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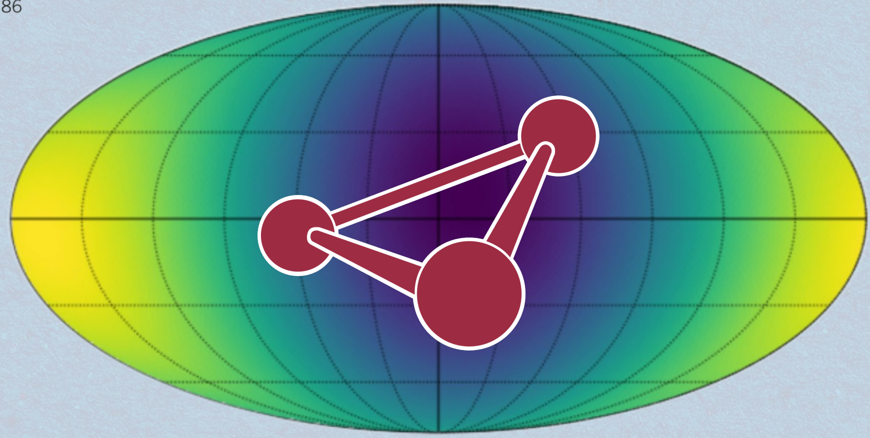

Doppler-boosted anisotropies of SGWB and LISA

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10th LISA Cosmology Working Group Workshop

Scope and objectives

- Main challenge of the search for SGWB signal with LISA:

  How do we distinguish a potential cosmological signal from instrumental noise ?

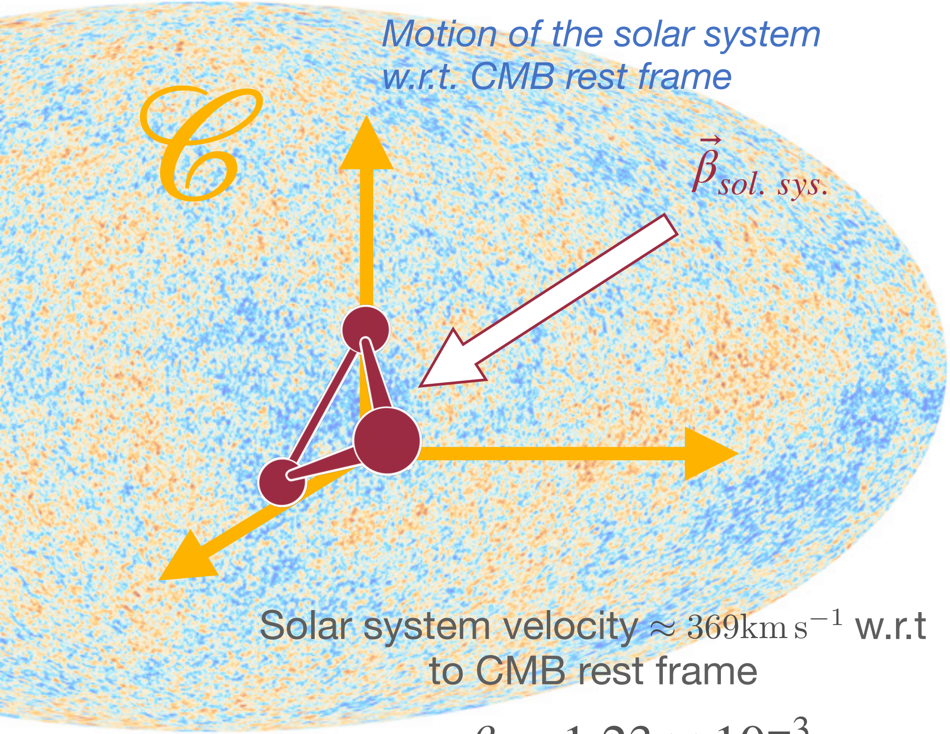
  (+ from galactic confusion noise, astrophysical background..)

  On what kind of evidence can we claim an apparent excess of power is cosmological ?

1. The instrument response projects differently noise and signal on data.
2. The signal has distinctive features not shared with the noise (anisotropy)

➡ **Kinematic anisotropy is a signature of an extragalactic origin**

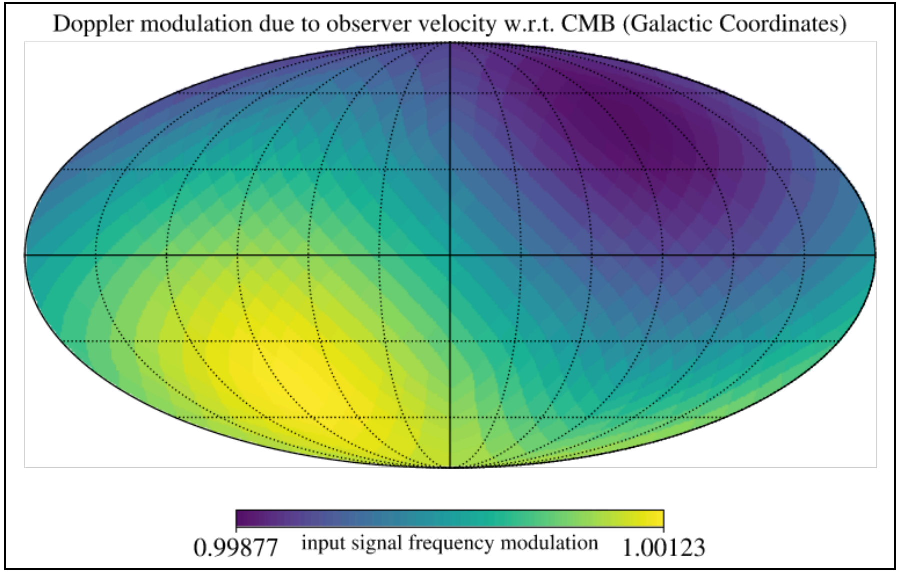
Principle: Doppler boosting of the SGWB



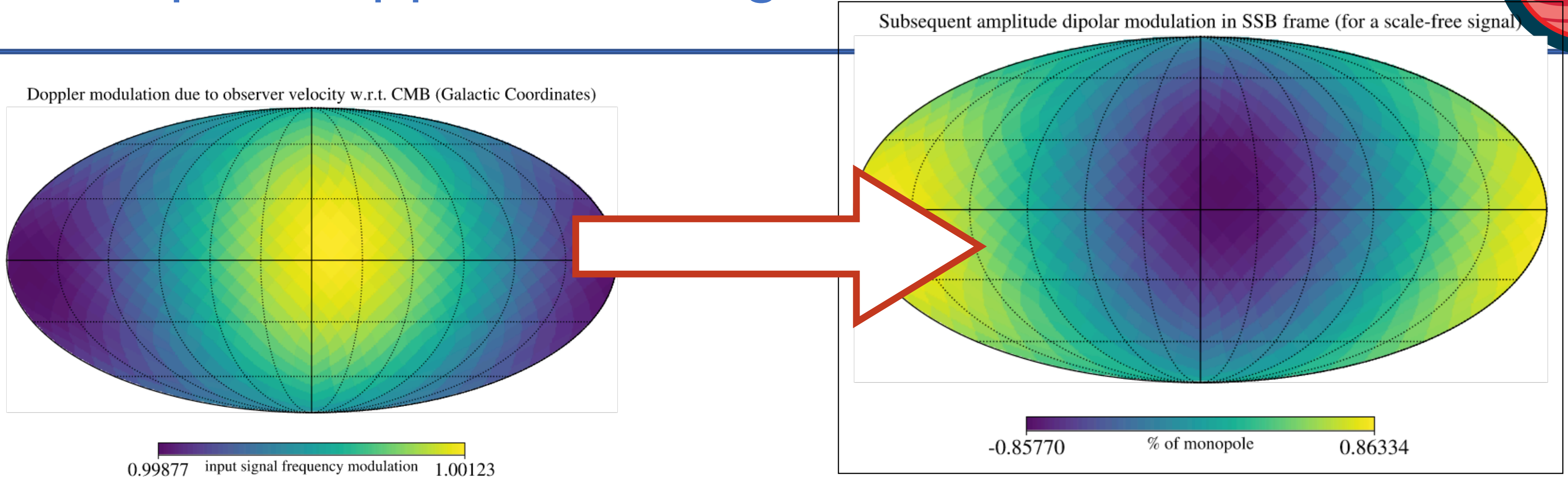
Motion of the solar system w.r.t. CMB rest frame

Solar system velocity $\approx 369\mathrm{km\,s^{-1}}$ w.r.t to CMB rest frame

$$\rightarrow \beta = 1.23 \times 10^{-3}$$



Principle: Doppler boosting of the SGWB



Doppler modulation due to observer velocity w.r.t. CMB (Galactic Coordinates)

Subsequent amplitude dipolar modulation in SSB frame (for a scale-free signal)

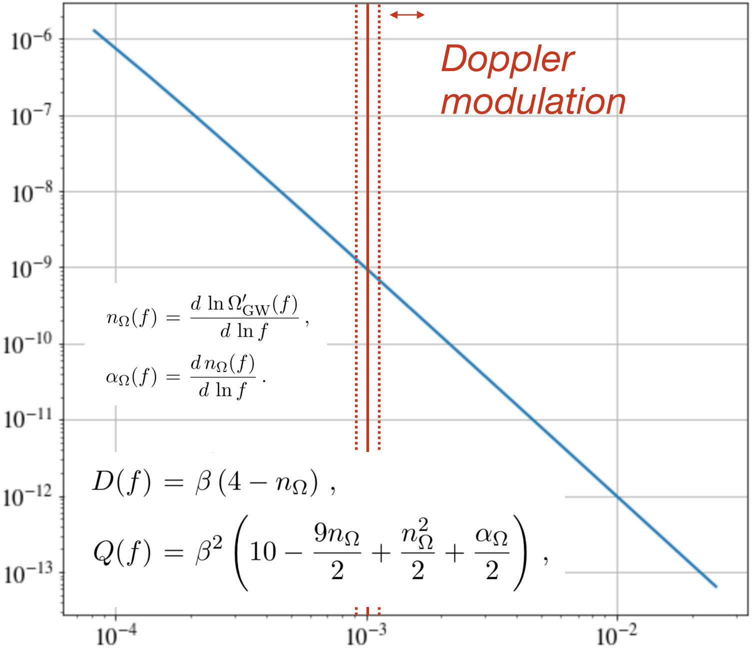
$$\mathscr{D} = \frac{1-\beta^2}{1-\beta\ \hat{n}\cdot\hat{v}}$$

$$\Omega_{GW}(f,\hat{\mathbf{n}}) = \mathscr{D}^4\ \Omega'_{GW}(f)\left(\mathscr{D}^{-1}f\,,\ \frac{\hat{\mathbf{n}}+\hat{\mathbf{v}}\left[\left(\gamma-\mathbf{1}\right)\xi-\gamma\beta\right]}{\gamma\left(\mathbf{1}-\beta\xi\right)}\right)$$

1. ***Cusin et al. 2022, "Doppler boosting the stochastic gravitational wave background"***

2. **Bartolo et al. 2022, « Probing anisotropies of the Stochastic Gravitational Wave Background with LISA »**

Principle: Doppler boosting of the SGWB

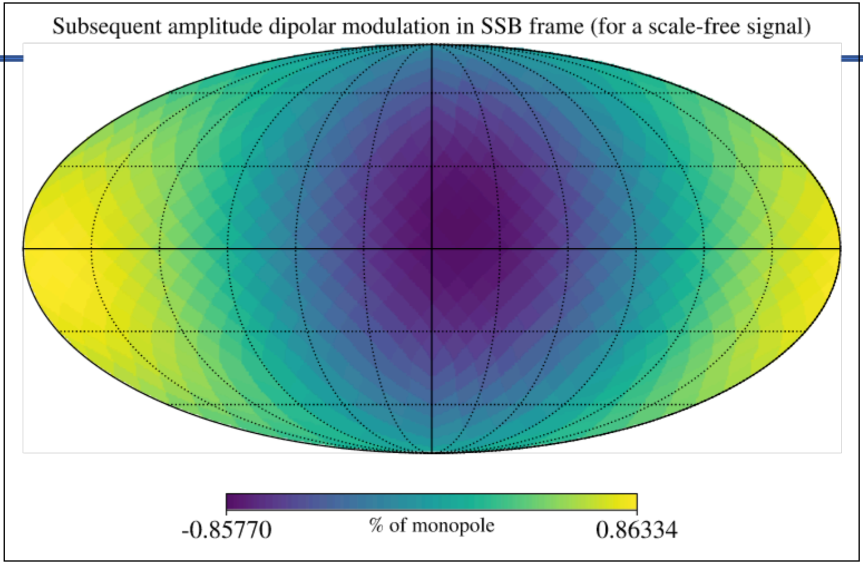


$$n_\Omega(f) = \frac{d\ln\Omega'_{\rm GW}(f)}{d\ln f}\,,$$

$$\alpha_\Omega(f) = \frac{d\,n_\Omega(f)}{d\ln f}\,.$$

$$D(f) = \beta\,(4 - n_\Omega)\,,$$

$$Q(f) = \beta^2\left(10 - \frac{9n_\Omega}{2} + \frac{n_\Omega^2}{2} + \frac{\alpha_\Omega}{2}\right)\,,$$



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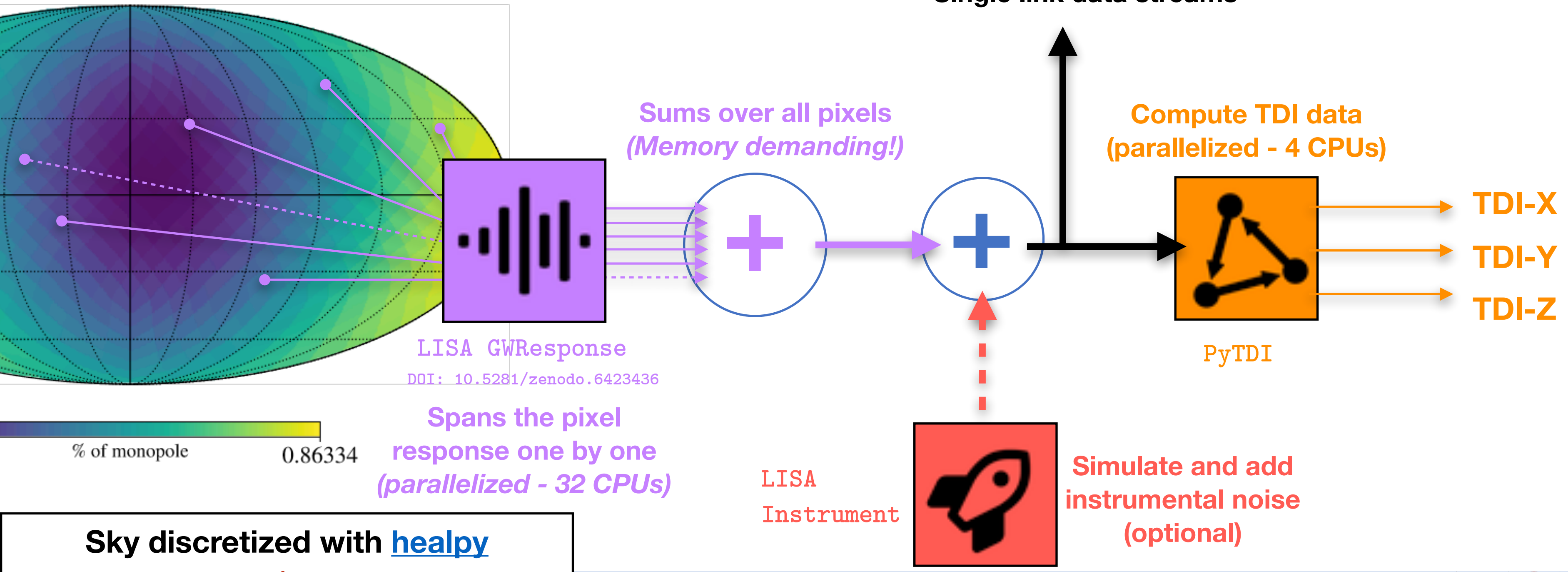
DATA generation: full **time-domain** simulation of GW anisotropic sky, via `LISA Simulation Suite`



Generate noise time series for each pixel

$\eta_{12}(t), \eta_{21}(t), \eta_{13}(t), \eta_{31}(t), \eta_{23}(t), \eta_{32}(t)$

Single link data streams

Sums over all pixels
(Memory demanding!)

Compute TDI data (parallelized - 4 CPUs)

TDI-X
TDI-Y
TDI-Z

`LISA GWResponse`
`DOI: 10.5281/zenodo.6423436`

`PyTDI`

Spans the pixel response one by one
(parallelized - 32 CPUs)

`LISA Instrument`

Simulate and add instrumental noise (optional)

Sky discretized with healpy
direction $\hat{n} \rightarrow$ ***pixel*** p

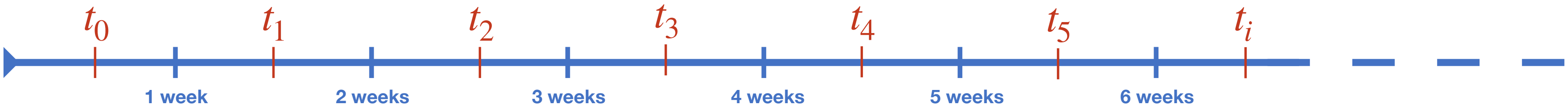
DATA generation: full **time-domain** simulation of GW anisotropic sky, via `LISA Simulation Suite`

- Physical assumptions:
  - ✓Pixel stochastic strain time series uncorrelated
  - ✓*Equal arm* or *keplerian* orbits.
  - ✓TDI 2.0
  - ✓**Arm propag. delays in TCB time**
  - ✓Secondary noise only (when noise on)

- Simulation settings
  - 3 years, sampling frequency = [0.05 Hz / 0.1 Hz]
  - Number of pixels: [12288 / 3072]
  - Cosmological signal: $\alpha = 0, \Omega = [10^{-12}, 10^{-7}]$

Map-making strategy: pre-processing the **DATA**

- Time-splitting the 3 years long TDI data streams.
—> *Will set the angular resolution of the analysis*



- Frequency averaging (data compression) :

$$\bar{\mathbf{D}}(t_i, f_j) \equiv \frac{1}{n_j} \sum_{k=j-\frac{n_j}{2}}^{j+\frac{n_j}{2}} \tilde{\mathbf{d}}(t_i, f_k)\tilde{\mathbf{d}}(t_i, f_k)^{\dagger} .$$

TDI X, Y, Z data streams

- DA problem: we're solving for the covariance C_d of the signal $\tilde{\mathbf{d}}$.

Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"

Baghi et al. 2023, "Uncovering gravitational-wave backgrounds from noises of unknown shape with LISA"

Decompose R in term of G and TDI transfer function. Point to Baghi et al. for details.

Covariance **MODEL** and max likelihood map-making strategy

[*Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"*]

- Covariance model:

$$\mathbf{C}_d(t_i, f_j) = \mathbf{A}(t_i, f_j, p)\ I(p)\ + \mathbf{N}(t_i, f_j)$$

LISA quadratic response

Pixel Map to solve for

Instrumental Noise

$$I(f, \hat{n}) = \Omega_{\mathrm{GW}}(f, \hat{n}) \frac{3H_0^2}{4\pi^2 f^3}$$

Sky discretized with healpy

direction $\hat{n} \rightarrow$ pixel p

- LISA quadratic response:

$$A(t_i, f_j, p) = R_+(t_i, f_j, p) \otimes R_+(t_i, f_j, p)^* \ + \ R_\times(t_i, f_j, p) \otimes R_\times(t_i, f_j, p)^*$$

$$R_P(t_i, f_j, p) = M_{TDI}(t_i, f_j)\ G_P(t_i, f_j, p)\ M_{TDI}(t_i, f_j)^\dagger$$

TDI matrix (phasing operators)

single link response (freq. domain, at time t_i)

TDI matrix (phasing operators)

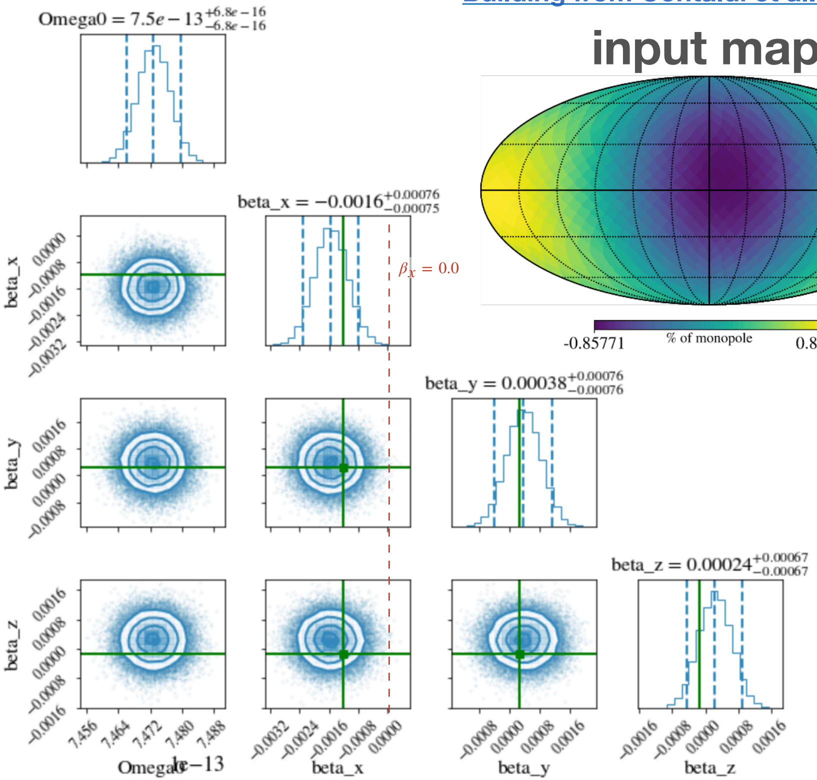
- log-Likelihood, Wishart statistics:

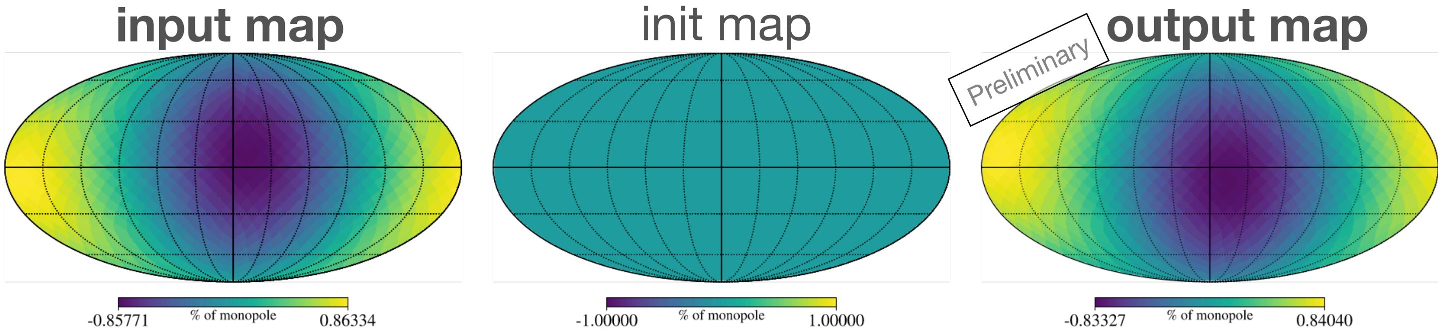
[*Baghi et al. 2023*]

$$\log \mathscr{L} = \sum_{t_i} \sum_{f_j} \left[-\mathrm{tr}(\mathbf{C}_d^{-1} \mathbf{D}(t_i, f_j)) - \nu \log |\, \mathbf{C}_d(t_i, f_j) \,| \right]$$

MCMC sampling the velocity: recovered sky maps

Building from Contaldi et al. 2020, extending to time-domain sim, spectrum averaging, and MCMC sampling





$$\Omega_{GW}(f,\hat{\mathbf{n}}) = \mathscr{D}^4\ \Omega'_{GW}(f)\left(\mathscr{D}^{-1}f\ ,\ \hat{\mathbf{n}}\right)$$

$$\mathscr{D} = \frac{1-\beta^2}{1-\beta\ \hat{n}\cdot\hat{v}}$$

← **Fitting for $\vec{\beta}$**

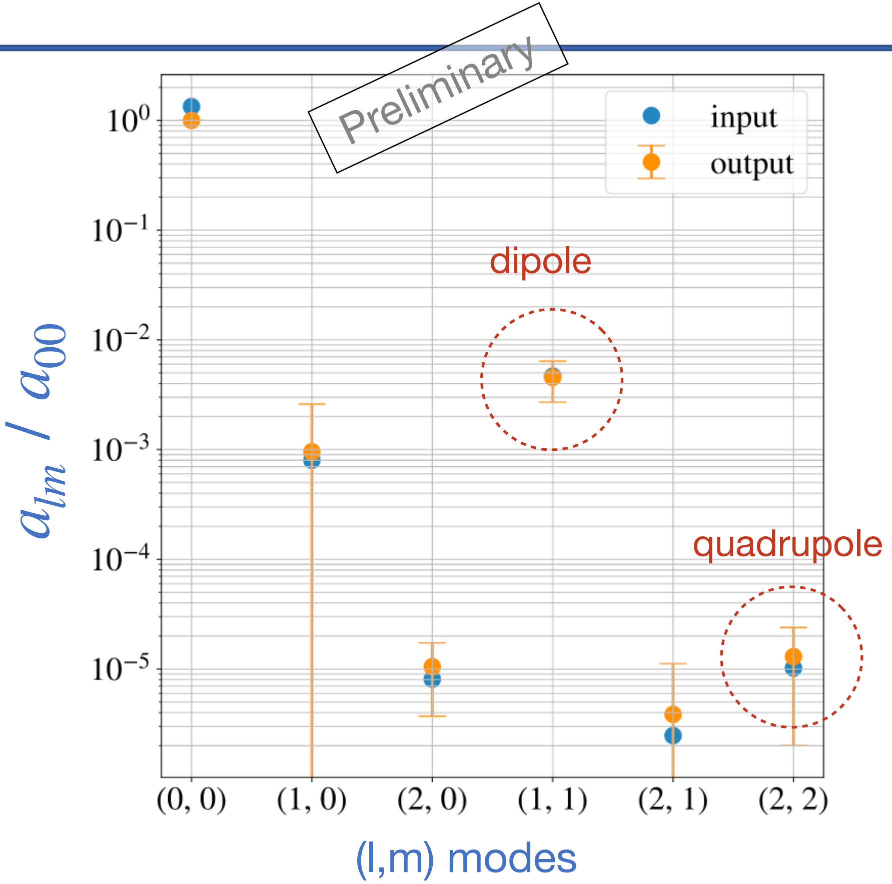
and

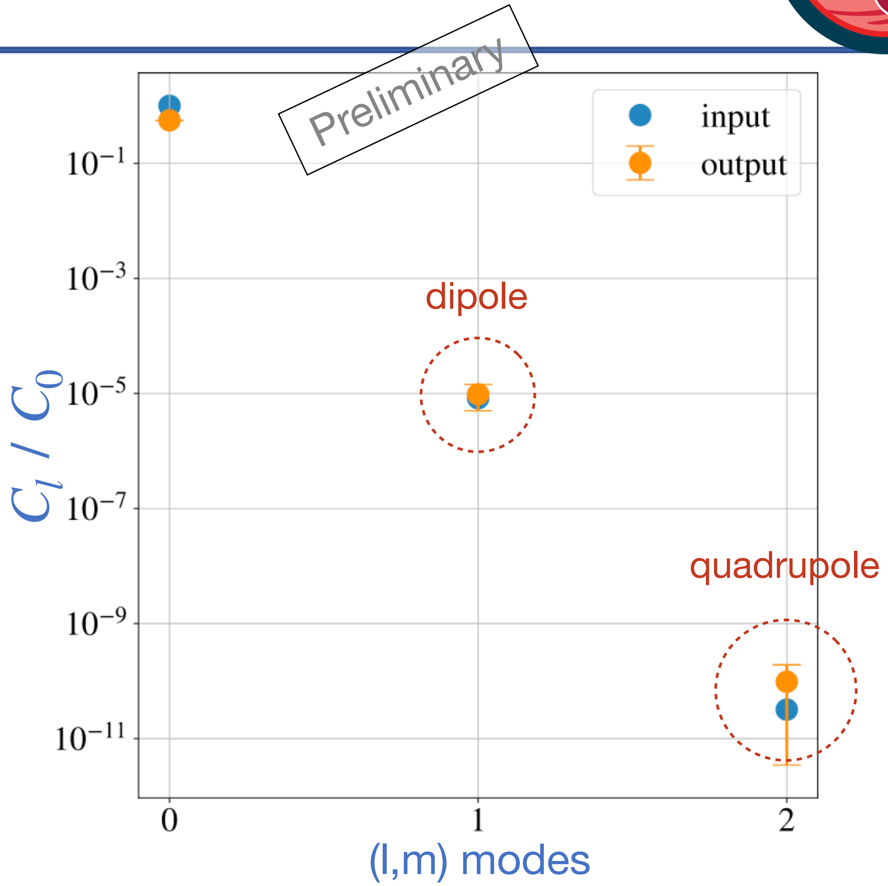
$\Omega'_{GW}(f_0 = 1mHz)$

Could also Taylor expand in power of β:

$$\Omega_{GW}(f,\mathbf{n}) = \Omega'_{GW}(f)\left[1 + M(f) + \mathbf{v}\cdot\mathbf{n}\ D(f) + \left((\mathbf{v}\cdot\mathbf{n})\right)^2 + \frac{1}{3}Q(f)\right]$$

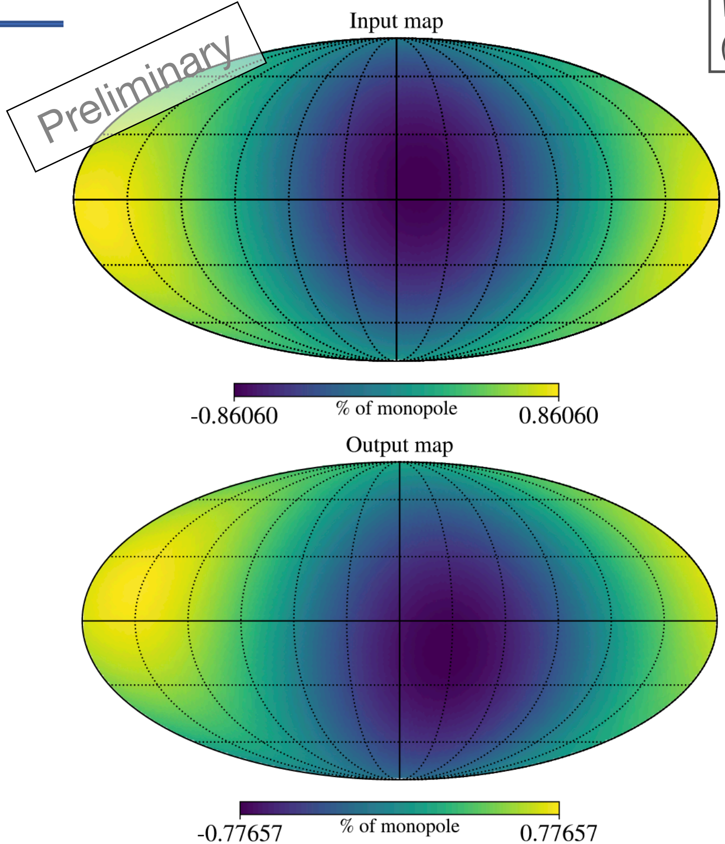
MCMC sampling the velocity: recovered angular spectrum

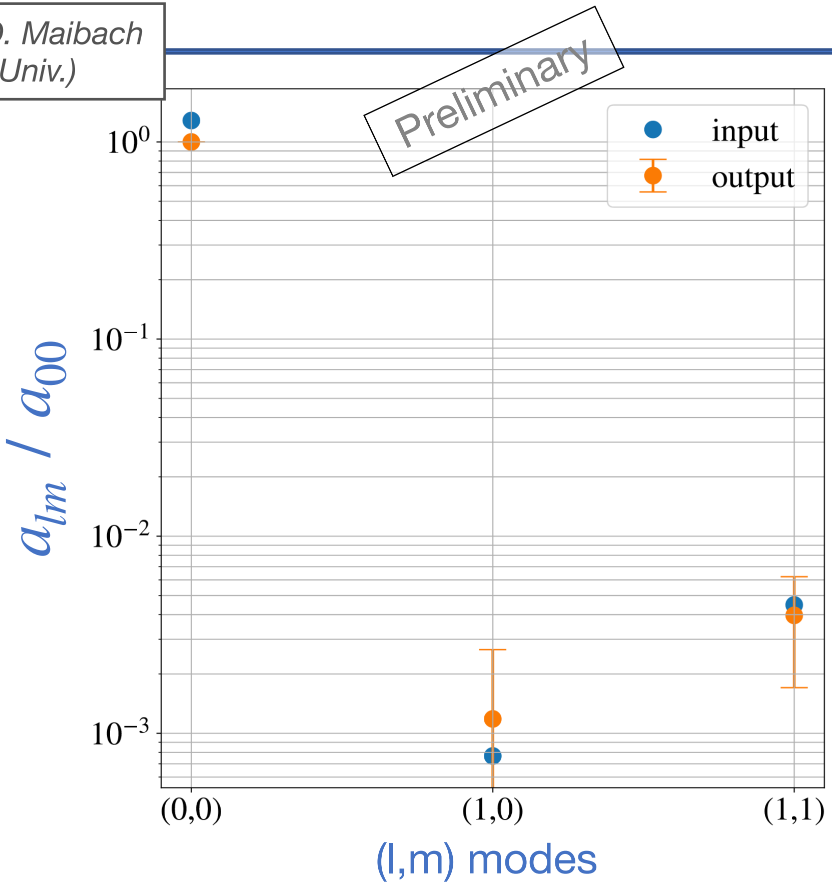




MCMC sampling the alms: recovered sky maps

Work from D. Maibach (Heidelberg Univ.)





Conclusion & Perspectives

- End-to-end simulation and analysis of an anisotropic GW sky with LISA.
- With up-to-date and most complete simulation tools of the consortium to date (LISA GWResponse, LISA Instrument, PyTDI)
- Validation of the method to recover kinematic anisotropy, induced on scale-free SGWB signal (spectral index $\alpha = 0$), **for noiseless instrument.**
- What’s next ?

  1. Apply the method to SGWB with **richer spectrum profiles** (broken power laws, peaks)
     Sharp spectrum transition, breaks, peaks…
     → can boost the SNR a lot (dipole AND quadrupole)

$$\boxed{\begin{aligned} D(f) &= \beta\,(4 - n_\Omega)\ , \\ Q(f) &= \beta^2 \left(10 - \frac{9n_\Omega}{2} + \frac{n_\Omega^2}{2} + \frac{\alpha_\Omega}{2}\right)\ , \end{aligned}}$$

  2. Apply the method to **the mapping of the galactic foreground** (on LDC data!)

Back slides

LISA response

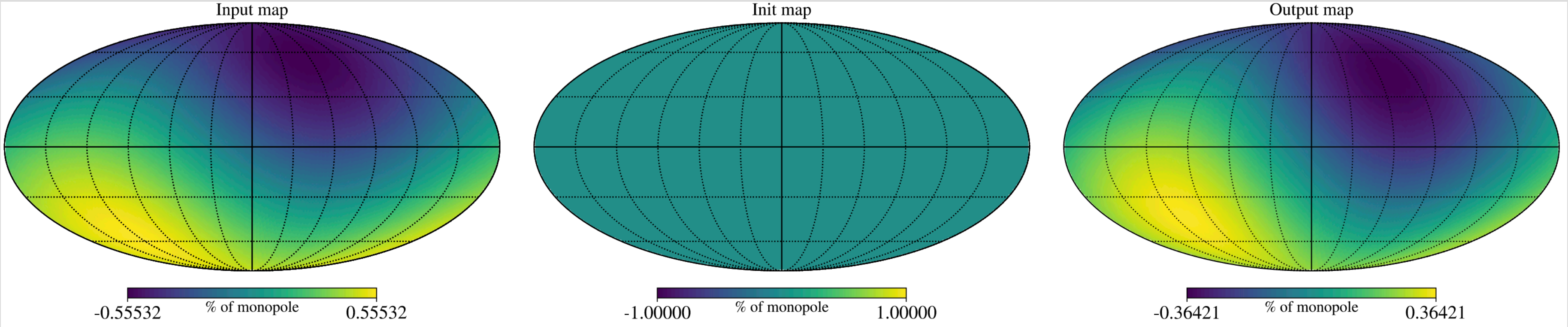
$$G_{lm,p}(f', t, \hat{\mathbf{k}}) = \frac{\xi_p\left(\hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k, \hat{\mathbf{n}}_{lm}\right)}{2\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_{lm}(t)\right)} \left[e^{-\frac{2\pi i f'}{c}\left(L_{lm}(t) + \hat{\mathbf{k}} \cdot \mathbf{x}_m(t)\right)} - e^{-\frac{2\pi i f'}{c} \hat{\mathbf{k}} \cdot \mathbf{x}_l(t)} \right]. \tag{B.5}$$

From Baghi et al. 2023

$$\begin{aligned} X_2 = X_1 &+ \mathbf{D}_{13121} y_{12} + \mathbf{D}_{131212} y_{21} + \mathbf{D}_{1312121} y_{13} + \mathbf{D}_{13121213} y_{31} \\ &- \left[\mathbf{D}_{12131} y_{13} + \mathbf{D}_{121313} y_{31} + \mathbf{D}_{1213131} y_{12} + \mathbf{D}_{12131312} y_{21}\right], \end{aligned}$$

$$\mathbf{D}_{ij} \tilde{x}(f) \approx \tilde{x}(f) e^{-2\pi i f L_{ij}}.$$

MCMC sampling the alms: **artificially rotated input** - *Sanity check*



Work from D. Maibach (Heidelberg Univ.)