Probing Primordial Black Holes and Inflation with LISA

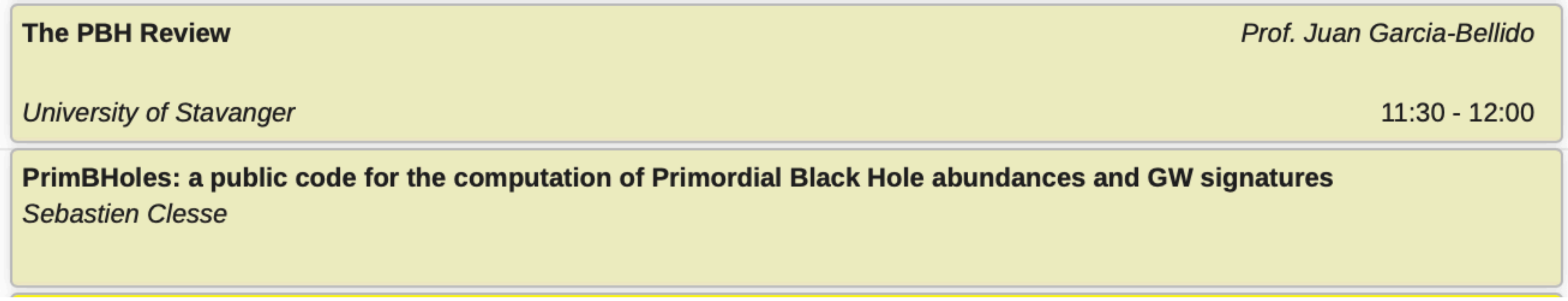
Gabriele Franciolini

08-06-2023

LISA CosmoWG
Workshop Stavanger

Outline

PBH overview:



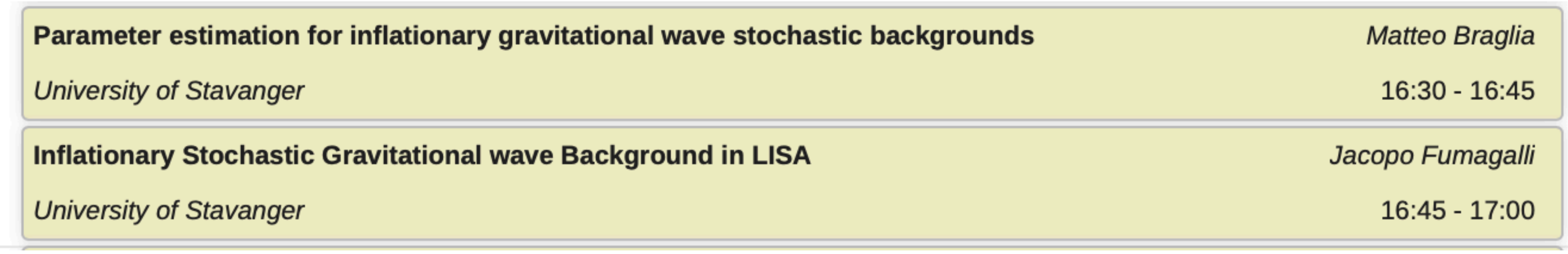
The PBH Review *Prof. Juan Garcia-Bellido*

University of Stavanger 11:30 - 12:00

PrimBHoles: a public code for the computation of Primordial Black Hole abundances and GW signatures

Sebastien Clesse

Inflationary SGWB overview:



Parameter estimation for inflationary gravitational wave stochastic backgrounds *Matteo Braglia*

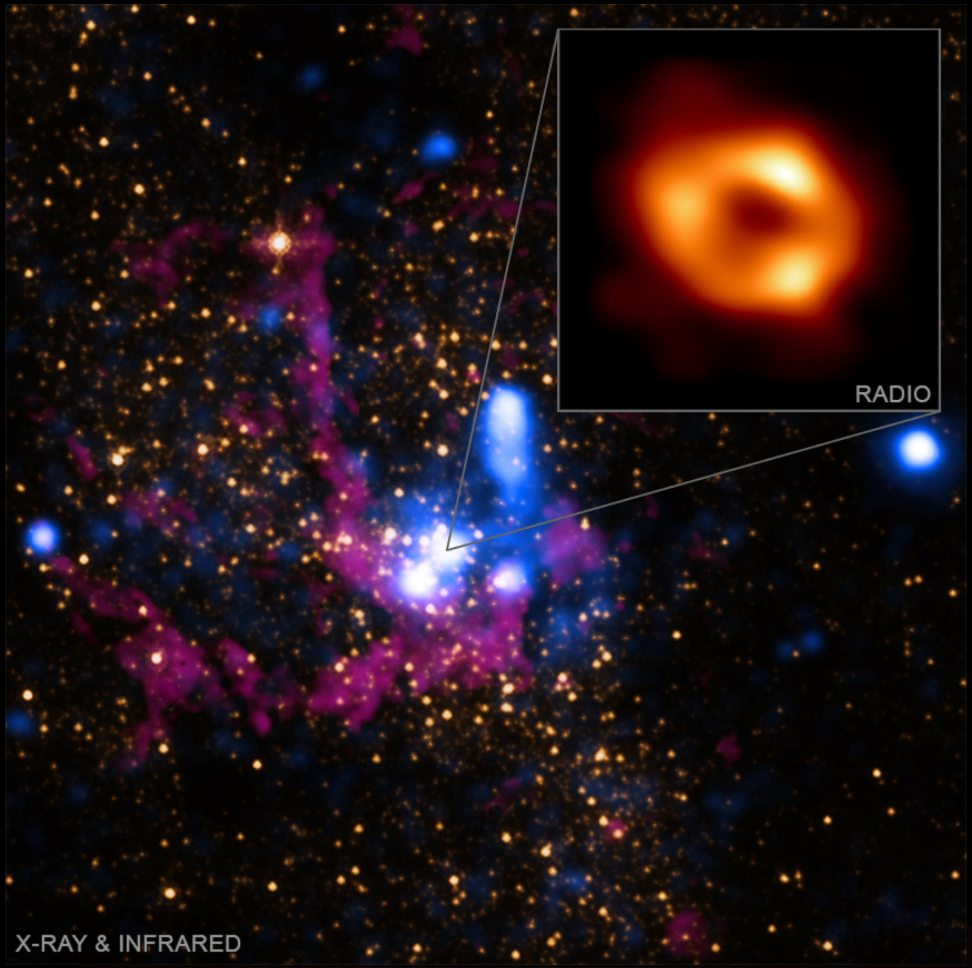
University of Stavanger 16:30 - 16:45

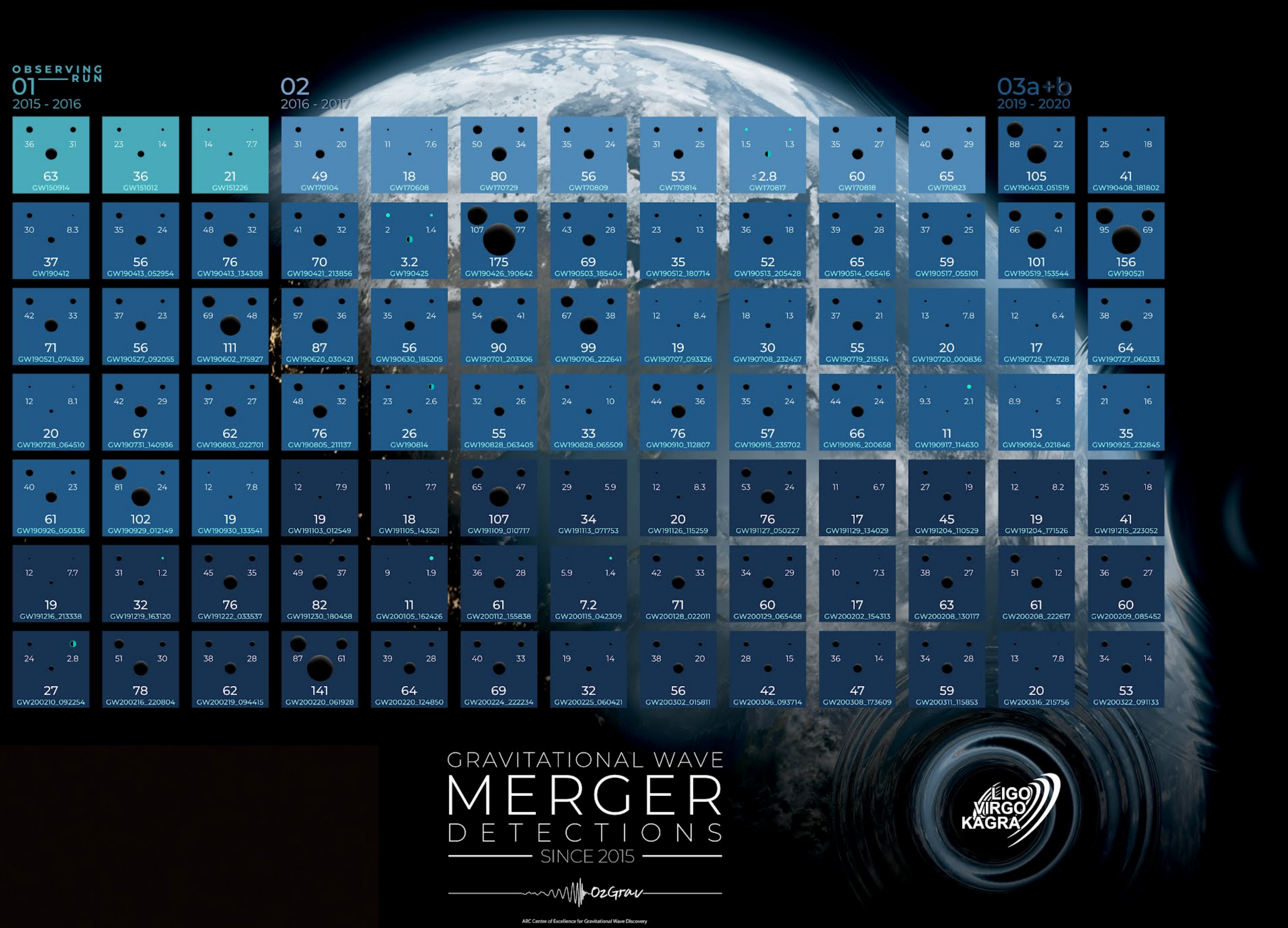
Inflationary Stochastic Gravitational wave Background in LISA *Jacopo Fumagalli*

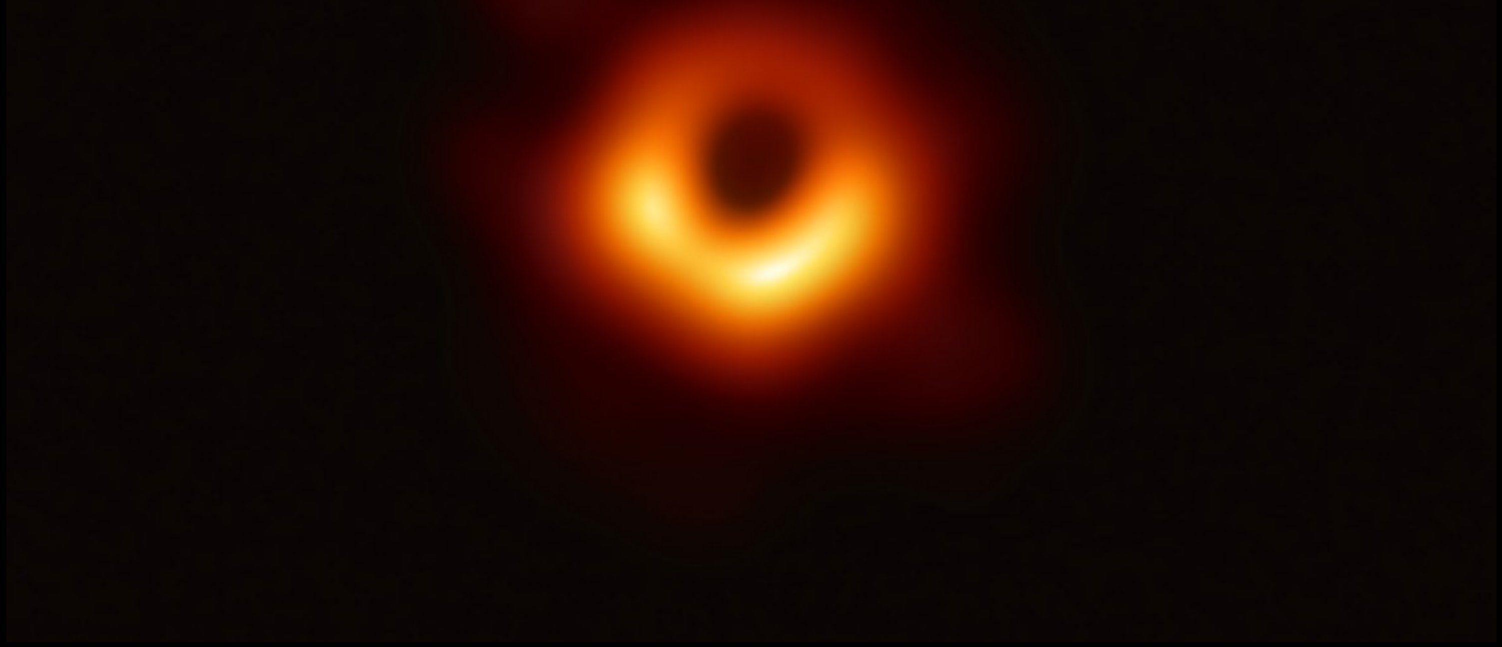
University of Stavanger 16:45 - 17:00

- *Connection between PBH (DM) and induced GWs*
- *The impact of NGs on the SGWB amplitude*
- *Reverse engineering inflationary dynamics*

Black holes populate our universe

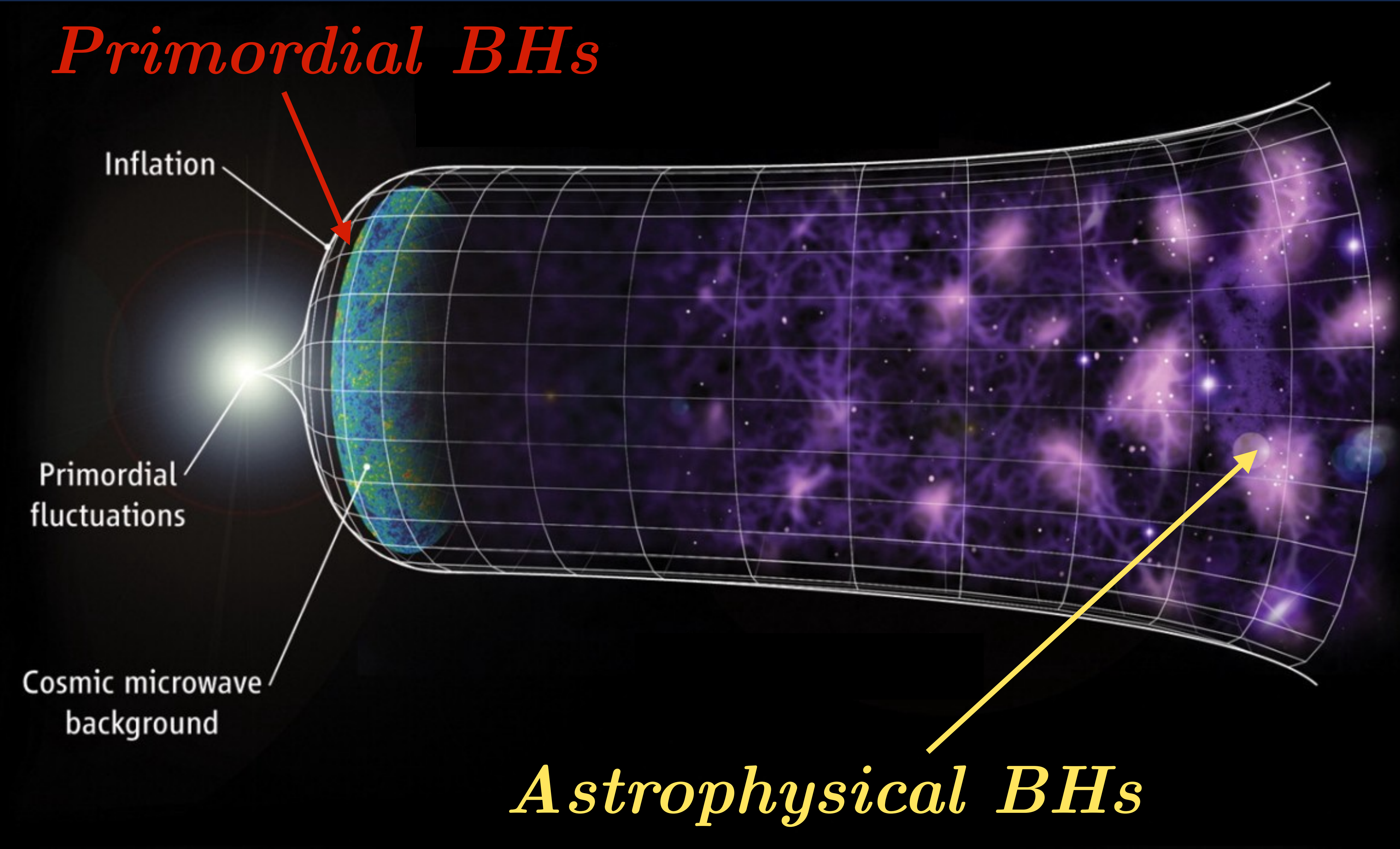






[Credits: LIGO/Virgo/Kagra, EHT, ...]

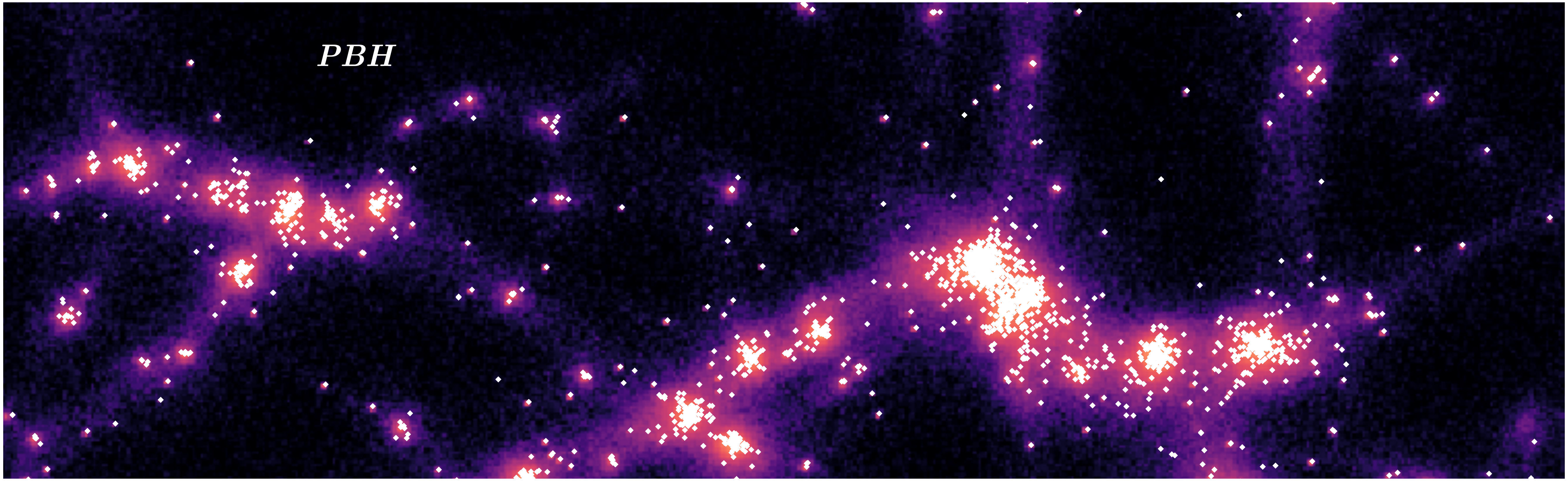
Is it possible to form them in the early universe?



Primordial BHs

Astrophysical BHs

PBH dark matter



D. Inman and Y. Ali-Haïmoud, Phys. Rev. D **100**, no.8, 083528 (2019) [arXiv:1907.08129]

Primordial black holes on large scales behave as a cold and collisionless fluid

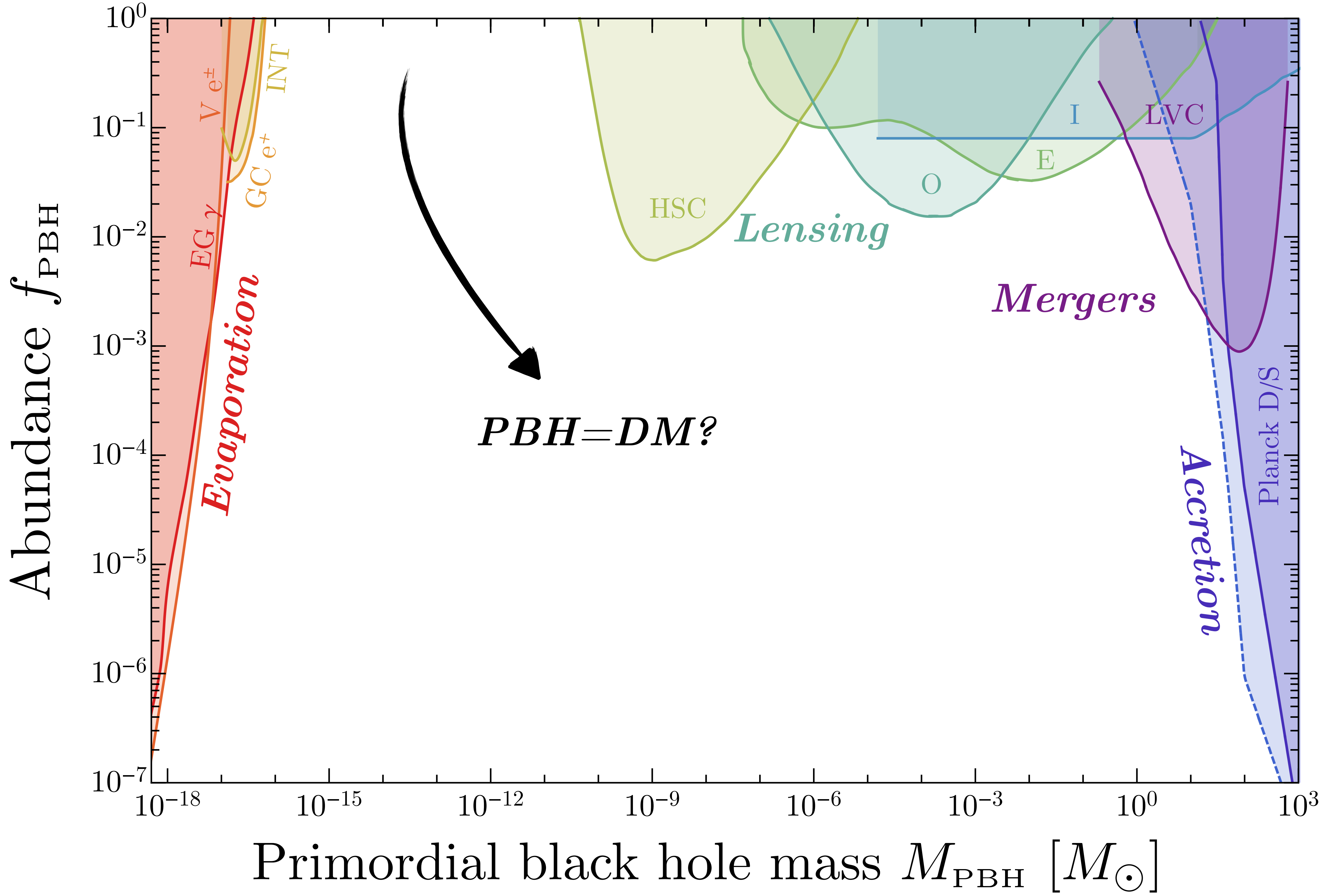
- ***PBH abundance expressed in terms of the dark matter***

$$\boxed{f_{\rm PBH} \equiv \Omega_{\rm PBH}/\Omega_{\rm DM}}$$

(can be thought as a proxy for the average PBH number density)

Constraints on the PBH DM abundance

Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]



****assuming narrow mass distribution***

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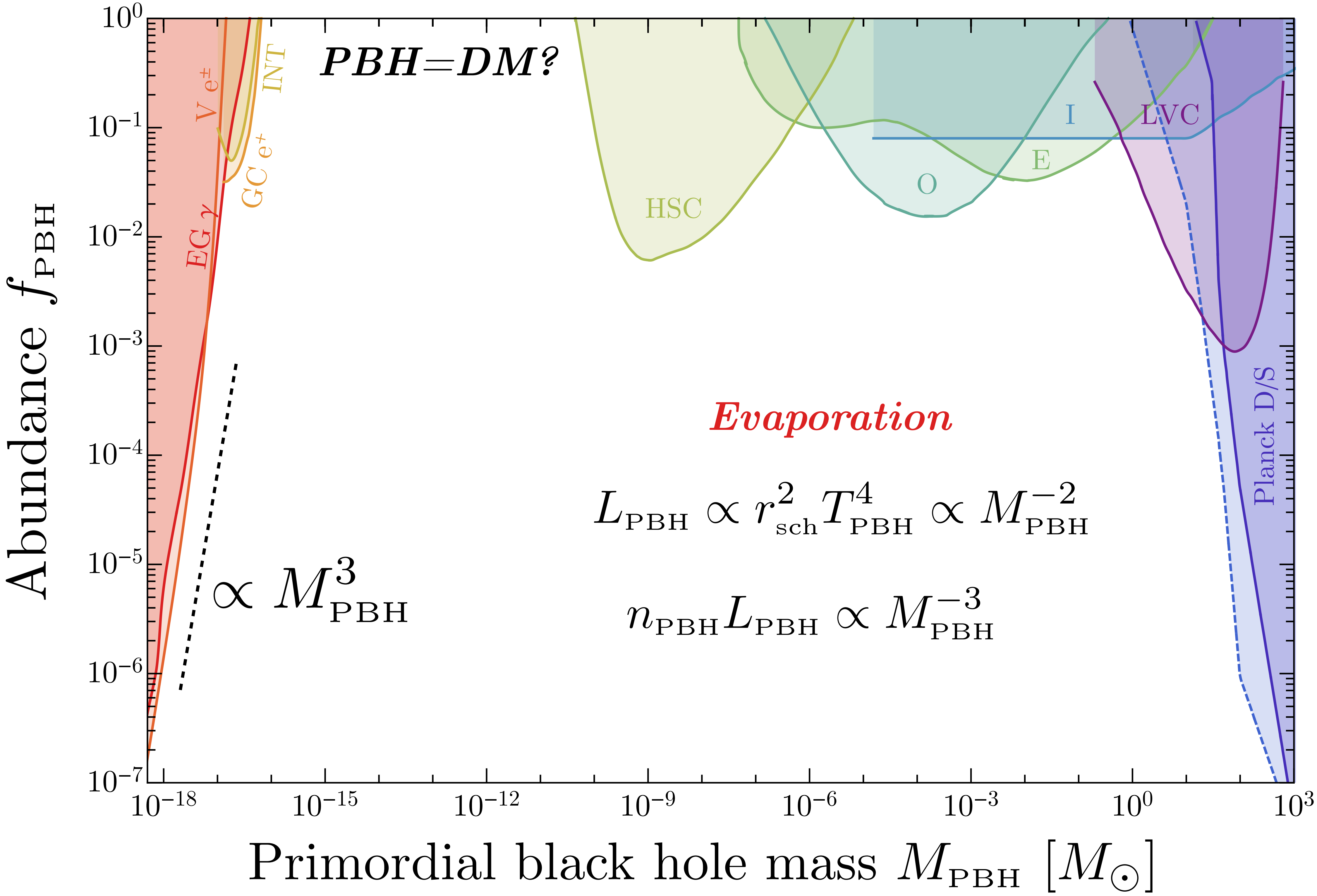
Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]



PBH=DM?

Evaporation

$$\propto M_{\rm PBH}^3$$

$$L_{\rm PBH} \propto r_{\rm sch}^2 T_{\rm PBH}^4 \propto M_{\rm PBH}^{-2}$$

$$n_{\rm PBH} L_{\rm PBH} \propto M_{\rm PBH}^{-3}$$

**assuming narrow mass distribution*

Constraints on the PBH DM abundance

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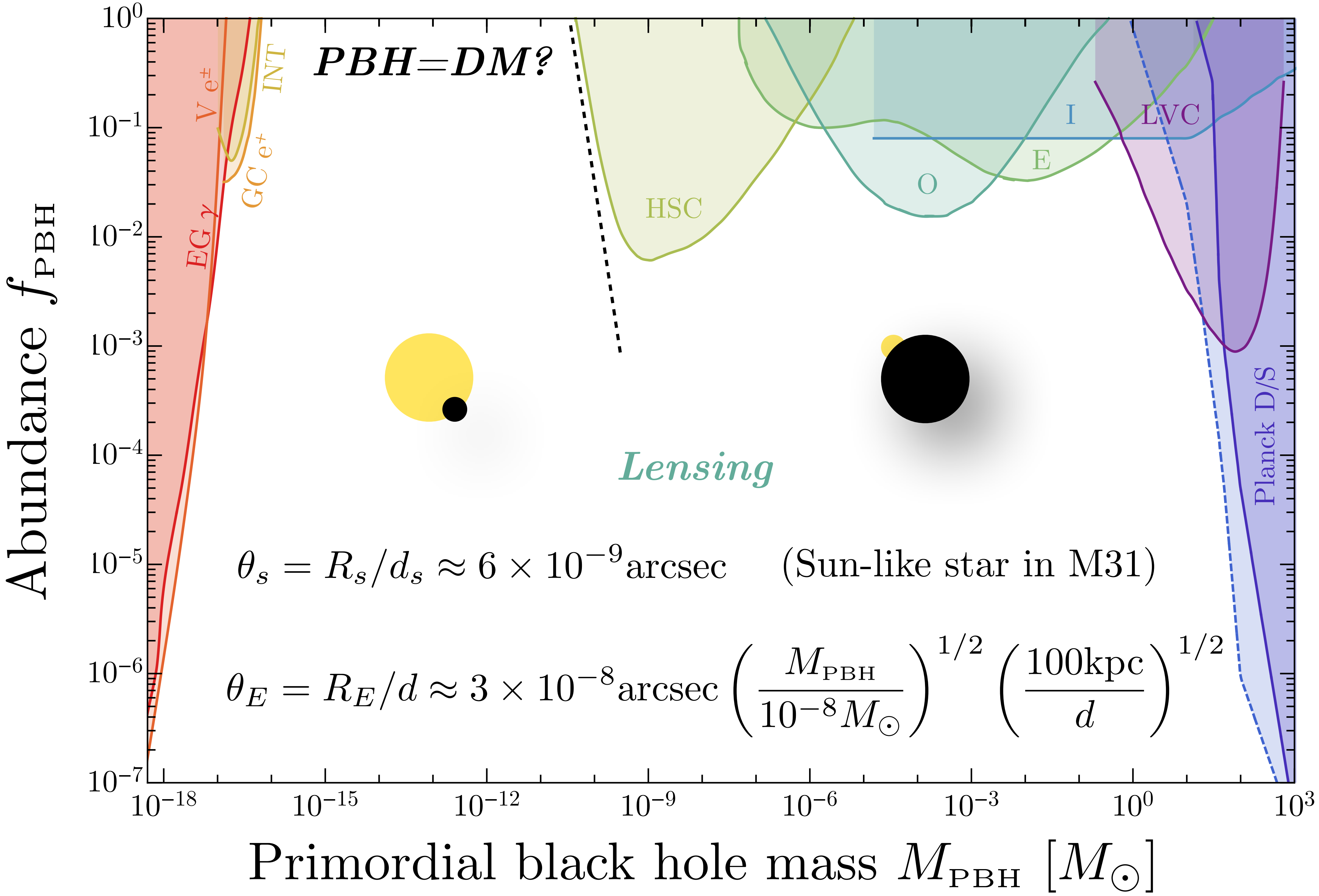


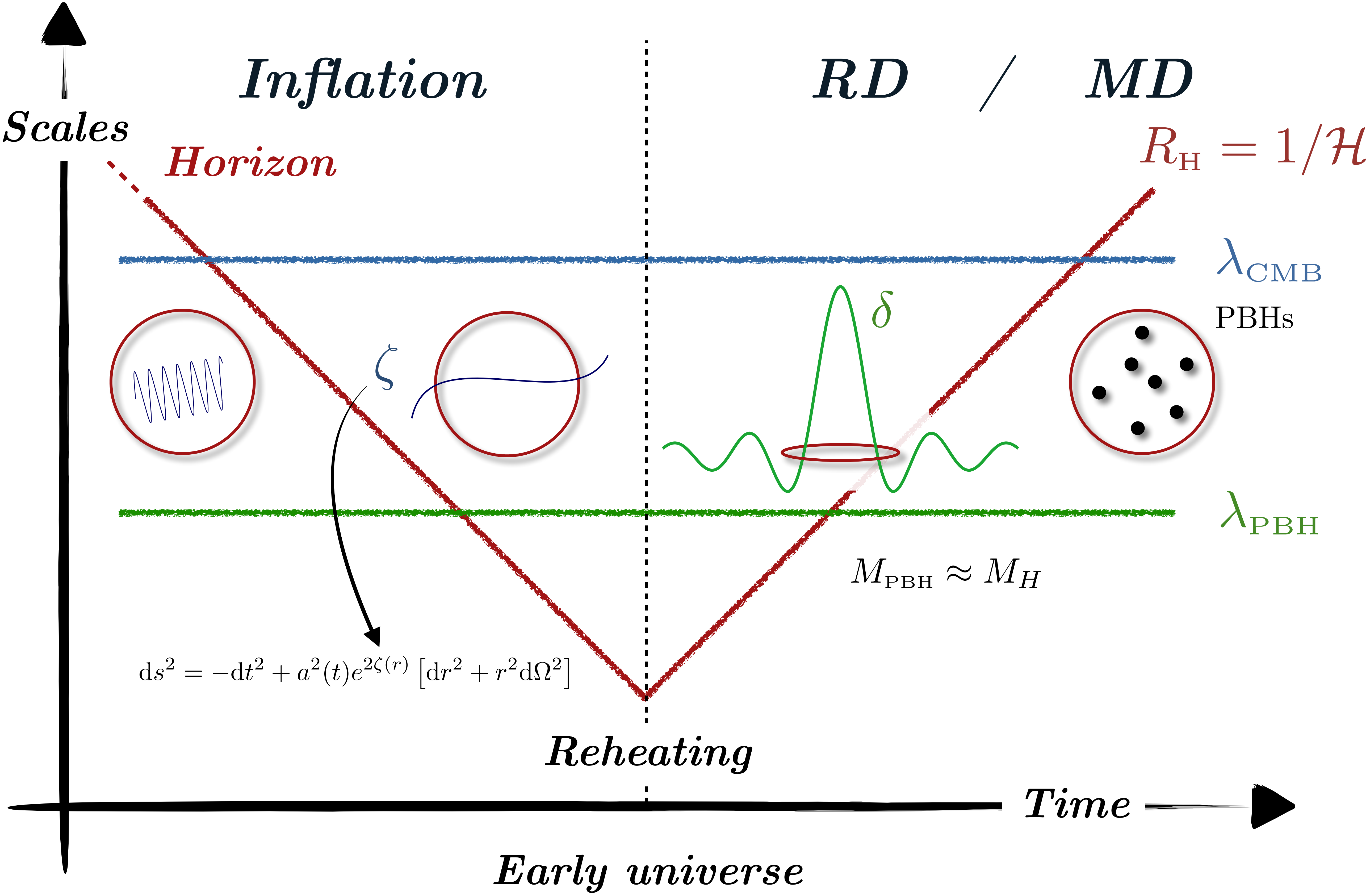
$$\theta_s = R_s/d_s \approx 6\times 10^{-9}\text{arcsec} \qquad \text{(Sun-like star in M31)}$$

$$\theta_E = R_E/d \approx 3\times 10^{-8}\text{arcsec}\left(\frac{M_{\rm PBH}}{10^{-8}M_\odot}\right)^{1/2}\left(\frac{100\text{kpc}}{d}\right)^{1/2}$$

***assuming narrow mass distribution*

Induced GWs as an “indirect” probe of PBHs

PBH formation timeline



Scales

Inflation

RD / MD

Horizon

$R_{\rm H} = 1/\mathcal{H}$

$\lambda_{\rm CMB}$

PBHs

ζ

δ

$\lambda_{\rm PBH}$

$M_{\rm PBH} \approx M_H$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)e^{2\zeta(r)}\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right]$$

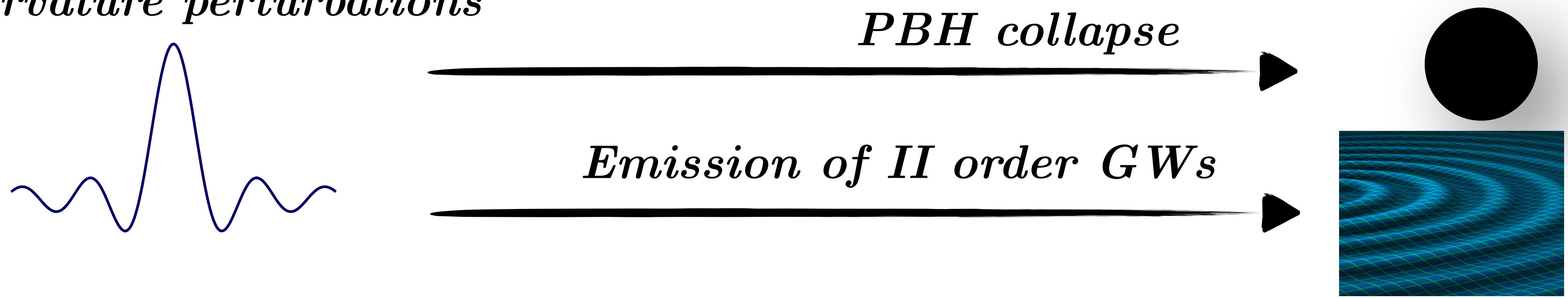
Reheating

Time

Early universe

Induced SGWB at second order

Large curvature perturbations



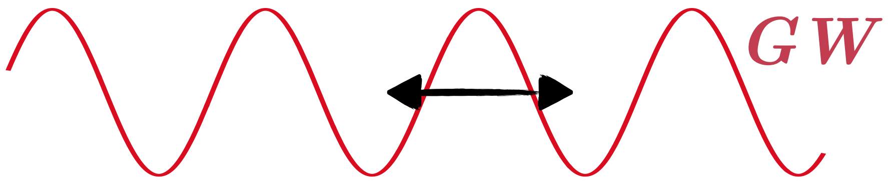
PBH collapse

Emission of II order GWs

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \approx \mathcal{S}_{ij}\left(\zeta\zeta\right)$$

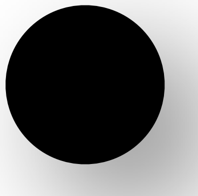
K. Tomita, Prog. Theor. Phys. 54, 730 (1975).
S. Matarrese, O. Pantano, and D. Saez, Phys. Rev. Lett. 72, 320 (1994), [arXiv:9310036].
V. Acquaviva, *et al.* Nucl. Phys. B 667, 119 (2003), [arXiv:0209156].
S. Mollerach, D. Harari, and S. Matarrese, Phys. Rev. D 69, 063002 (2004), [arXiv:0310711].
K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev. D 75, 123518 (2007), [arXiv:0612013].
...

Mass and frequency related by the Hubble horizon at formation



$$f_{\rm GW} \approx 3\cdot 10^{-9}{\rm Hz}\left(\frac{m_{\rm PBH}}{M_\odot}\right)^{-1/2}$$

PBH

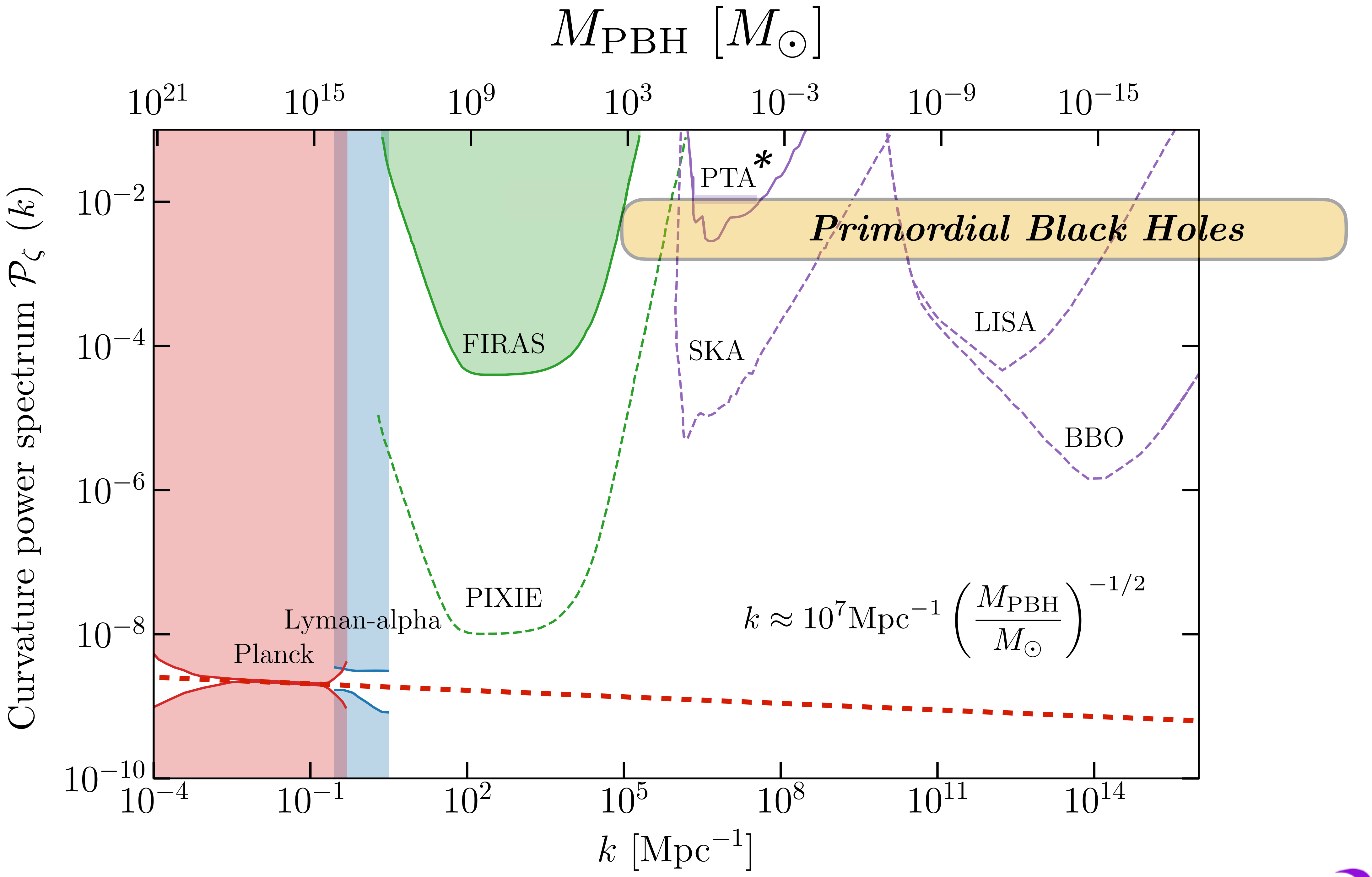


R.Saito and J.Yokoyama, Phys. Rev. Lett. **102** (2009), 161101 [arXiv:0812.4339]
J. Garcia-Bellido, M. Peloso and C. Unal, JCAP **09** (2017), 013 [arXiv:1707.02441]
N. Bartolo, *et al*, Phys. Rev. Lett. **122** (2019) no.21, 211301 [arXiv:1810.12218]
...

LISA

$${\rm mHz} \leftrightarrow 10^{-12} M_\odot$$

No PBH overproduction constrain the power spectrum at small scales

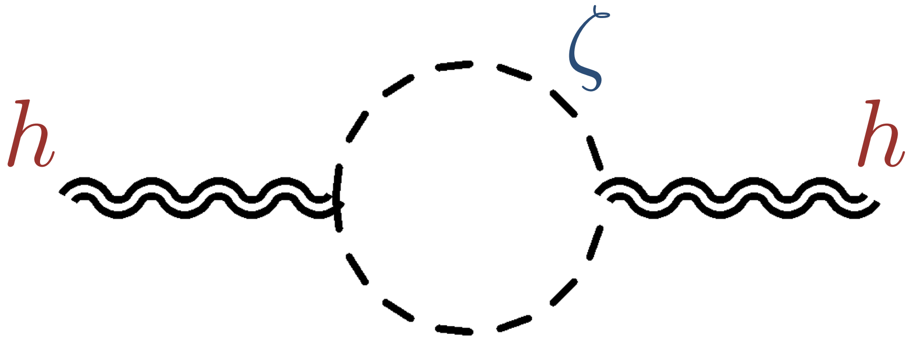


A. M. Green and B. J. Kavanagh, J. Phys. G **48** (2021) no.4, 043001 [arXiv:2007.10722]

Characterisation of the SGWB: spectrum

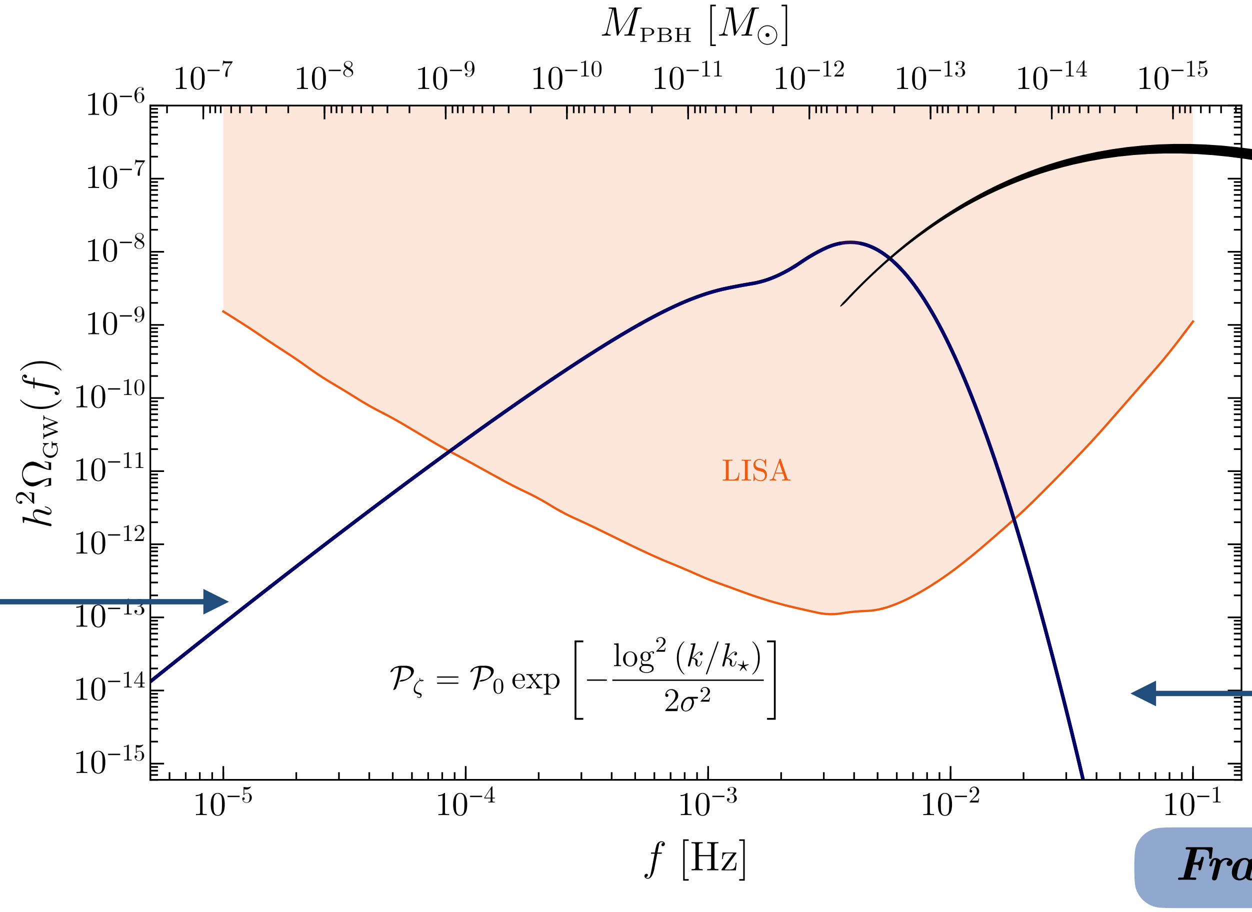
Power spectrum of GWs:

$$\left\langle h^{\lambda_1}(\eta,\vec{k}_1)h^{\lambda_2}(\eta,\vec{k}_2)\right\rangle' \approx \mathcal{P}_\zeta\mathcal{P}_\zeta$$



At second order in comoving curvature perturbation, after averaging over the fast oscillating pieces:

$$\Omega_{\rm GW}(\eta,k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2}\int\frac{d^3p}{(2\pi)^3}\frac{p^4\left[1-\mu^2\right]^2}{p^3\,|\vec{k}-\vec{p}|^3}\boxed{\mathcal{P}_\zeta(p)\,\mathcal{P}_\zeta\left(|\vec{k}-\vec{p}|\right)}\boxed{\mathcal{I}^2\left(\vec{k},\vec{p}\right)}$$



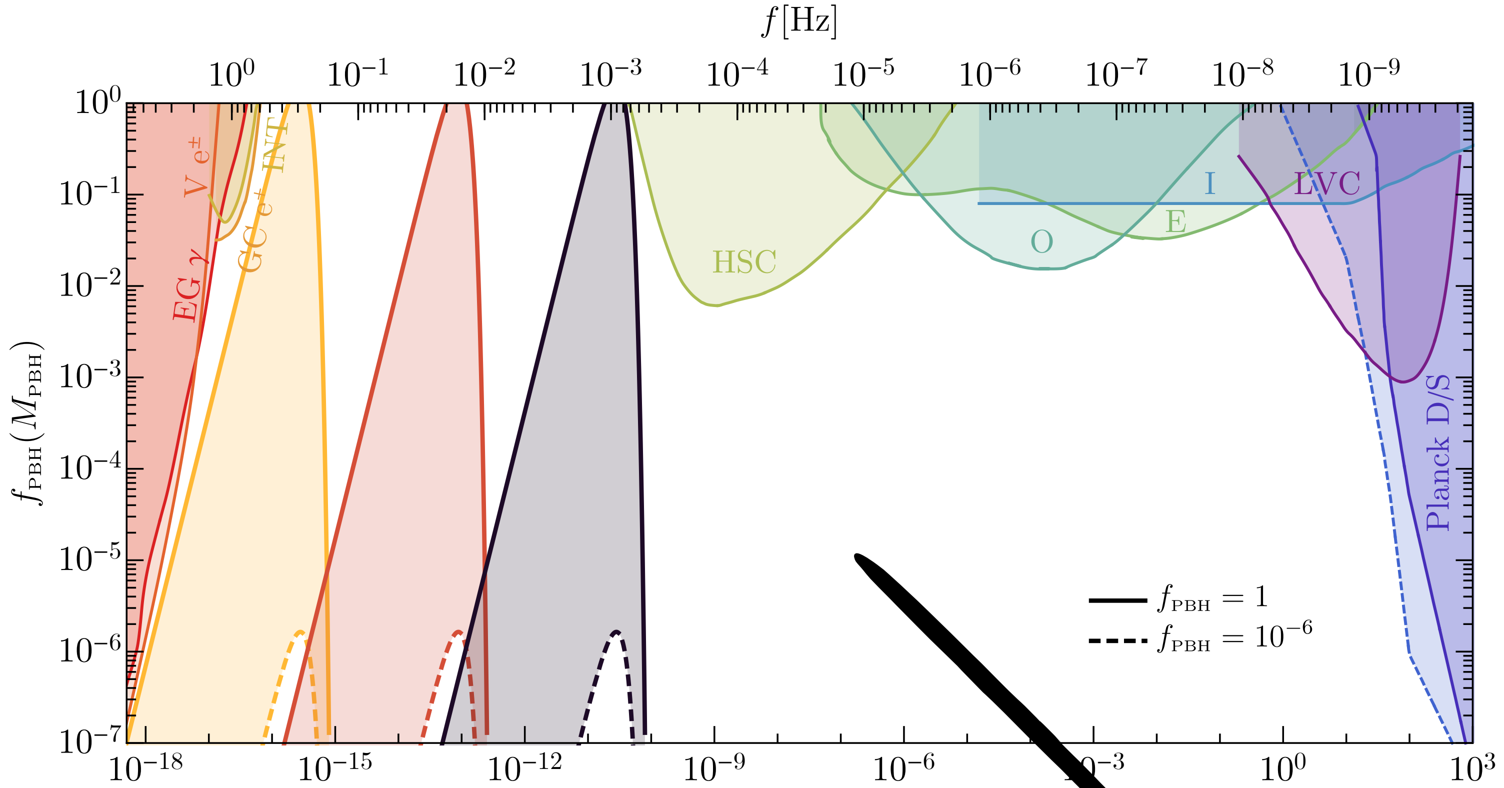
Double peak feature for narrow spectra

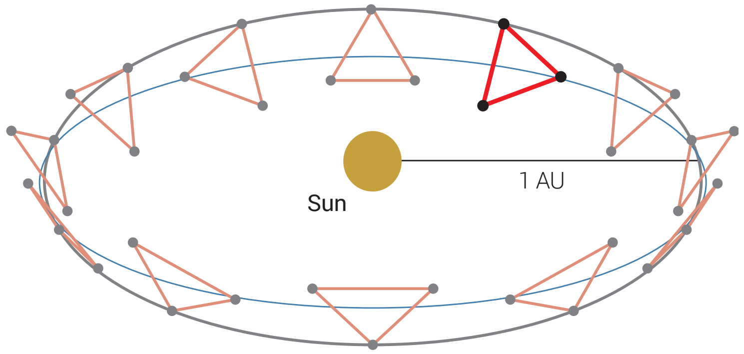
$\propto k^3$

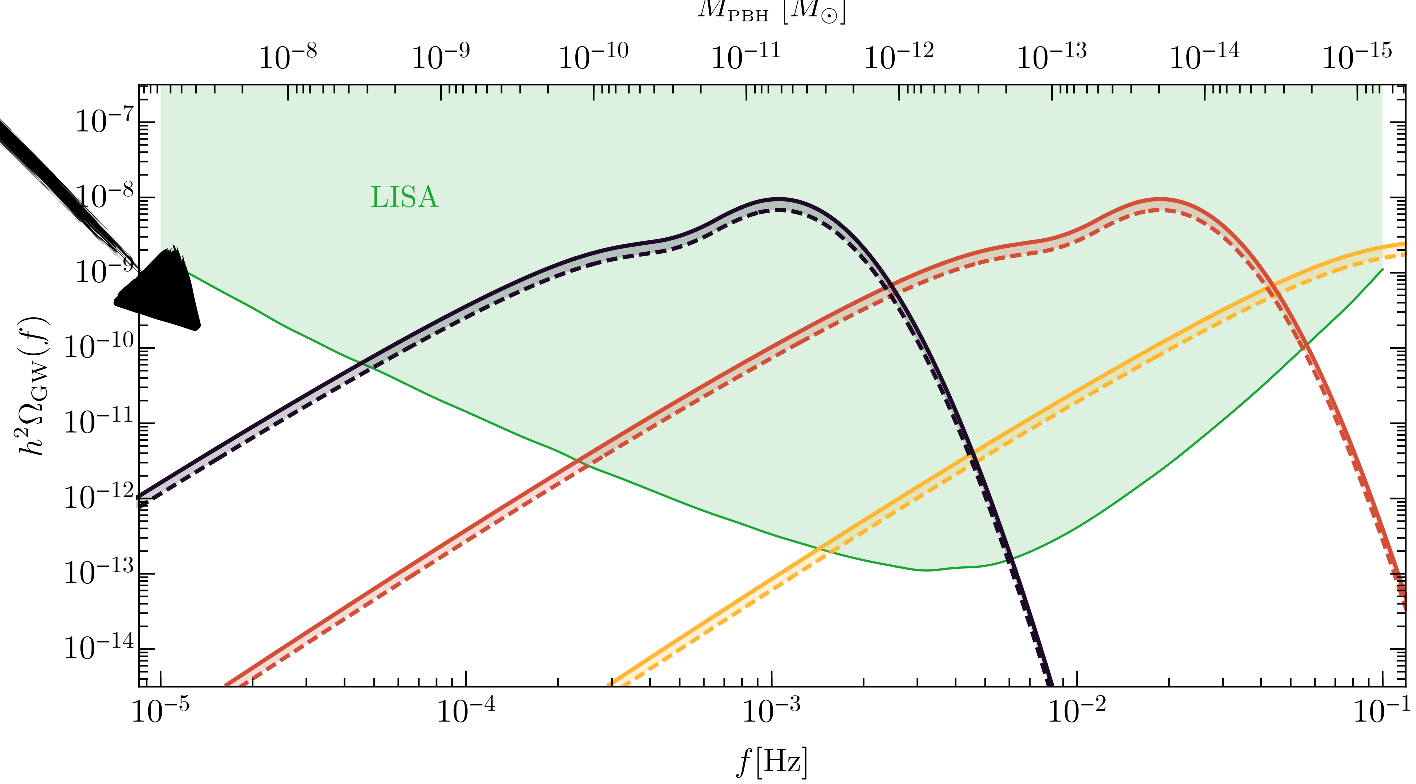
$\propto \mathcal{P}_\zeta^2(k \gg k_*)$

LISA: asteroidal mass range

N. Bartolo, V. De Luca, G. Franciolini, A. Lewis, M. Peloso and A. Riotto, Phys. Rev. Lett. **122** (2019) no.21, 211301 [arXiv:1810.12218]



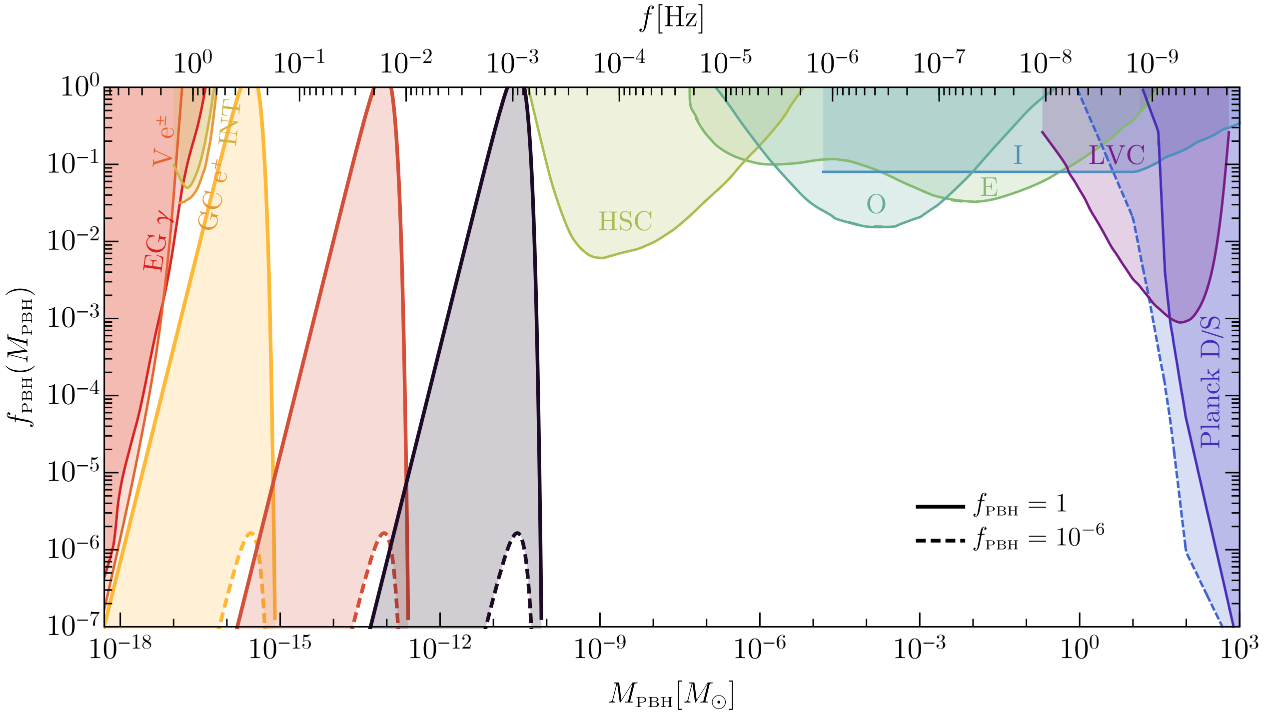


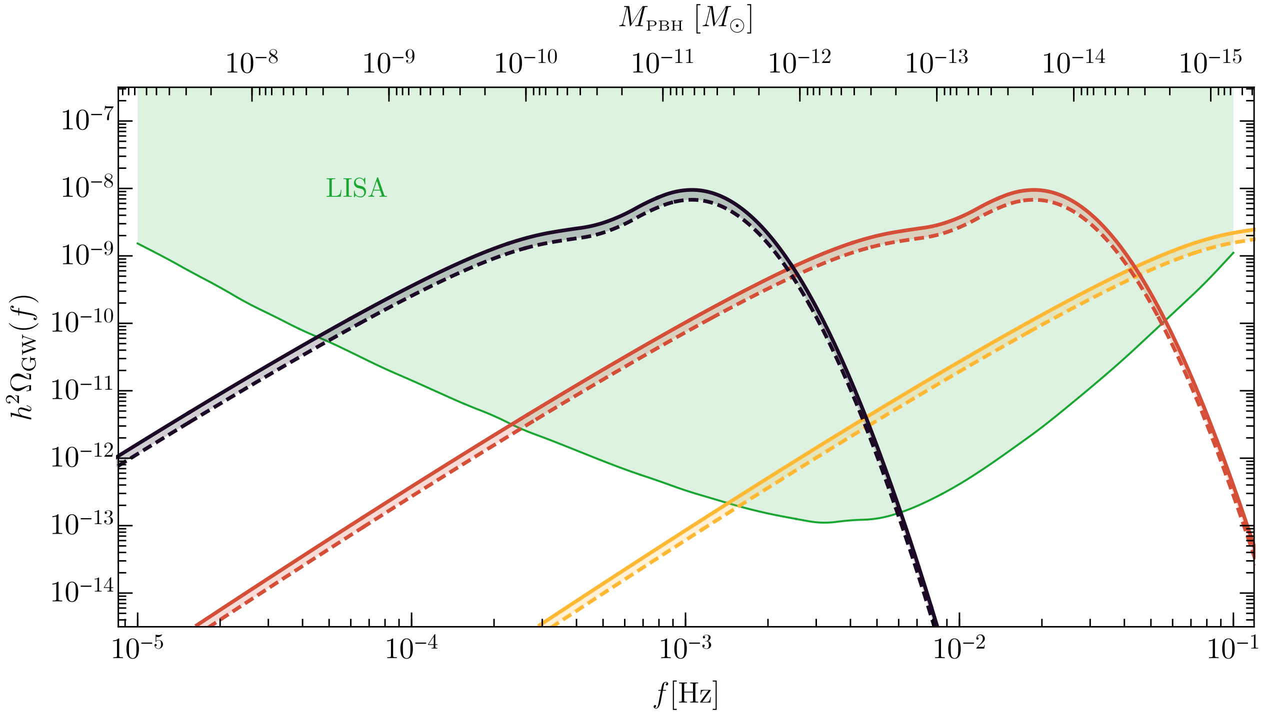


LISA can rule out standard formation scenarios for PBH dark matter in the open mass range

Abundance strongly sensitive to the amplitude

N. Bartolo, V. De Luca, G. Franciolini, A. Lewis, M. Peloso and A. Riotto, Phys. Rev. Lett. **122** (2019) no.21, 211301 [arXiv:1810.12218]





$$f_{\rm PBH} \sim e^{-\delta_c^2/2\mathcal{A}^2}$$

$$\Omega_{\rm GW} \sim \mathcal{A}^4$$

Abundance strongly sensitive to the amplitude

Necessity to control the model to set reliable bounds

Induced GWs as an “indirect” probe of PBHs

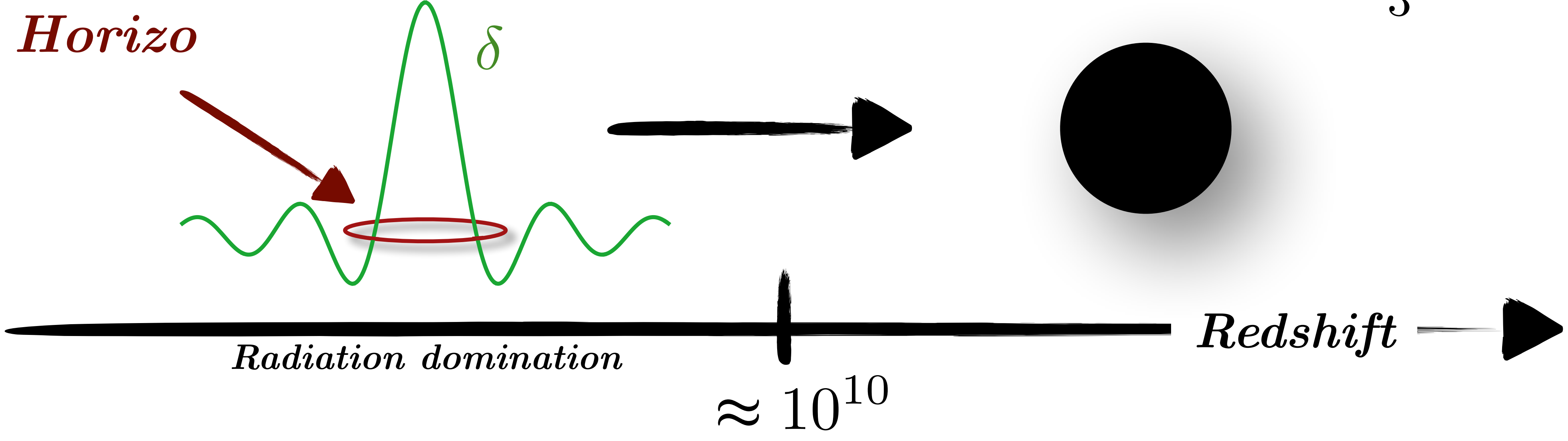
Part I: impact of NGs

PBH abundance

Review: M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Class. Quant. Grav. **35**, no.6, 063001 (2018) [arXiv:1801.05235]

Density perturbations

$$M_{\rm PBH} \approx M_{\rm H} = \overline{\rho}(\eta_{\rm H}) \frac{4\pi}{3} R_{\rm H}^3$$



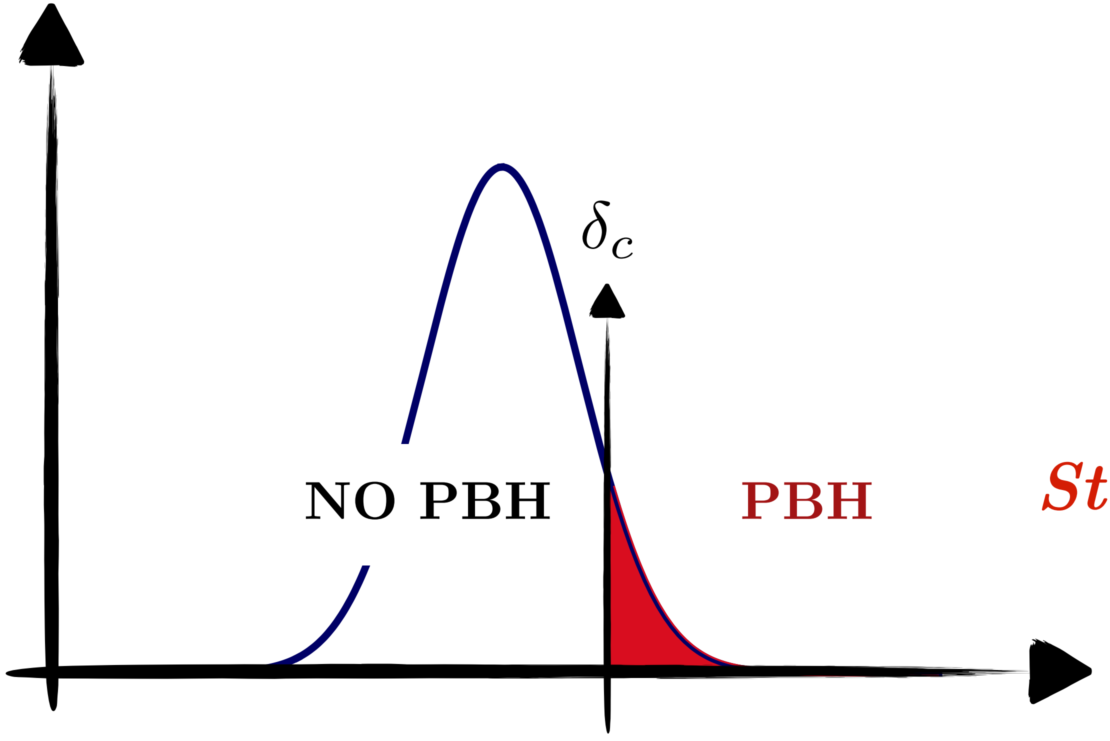
Formation criterion for horizon-crossing perturbations:

$$\delta \geq \delta_c \; (\approx 0.5)$$

M. Shibata and M. Sasaki, Phys. Rev. D **60**, 084002 (1999) [arXiv:gr-qc/9905064]
T. Harada, C. M. Yoo and K. Kohri, Phys. Rev. D **88**, no.8, 084051 (2013) [arXiv:1309.4201]
C. Germani and I. Musco, Phys. Rev. Lett. **122**, no.14, 141302 (2019) [arXiv:1805.04087]
...

Strong sensitivity to non-Gaussian corrections

$$\mathcal{C}(r_m) = \frac{\delta M(r,t)}{M_b(r,t)} = \frac{1}{V_b(r,t)} \int_{S_R^2} d^3\vec{x}\, \delta\rho(\vec{x},t)$$



Density contrast/compaction function

Non-Gaussianties vs PBH abundance

V.De Luca, G.Franciolini, A.Kehagias, M.Peloso, A.Riotto and C.Ünal, JCAP **07** (2019), 048 [arXiv:1904.00970]
S. Young, I. Musco and C. T. Byrnes, JCAP **11** (2019), 012 [arXiv:1904.00984]
...

- **Non-linearities:**

$$\delta(\vec{x},t) = -\frac{8}{9a^2H^2}e^{-5\zeta(\vec{x})/2}\nabla^2 e^{\zeta(\vec{x})/2} + \dots \qquad \text{(on super-Hubble scales)}$$

- ***Primordial Non-Gaussianity:***

$$\zeta = F(\zeta_{\rm G})$$

power-series expansion

$$\zeta = \zeta_{\rm G} + \tfrac{3}{5}f_{\rm NL}\zeta_{\rm G}^2 + \tfrac{9}{25}g_{\rm NL}\zeta_{\rm G}^3 + \dots$$

curvaton

$$\zeta = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$

ultra slow-roll

$$\zeta = -\frac{1}{3}\ln\left(1-3\zeta_{\rm G}\right)$$

ultra slow-roll
with an upward step

$$\zeta = -\tfrac{2}{|h|}\left[\sqrt{1-|h|\zeta_{\rm G}}-1\right]$$

$$\mathcal{C}(r) = \mathcal{C}_{\rm G}(r)\,\frac{dF}{d\zeta_{\rm G}} - \frac{1}{4\Phi}\mathcal{C}_{\rm G}^2(r)\left(\frac{dF}{d\zeta_{\rm G}}\right)^2$$

PBH abundance: Violation of perturbativity

G. Ferrante, G. Franciolini, A. Iovino, Junior. and A. Urbano, Phys. Rev. D **107** (2023) no.4, 043520 [arXiv:2211.01728]
(see also: A.Gow, H.Assadullahi, J.Jackson, K.Koyama, V.Vennin,D.Wands, EPL 142 (2023) no.4, 49001 [arXiv:2211.08348])

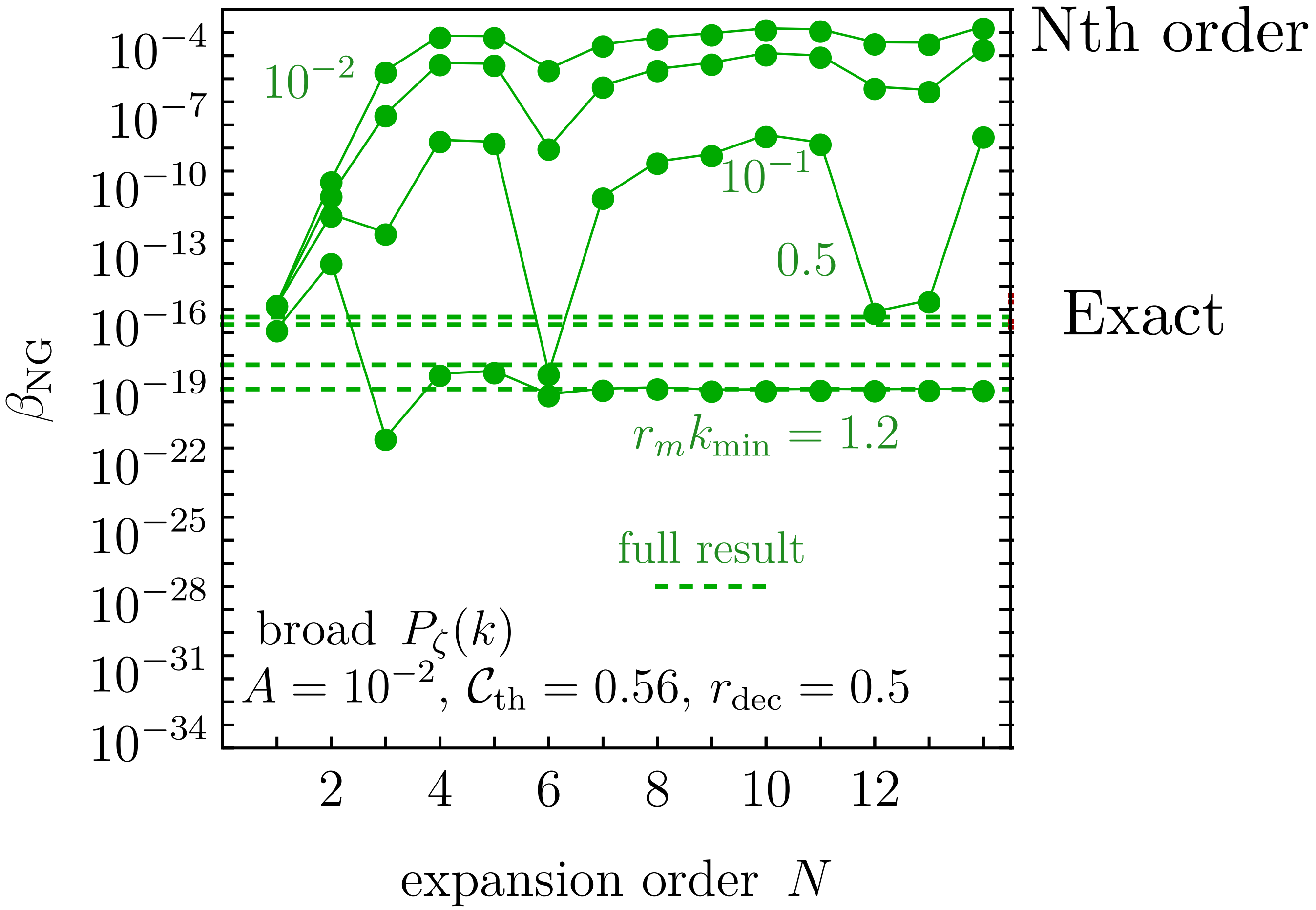
Threshold statistics on the compaction function

$$\beta_{\rm NG} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C}-\mathcal{C}_{\rm th})^{\gamma} \mathrm{P_G}(\mathcal{C}_{\rm G},\zeta_{\rm G}) d\mathcal{C}_{\rm G} d\zeta_{\rm G}\,,$$

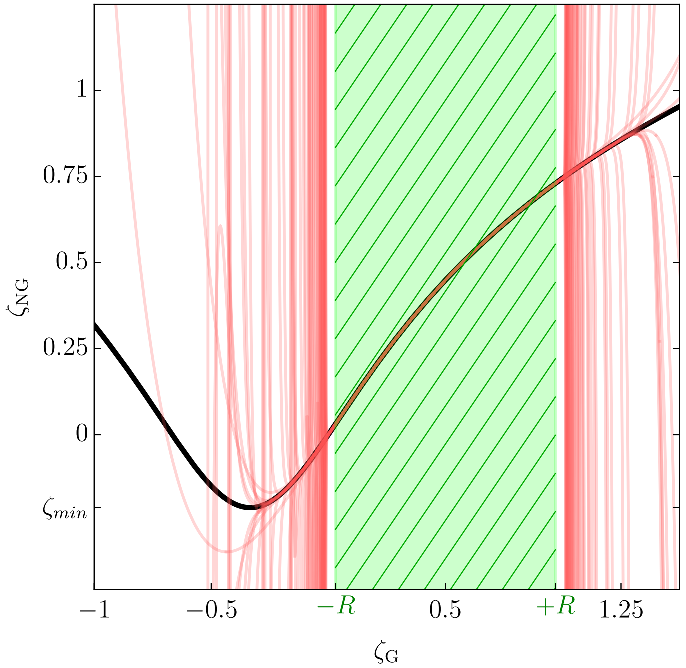
$$\mathrm{P_G}(\mathcal{C}_{\rm G},\zeta_{\rm G}) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}} \exp\left(-\frac{\zeta_{\rm G}^2}{2\sigma_r^2}\right) \exp\left[-\frac{1}{2(1-\gamma_{cr}^2)}\left(\frac{\mathcal{C}_{\rm G}}{\sigma_c}-\frac{\gamma_{cr}\zeta_{\rm G}}{\sigma_r}\right)^2\right]$$

$$\mathcal{D} = \{\mathcal{C}_{\rm G},\, \zeta_{\rm G} \in \mathbb{R} \;:\; \mathcal{C}(\mathcal{C}_{\rm G},\zeta_{\rm G}) > \mathcal{C}_{\rm th} \;\wedge\; \mathcal{C}_1(\mathcal{C}_{\rm G},\zeta_{\rm G}) < 2\Phi\}\,,$$

Example: curvaton model with broad spectrum



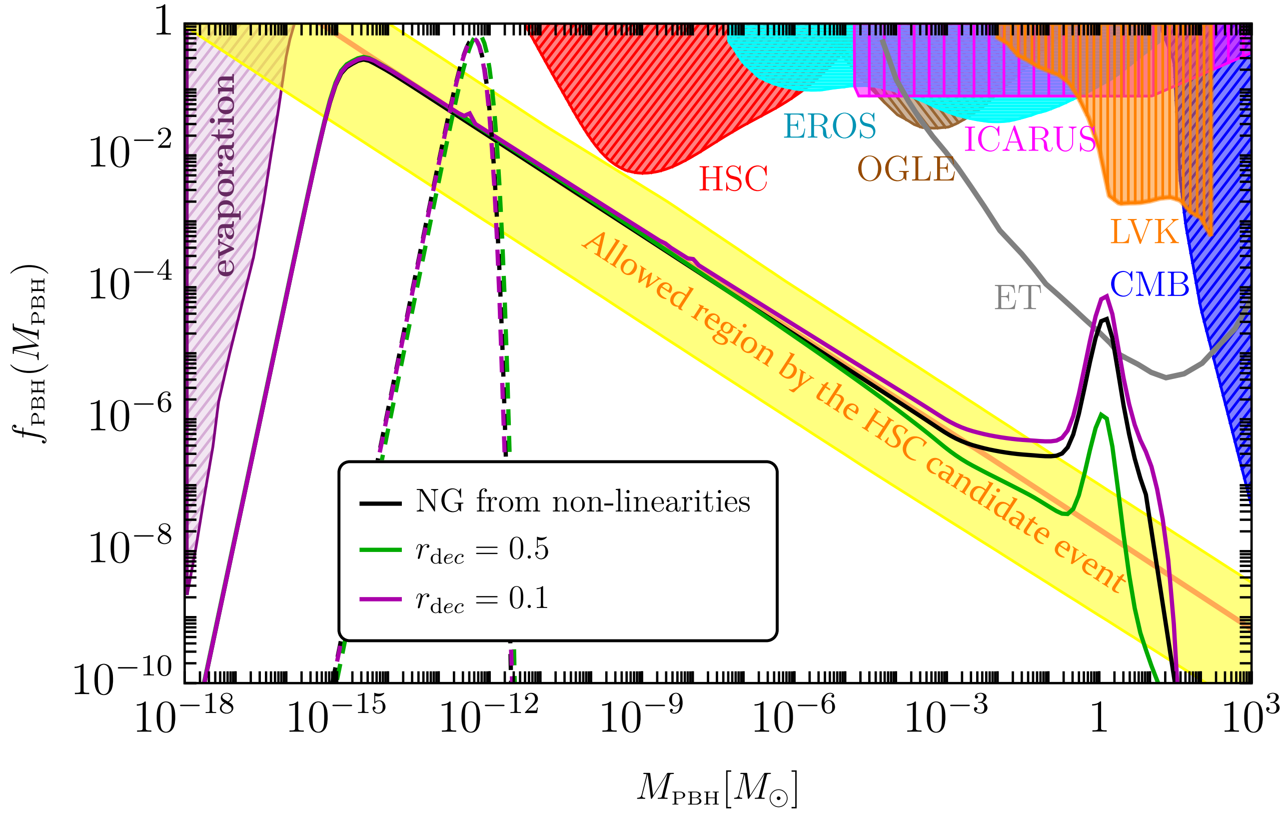
$$\sum_{n=1}^{\infty} c_n(r_{\rm dec})\zeta_{\rm G}^n = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$



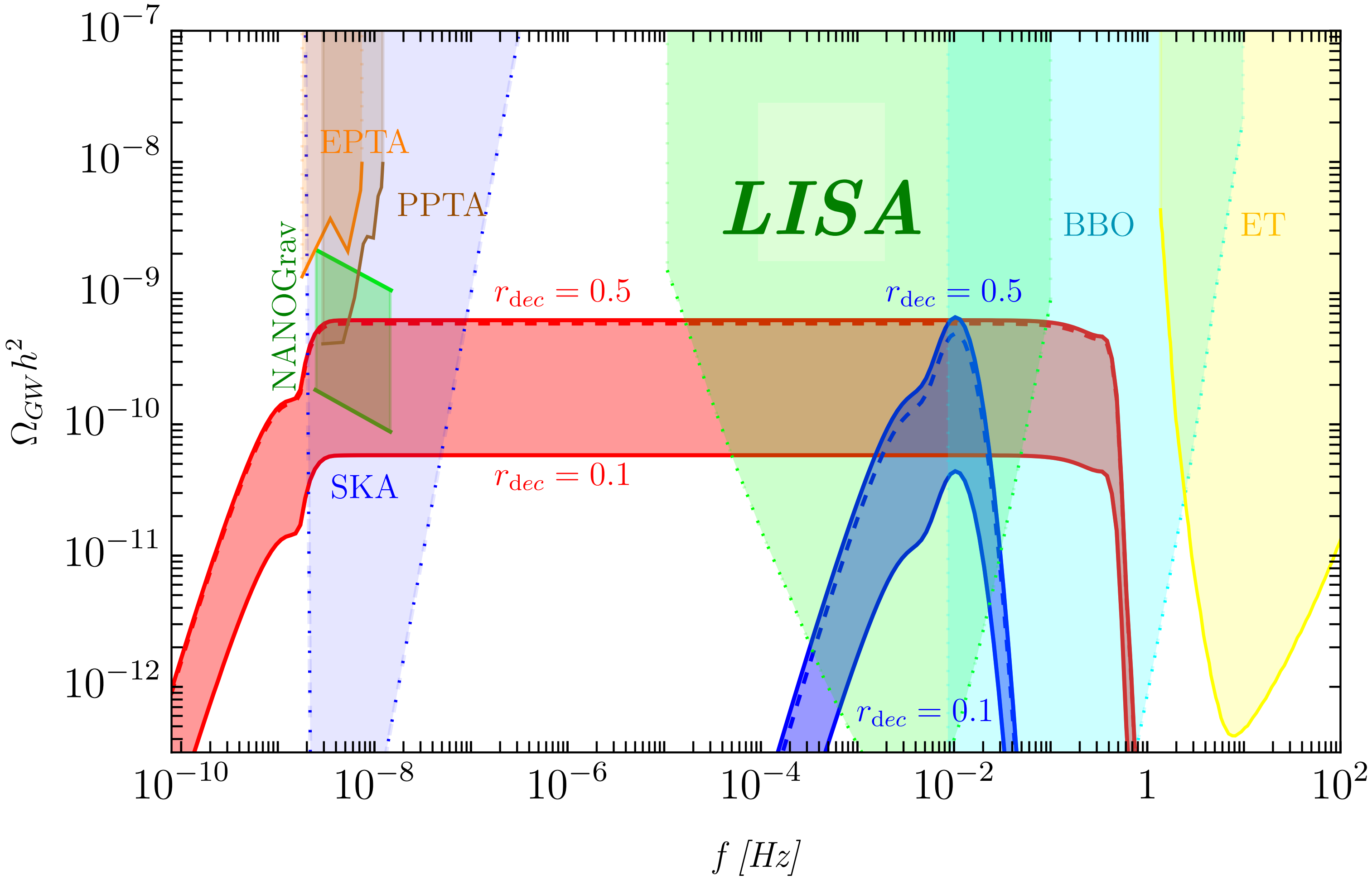
Over-threshold perturbations computed outside the radius of convergence

Impact on phenomenology

Curvaton scenarios with an abundance O(1)



SGWB at LISA

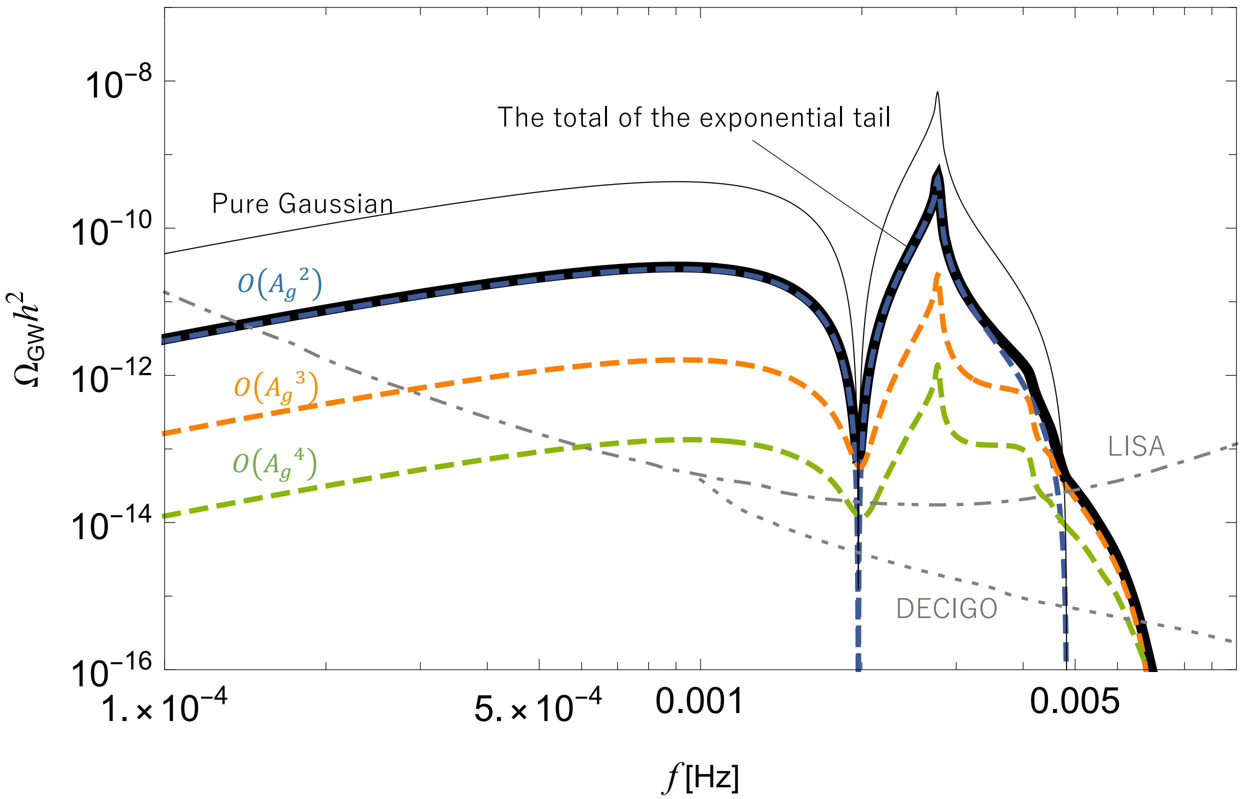


Can we constrain NGs from the spectrum?

(first NG orders sufficient for the computation of the spectrum)

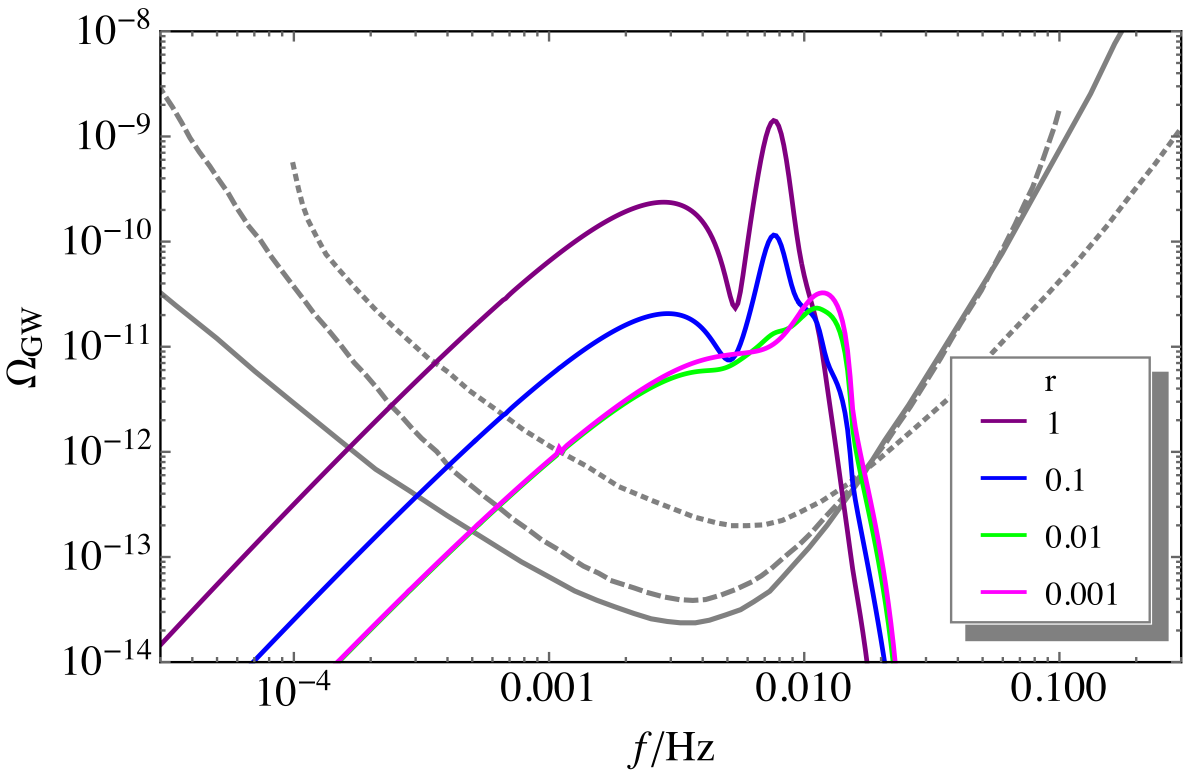
Result for narrow PS

NGs USR: $\zeta = -\frac{1}{3}\ln(1-3\zeta_{\rm G})$



K. Abe, R. Inui, Y. Tada, S. Yokoyama, JCAP **05** (2023), 044 [arXiv:2209.13891]

Curvaton: $F_{\rm NL} = 3/(4r)$



S. Pi and M. Sasaki, [arXiv:2112.12680]

- *Additional high-frequency features (?)*

See also:
C. Unal, Phys. Rev. D **99** (2019) no.4, 041301 [arXiv:1811.09151]
R.g.Cai, S.Pi and M.Sasaki, Phys. Rev. Lett. **122** (2019) no.20, 201101 [arXiv:1810.11000]
P. Adshead, K. D. Lozanov and Z. J. Weiner, JCAP **10** (2021), 080 [arXiv:2105.01659]

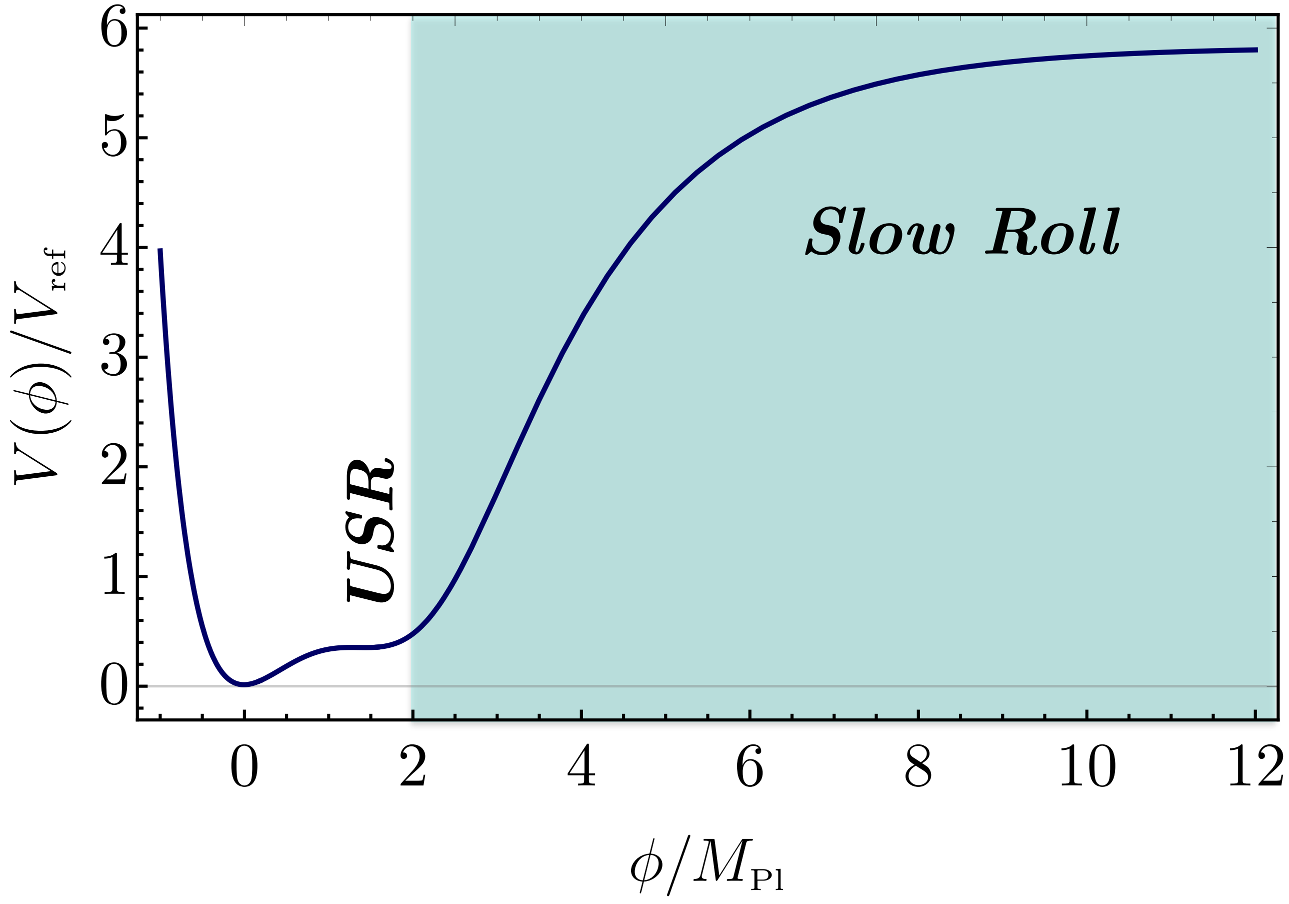
Induced GWs as an "indirect" probe of PBHs

and inflation

Part II: reverse engineering the inflationary potential

Single field formation scenario

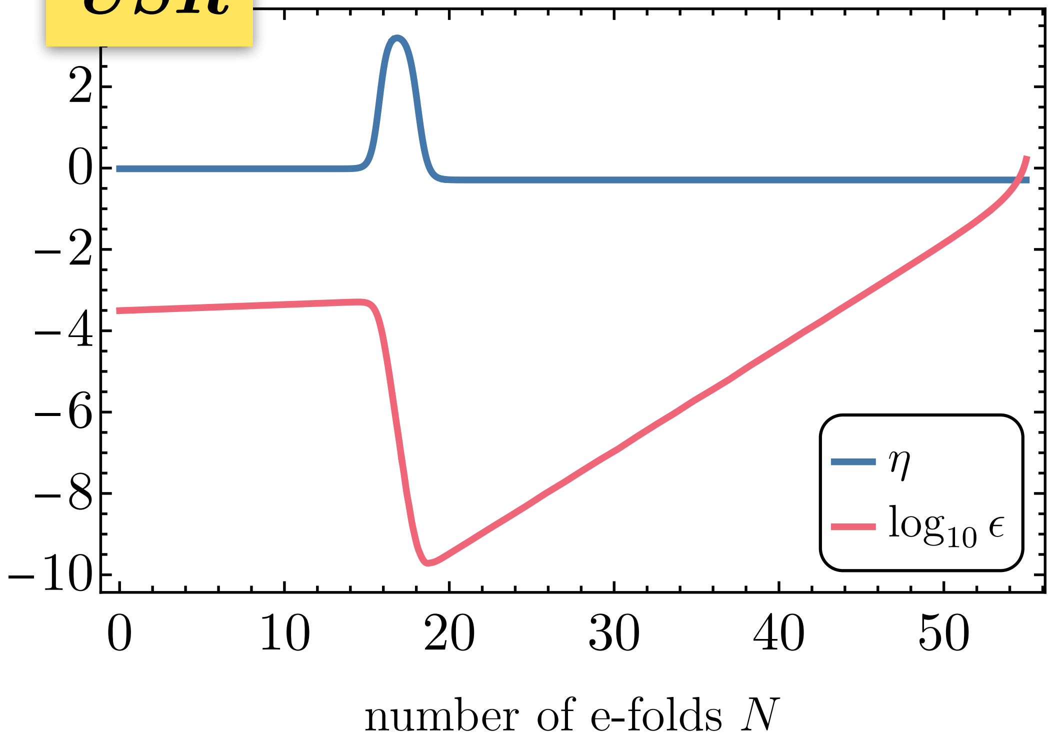
Inflection point in the inflaton potential



SR

$$\mathcal{P}_\zeta = \left.\frac{8\pi G H^2}{\epsilon}\right|_{aH=k}$$

USR



USR phase requires careful treatment of perturbations beyond slow roll approx.

$\ddot\phi + 3H\dot\phi \approx 0$ ***Deceleration becomes dominant***

$$\epsilon = -\dot H/H^2 \propto a^{-6}$$

$$\eta = -\ddot H/(2H\dot H) \sim 3$$

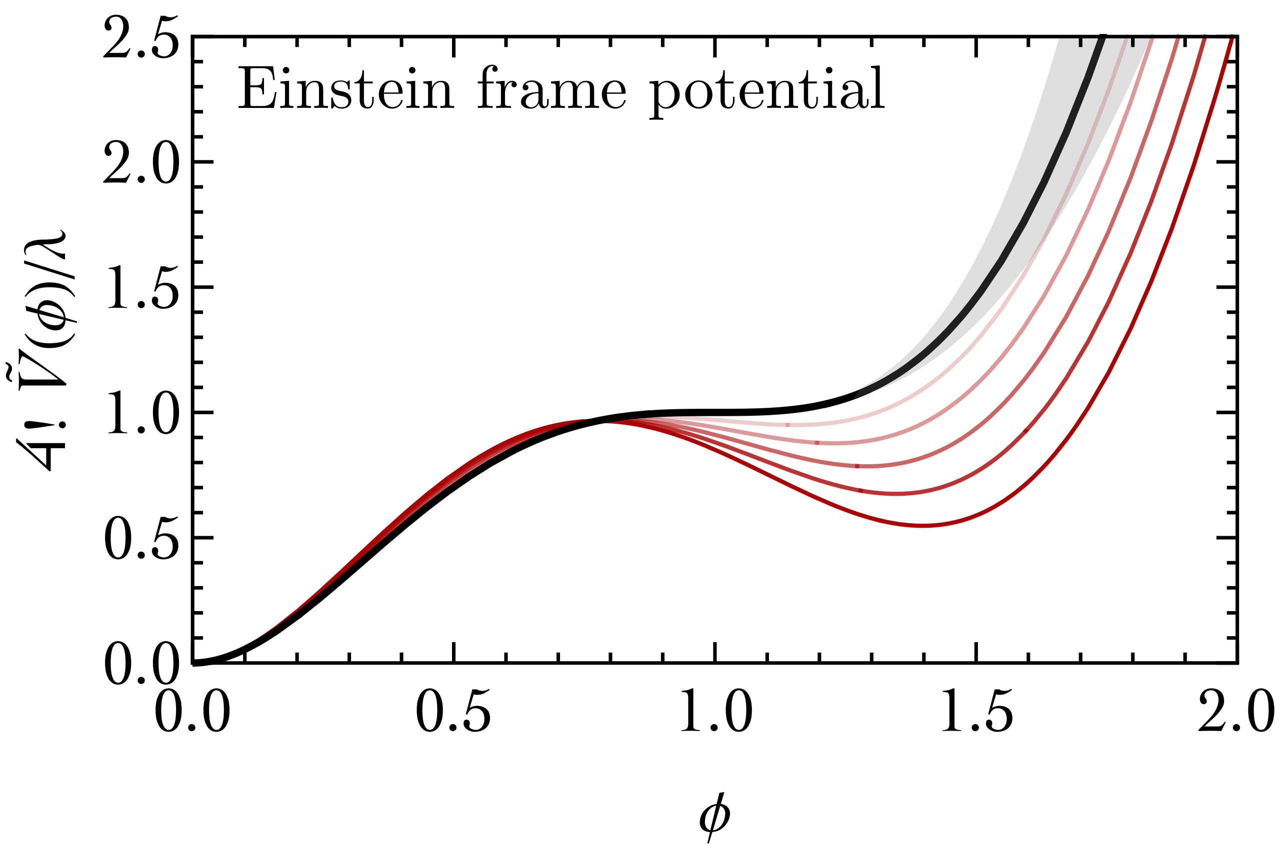
Single field formation scenario

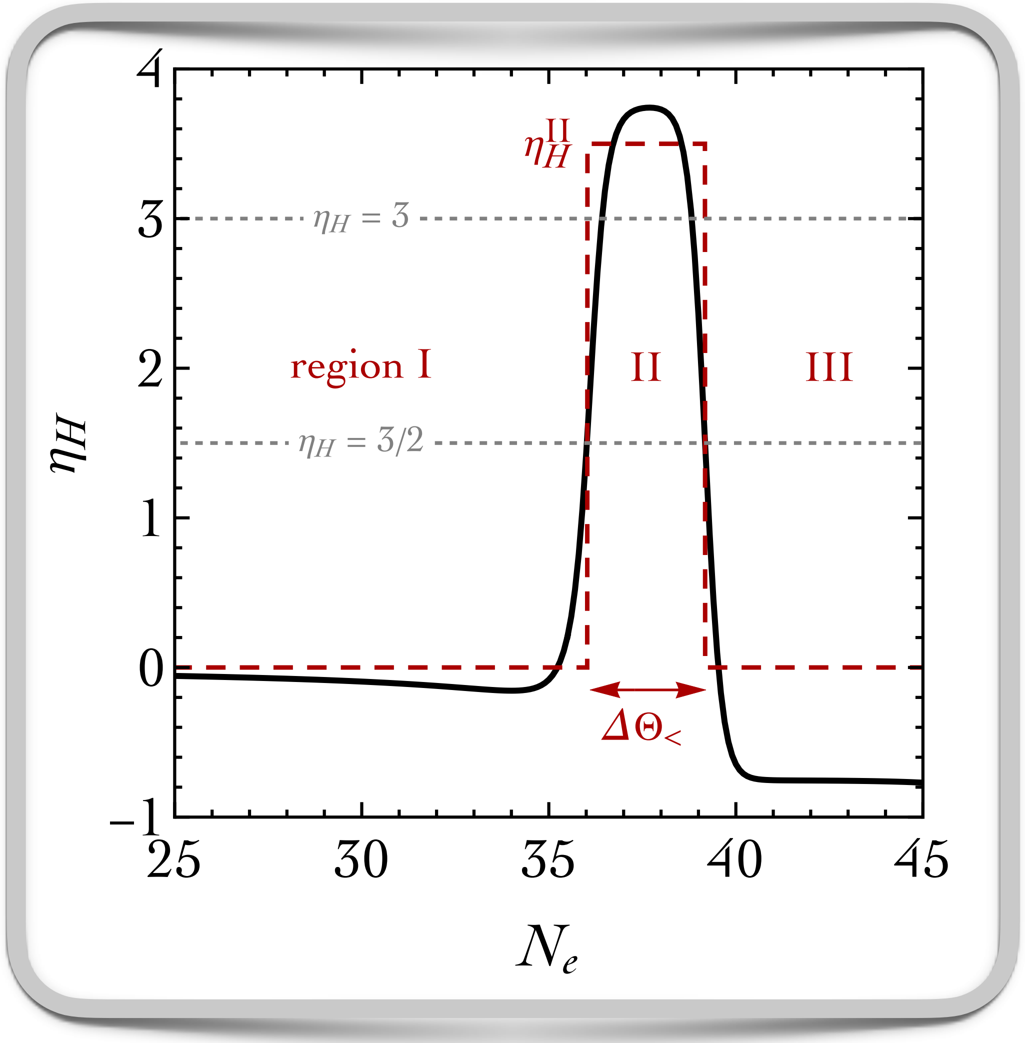
Example: polinomial inflation

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \left(M_P^2 + \xi\, \phi^2\right) R + \frac{1}{2} g_{\mu\nu}\, \partial^\mu \phi\, \partial^\nu \phi - V(\phi) \right)$$

$$V(\phi) = a_2 \phi^2 + a_3 \phi^3 + a_4 \phi^4$$

G. Ballesteros, J. Rey, M. Taoso and A. Urbano, JCAP **07** (2020), 025 [arXiv:2001.08220]





See also:

J. Garcia-Bellido and E. Ruiz Morales, Phys. Dark Univ. **18** (2017), 47-54 [arXiv:1702.03901]

G.Ballesteros, M.Taoso, Phys.Rev.D 97, no.2, 023501 (2018) [arXiv:1709.05565]

Reverse engineering inflationary dynamics

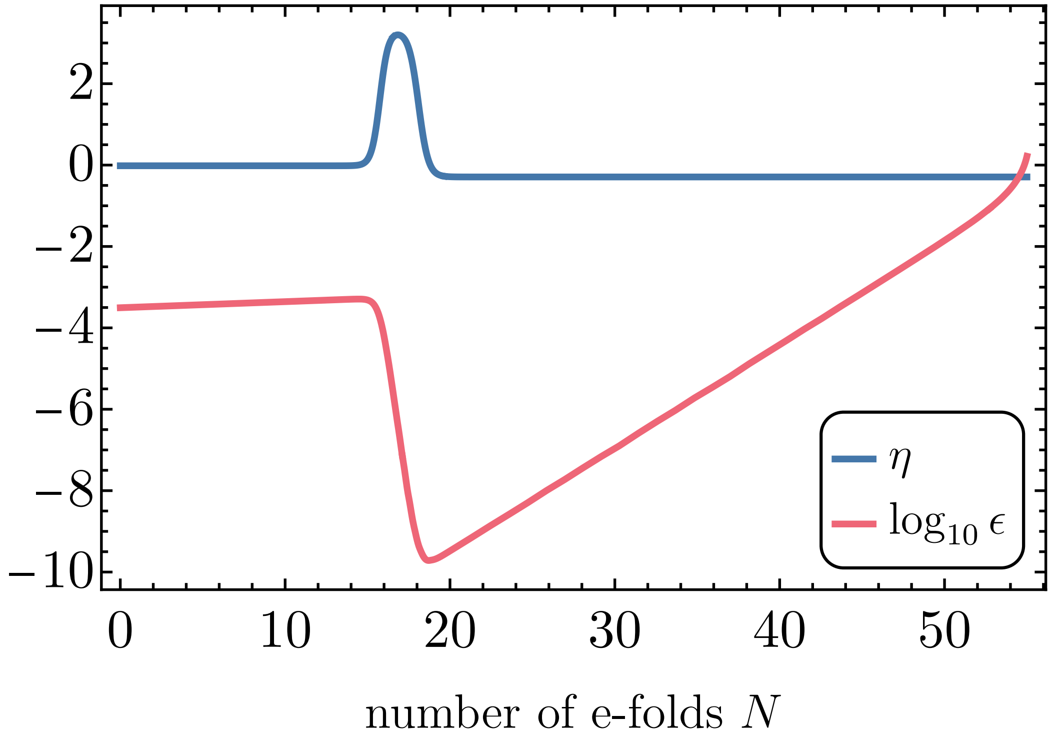
G. Franciolini and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123519 [arXiv:2207.10056]
(see also A.Karam, N.Koivunen, E.Tomberg, V.Vaskonen and H.Veermäe, JCAP **03** (2023), 013 [arXiv:2205.13540])

Reverse engineering starting from modelling Hubble parameter evolution to include USR

$$\eta(N) = \frac{1}{2}\left\{ \left[\eta_{\rm I} - \eta_{\rm II} + (\eta_{\rm II} - \eta_{\rm I}) \tanh\left(\frac{N - N_{\rm I}}{\delta N_{\rm I}}\right)\right] + \left[\eta_{\rm II} + \eta_{\rm III} + (\eta_{\rm III} - \eta_{\rm II}) \tanh\left(\frac{N - N_{\rm II}}{\delta N_{\rm II}}\right)\right] \right\}$$

$$\epsilon - \frac{1}{2}\frac{d\log\epsilon}{dN} = \eta$$

Compute the primordial power spectrum with Muchanov-Sasaki equation* to match observations



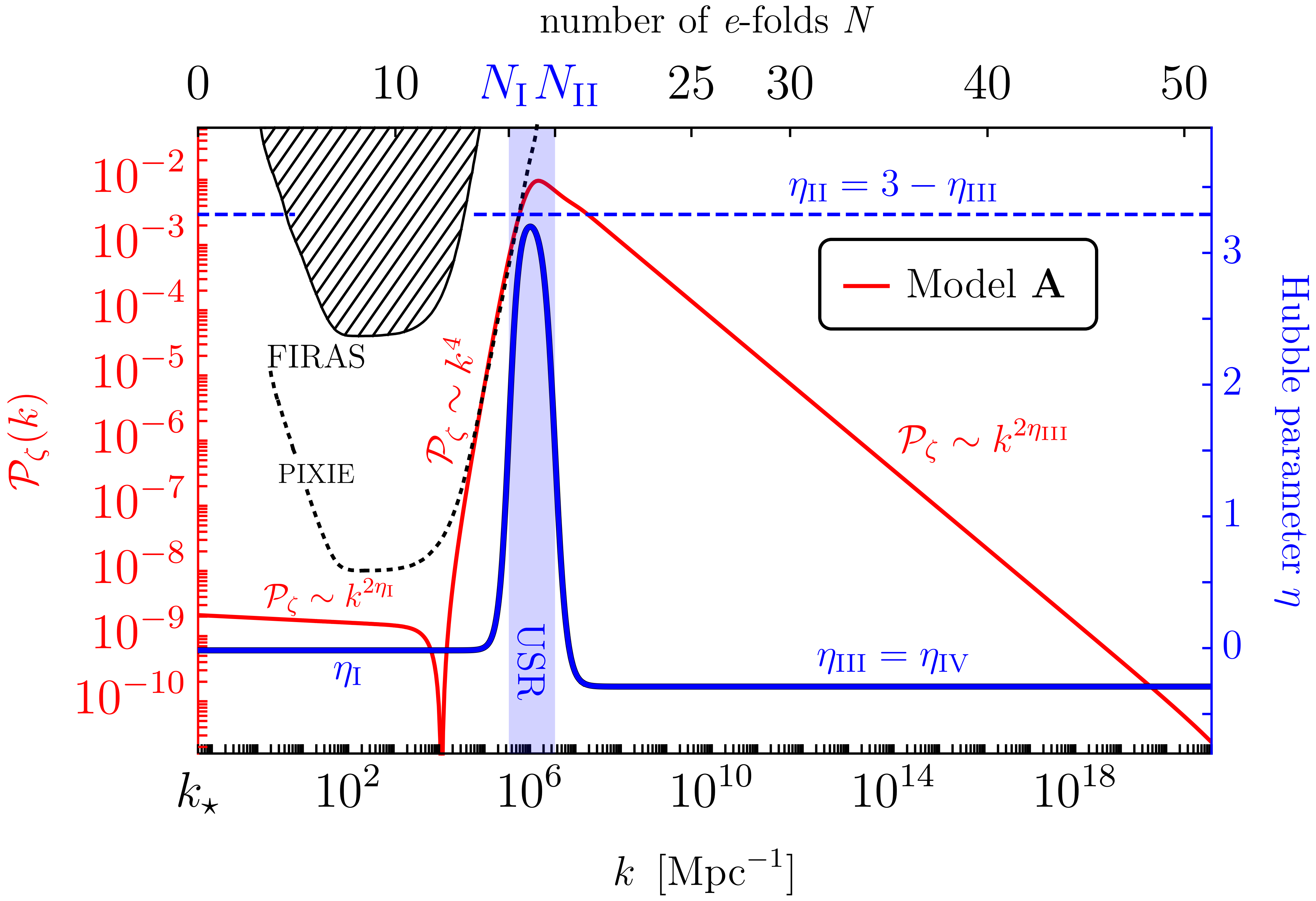
To arrive at the inflaton potential

$$V(N) = V(N_{\rm ref}) \exp\left\{-2\int_{N_{\rm ref}}^{N} dN' \left[\frac{\epsilon(3-\eta)}{3-\epsilon}\right]\right\}$$

$$\phi(N) = \phi(N_{\rm ref}) - \int_{N_{\rm ref}}^{N} dN' \sqrt{2\epsilon}$$

Data driven reverse engineering

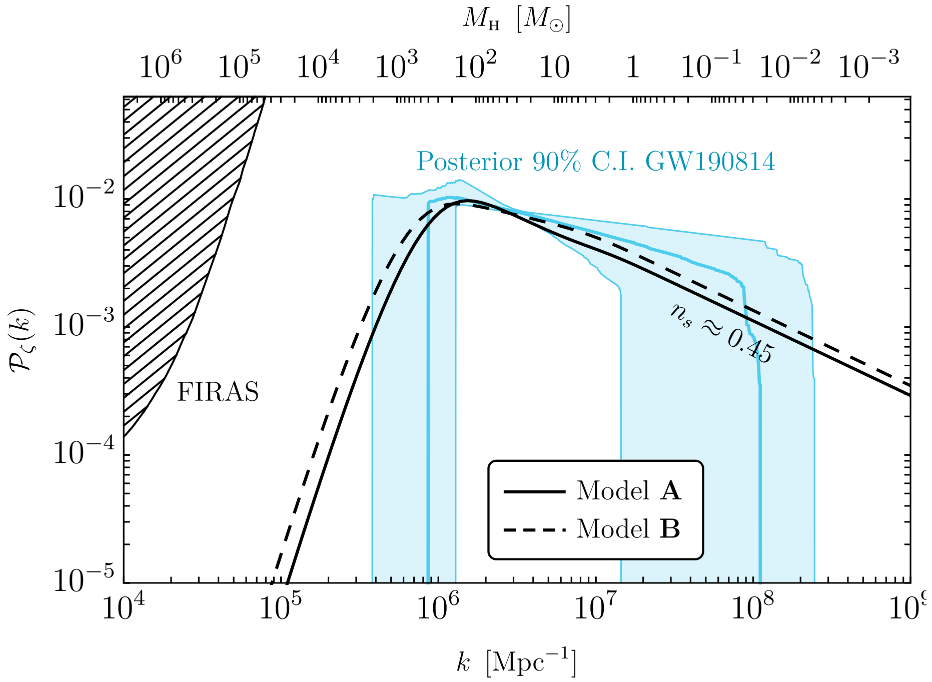
G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]



(example scenario from the LVK bounds)

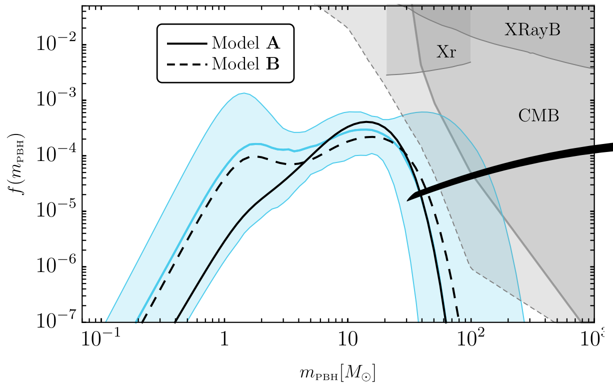
Constraints on the inflationary dynamics

G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]

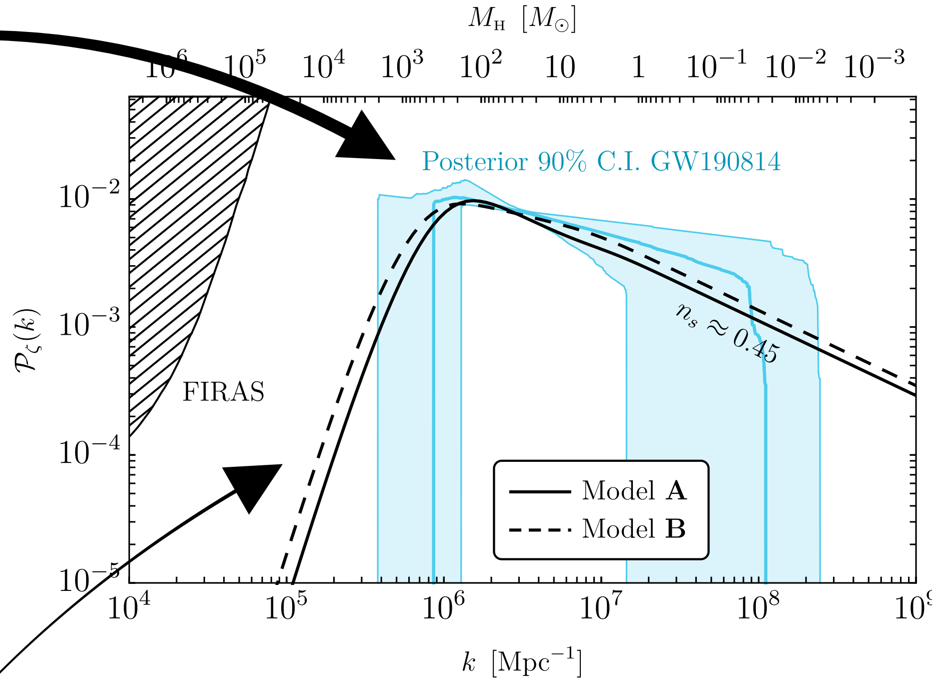


Constraints on the inflationary dynamics

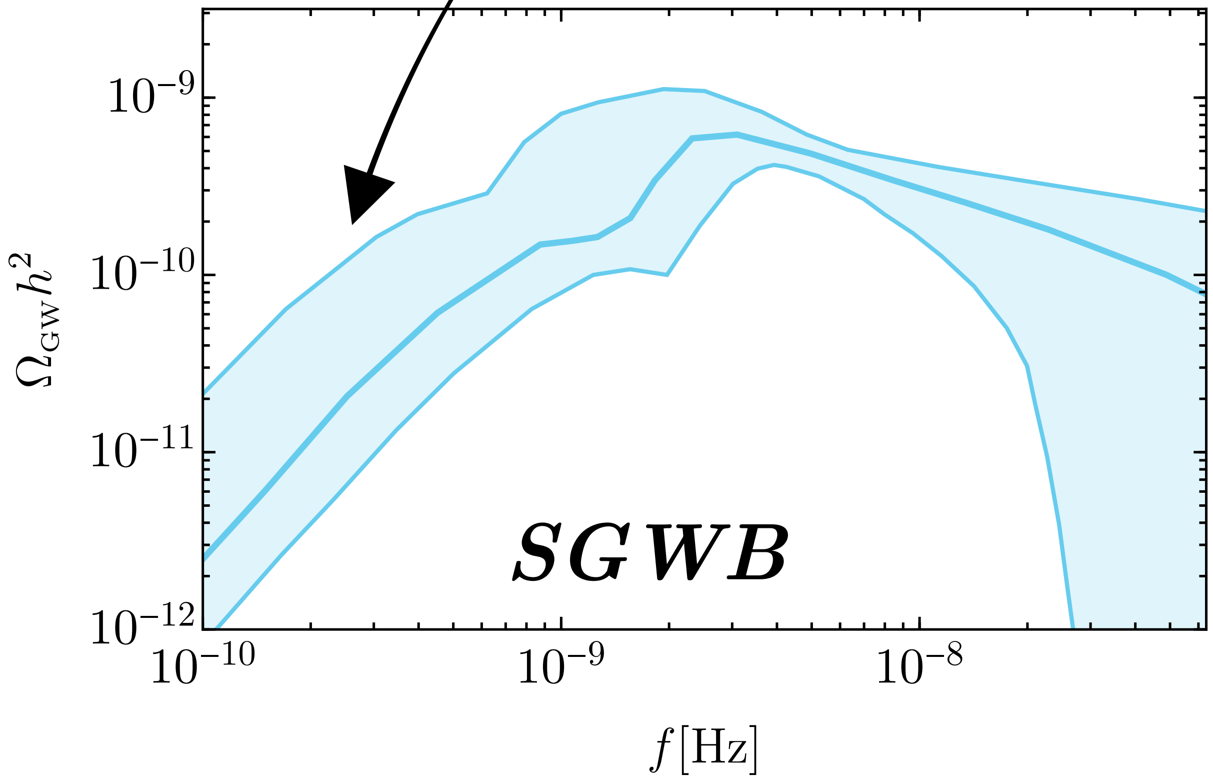
G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]



Mass function

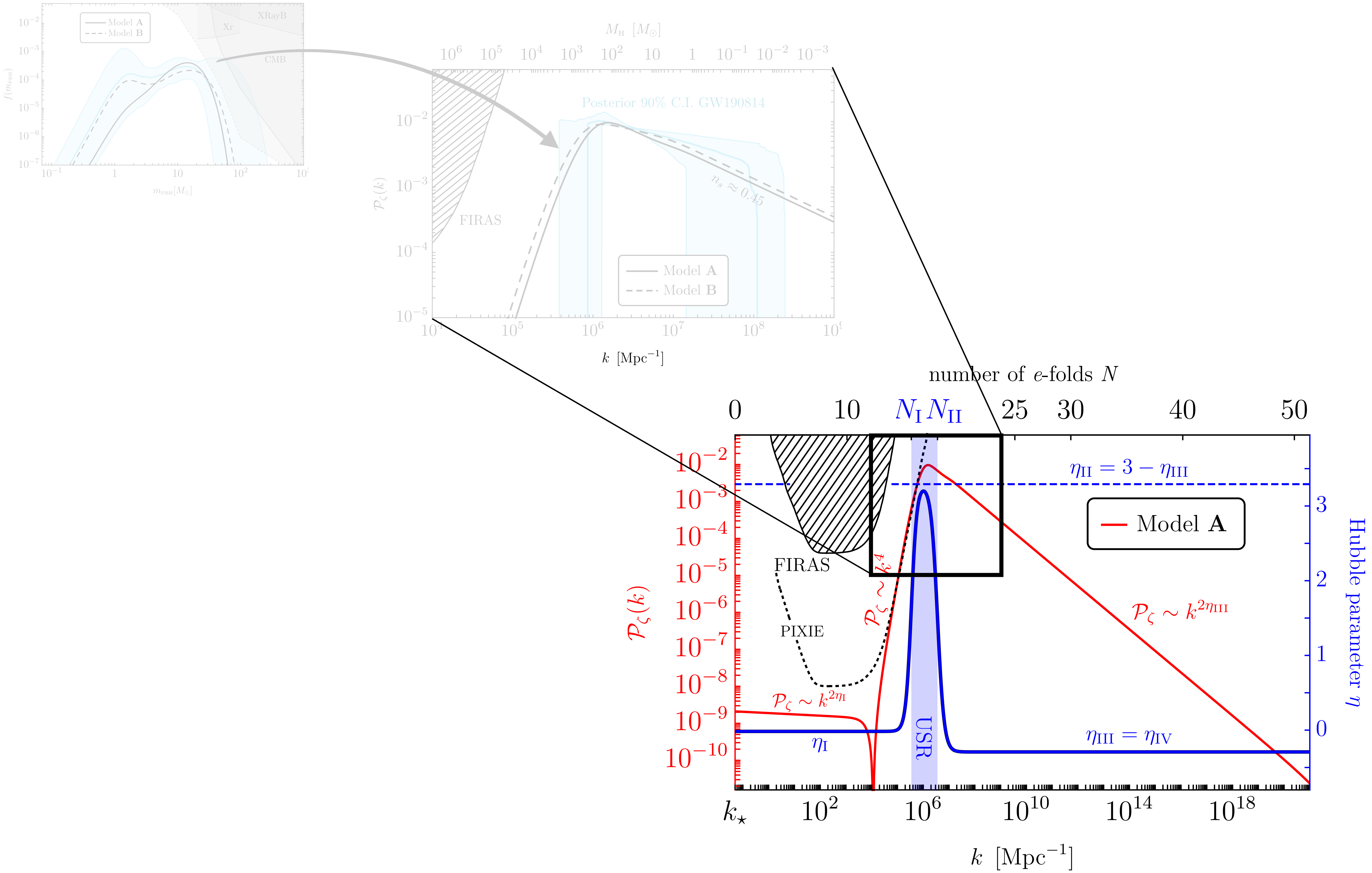


$\left(\textit{Small scale primordial power spectrum needed to interpret GW190814 as a primordial binary} \right)$

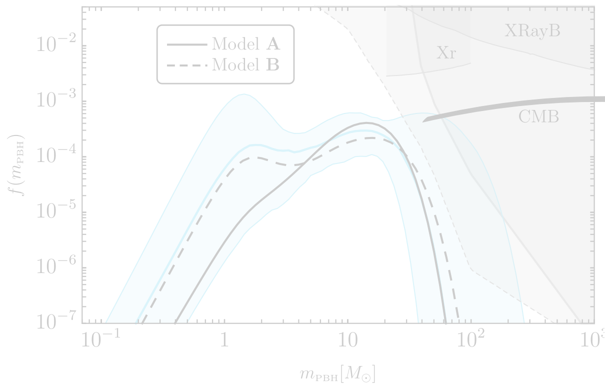


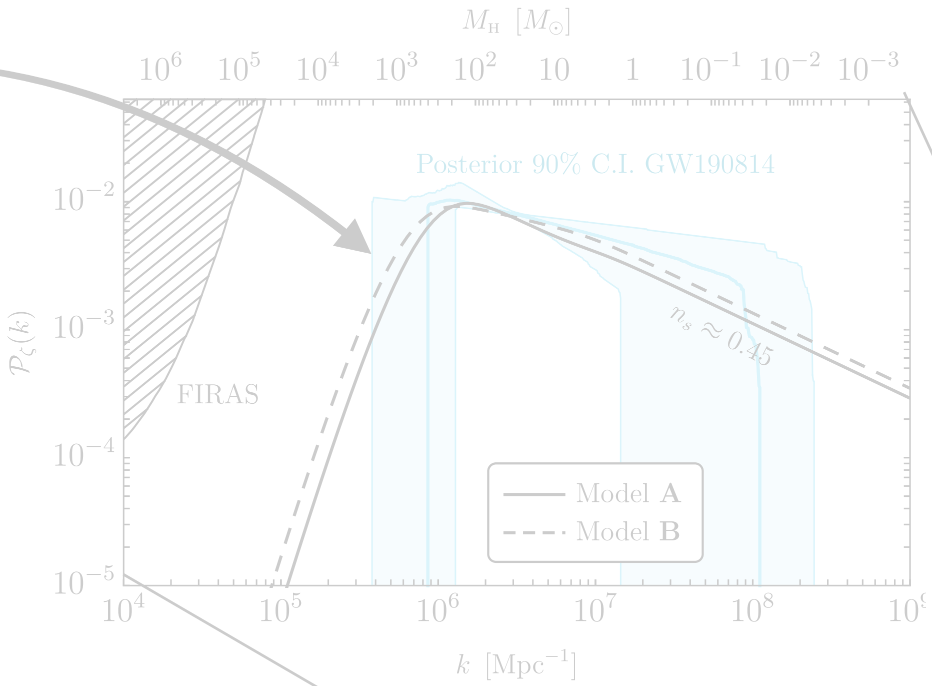
SGWB

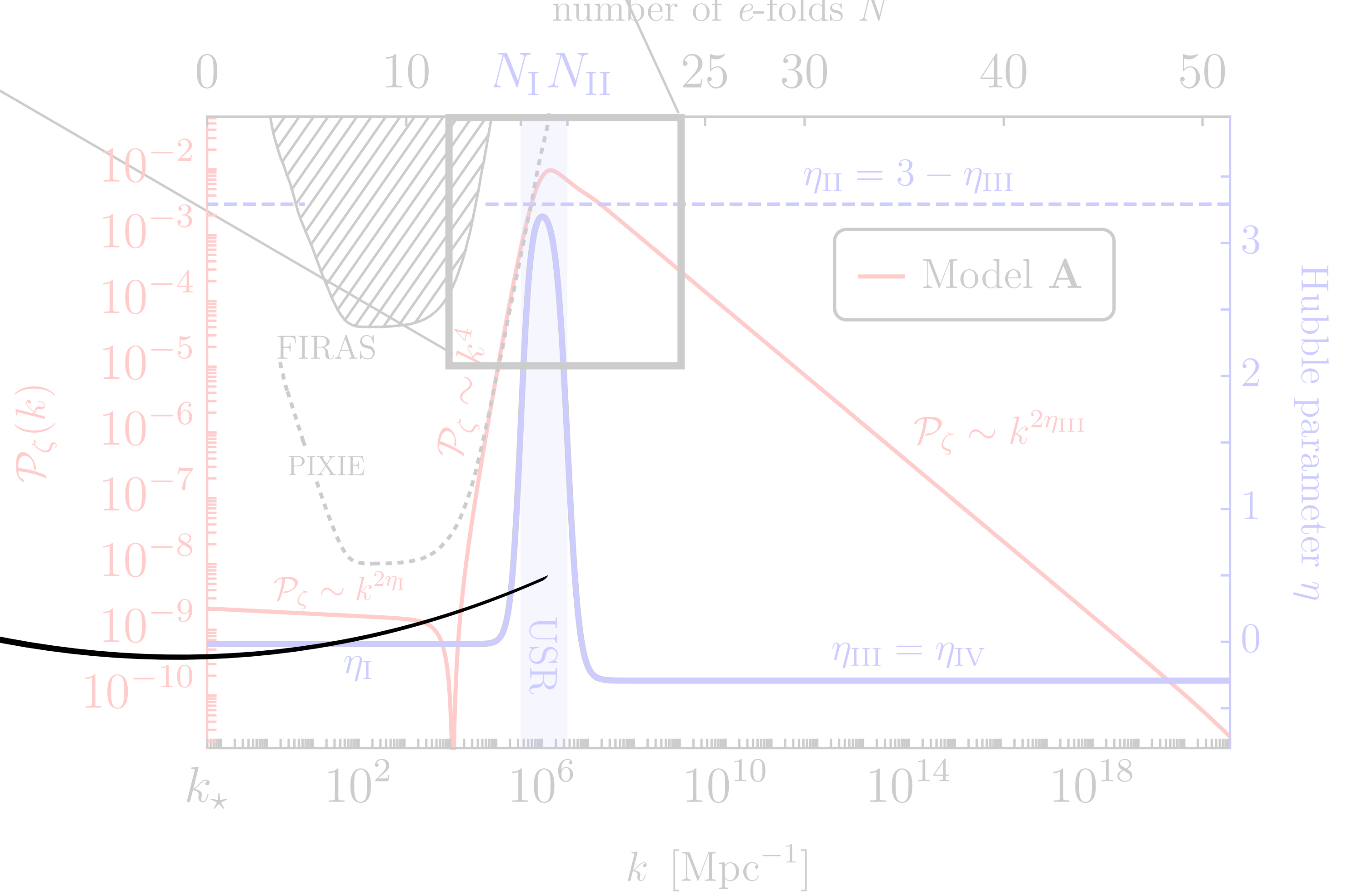
Constraints on the inflationary dynamics



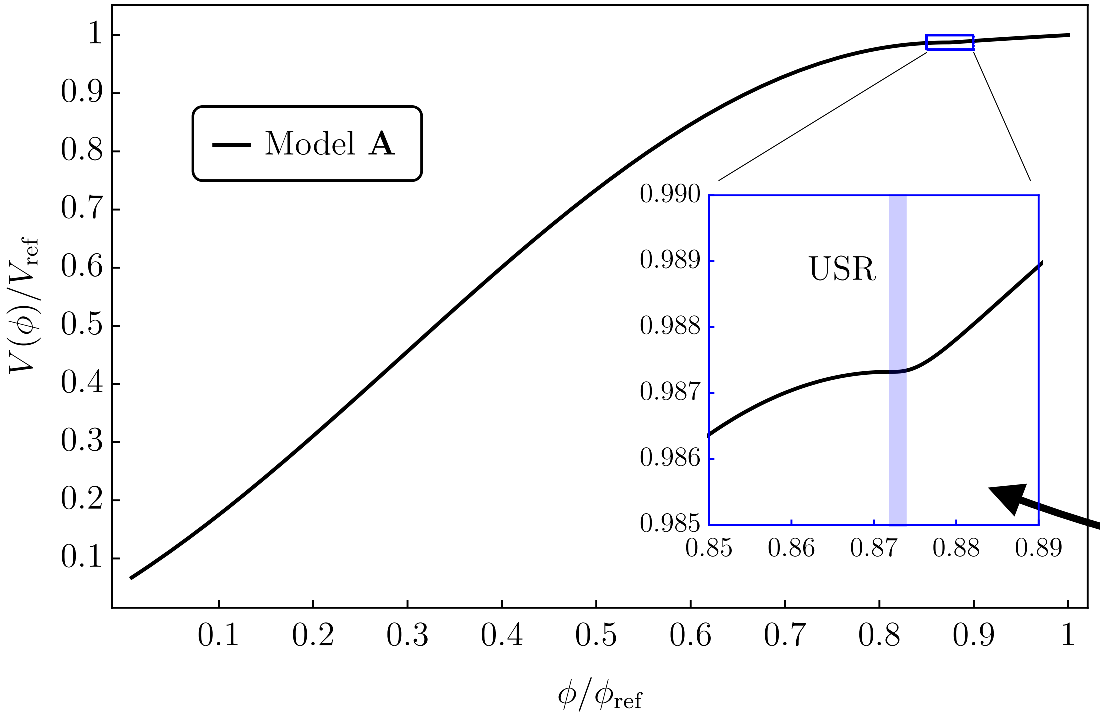
Constraints on the inflationary dynamics





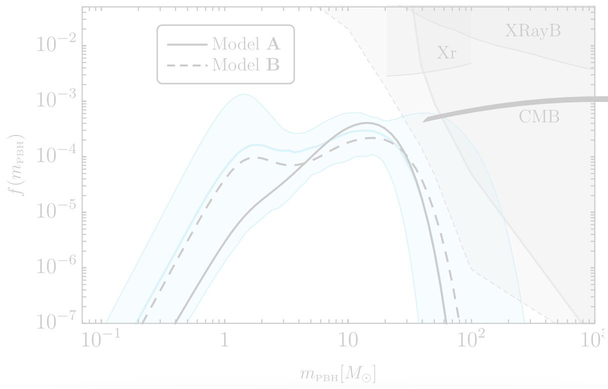


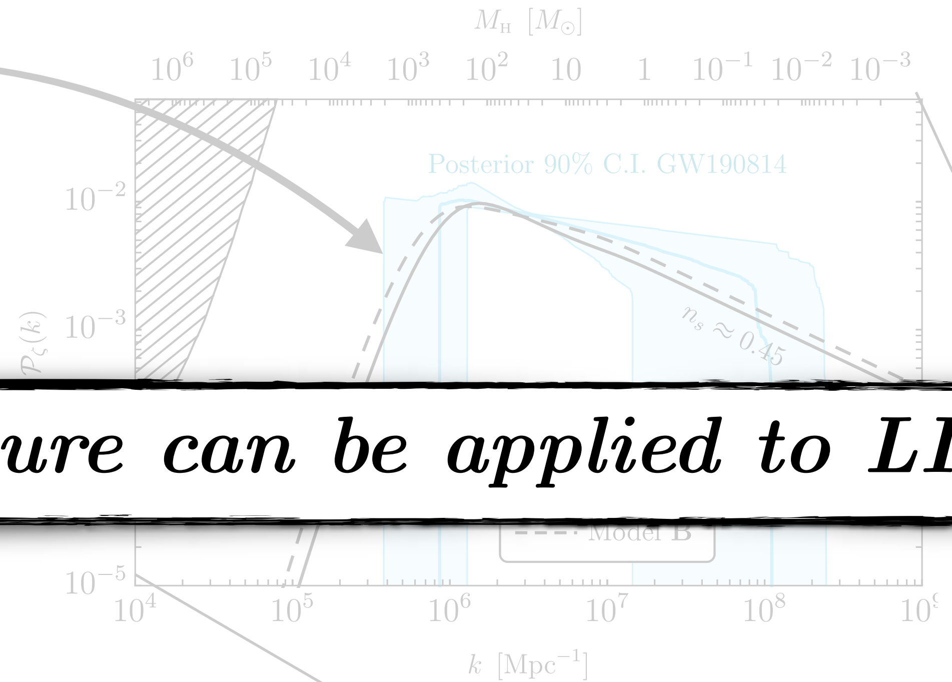
Inflaton potential



Can we reproduce this potential within a particle physics model of inflation?

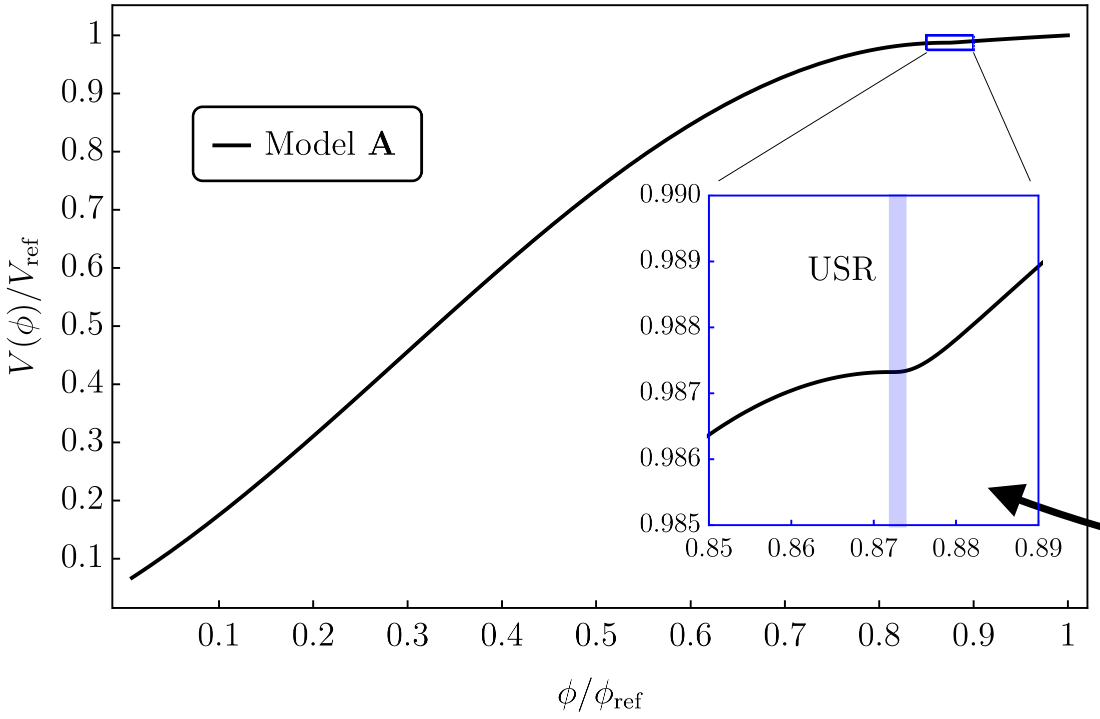
Constraints on the inflationary dynamics

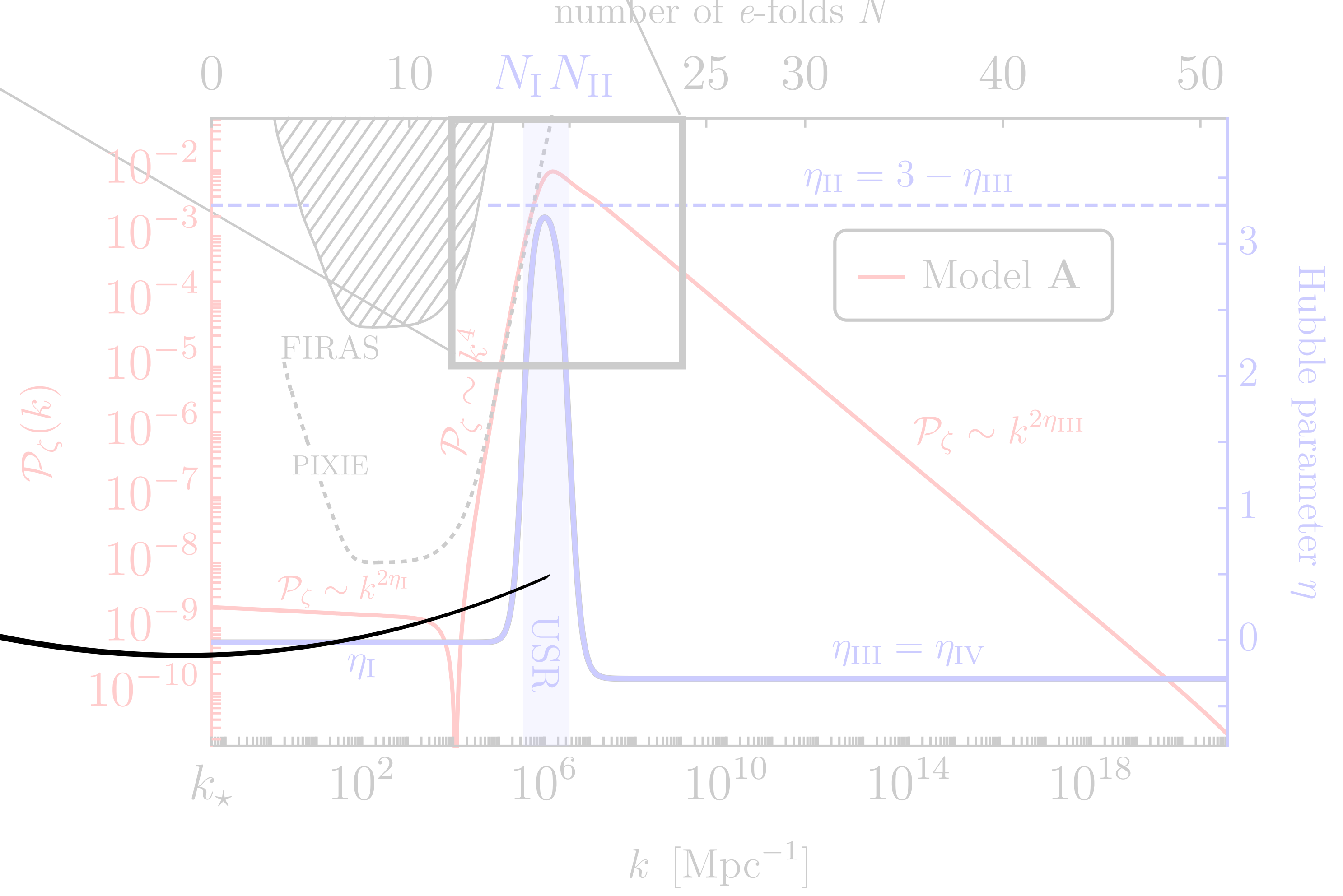




Same procedure can be applied to LISA results/forecasts

Inflaton potential





Can we reproduce this potential within a particle physics model of inflation?

Conclusions

- *LISA will be able to test the formation of asteroidal mass PBHs as dark matter, potentially closing the remaining window*
- *Non-Gaussianities have large impact on the constraints, and it requires non-perturbative computation of the abundance.*
- *One can reverse engineer inflationary dynamics compatible with an eventual SGWB detection*

Outlook

- *Develop solid tests to distinguish signals of different nature*
- *Explore different PBH formation mechanisms (early matter era, phase transitions, ...)*
- *Contribute to the science case of LISA, Einstein Telescope experiments*

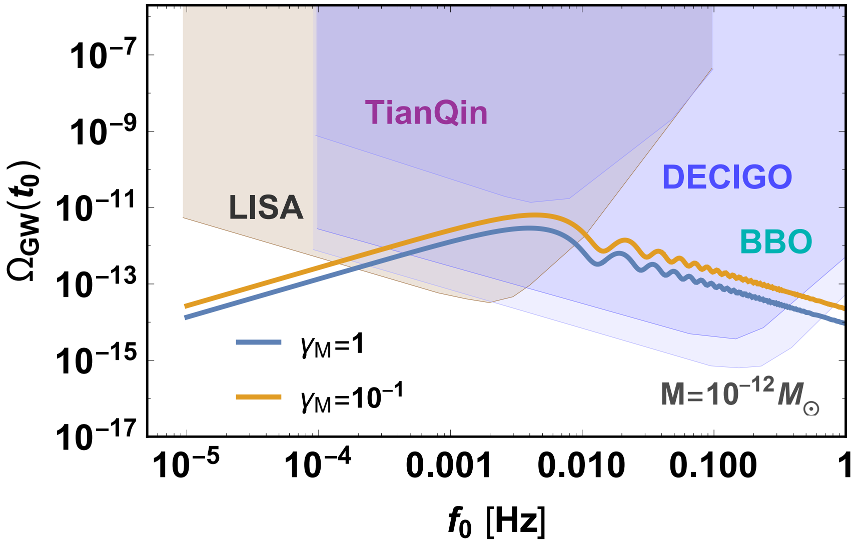
Thanks!

Backup

What about alternative formation scenarios?

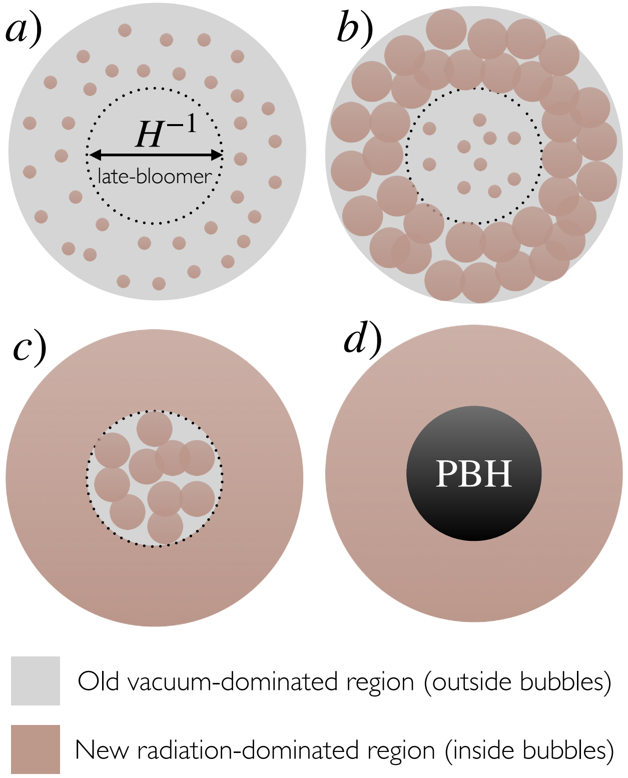
- ***Early Matter dominated era:***

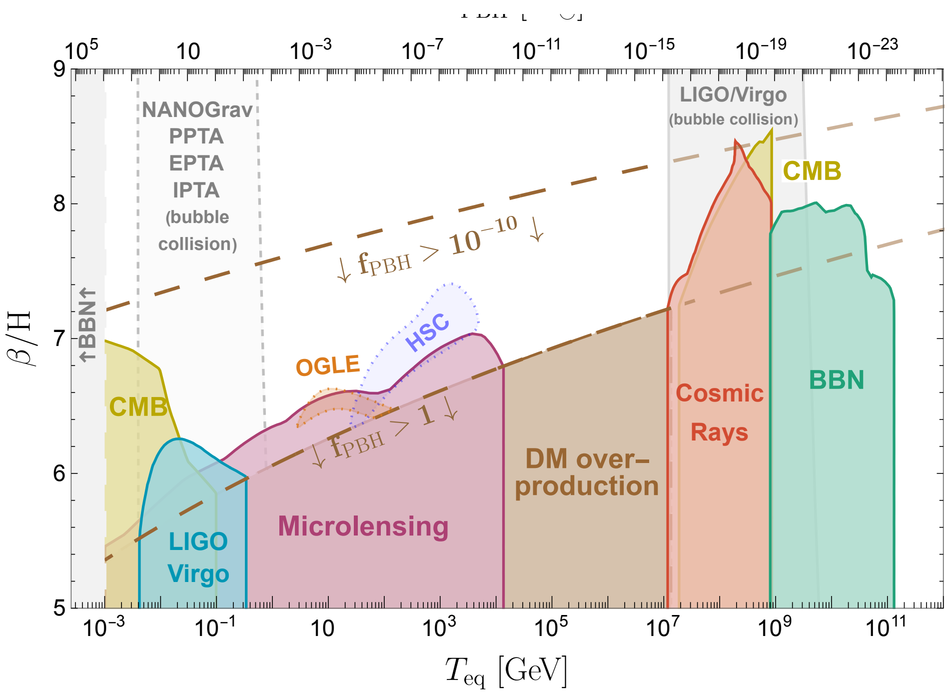
I.Dalianis and C.Kouvaris, JCAP **07** (2021), 046 [arXiv:2012.09255]



- ***Phase transitions:***

J. Liu *et al*,Phys. Rev. D **105** (2022) no.2, L021303 [arXiv:2106.05637]
Lewicki, Toczek, Vaskonen, 2305.04924
Y.Gouttenoire and T.Volansky, [arXiv:2305.04942]





+ ***Cosmic strings, etc. ...***

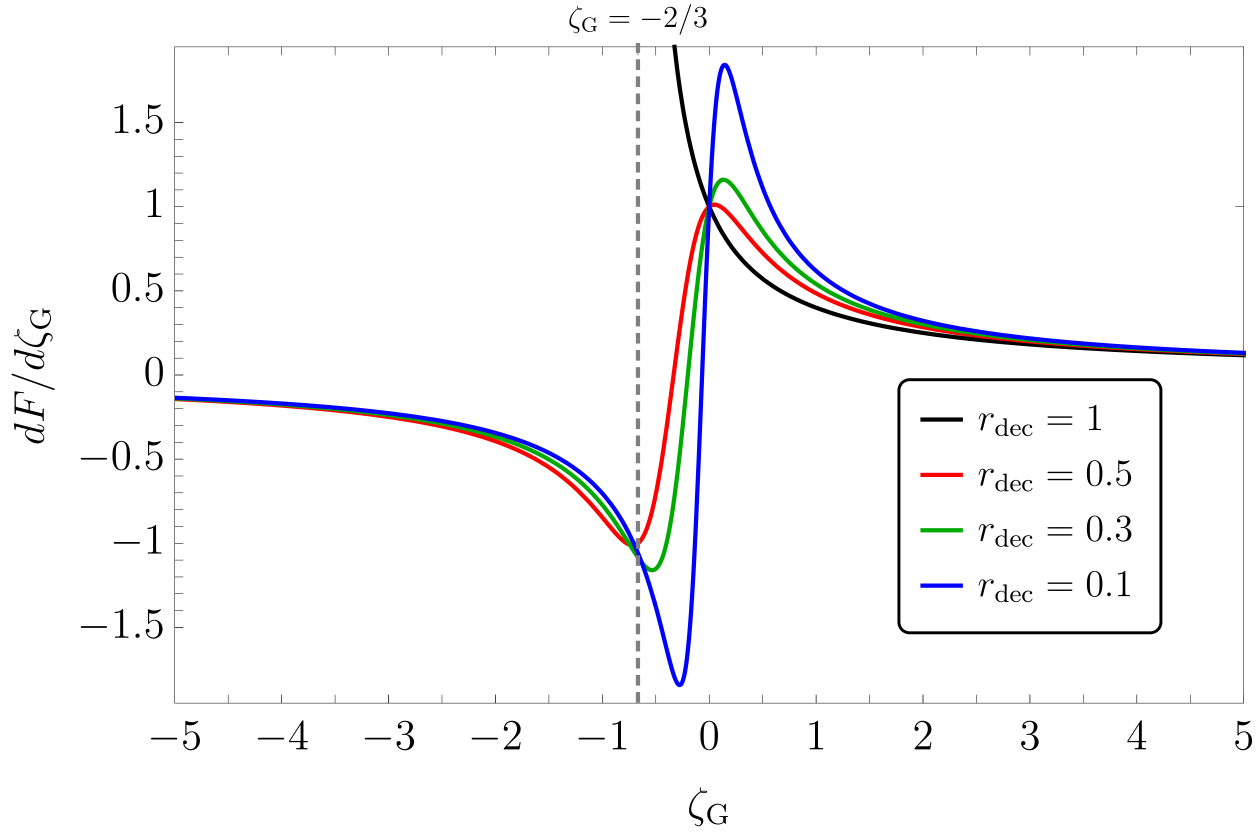
Curvaton model - NGs

G. Ferrante, G. Franciolini, A. Iovino, Junior. and A. Urbano, Phys. Rev. D **107** (2023) no.4, 043520 [arXiv:2211.01728]

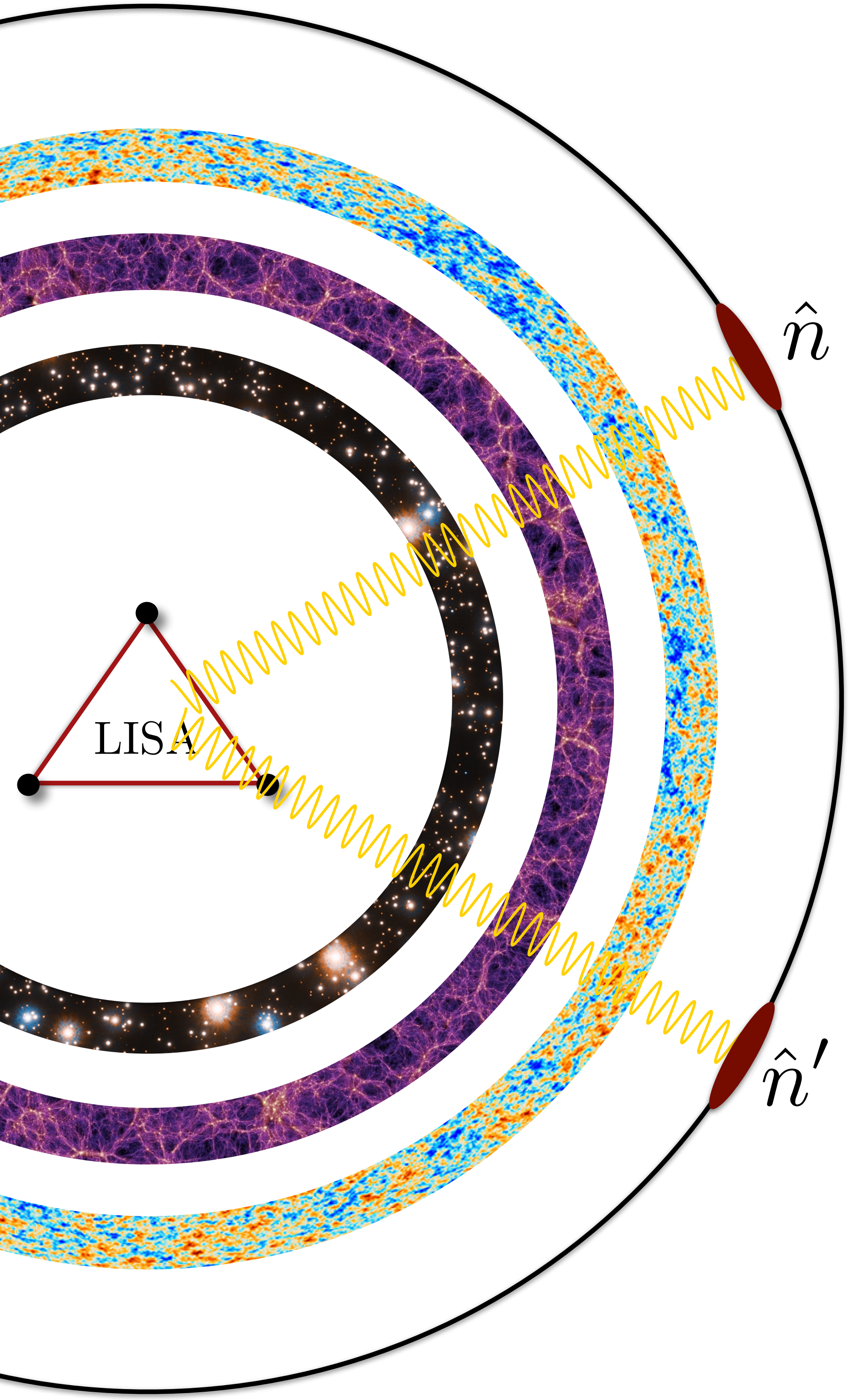
$$\zeta = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]\,,$$

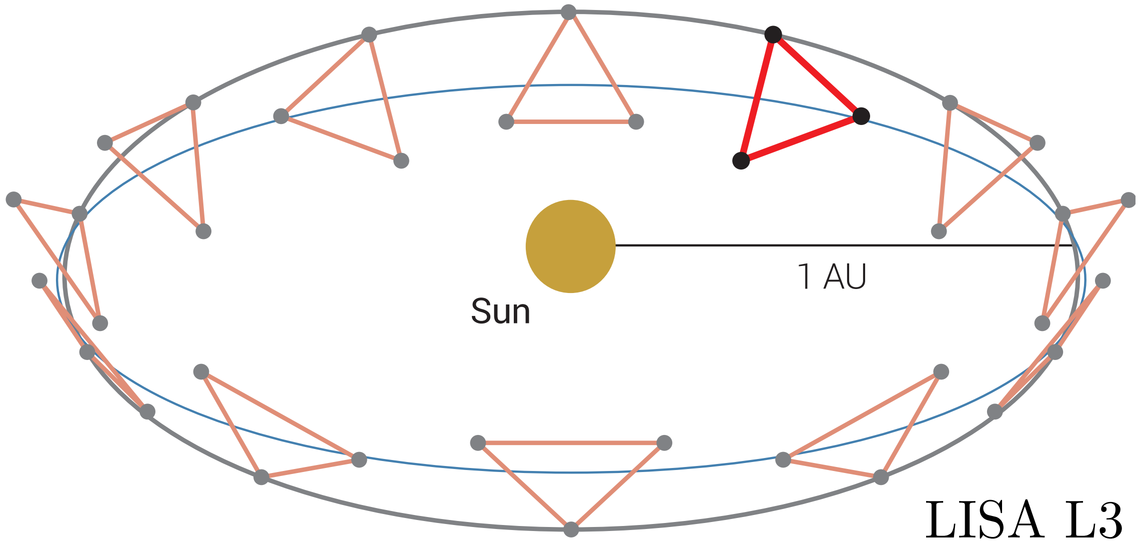
$$\begin{aligned}X(r_{\rm dec},\zeta_{\rm G}) \equiv &\frac{1}{\sqrt{2(3+r_{\rm dec})^{1/3}}}\left\{\sqrt{\frac{-3+r_{\rm dec}(2+r_{\rm dec})+[(3+r_{\rm dec})P(r_{\rm dec},\zeta_{\rm G})]^{2/3}}{(3+r_{\rm dec})P^{1/3}(r_{\rm dec},\zeta_{\rm G})}}\right.\\ &\left.+\sqrt{\frac{(1-r_{\rm dec})}{P^{1/3}(r_{\rm dec},\zeta_{\rm G})}-\frac{P^{1/3}(r_{\rm dec},\zeta_{\rm G})}{(3+r_{\rm dec})^{1/3}}+\frac{(2r_{\rm dec}+3\zeta_G)^2P^{1/6}(r_{\rm dec},\zeta_{\rm G})}{r_{\rm dec}\sqrt{-3+r_{\rm dec}(2+r_{\rm dec})+[(3+r_{\rm dec})P(r_{\rm dec},\zeta_{\rm G})]^{2/3}}}}\right\}\,,\end{aligned}$$

$$P(r_{\rm dec},\zeta_{\rm G}) \equiv \frac{(2r_{\rm dec}+3\zeta_{\rm G})^4}{16r_{\rm dec}^2}+\sqrt{(1-r_{\rm dec})^3(3+r_{\rm dec})+\frac{(2r_{\rm dec}+3\zeta_{\rm G})^8}{256r_{\rm dec}^4}}\,.$$



Additional probe of abundance anisotropies





LISA L3 proposal (2017)

In addition LISA will probably have an angular resolution up to around $\ell \approx 10$

$$\Omega(\eta_0, k, \hat{n}) \equiv \Omega(\eta_0, k) + \delta\Omega(\eta_0, k, \hat{n})$$

Small perturbations (direction dependent) in the abundance

1) Anisotropies at **emission**
2) Anisotropies due to **propagation**

Bartolo et al. [1908.00527]

GW anisotropies vs Isocurvature bounds

N. Bartolo, *et al* JCAP **02** (2020), 028 [arXiv:1909.12619]

Anisotropies at emission: $\langle\, \delta\Omega(\eta_{\rm e},\vec{x})\, \delta\Omega(\eta_{\rm e},\vec{y})\,\rangle \propto \left(\frac{1}{k_*|\vec{x}-\vec{y}|}\right)^2$

At unreachable scales: $k_* \approx 1/R_{\rm H}(\eta_{\rm e})$

One can have large-scale initial anisotropies due to Non-Gaussianities

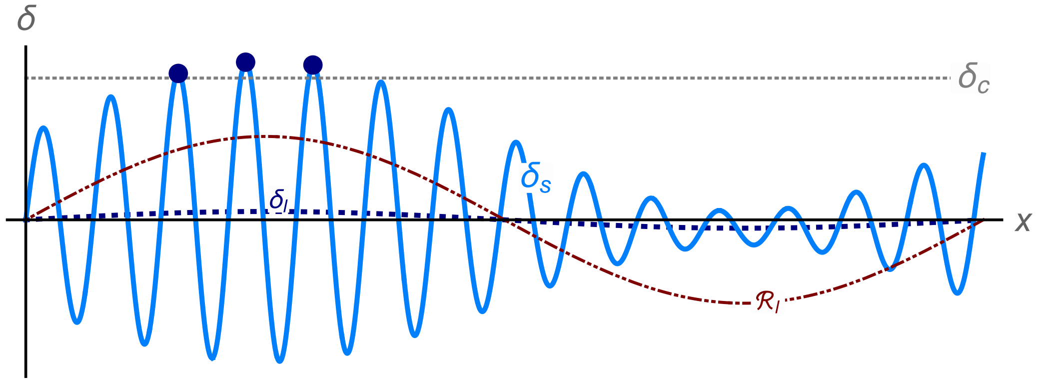
$$\zeta_{\rm NG}(\vec{x}) = \zeta(\vec{x}) + \frac{3}{5} f_{\rm NL}\left(\zeta^2(\vec{x}) - \langle\zeta^2(\vec{x})\rangle\right)$$

$$\Omega_{\rm GW}(\eta,\vec{x},k) = \bar{\Omega}_{\rm GW}(\eta,\, k)\left[1 + \frac{24}{5} f_{\rm NL}\int \frac{d^3q}{(2\pi)^3}\, {\rm e}^{i\vec{q}\cdot\vec{x}}\, \zeta_L(\vec{q})\right]$$

This also modulate PBH abundance = isocurvature modes

$$S := \delta_{\rm PBH} - \frac{3}{4}\delta.$$

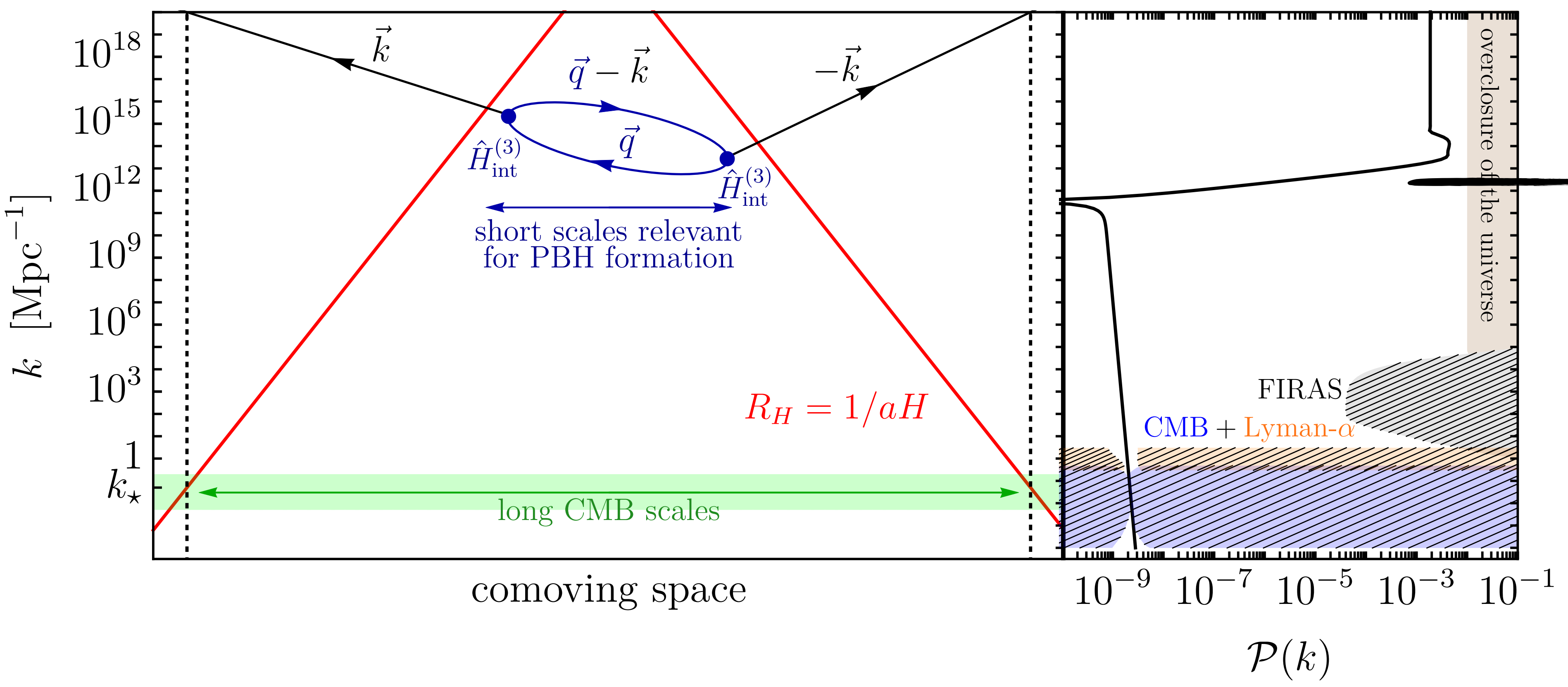
$$\mathcal{P}_S(k_{\rm CMB}) \simeq \left(2 f_{\rm NL}\nu^2\right)^2 \mathcal{P}_{\mathcal{R}}(k_{\rm CMB}).$$



$$|f_{\rm NL}|\nu^2 \lesssim \begin{cases} \frac{\sqrt{0.0025}}{2} = 0.025. & (f_{\rm NL} > 0) \\ \frac{\sqrt{0.0087}}{2} = 0.047. & (f_{\rm NL} < 0) \end{cases}$$

Planck constraints

Loops in single field models with USR phase



USR enhancement

$$S[\zeta] = S^{(2)}[\zeta] + S_{\rm bulk}[\zeta] = M_{\rm pl}^2 \int {\rm d}\tau\ {\rm d}^3x\ a^2\epsilon \left[(\zeta')^2 - (\partial_i\zeta)^2 + \frac{1}{2}\eta'\zeta'\zeta^2\right].$$

$$\zeta''_{\mathbf{p}} + \frac{(a^2\epsilon)'}{a^2\epsilon}\zeta'_{\mathbf{p}} + \frac{(a^2\epsilon\eta')'}{4a^2\epsilon}\int \frac{{\rm d}^3k}{(2\pi)^3}\zeta_{\mathbf{k}}\zeta_{\mathbf{p}-\mathbf{k}} = 0,$$

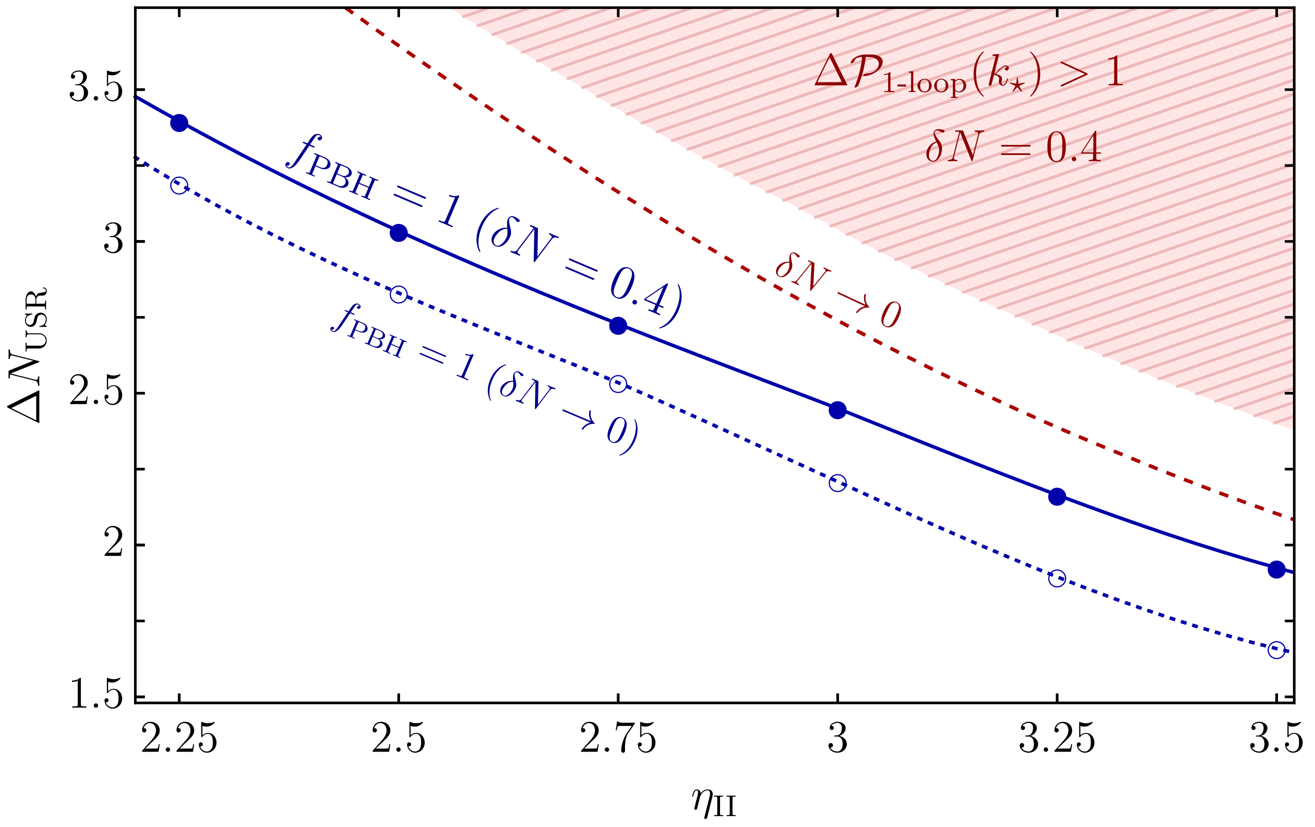
Tree level ***Dangerous?*** ***Poisson suppressed***

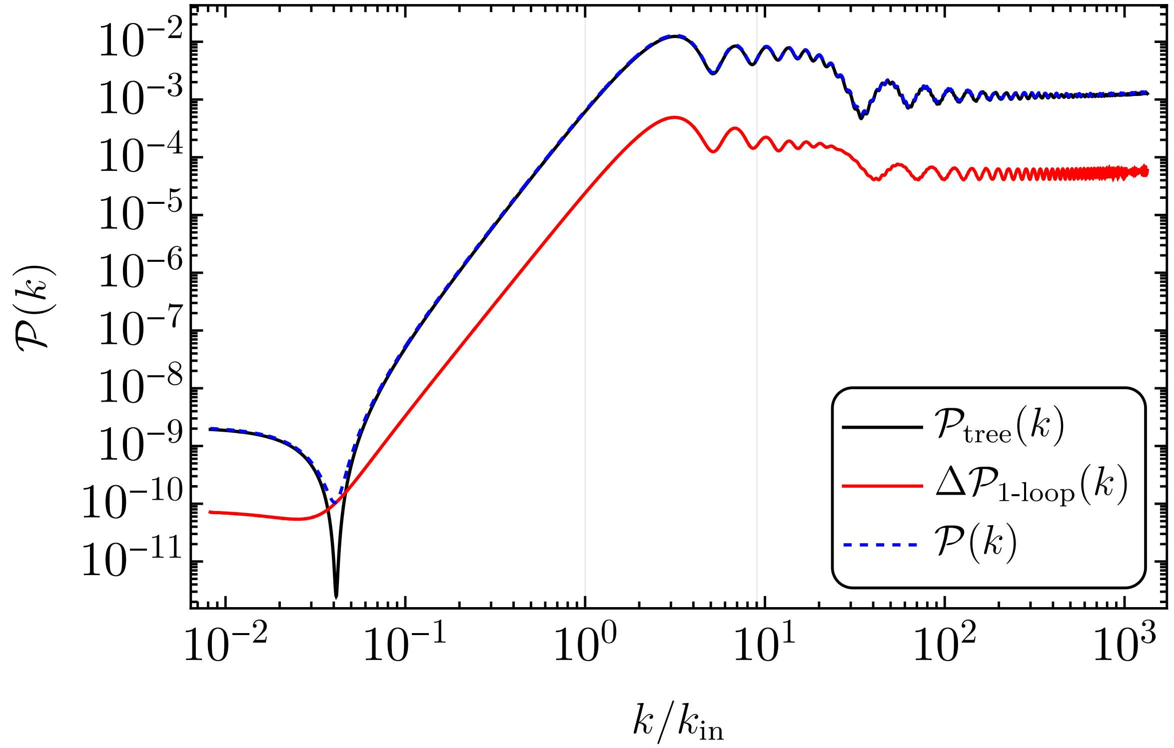
CMB PS: $\langle\!\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle\!\rangle = \langle\!\langle \zeta^f_{\mathbf{p}}(\tau_0)\zeta^f_{-\mathbf{p}}(\tau_0)\rangle\!\rangle + 2\langle\!\langle \zeta^s_{\mathbf{p}}(\tau_0)\zeta^f_{-\mathbf{p}}(\tau_0)\rangle\!\rangle + \langle\!\langle \zeta^s_{\mathbf{p}}(\tau_0)\zeta^s_{-\mathbf{p}}(\tau_0)\rangle\!\rangle.$

J. Kristiano and J. Yokoyama, (2022), [arXiv:2211.03395,2303.00341]
A. Riotto, (2023), [arXiv:2301.00599,2303.01727]
H. Firouzjahi, (2023), [arXiv:2303.12025]
S. Choudhury, M. R. Gangopadhyay, and M. Sami, (2023), [arXiv:2301.10000]
G. Tasinato, A large $|\eta|$ approach to single field inflation, [arXiv:2305.11568].
S. L. Cheng, D. S. Lee and K. W. Ng, [arXiv:2305.16810].
J. Fumagalli, [arXiv:2305.19263]

Loops in single field models with USR phase

Realistic models: no violation of perturbativity from USR modes





Loop correction to any scale of around few percent

G. Franciolini, A. Iovino, Junior., M. Taoso and A. Urbano, [arXiv:2305.03491]

Still open questions remain...