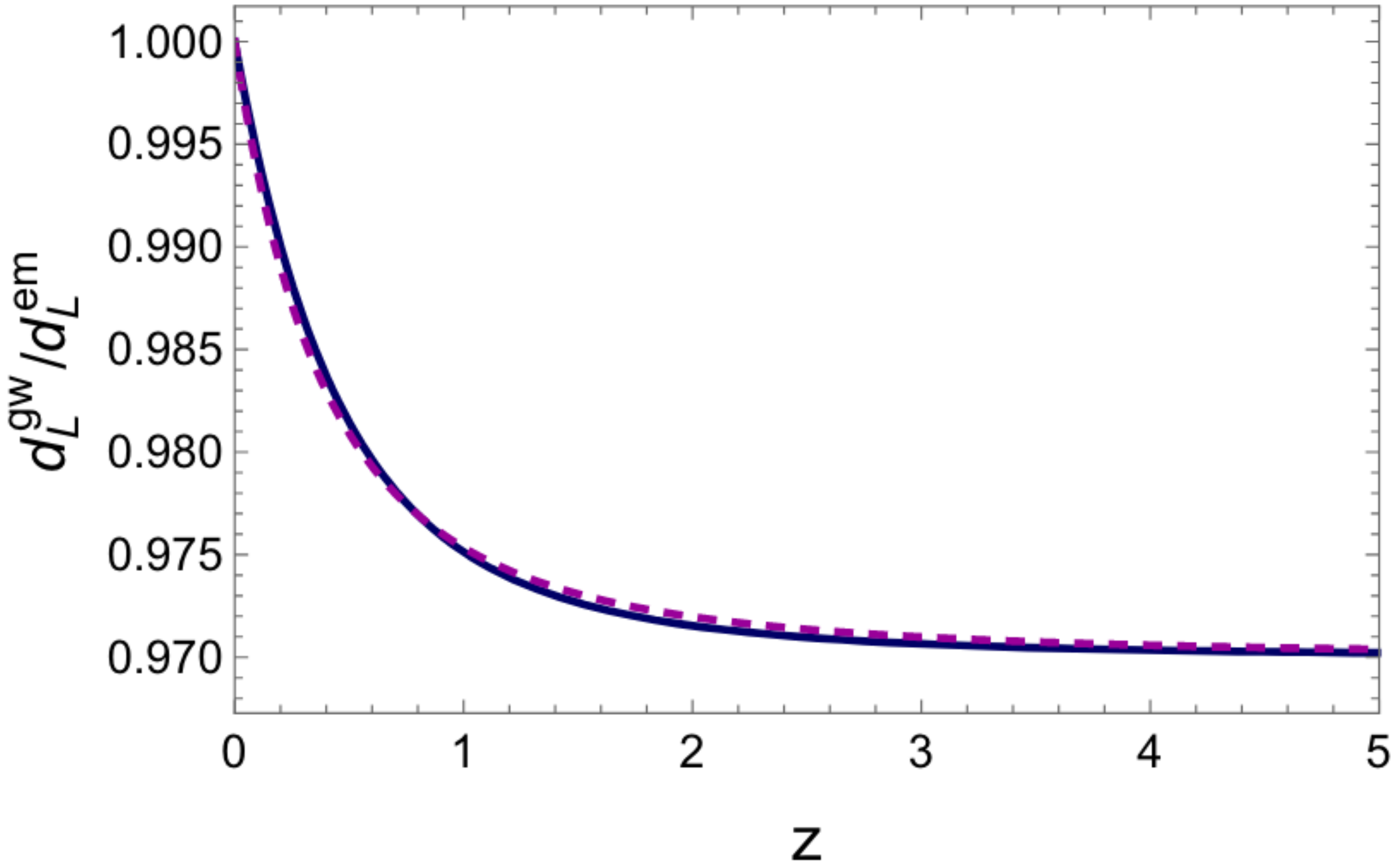
at lower redshift, expand the formula:

$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} = 1 - z\delta(0) + \mathcal{O}(z^2)\,,$$

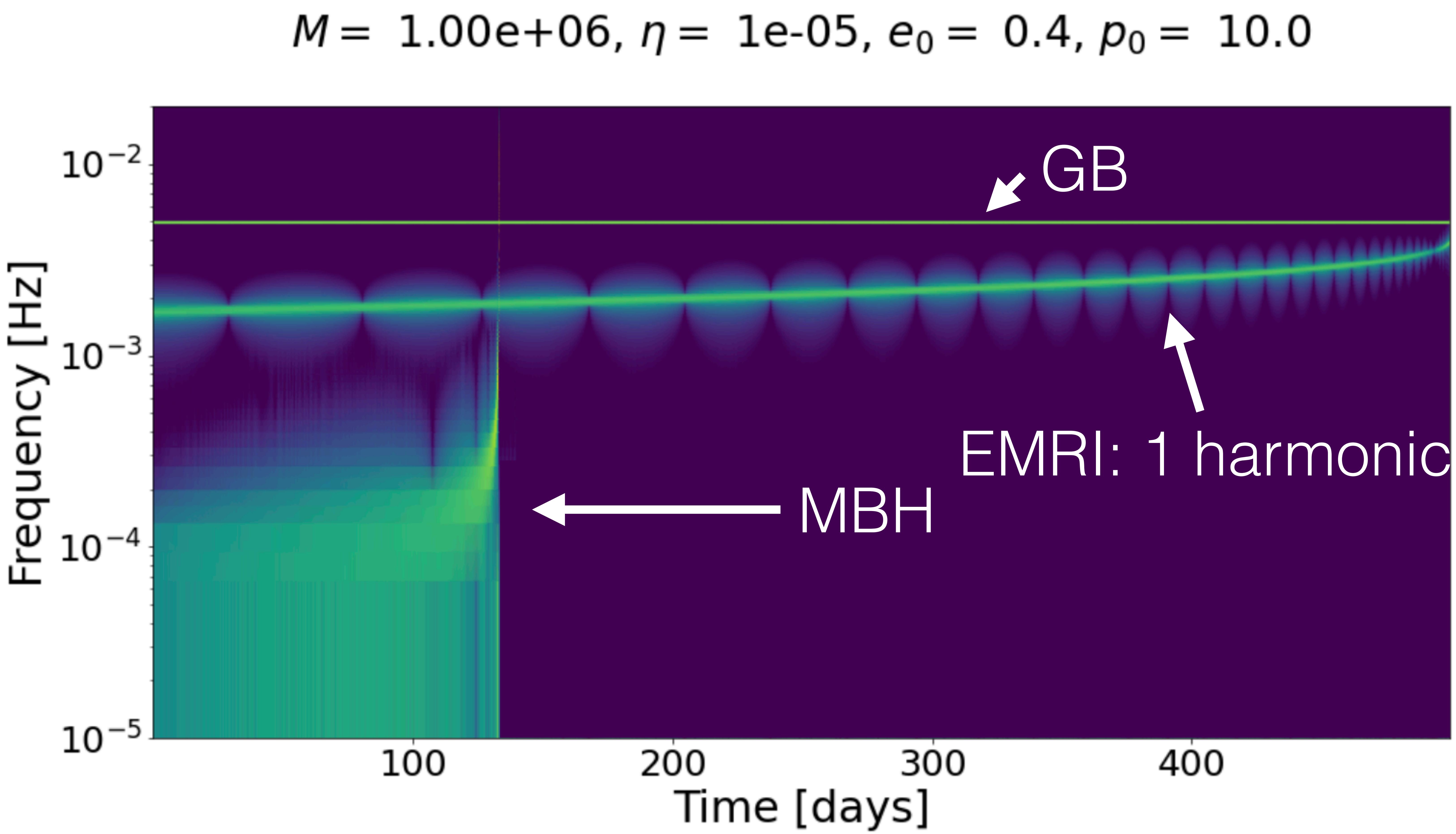
at very low redshift we are actually sensitive to $\delta(0) \equiv \delta(z=0)$

$$\delta(z) = \frac{n(1-\Xi_0)}{1-\Xi_0+\Xi_0(1+z)^n}$$

$$\delta(0) = n(1-\Xi_0)$$

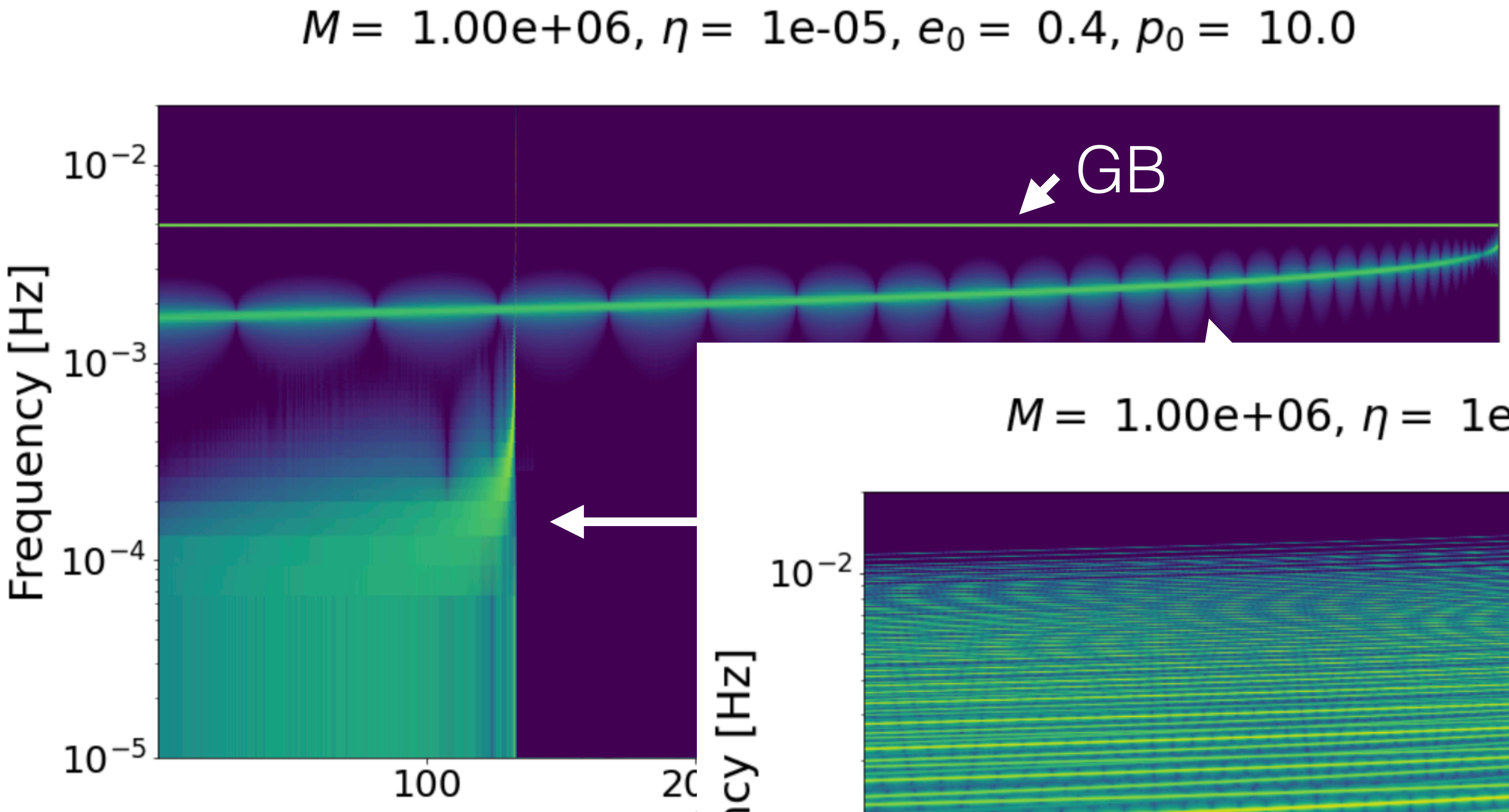


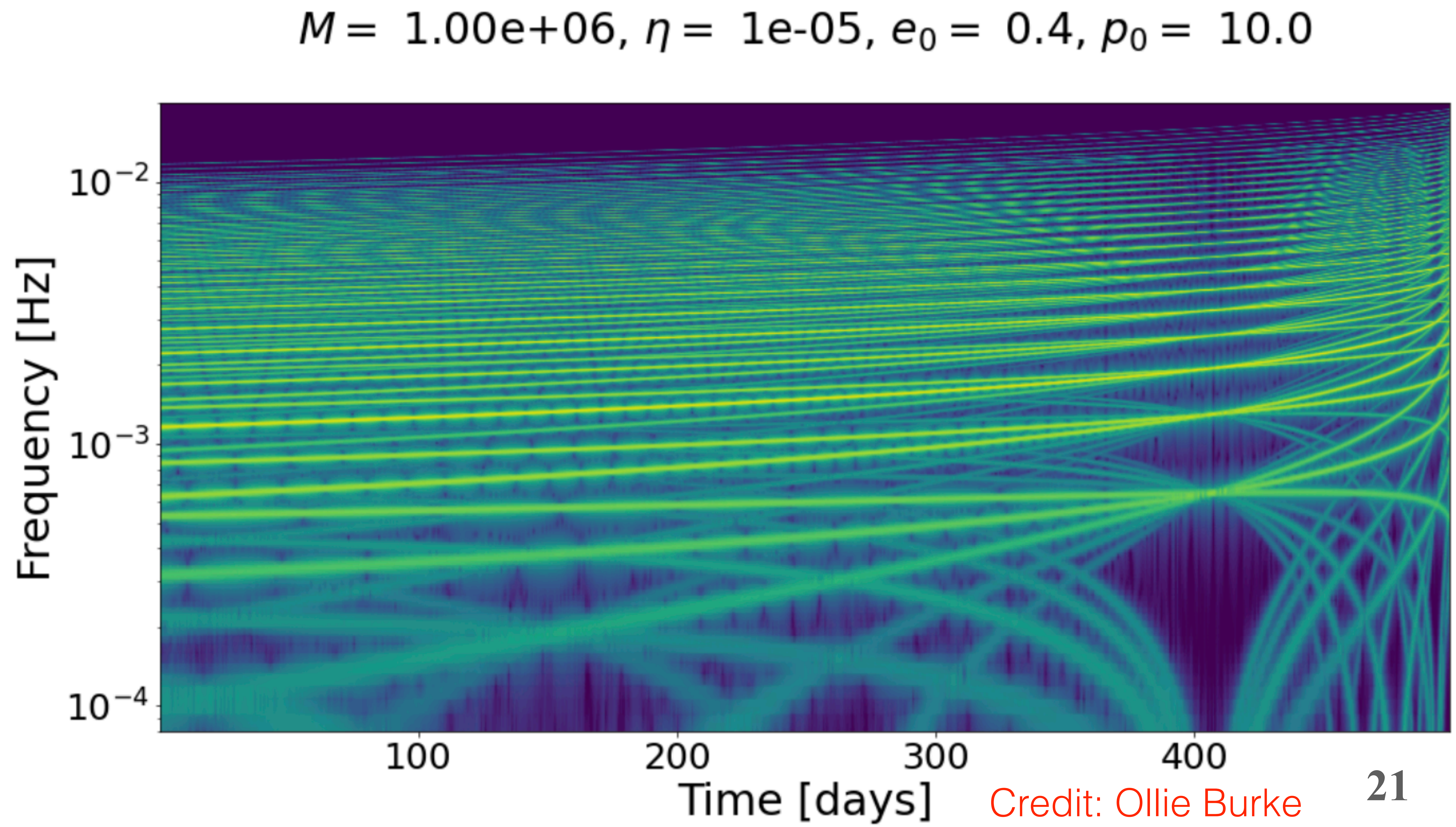
Detection of a single EMRI



Credit: Ollie Burke

Detection of a single EMRI





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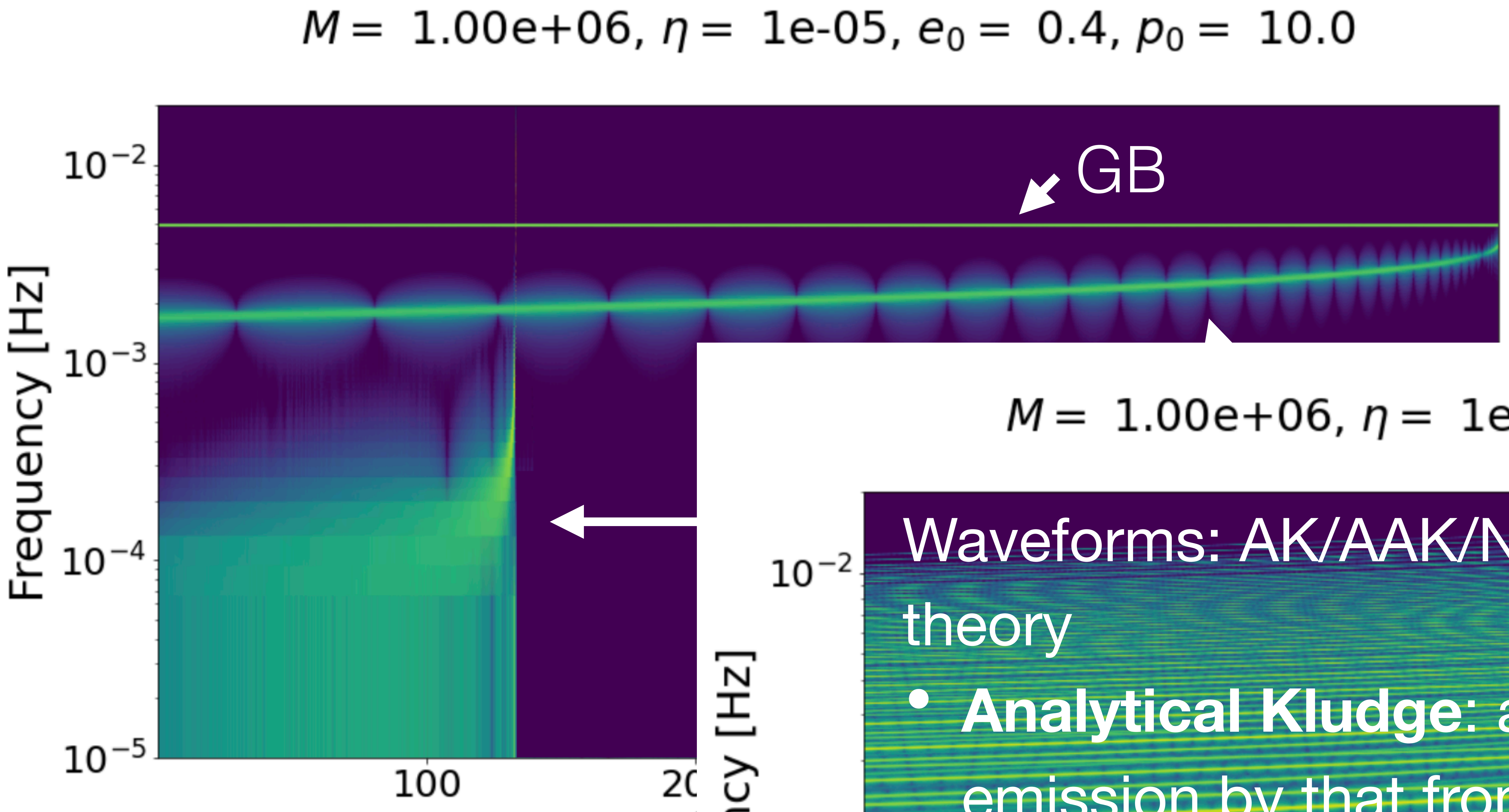
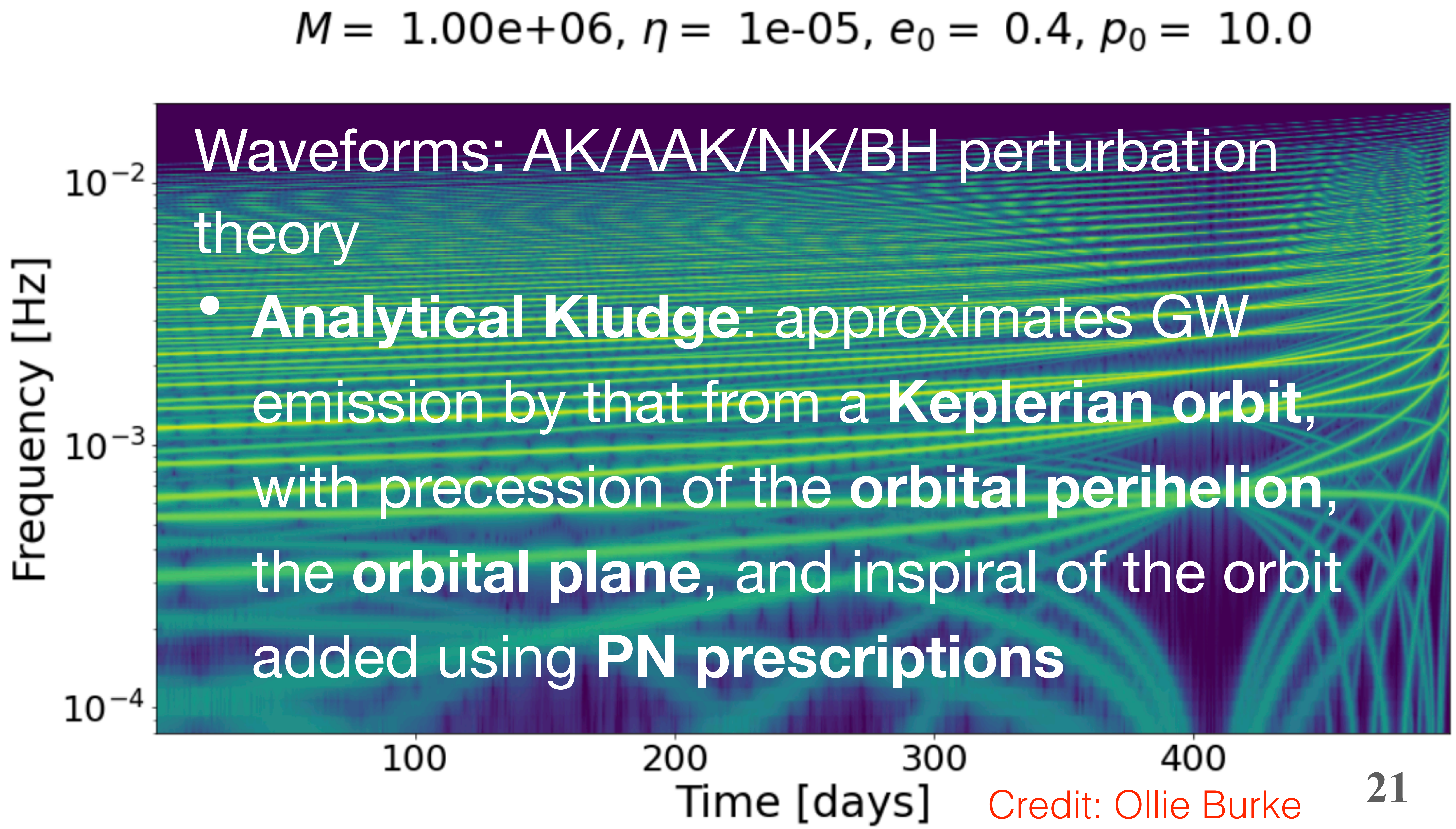
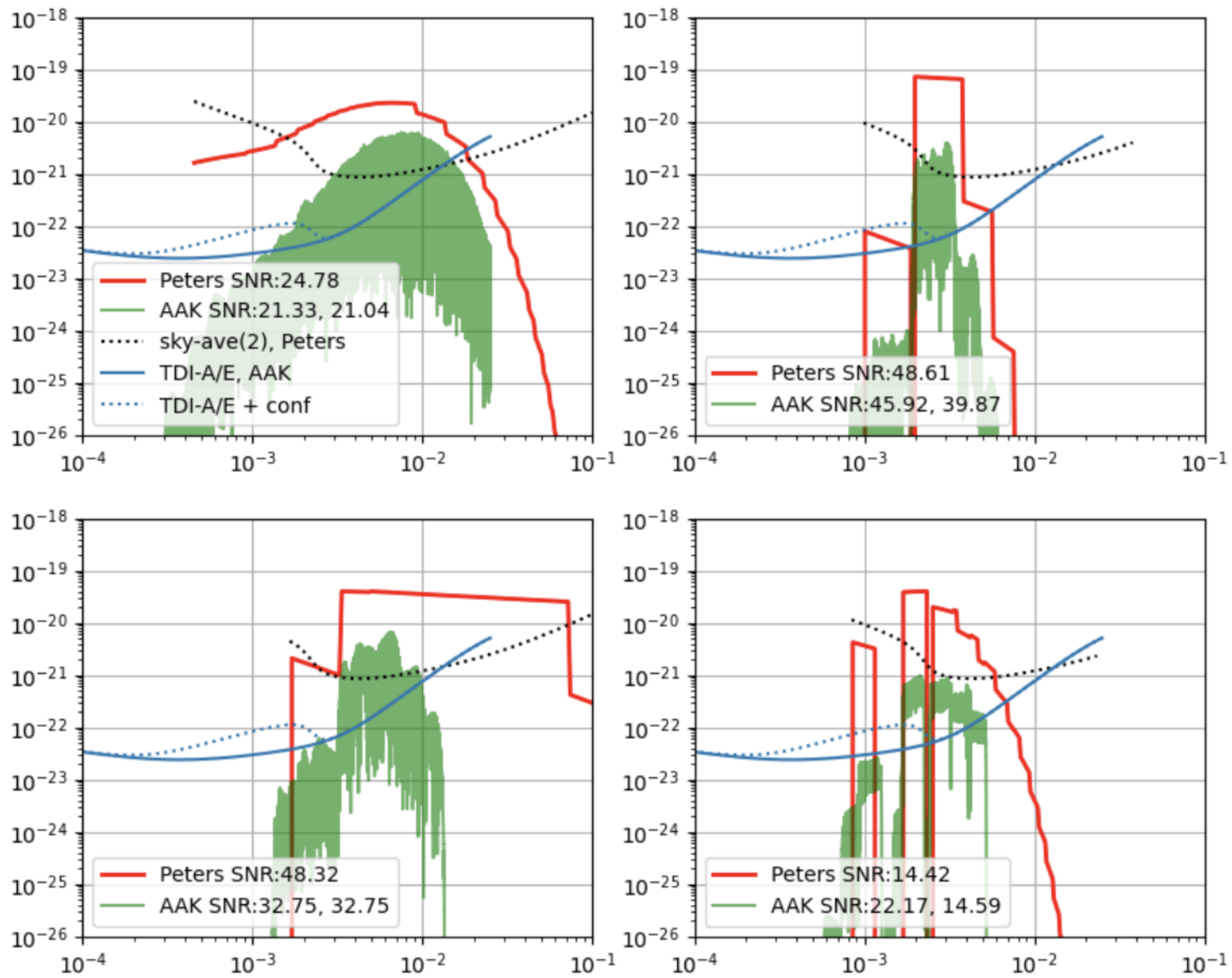




Waveforms: AK/AAK/NK/BH perturbation theory

- **Analytical Kludge**: approximates GW emission by that from a **Keplerian orbit**, with precession of the **orbital perihelion**, the **orbital plane**, and inspiral of the orbit added using **PN prescriptions**

Credit: Ollie Burke



Peters SNR:24.78
AAK SNR:21.33, 21.04
sky-ave(2), Peters
TDI-A/E, AAK
TDI-A/E + conf
Peters SNR:48.61
AAK SNR:45.92, 39.87
Peters SNR:48.32
AAK SNR:32.75, 32.75
Peters SNR:14.42
AAK SNR:22.17, 14.59