Computation of Stochastic Background from Extreme Mass Ratio Inspirals populations for LISA

https://arxiv.org/abs/2302.07043

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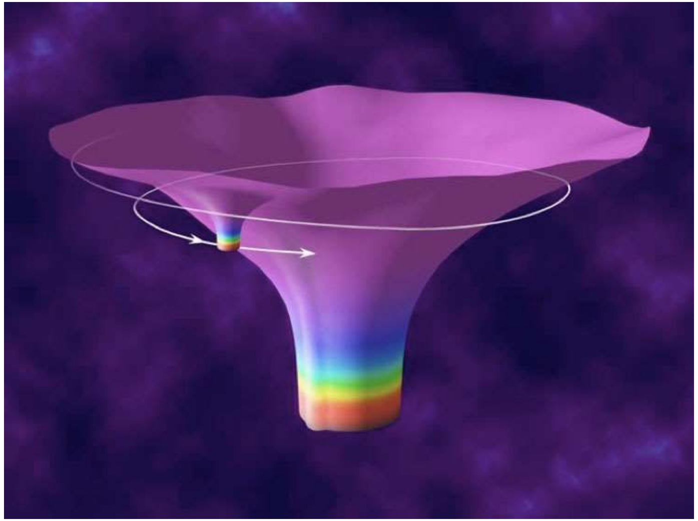
INTRODUCTION

EMRIs are relativistic binaries characterized by a tiny mass ratio ranging between [1e-9,1e-4].

EMRIs complete ~ 1e4 cycles in LISA’s sensitive band.

The vast majority of EMRIs is not expected to be individually detectable.



ANALYSIS SCHEME

I. EMRIs CATALOGS
    A. Astrophysical Population
    B. Sources Selection

II. Computation of the GW signal

III. Injection in LISA

IV. GWB Evaluation

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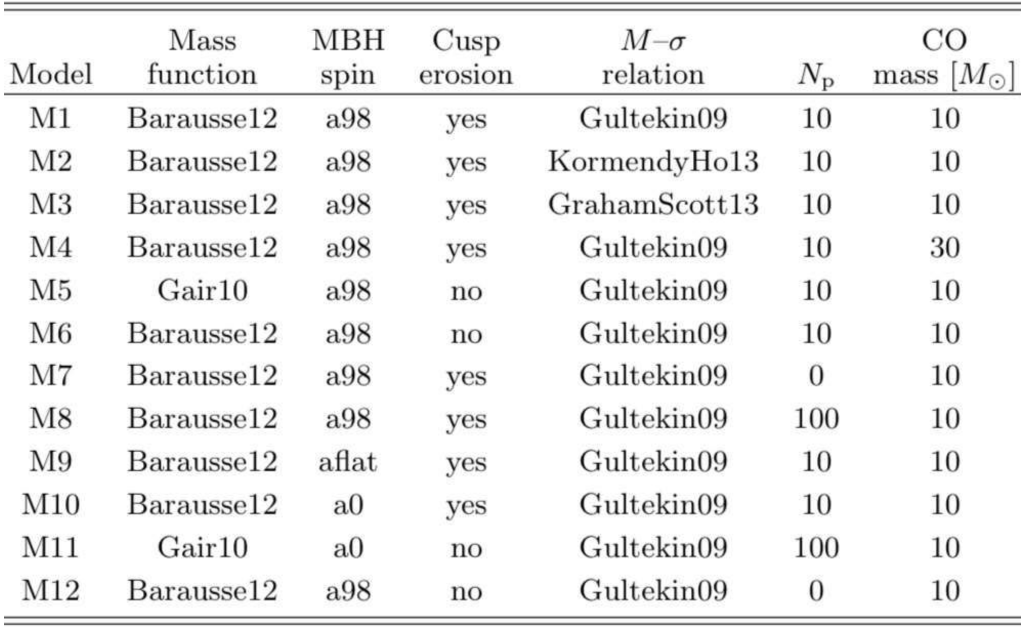
I. EMRI CATALOGS - ASTROPHYSICAL POPULATION



Model	Mass function	MBH spin	Cusp erosion	M–σ relation	$N_{\rm p}$	CO mass $[M_\odot]$
M1	Barausse12	a98	yes	Gultekin09	10	10
M2	Barausse12	a98	yes	KormendyHo13	10	10
M3	Barausse12	a98	yes	GrahamScott13	10	10
M4	Barausse12	a98	yes	Gultekin09	10	30
M5	Gair10	a98	no	Gultekin09	10	10
M6	Barausse12	a98	no	Gultekin09	10	10
M7	Barausse12	a98	yes	Gultekin09	0	10
M8	Barausse12	a98	yes	Gultekin09	100	10
M9	Barausse12	aflat	yes	Gultekin09	10	10
M10	Barausse12	a0	yes	Gultekin09	10	10
M11	Gair10	a0	no	Gultekin09	100	10
M12	Barausse12	a98	no	Gultekin09	0	10

Distribution of MBHs which could be potential EMRI hosts

$$\boxed{\frac{dN^3}{dMdadz}} * \boxed{p_0\left(M, z\right)} * \boxed{R(M, a)}$$

MBH distribution in M, a, z.

Probability that a MBH did not suffer a merger within its cusp regrowth time.

Rate at which COs are captured.

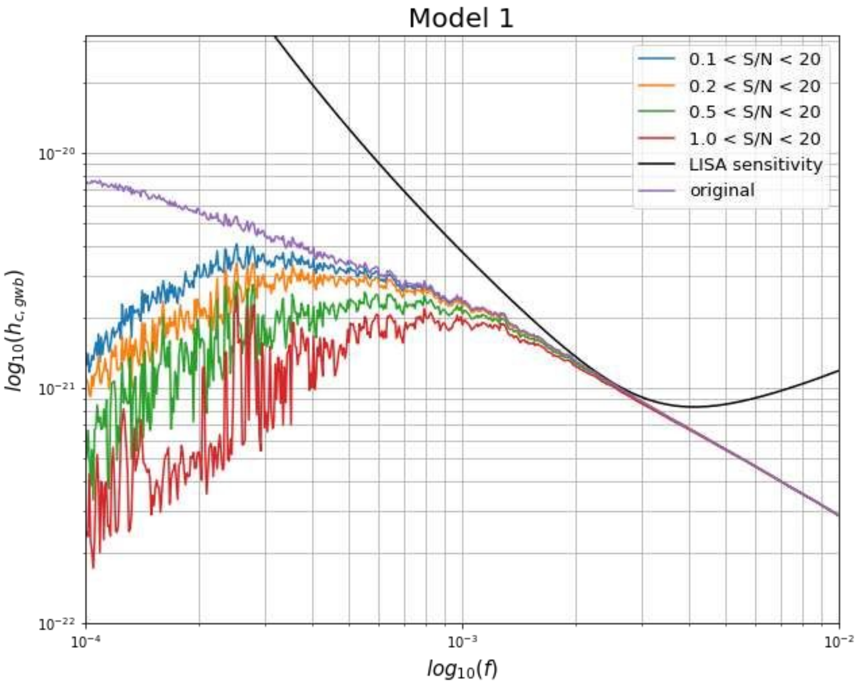
Babak et al. (2017) https://arxiv.org/pdf/1703.09722.pdf

I. EMRI CATALOGS - SOURCES SELECTION

The catalogs are composed by a variable number of sources which spans from ten thousand up to several millions.

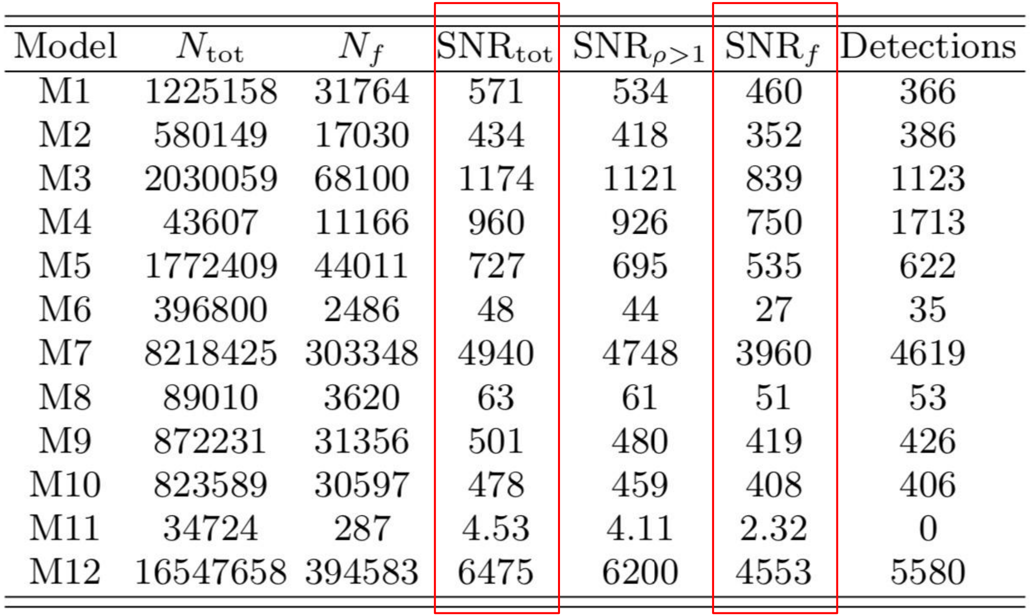
A complete (AAK + LISA RESPONSE) computation of GW signal requires ≈ $100s$ (without GPU).

We remove the dimmest sources with individual SNR < 1 (Bonetti et al. (2020) https://arxiv.org/pdf/2007.14403.pdf)

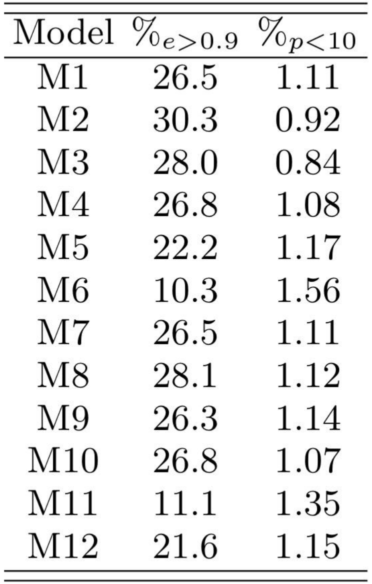


I. EMRI CATALOGS - SOURCES SELECTION

We subtract also the sources with $e0 > 0.9$ and $p0 < 10$…



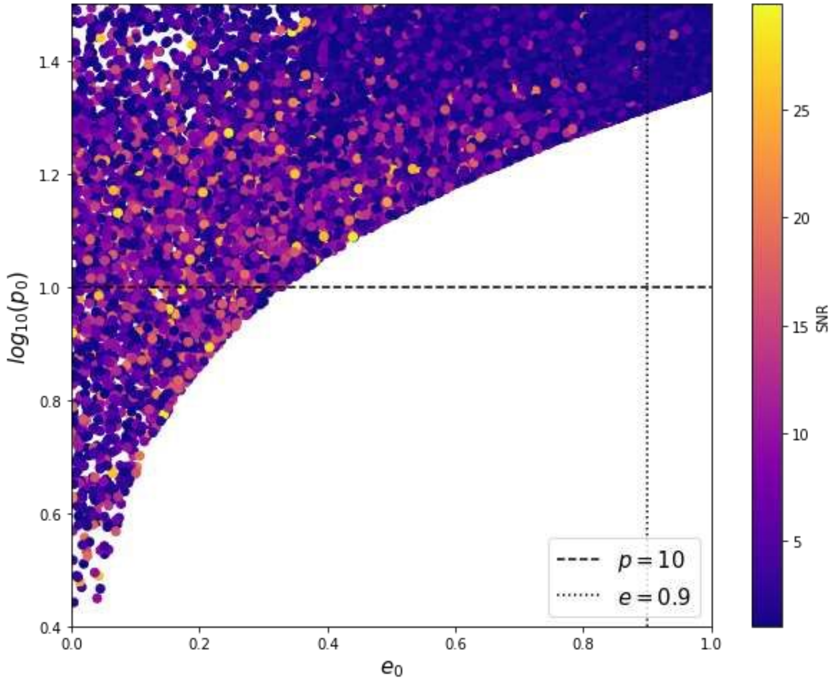
Model	$N_{\rm tot}$	N_f	$\mathrm{SNR}_{\rm tot}$	$\mathrm{SNR}_{\rho>1}$	SNR_f	Detections
M1	1225158	31764	571	534	460	366
M2	580149	17030	434	418	352	386
M3	2030059	68100	1174	1121	839	1123
M4	43607	11166	960	926	750	1713
M5	1772409	44011	727	695	535	622
M6	396800	2486	48	44	27	35
M7	8218425	303348	4940	4748	3960	4619
M8	89010	3620	63	61	51	53
M9	872231	31356	501	480	419	426
M10	823589	30597	478	459	408	406
M11	34724	287	4.53	4.11	2.32	0
M12	16547658	394583	6475	6200	4553	5580

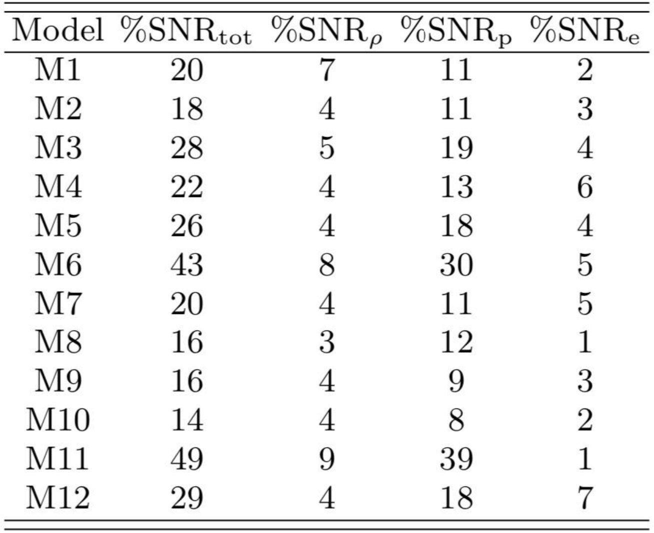


Model	$\%_{e>0.9}$	$\%_{p<10}$
M1	26.5	1.11
M2	30.3	0.92
M3	28.0	0.84
M4	26.8	1.08
M5	22.2	1.17
M6	10.3	1.56
M7	26.5	1.11
M8	28.1	1.12
M9	26.3	1.14
M10	26.8	1.07
M11	11.1	1.35
M12	21.6	1.15

I. EMRI CATALOGS - SOURCES SELECTION

On average, we recover almost ≈80% of the original SNR using only ≦5% of the sources.





Model	$\%SNR_{tot}$	$\%SNR_{\rho}$	$\%SNR_{p}$	$\%SNR_{e}$
M1	20	7	11	2
M2	18	4	11	3
M3	28	5	19	4
M4	22	4	13	6
M5	26	4	18	4
M6	43	8	30	5
M7	20	4	11	5
M8	16	3	12	1
M9	16	4	9	3
M10	14	4	8	2
M11	49	9	39	1
M12	29	4	18	7

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II. COMPUTATION OF THE WAVEFORM

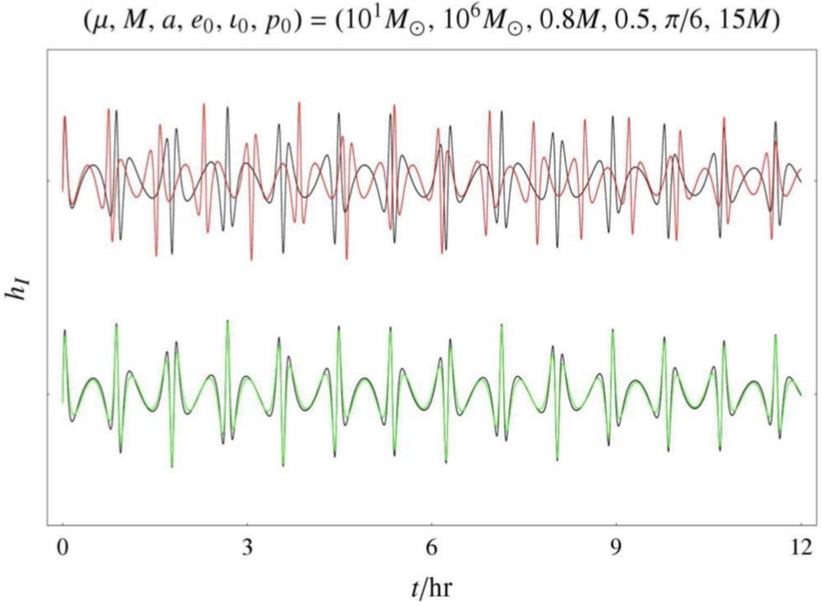
We compute the GW signal through the Augmented Analytic Kludge + 5PN Trajectory.
FastEMRIWaveform (FEW)
https://bhptoolkit.org/FastEMRIWaveforms/html/index.html

AAK is a combination of the AK and numerical Kludge (NK). The main idea is to map the parameters of the AK model to match the frequencies of NK waveforms.

$$\underbrace{f_{LT}\left(\tilde{M},\tilde{a},\tilde{p}\right)}_{\text{AK}} = \underbrace{\omega_\theta(M,a,p) - \omega_\phi(M,a,p)}_{\text{NK}}$$

$$f_{prec}\left(\tilde{M},\tilde{a},\tilde{p}\right) = \omega_\phi(M,a,p) - \omega_r(M,a,p)$$

$$f_{orb}\left(\tilde{M},\tilde{a},\tilde{p}\right) = \omega_r(M,a,p)$$


$(\mu, M, a, e_0, \iota_0, p_0) = (10^1 M_\odot, 10^6 M_\odot, 0.8M, 0.5, \pi/6, 15M)$

Chua et al. (2017) https://arxiv.org/pdf/1705.04259.pdf

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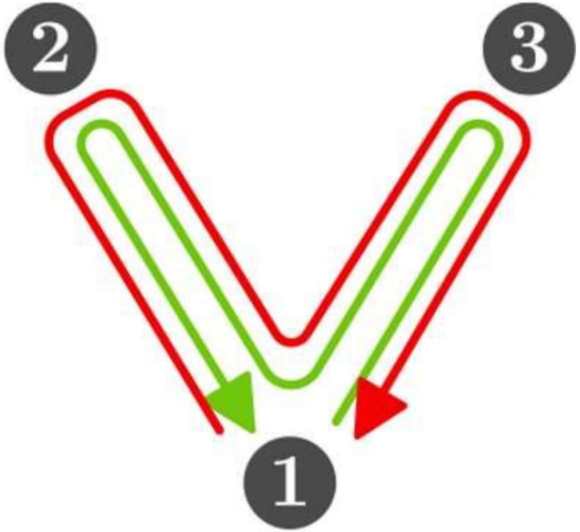
III. INJECTION IN LISA

There exist different versions of TDI: we are dealing with static, unequal, and non-flexing arms.

$$L_i(t) = L_i \,,\; L_i \;= L_{i'}$$

A possible realization of TDI defines three variables X, Y and Z, corresponding to pairwise Michelson-like interferometer.

$$\begin{aligned} X(t) = &\boxed{[(y_1 + y_{3,2}) + (y_{1,22} + y_{2,322})]} \\ &-\boxed{[(y_1 + y_{2,3}) + (y_{1,33} + y_{3,233})]} \end{aligned}$$



$$A(t) = \frac{Z - X}{\sqrt{2}}$$

$$E(t) = \frac{X - 2Y + Z}{\sqrt{6}}$$

$$T(t) = \frac{X + Y + Z}{\sqrt{3}}$$

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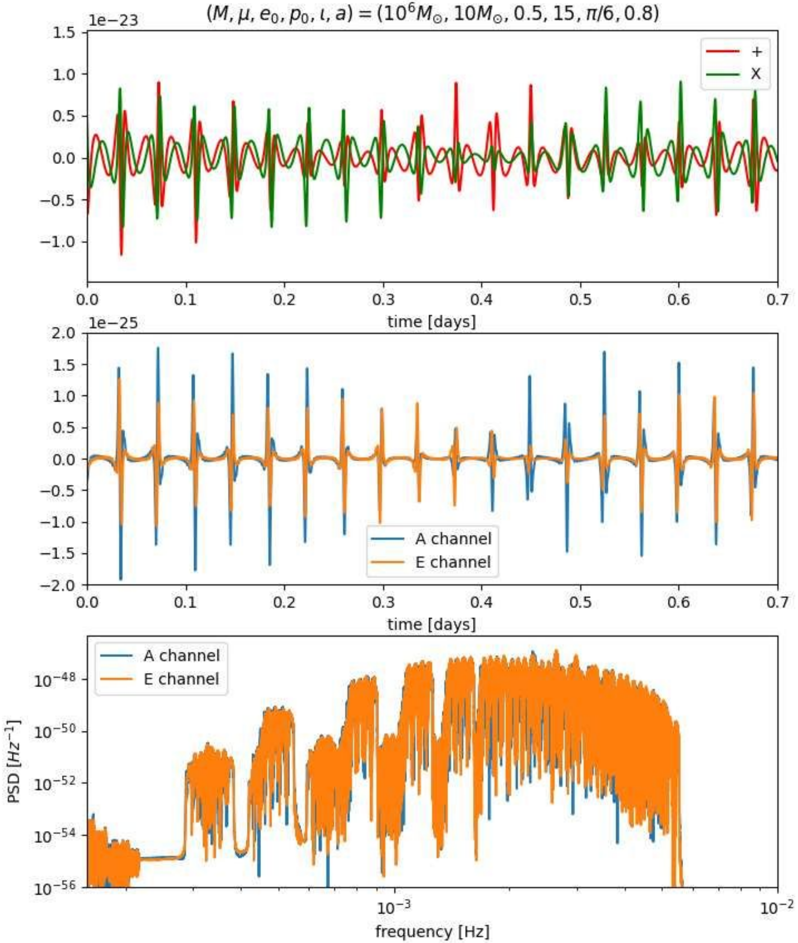


Summarizing…

Computation of GW strain in time domain with AAK considering $Tobs = 4yrs$.

Computation of TDI variables in time domain (Optimal Combination).

Conversion of TDI variables in frequency domain through the Fourier transform.

IV. EVALUATION OF STOCHASTIC BACKGROUND

Karnesis et al. (2021) https://arxiv.org/pdf/2103.14598.pdf

1. Estimation of confusing noise Sn,k using running median on the power spectrum of the data.

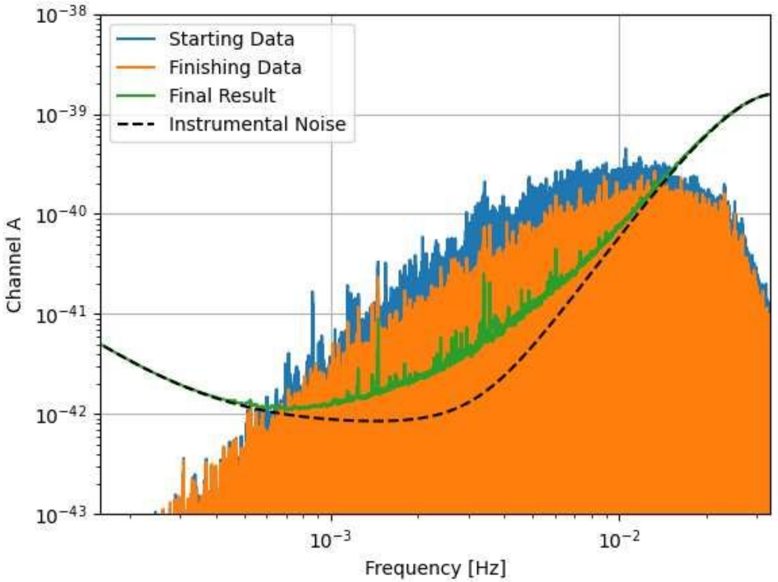
$$S_{n,k} = S_{LISA} + S_{SGWB,k}$$

2. Computation of SNR, ϱ, using Sn,k:

$$\rho = \sqrt{(h_i|h_i)},\ i = A,\ E$$

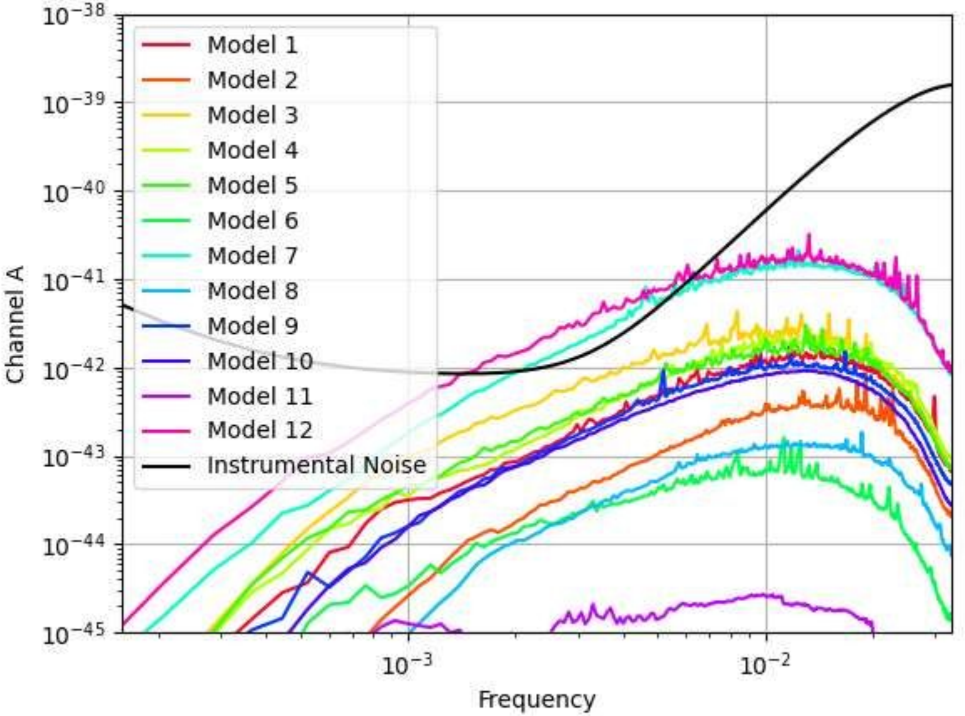
$$(a|b) = 2\int_0^\infty df\,\frac{a(f)\cdot\bar{b}(f) + \bar{a}(f)\cdot b(f)}{S_{n,k}}$$

3. Subtraction of brightest sources with ϱ > 20. Back to point (1) and evaluation of Sn,k+1.



The code stops when there are no more sources removable.

RESULTS



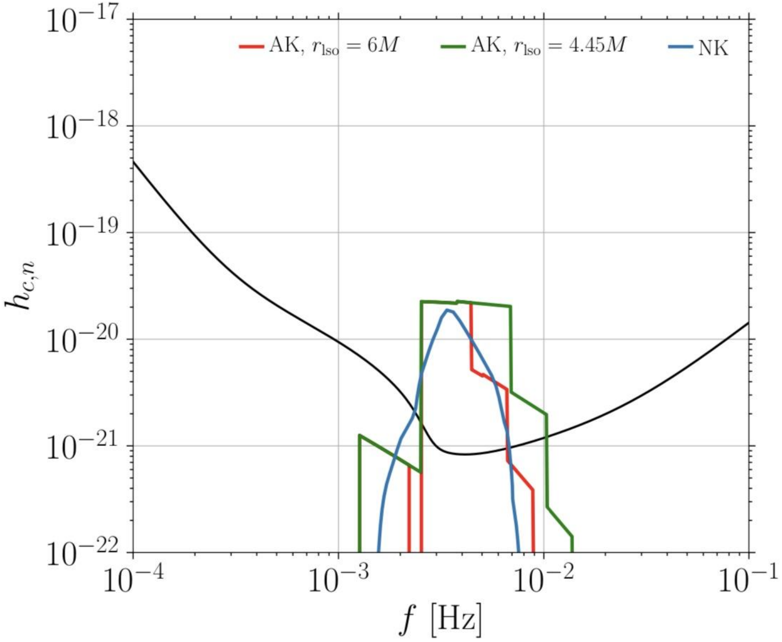
Bonetti et al. formalism

Model	Detections	$\mathrm{SNR}_{\mathrm{GWB}}$	SNR_f
M1	139	180	460
M2	42	40	352
M3	346	441	839
M4	516	235	750
M5	188	252	535
M6	13	21	27
M7	724	980	3960
M8	19	22	51
M9	108	160	419
M10	97	136	408
M11	0	1.44	2.32
M12	891	1146	4553

RESULTS

We underestimate the SNR by a factor of 2-4:

- Waveform Choice
- SNR Computation



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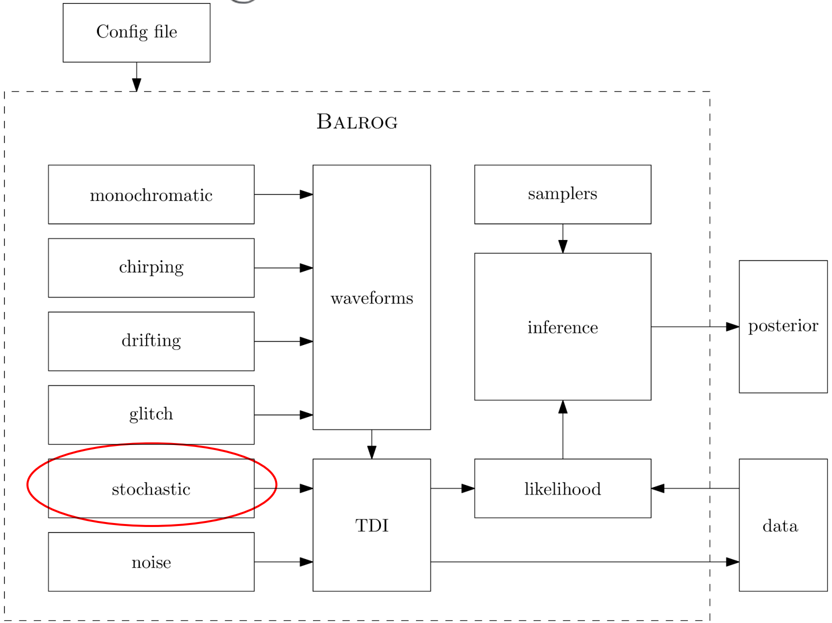
Bonetti et al. (2020) https://arxiv.org/pdf/2007.14403.pdf

CONCLUSIONS

- Realistic realizations of EMRI GWB with state-of-the-art LISA response and waveform.
- All the astrophysical models, except for M11, provide a detectable GWB of EMRIs.
- Some GWB could be foregrounds: M7 and M12 are above LISA noise level in $[10^{-3},10^{-2}]$Hz.

OUTLOOKS



Config file

BALROG

monochromatic

chirping

drifting

glitch

stochastic

noise

waveforms

TDI

samplers

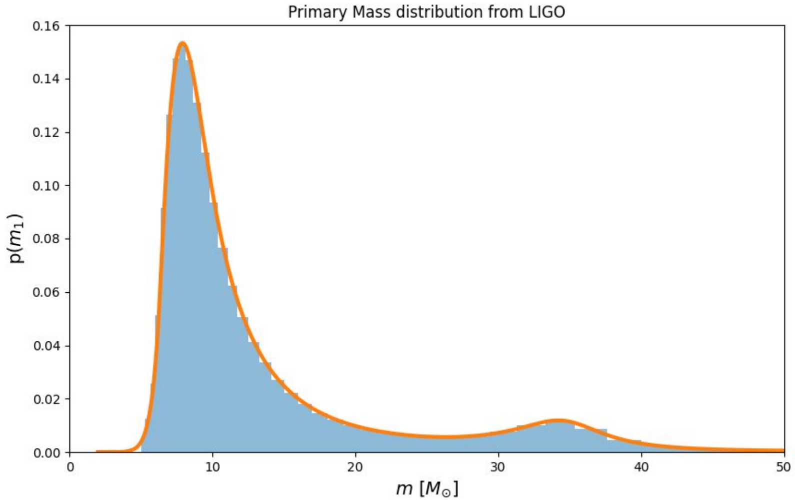
inference

likelihood

posterior

data

Credits: R. Buscicchio, Balrog developers

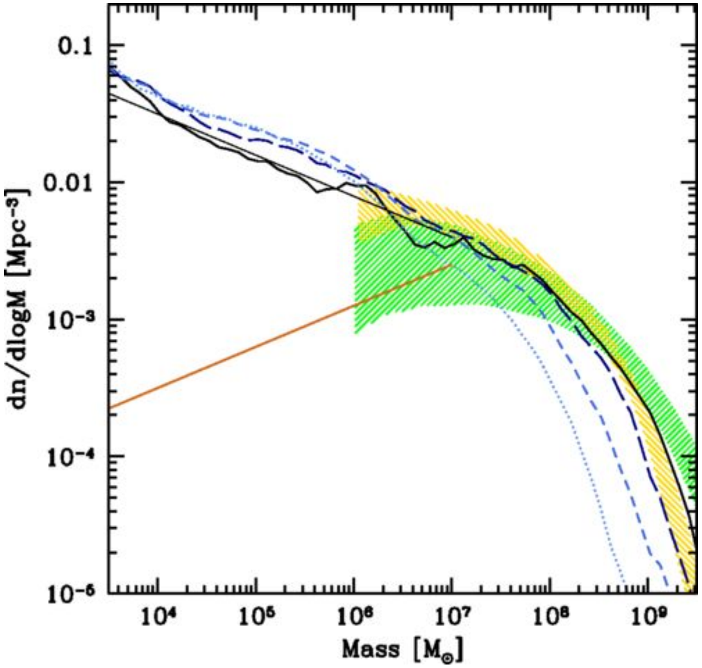


Credits: M. Piarulli

Thank You for the attention!

EMRI CATALOGS - ASTROPHYSICAL POPULATION

$$\boxed{\frac{dN^3}{dMdadz}}$$



Ingredients:

1. MBH mass fuction
   - ‘Barausse12’ self consistent model based on Pop III star formation and evolution
   - ‘Gair10’ phenomenological model
2. Spin Distribution
   - ‘a0’ non spinning MBH
   - ‘aflat’ flat spin distribution in [0,1]
   - ‘a98’ maximally spin distribution with median a=0.98

Babak et al. (2017) https://arxiv.org/pdf/1703.09722.pdf

EMRI CATALOGS - ASTROPHYSICAL POPULATION

$p_0\left(M, z\right)$

- Assuming Poissonian statistics for the mergers.

$$p_o(M, z) \; = \; \exp\left[-N_m(M, z)\right]$$

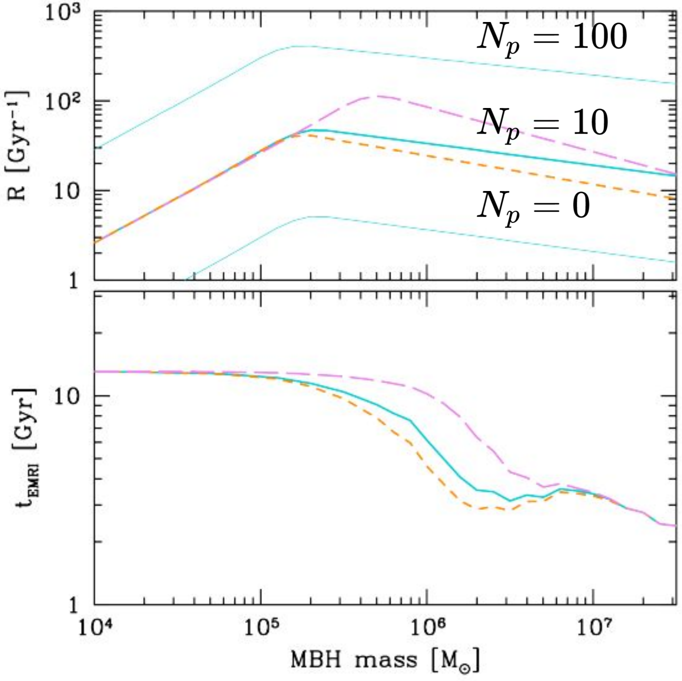
- N_m is the mean number of mergers experienced by an individual MBH of mass M observed at redshift z in its cusp regrowth time.

- Regrowth time depends on M-**σ** relation. $t_{cusp} = 0.25\, t_{relax}$

  - ‘Gultekin09’ —> t_{cusp} = 6Gyr
  - ‘GrahamScott’ —> t_{cusp} = 2Gyr
  - ‘KormendyHo’ —> t_{cusp} = 10Gyr

EMRI CATALOGS - ASTROPHYSICAL POPULATION

$$\boxed{R(M,a)}$$



- Amaro Seoane and Preto (2011):

$$R_0 = 300\left(\frac{M}{10^6 M_\odot}\right)^{-0.19} Gyr^{-1}$$

- The rate has been corrected including the correction for direct plunges.