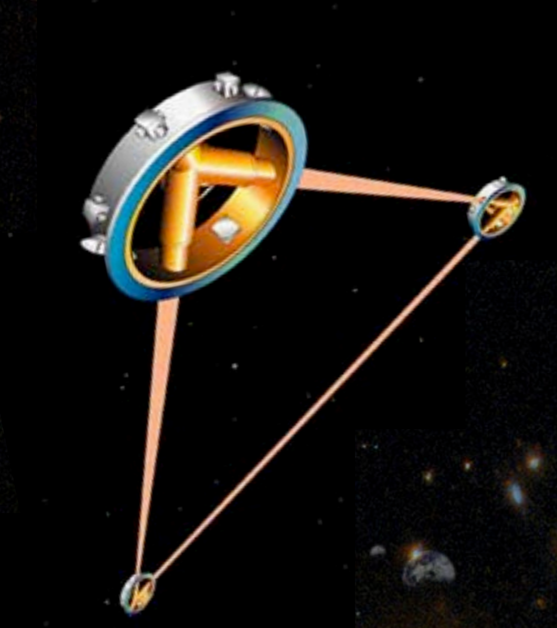
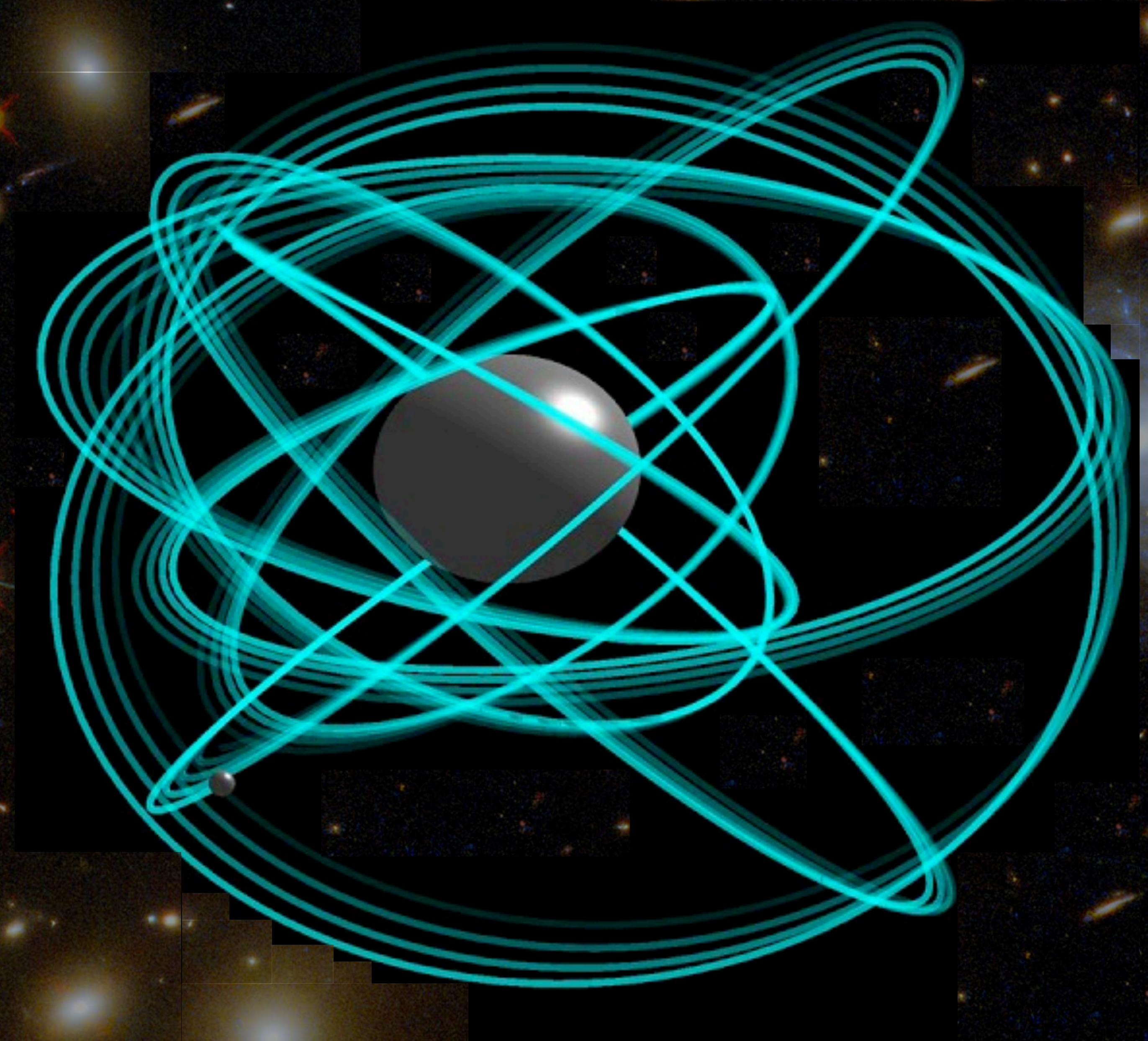
Gravitational wave cosmology with LISA standard sirens



10th LISA CosWG Workshop - Stavanger - 07/06/2023



Danny Laghi

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Laboratoire des 2 Infinis - Toulouse

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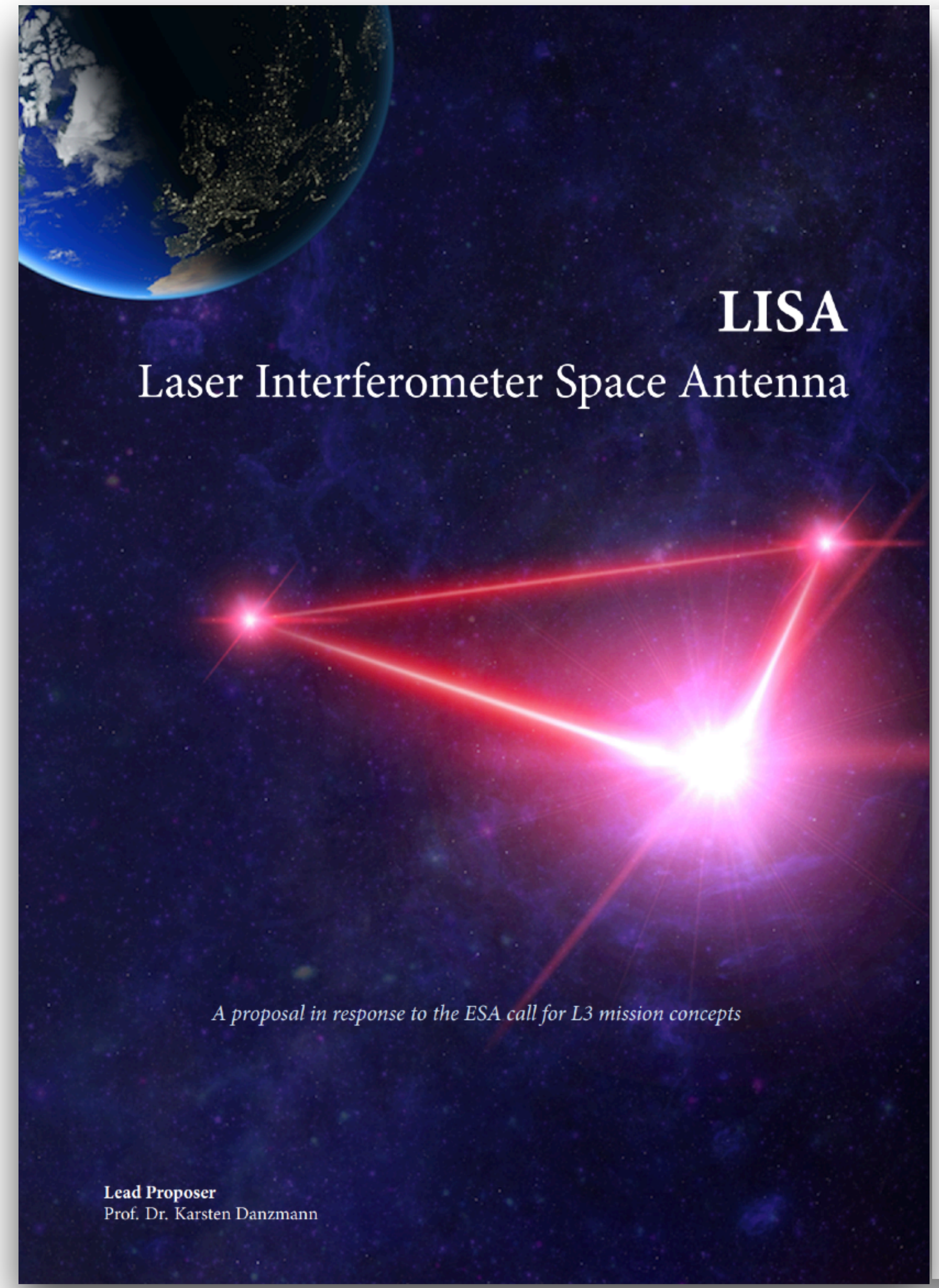
Credits: Steve Drasco/ESA-HUBBLE/NASA

* This research is directly supported by the CNES

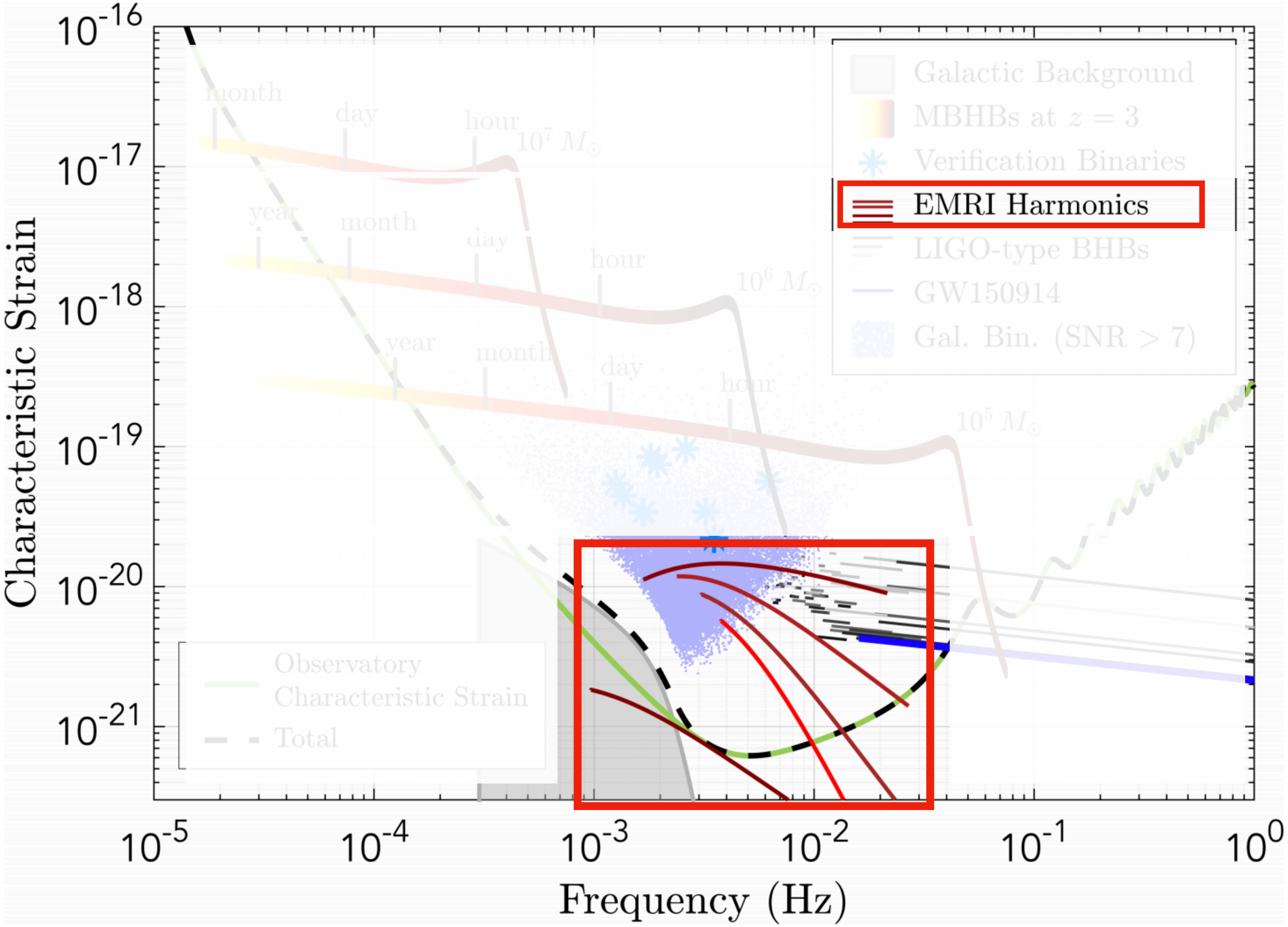


LISA Mission Proposal: arXiv:1702.00786

Contents

Measure the **cosmological parameters** $\Omega = \{H_0, \Omega_m, \ldots\}$

LASER INTERFEROMETER SPACE ANTENNA



Amaro-Seoane et al. (2017)

EXTREME MASS-RATIO INSPIRALS

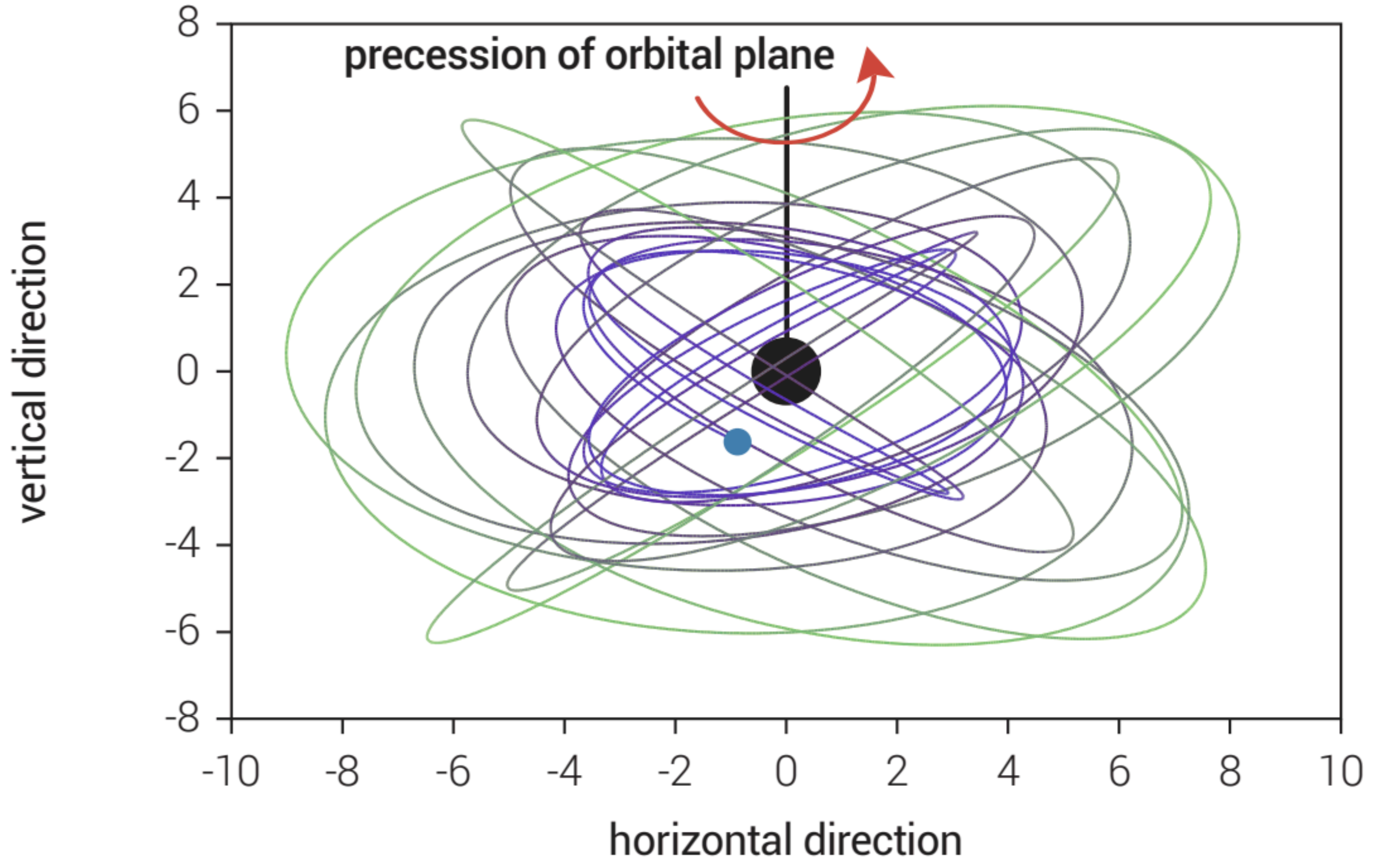
Binary systems with **mass-ratio** $q \sim 10^{-6} - 10^{-3}$

- **Massive BH** $(10^4 M_\odot - 10^7 M_\odot)$
- **Compact object** $(10 M_\odot)$

Slow inspiral, $10^4 - 10^5$ orbital cycles in the final year before plunge

✅ **Extremely accurate** measurements of the system parameters

❌ No EM counterpart



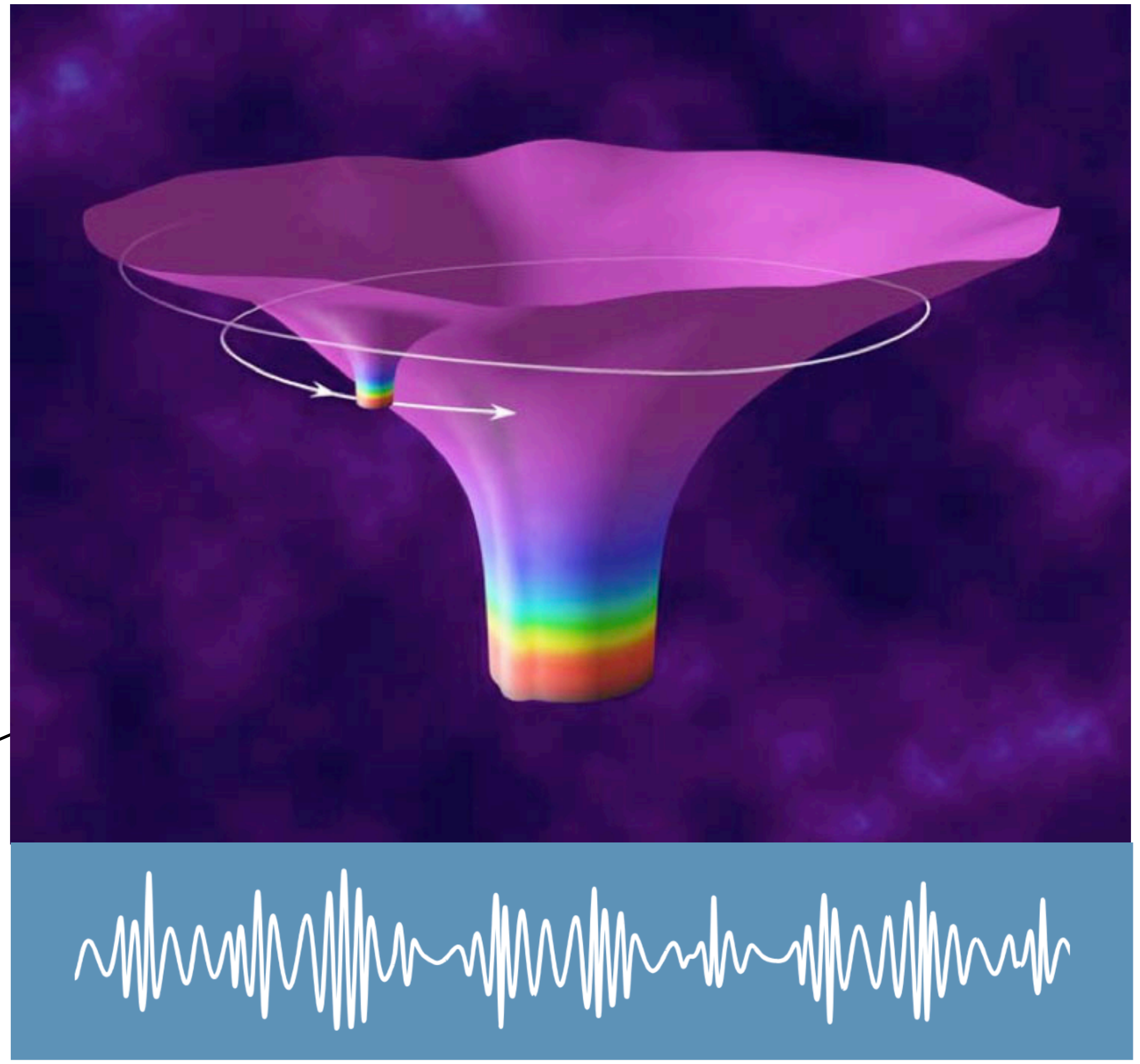


Figure 4: An artist's impression of the spacetime of an extreme-mass-ratio inspiral and a representative waveform of the expected gravitational waves. A smaller black hole orbits around a supermassive black hole. *Credit: NASA.*

eLISA White Paper, arXiv:1305.5720

$$q = \frac{m_2}{m_1}$$

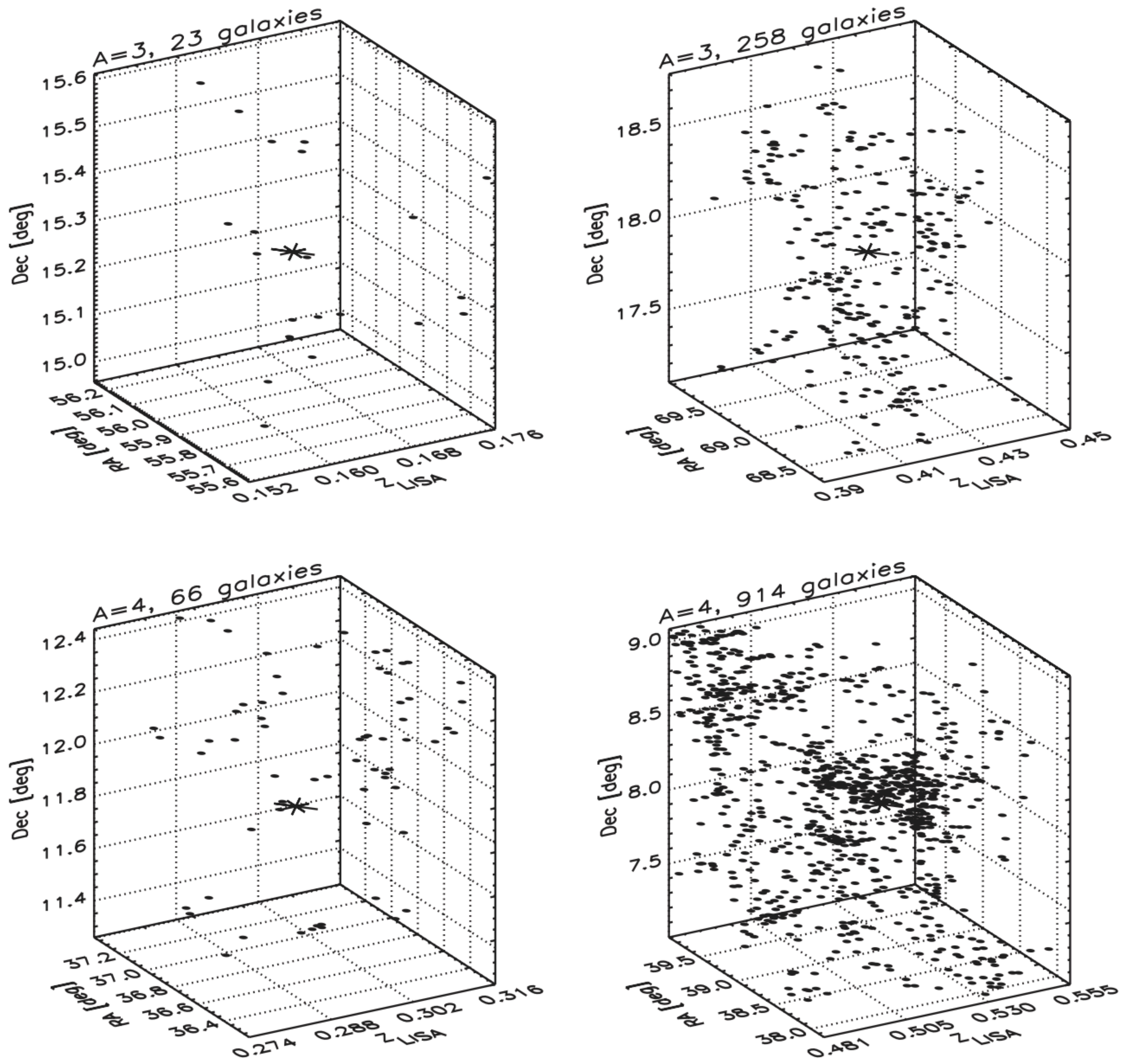
PREVIOUS STUDIES ON EMRIs AS DARK SIRENS

Macleod, Hogan, *PRD* (2008):

H_0 at 1% with 20 EMRIs at $z < 0.5$

BUT

- assume only linear cosmic expansion
- assume old 5 Gm LISA configuration
- no PE on the GW signals
- no Bayesian inference framework



Macleod, Hogan, PRD (2008)

'PE' = Parameter estimation

EMRIs AS DARK STANDARD SIRENS

Measure
EMRI GW

Obtain
d_L and (θ, ϕ)

Measure
galaxy redshifts

Obtain
galaxy catalog

cross-match

Estimate
EMRI redshift

$$d_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda (1+z')^{3(1+w_0+w_a)} e^{-3\frac{w_a z'}{1+z'}}}}$$

Bayesian analysis ➡ Measure $H_0, \Omega_m, \ldots$

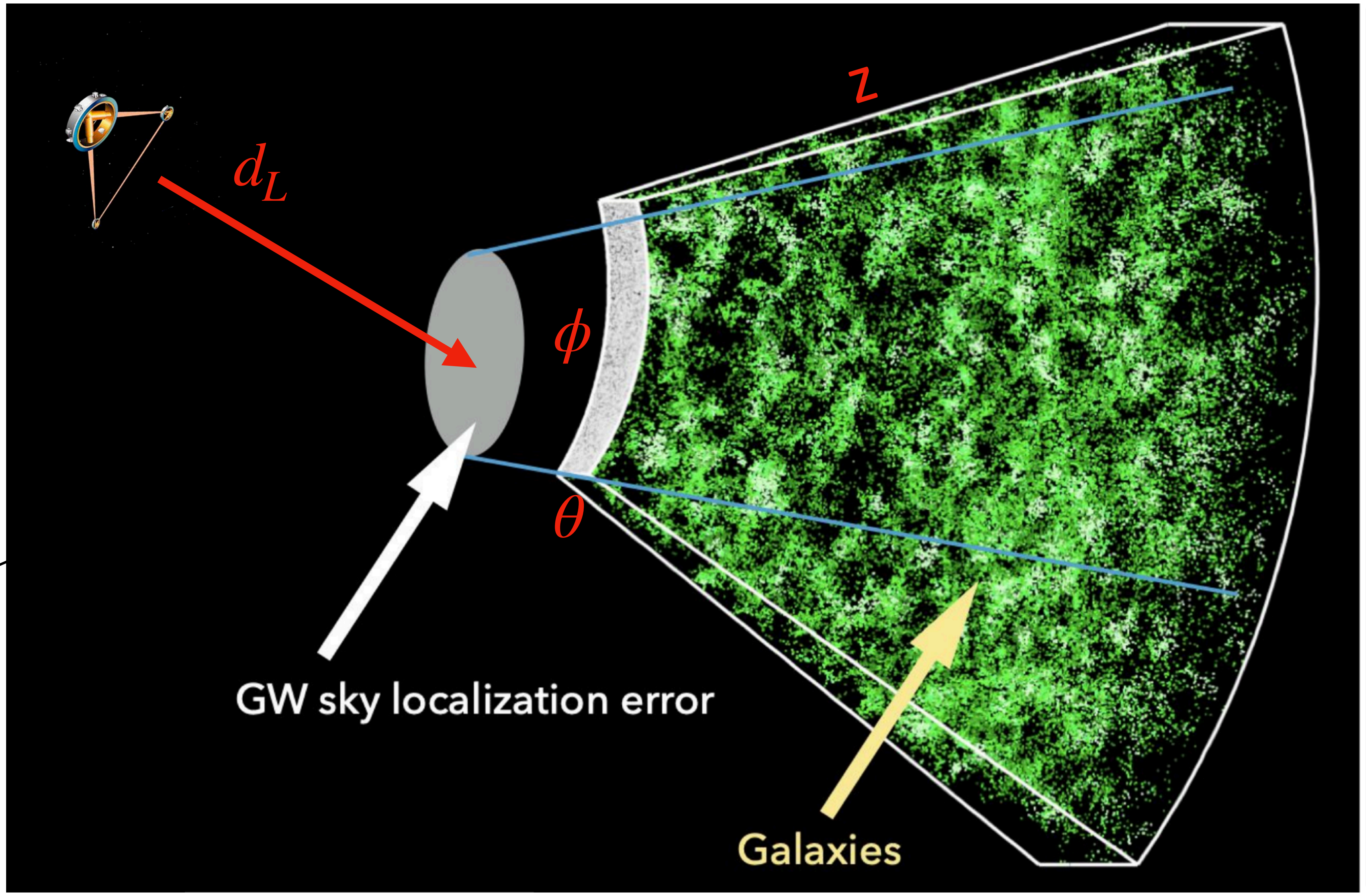


Image credit: Jeremy Tinker and the SDSS-III collaboration

HOW MANY EMRIs WILL WE OBSERVE?

EMRI rates span 2-3 orders of magnitudes, reflecting variations in:

- MBH population: semi-analytic models, realistic/pessimistic
- Stellar clusters distributions around MBHs
- EMRI's orbit parameters
- …

Model	Mass function	MBH spin	Cusp erosion	M–σ relation	N_p	CO mass $[M_\odot]$	EMRI rate $[\mathrm{yr}^{-1}]$ Total	Detected (AKK)	Detected (AKS)
M1	Barausse12	a98	yes	Gultekin09	10	10	1600	294	189
M2	Barausse12	a98	yes	KormendyHo13	10	10	1400	220	146
M3	Barausse12	a98	yes	GrahamScott13	10	10	2770	809	440
M4	Barausse12	a98	yes	Gultekin09	10	30	520 (620)	260	221
M5	Gair10	a98	no	Gultekin09	10	10	140	47	15
M6	Barausse12	a98	no	Gultekin09	10	10	2080	479	261
M7	Barausse12	a98	yes	Gultekin09	0	10	15800	2712	1765
M8	Barausse12	a98	yes	Gultekin09	100	10	180	35	24
M9	Barausse12	aflat	yes	Gultekin09	10	10	1530	217	177
M10	Barausse12	a0	yes	Gultekin09	10	10	1520	188	188
M11	Gair10	a0	no	Gultekin09	100	10	13	1	1
M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279

Babak et al., PRD (2017)

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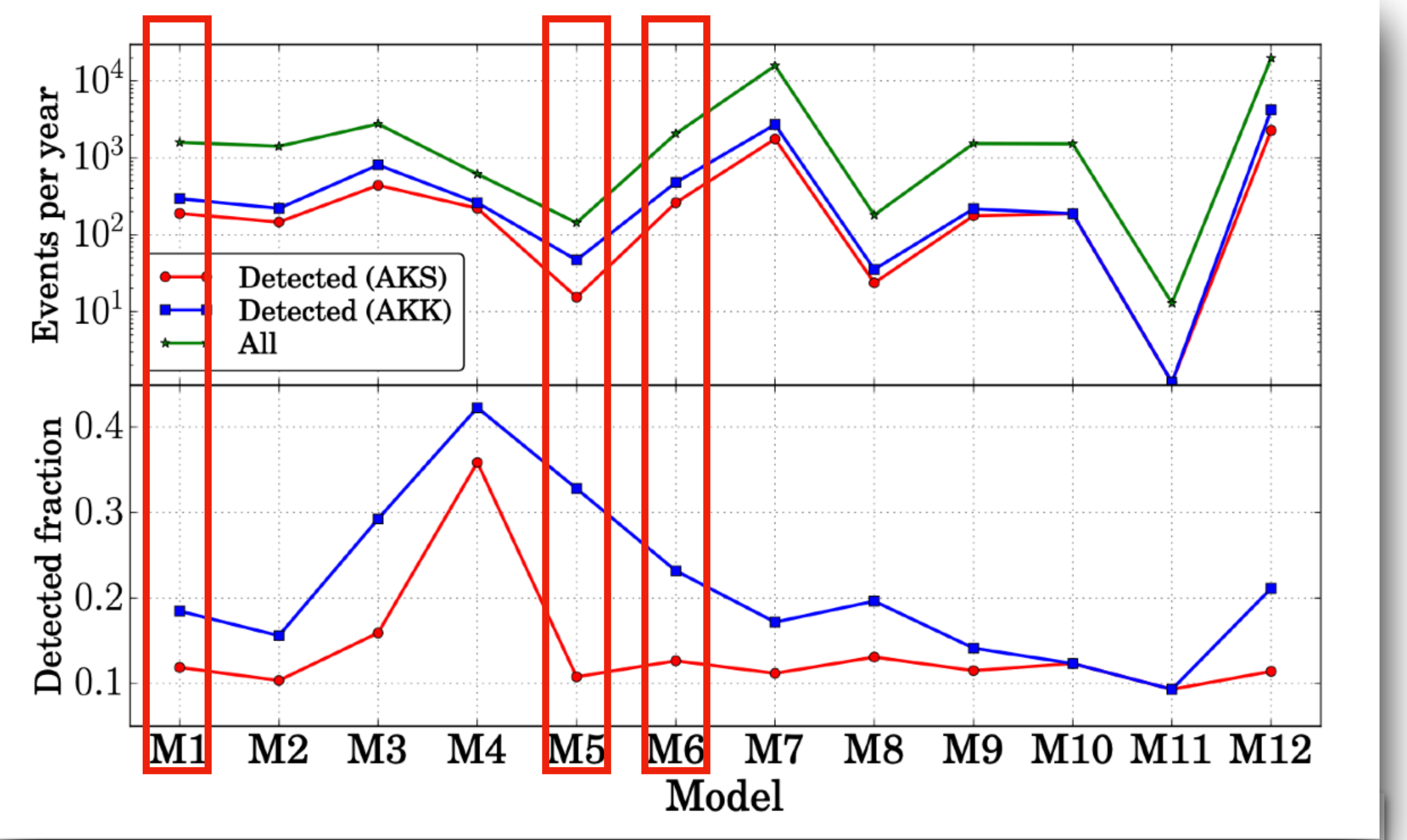

fiducial → M1
pessimistic → M5
optimistic → M6

Model	Mass function	MBH spin	Cusp erosion	M–σ relation	N_p	CO mass $[M_\odot]$	EMRI rate $[yr^{-1}]$ Total	Detected (AKK)	Detected (AKS)
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M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279

Babak et al., PRD (2017)

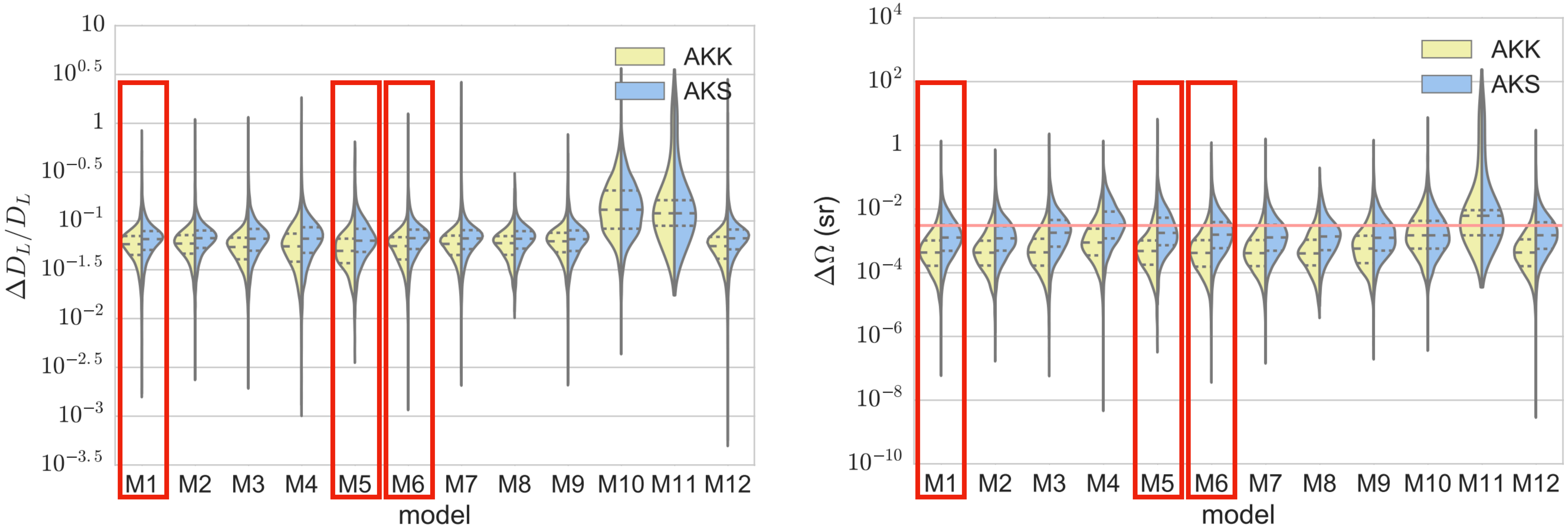
HOW WELL CAN WE LOCALIZE EMRIs?

EMRI PE: catalogs of Babak2017 provide **best estimates** and **uncertainties** for:

$$d_L \pm \sigma_{d_L} \qquad \phi \pm \sigma_\phi \qquad \theta \pm \sigma_\theta$$

$\Delta d_L/d_L \sim 10^{-1}$

$\Delta\Omega/\Omega \sim 10\ \mathrm{deg}^2$



Babak et al., PRD (2017)

OUR DATA: LOCALISATION ERROR VOLUMES

Flux-limited, full-sky galaxy simulations of Henriques et al., *MNRAS* (2012) based on the **Millennium Run**

Springel et al., *Nature* (2005)

- For a given cosmology:

$$\hat{d}_L \pm \Delta\hat{d}_L \longrightarrow z \pm \Delta z$$

- Assuming cosmological priors:

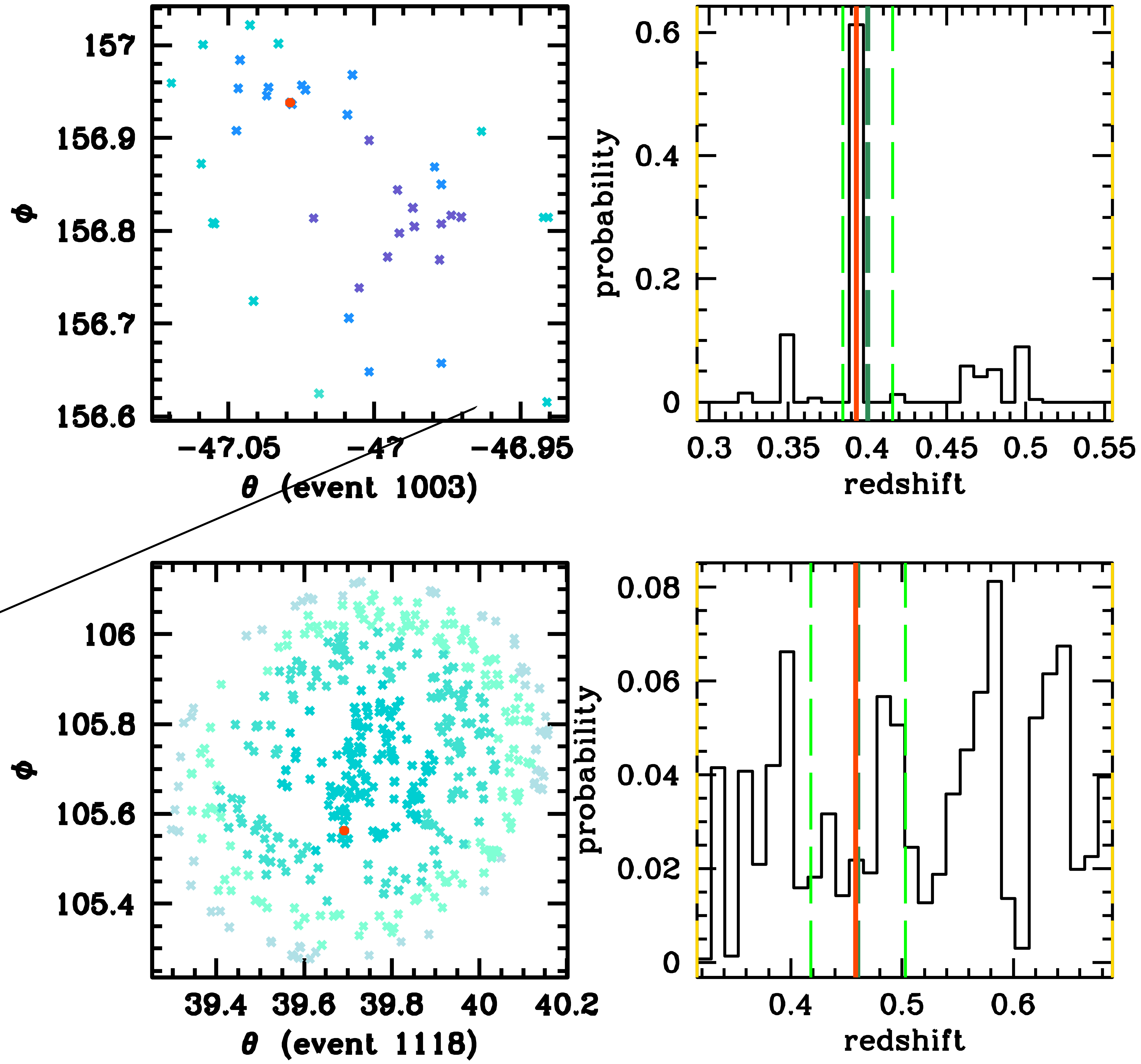
$$[z^-, z^+]$$

- Accounting for galaxy peculiar velocities:

$$[z^- - \Delta z^-_{v_p}, z^+ + \Delta z^+_{v_p}]$$

- EMRI localisation volume:

$$\Delta\Omega_{sky} \times [z^- - \Delta z^-_{v_p}, z^+ + \Delta z^+_{v_p}]$$

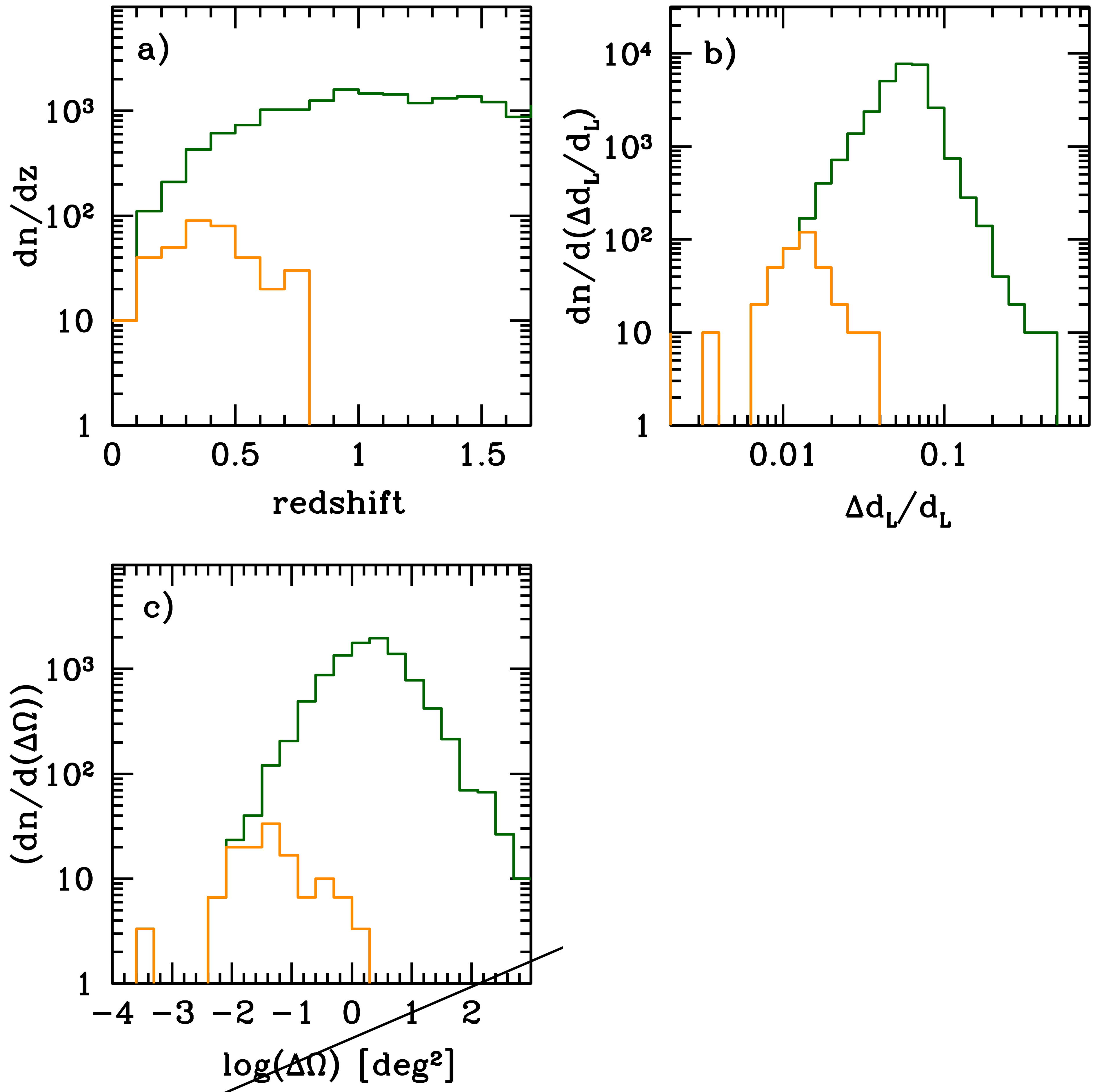


Laghi, Tamanini, Del Pozzo, Sesana, Gair, Babak, Izquierdo-Villalba, *MNRAS* (2021)

SELECTING EVENTS

Green: 10yrs 3k EMRIs!

M1 model



Laghi et al., *MNRAS* (2021)

SELECTING EVENTS

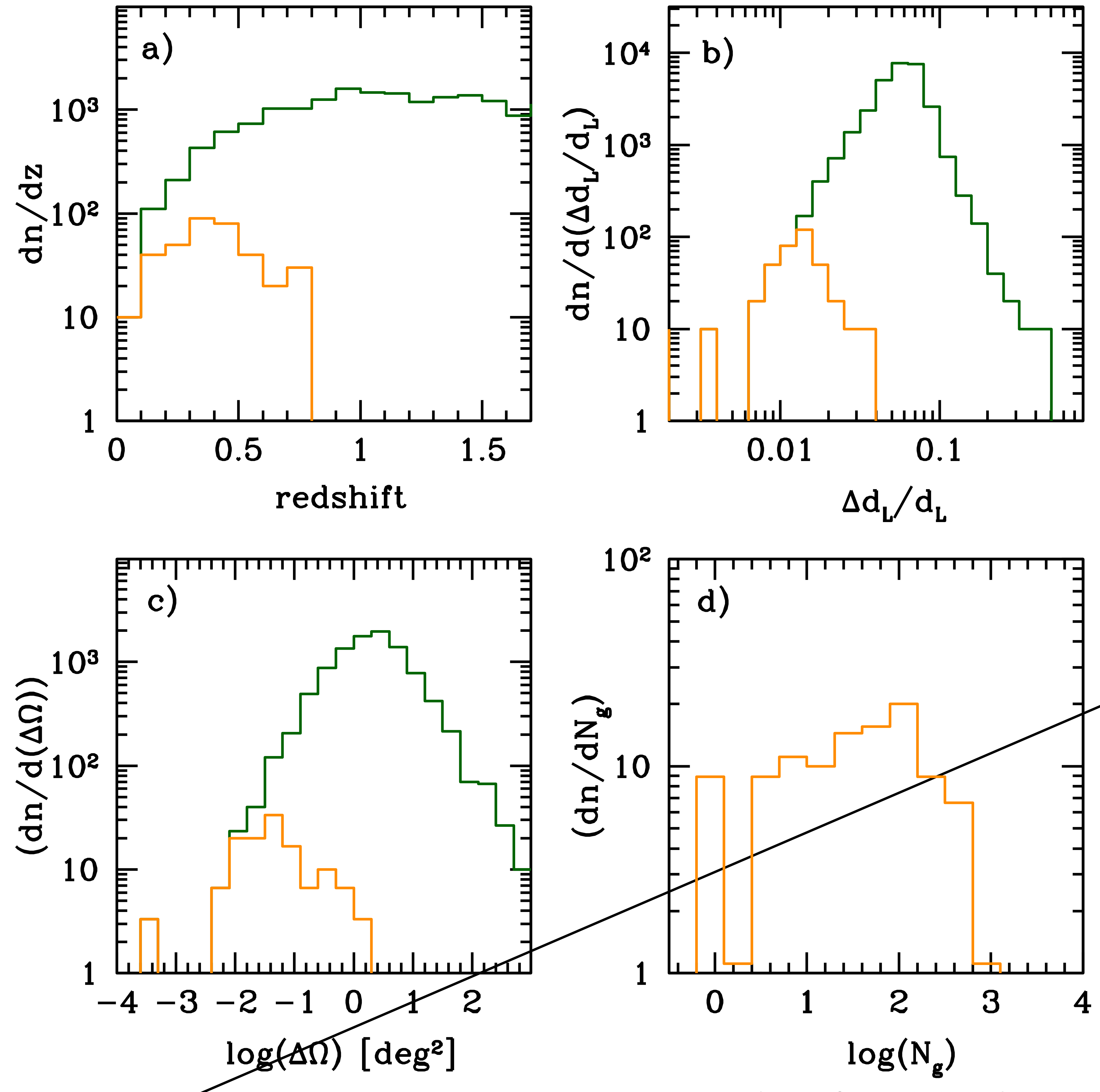
$\Delta d_L / d_L \lesssim 0.04$

$\Delta\Omega_{sky} \lesssim 2\ \mathrm{deg}^2$

Green: 10yrs 3k EMRIs!

Orange: 10yrs & SNR>100

M1 model



number of candidate hosts within the 3D error volume

Laghi et al., *MNRAS* (2021)

Require **SNR>100**:

- Well-localised, most-informative events
- Few hosts per error-box

(SNR>100)	Events (4 yr)	Events (10 yr)
M5 (pessimistic)	O(2)	O(5)
M1 (fiducial)	O(10)	O(30)
M6 (optimistic)	O(30)	O(70)

COSMOLOGICAL INFERENCE SCHEME

sky-position-dependent w_j

cosmological model $\mathscr{H}$:

$$d(\Omega, z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda (1+z')^{3(1+w_0+w_a)} e^{-3\frac{w_a z'}{1+z'}}}}$$

GW likelihood:

3D Gaussian distribution (from EMRI PE)

GW redshift prior:

LISA 3D error volume + galaxy catalog

$$p(\Omega \,|\, D, \mathscr{H}) \propto p(\Omega \,|\, \mathscr{H}) \prod_{i=1}^{N} \int dz_{\mathrm{GW}} \, \mathscr{N}(d(z_{\mathrm{GW}}, \Omega) - \bar{d}_L, \sigma_{d_L}^2) \sum_{j=1}^{N_{\mathrm{hosts}}} w_j \, \mathscr{N}(z_j - z_{\mathrm{GW}}, \sigma_{v_p}^2)$$

Posterior **Prior** **Quasi-likelihood**

Nested sampling
`[CPNest]`
Parallelised

Del Pozzo, Laghi [github.com/wdpozzo/cosmolisa] **`cosmoLISA`**

cosmoLISA

Quick facts

- Bayesian inference of cosmological parameters with LISA (and 3G detectors)
- Forecasts with **dark sirens** or **bright sirens** (GW data and galaxy catalog pre-processed)
- Sources: EMRI, MBHB, …
- **The code is public** [Del Pozzo, Laghi: `github.com/wdpozzo/cosmolisa`]

Implementation

- Modules written in **cython** (likelihood, libraries from LALCosmology) to speed up the inference
- **Nested sampling** algorithm (CPNest) optimised for multithreading

Ongoing development

- GW **selection effects** and **incompleteness** of galaxy catalogs
- **Joint** inference of **cosmological** (beyond H0) + **source population** parameters

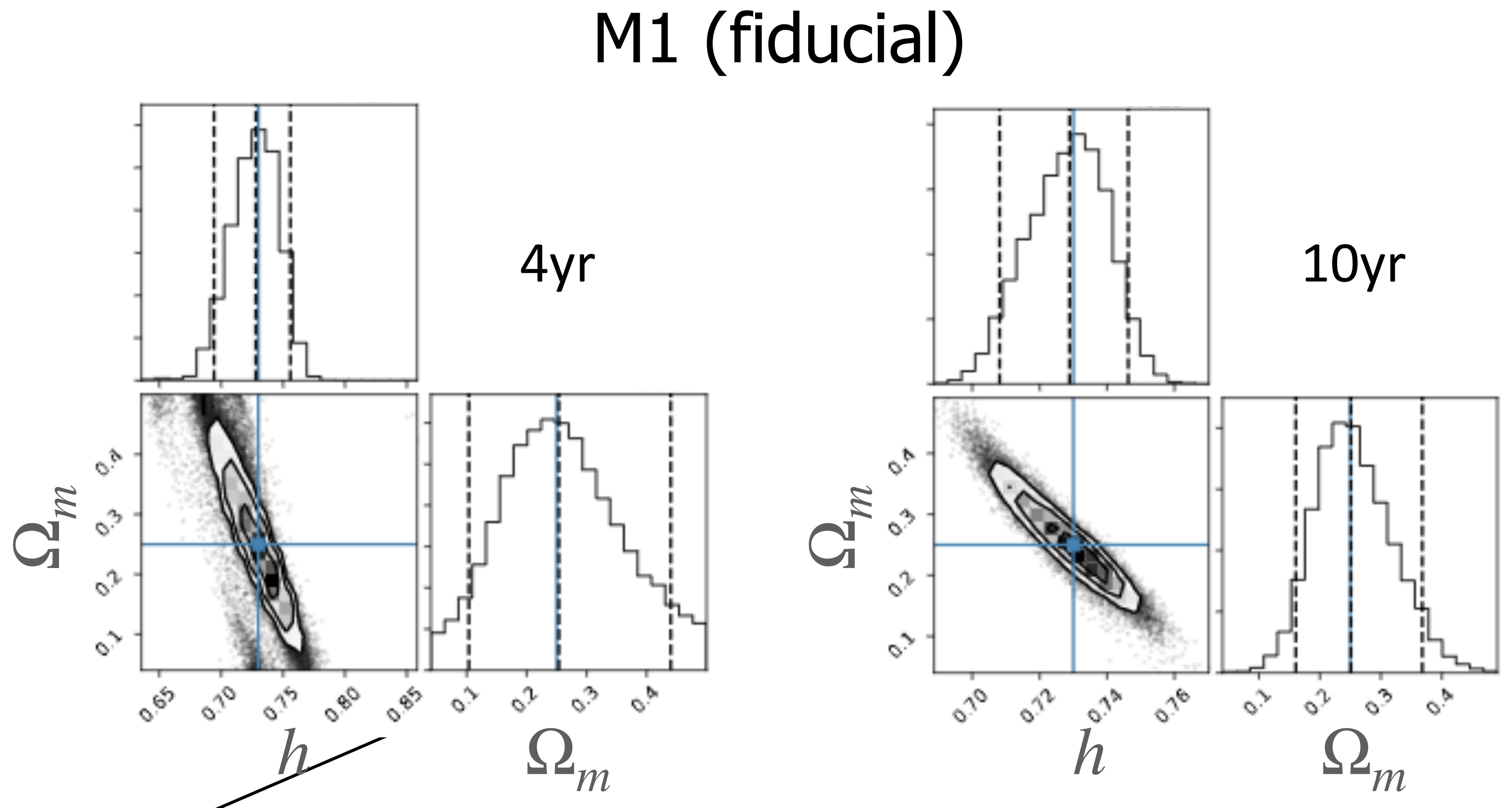
RESULTS: ΛCDM

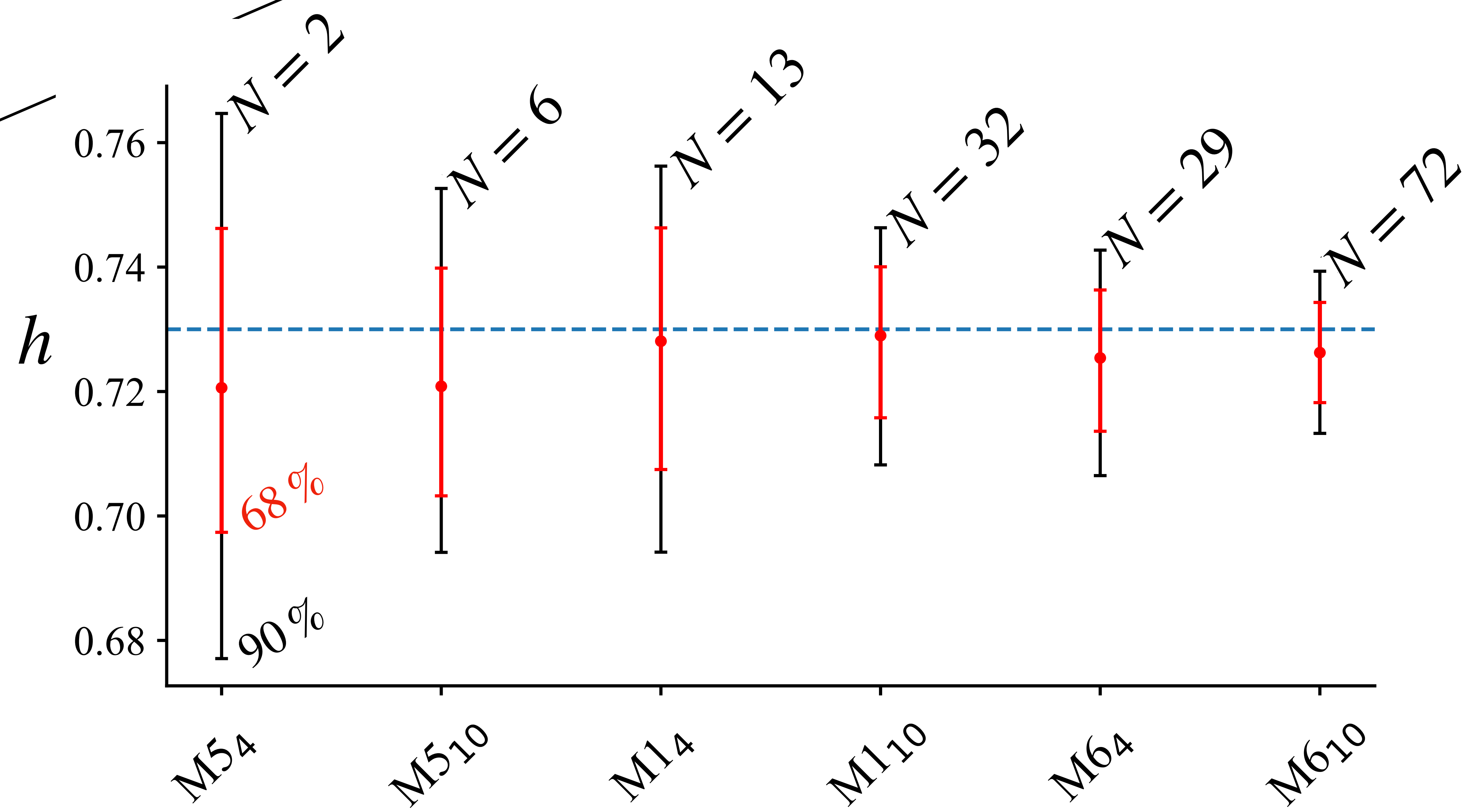
$h = H_0/100\,\mathrm{km}^{-1}\mathrm{s\ Mpc}$

EMRIs will be very good probes of H_0

h accuracy (68% CI)
1-6%

Ω_m accuracy (68% CI)
25% at most





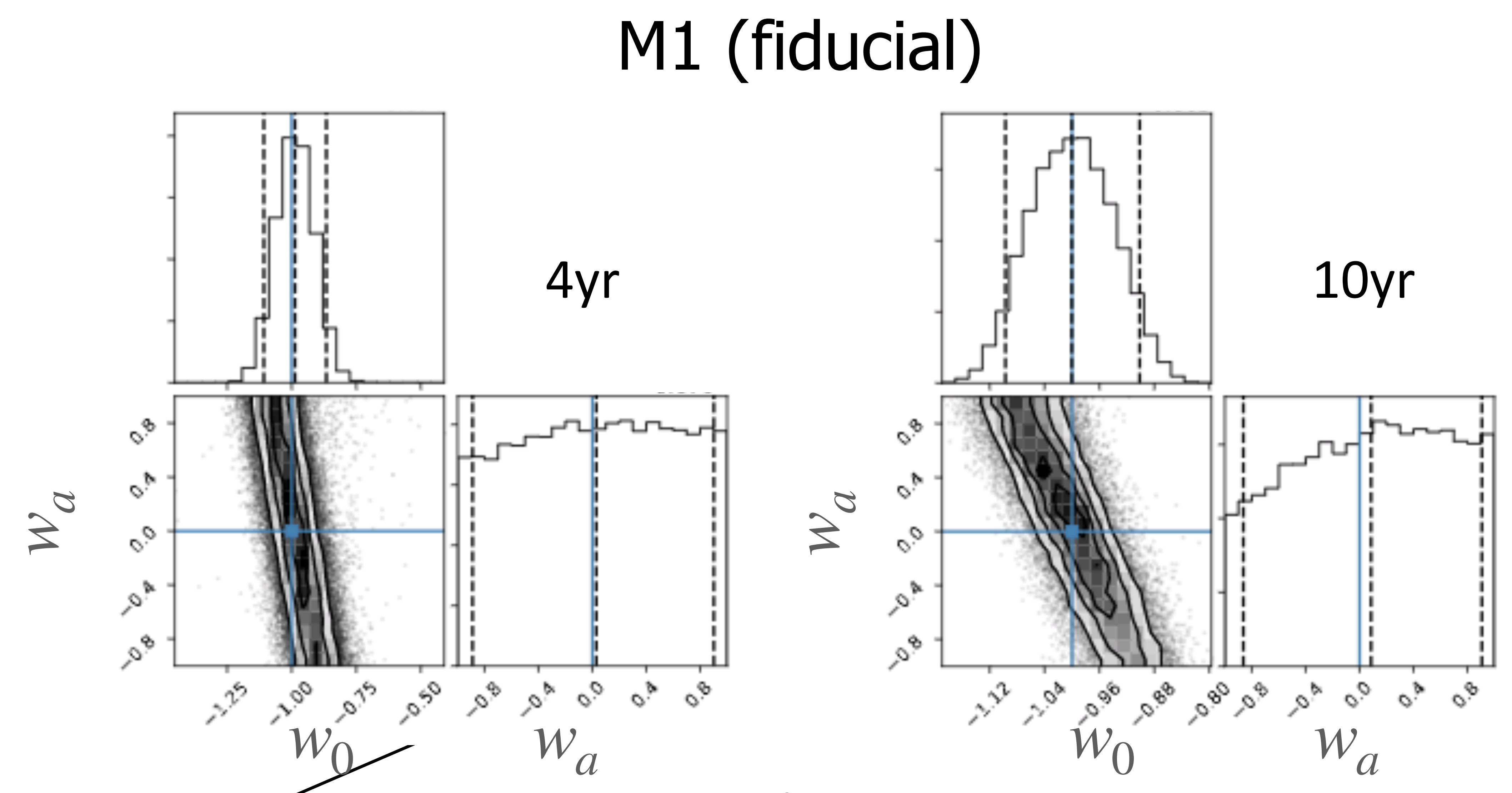
Laghi et al., *MNRAS* (2021)

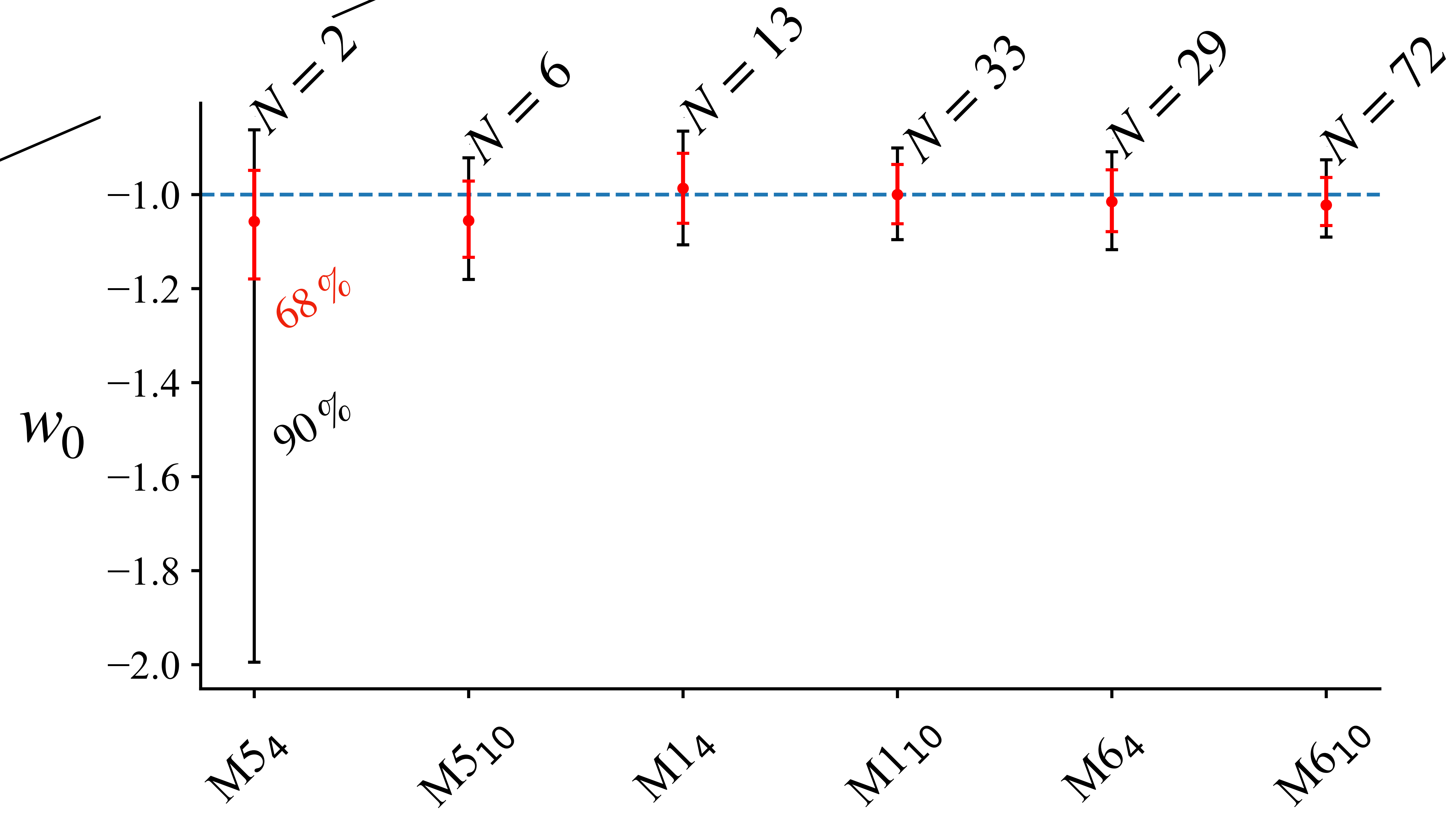
RESULTS: DARK ENERGY

EMRIs can constrain w_0

$$w(z) = w_0 + w_a z/(1+z)$$

w_0 accuracy (68% CI)
10% at least





Laghi et al., *MNRAS* (2021)

Modified gravity beyond CPL:
see talk by **C. Liu**

MASSIVE BLACK HOLE BINARIES AS BRIGHT STANDARD SIRENS

Measure MBHB GW

See previous talk by **A. Mangiagli**

Obtain d_L and (θ, ϕ)

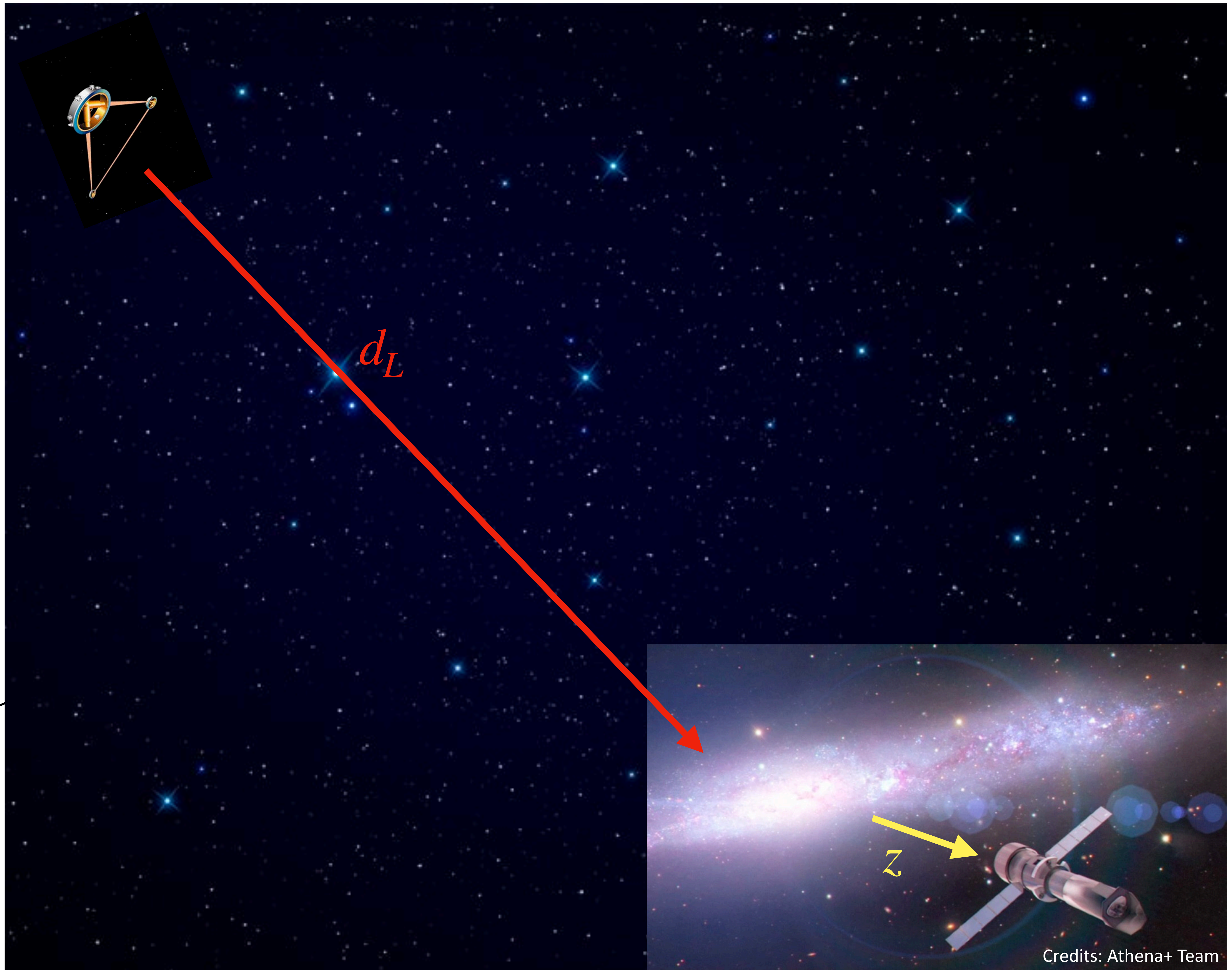
Measure EM counterpart

Obtain redshift

$$d_L = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda (1+z')^{3(1+w_0+w_a)} e^{-3\frac{w_a z'}{1+z'}}}}$$

Bayesian analysis

Measure $H_0, \Omega_m, \ldots$

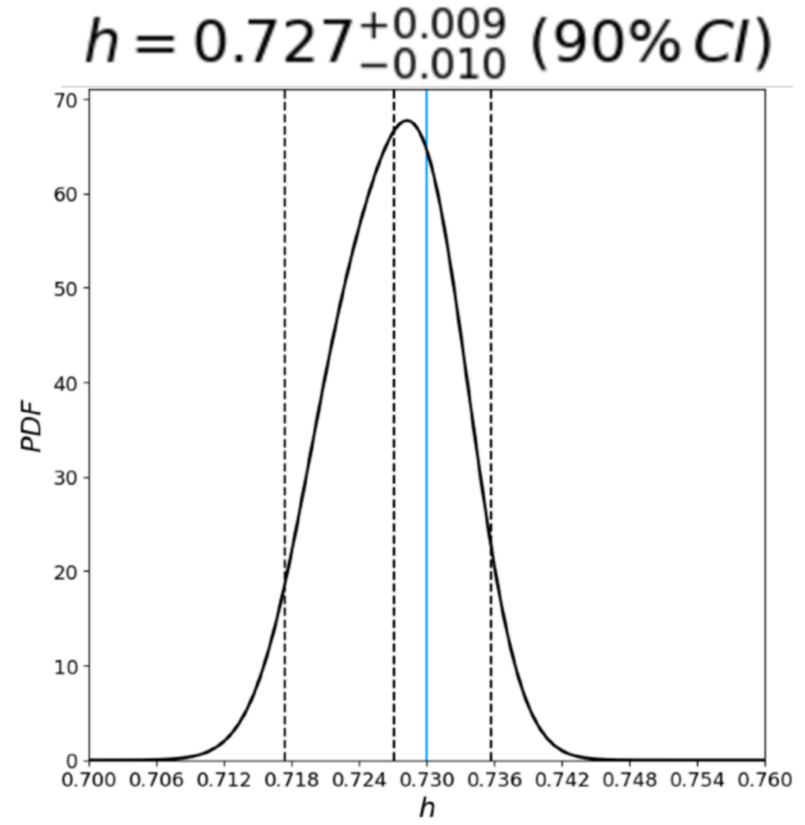


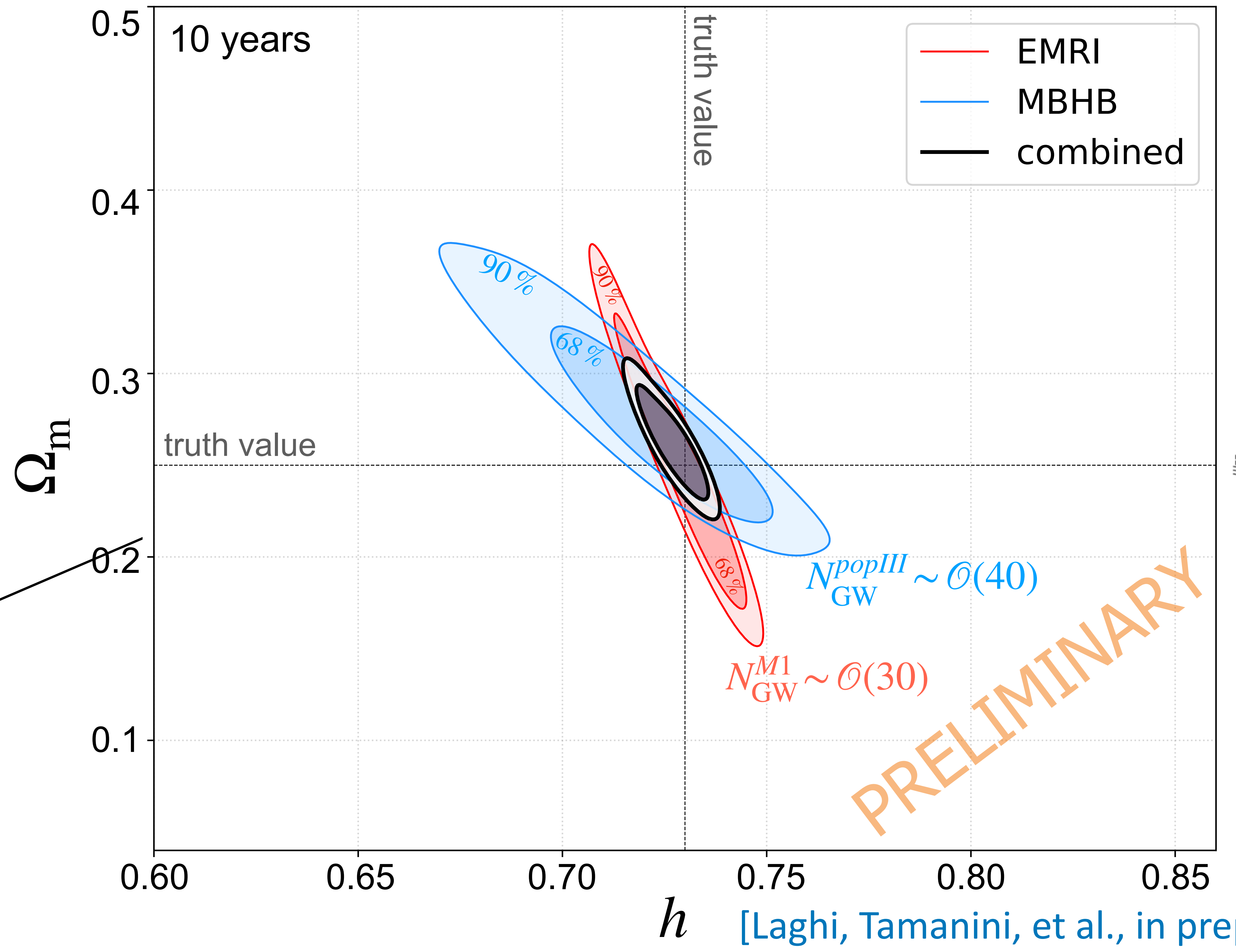
JOINT ANALYSIS: EMRIs + MBHBs

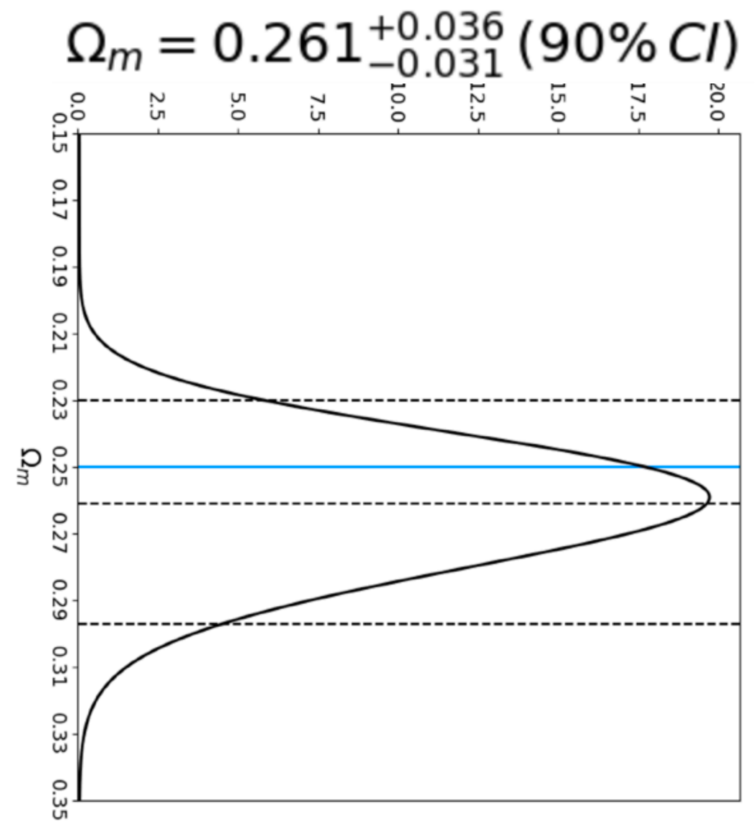
Combine them to help break degeneracies!

H_0 at <1% ?

Ω_m at <10% ?







[Laghi, Tamanini, et al., in preparation]

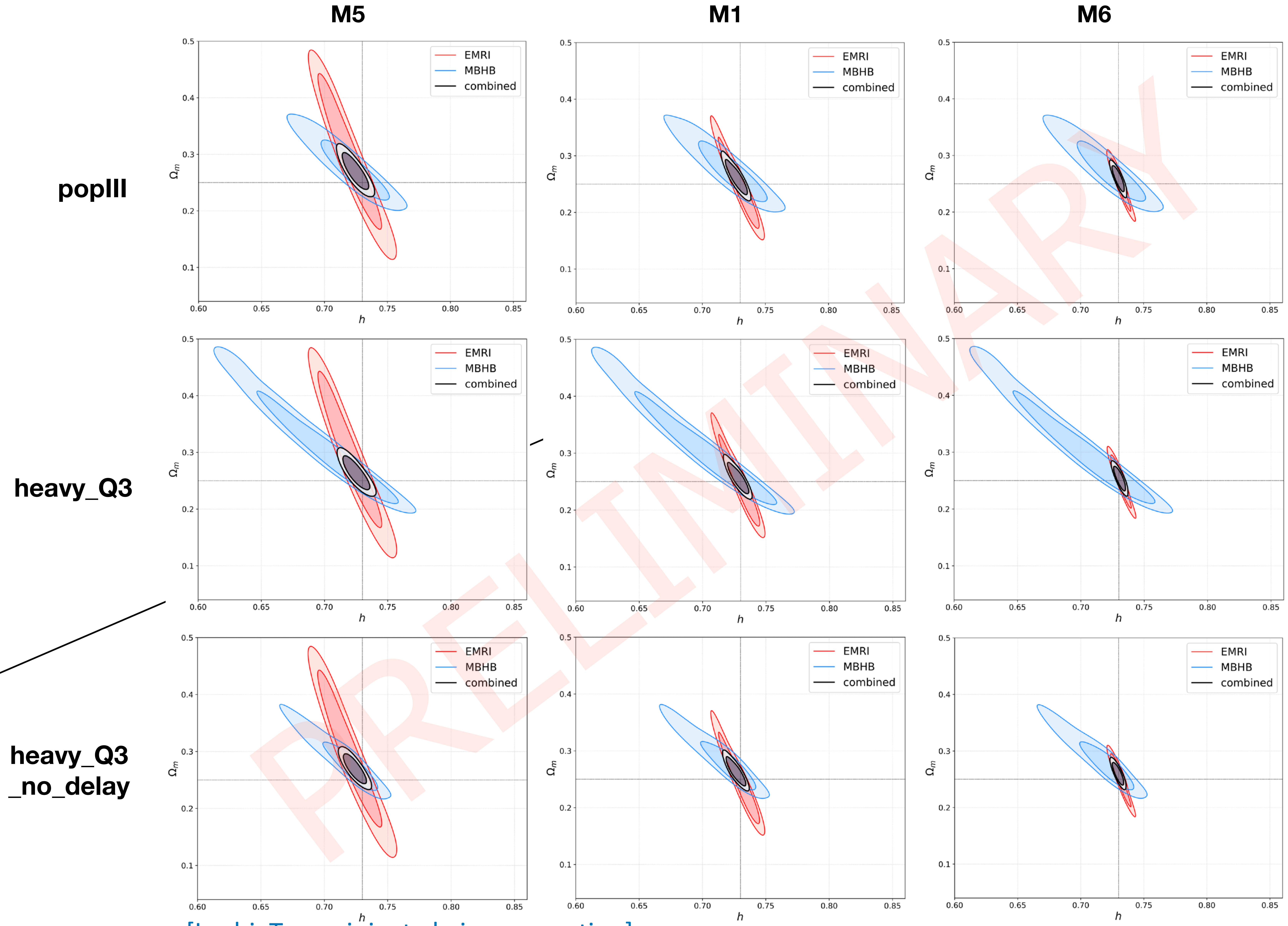
FORECASTS FOR DIFFERENT EMRIs & MBHB MODELS

- New MBHB+EM catalogs (see previous talk by A. Mangiagli)

- New EMRI catalogs [Liu, Burke, Chua, Kejriwal, DL, Tamanini, et al., in preparation]:
  - Updated LISA sensitivity curve
  - AAK EMRI waveform (FEW package) [Kats+2021]

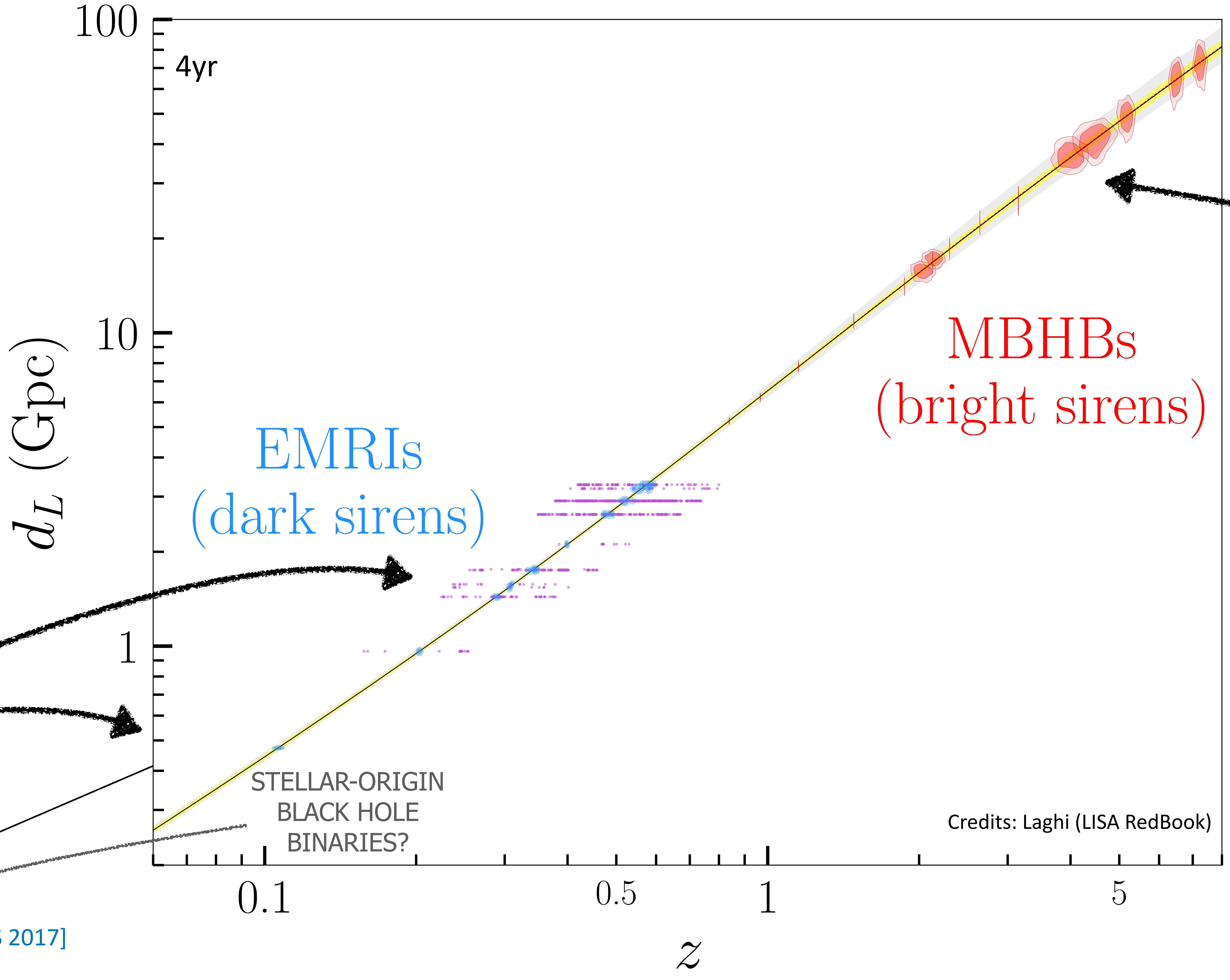
How many detected EMRIs?
How many informative events?

See also: [Pozzoli+ 2302.07043]



[Laghi, Tamanini, et al., in preparation]

LISA STANDARD SIRENS



"Bright sirens"
with
EM counterpart

MBHB+EM catalog
(from A. Mangiagli)

EMRI catalog
(from Babak2017)

"Dark sirens"
without
EM counterpart

[Del Pozzo, Sesana, Klein, MNRAS 2017]