SGWB anisotropies from inflation with non-Bunch-Davies states

Shingo Akama (Jagiellonian University)

with Shin'ichi Hirano and Shuichiro Yokoyama

LISA Cosmology Working Group Workshop

Non-Bunch-Davies states

- **States which get excited from the Bunch-Davies vacuum one**

(Minkowski vacuum)

$$u_k = \alpha_k u_k^{(\mathrm{BD})} + \beta_k u_k^{*(\mathrm{BD})}, \quad \lim_{\eta \to -\infty} u_k^{(\mathrm{BD})} = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

- **Motivation : Why non-Bunch-Davies?**

- Probe of initial states of perturbations

Must the initial state be the Bunch-Davies one?

- Test of models having dynamical transition

(e.g., inflation with multiple stages, ..)

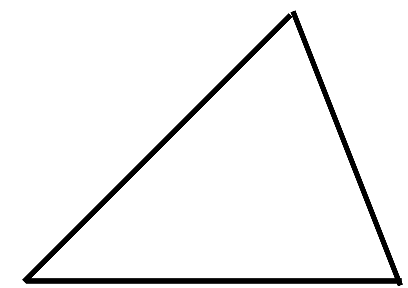
etc.

Primordial non-Gaussianities

Consequences of non-Bunch-Davies states

- Enhanced non-G for folded triangles

X. Chen et al, (2007), R. Holman and A. J. Tolley (2008), S. Kundu (2014), SA et al, (2020), …

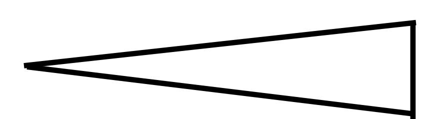


impacts on CMB bispectra

(scalar-scalar-scalar→Holman and Tolley (2008),…
tensor-tensor-tensor→SA and Tahara (2306.XXXX))

- Enhanced non-G for squeezed triangles

X. Chen et al, (2007), R. Holman and A. J. Tolley (2008), S. Kundu (2014), SA et al, (2020), …



impacts on SGWB (this talk)

We focus on tensor non-Gaussianities originating from tensor-tensor-scalar 3-pt function.

Primordial non-Gaussianities

Primordial non-Gaussianities

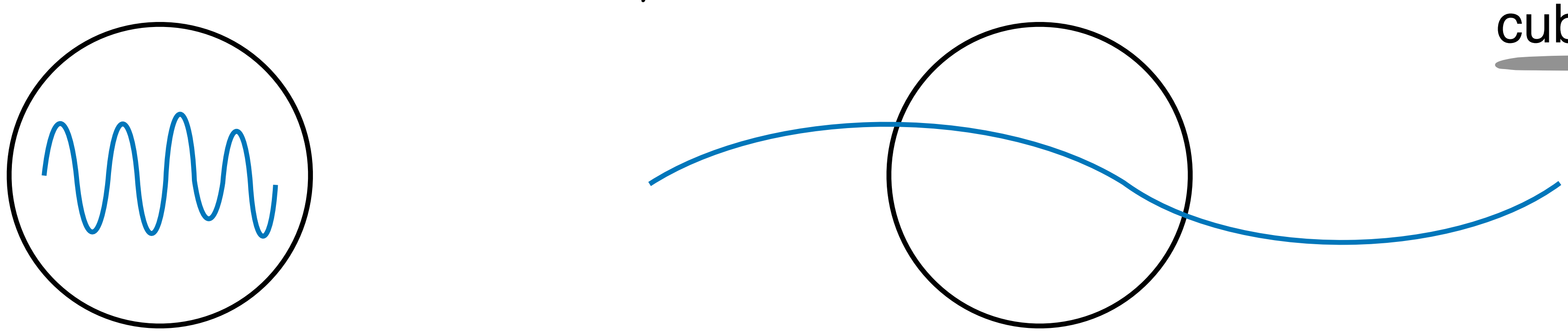
Gaussian: 2-pt function

non-Gaussian: Higher-point function (**3-pt**,..)

Formula for calculating the 3-pt function (in-in formalism)

$$\langle h(k_1)h(k_2)\psi(k_3)\rangle = -i\int_{\eta_0}^{0} \mathrm{d}\eta a(\eta)\langle[h(0,k_1)h(0,k_2)\psi(0,k_3), H_{\rm int}(\eta)]\rangle$$

cubic interaction (tts)

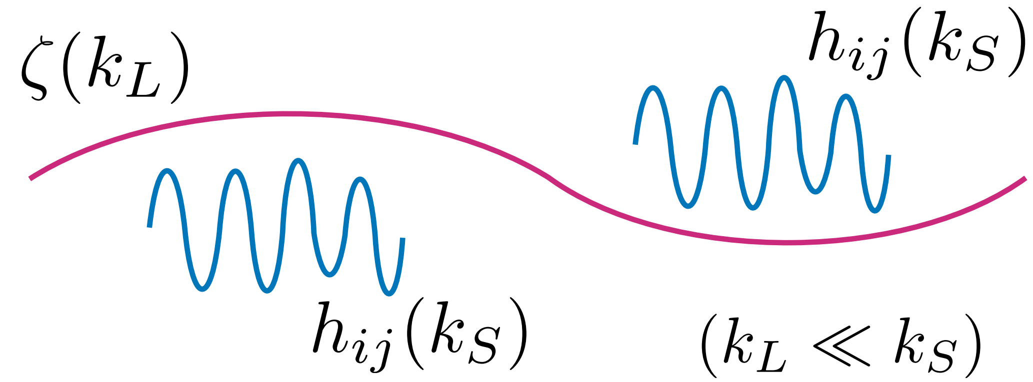


$\eta = \eta_0$ $\eta = 0$

"non-Gaussianities generated due to the cubic interactions

from $\eta = \eta_0$ to $\eta = 0$ (end of inflation)."

Primordial non-Gaussianities



Generation of squeezed non-Gaussianity
(we focus on the scalar-tensor-tensor interaction)

1. Bunch-Davies (BD) state

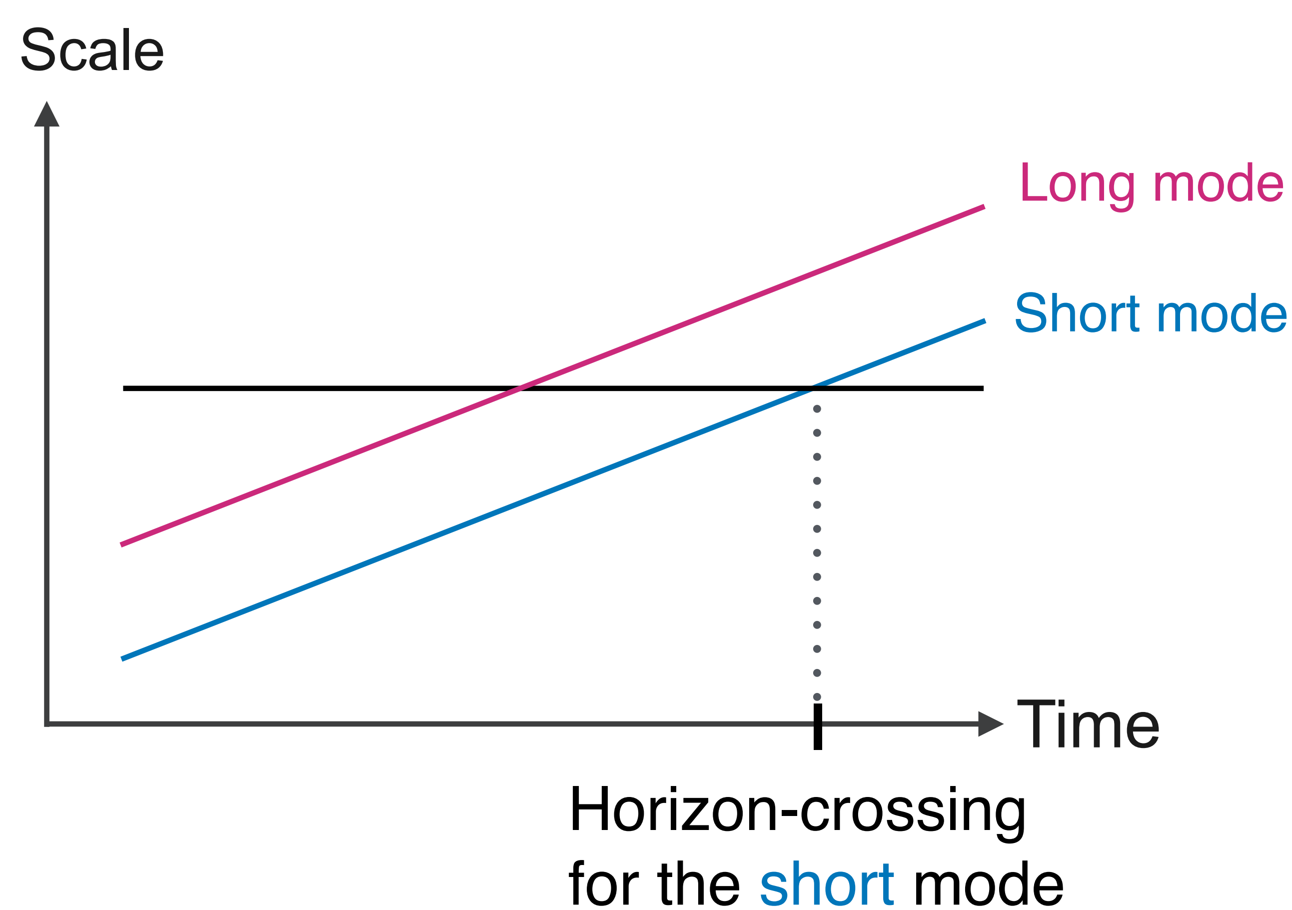
$$\langle h(k_S)h(k_S)\zeta(k_L)\rangle|_{\rm BD}$$
$$\sim \int_{\eta_0}^{0} \mathrm{d}\eta(-\eta)^n e^{-i(k_L+k_S+k_S)\eta}$$

Oscillation stops around $\eta_* \sim -1/k_S$

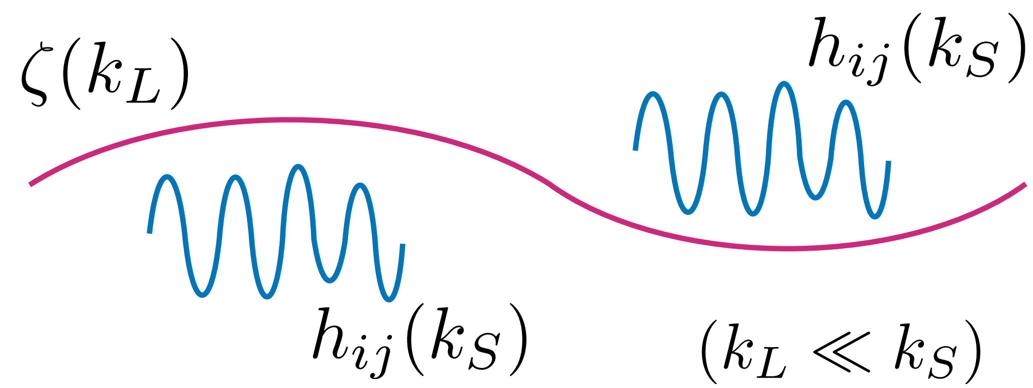
WKB approximation is broken for the short mode.

↓

Non-Gaussianity is generated after the short mode crosses the horizon.



Primordial non-Gaussianities



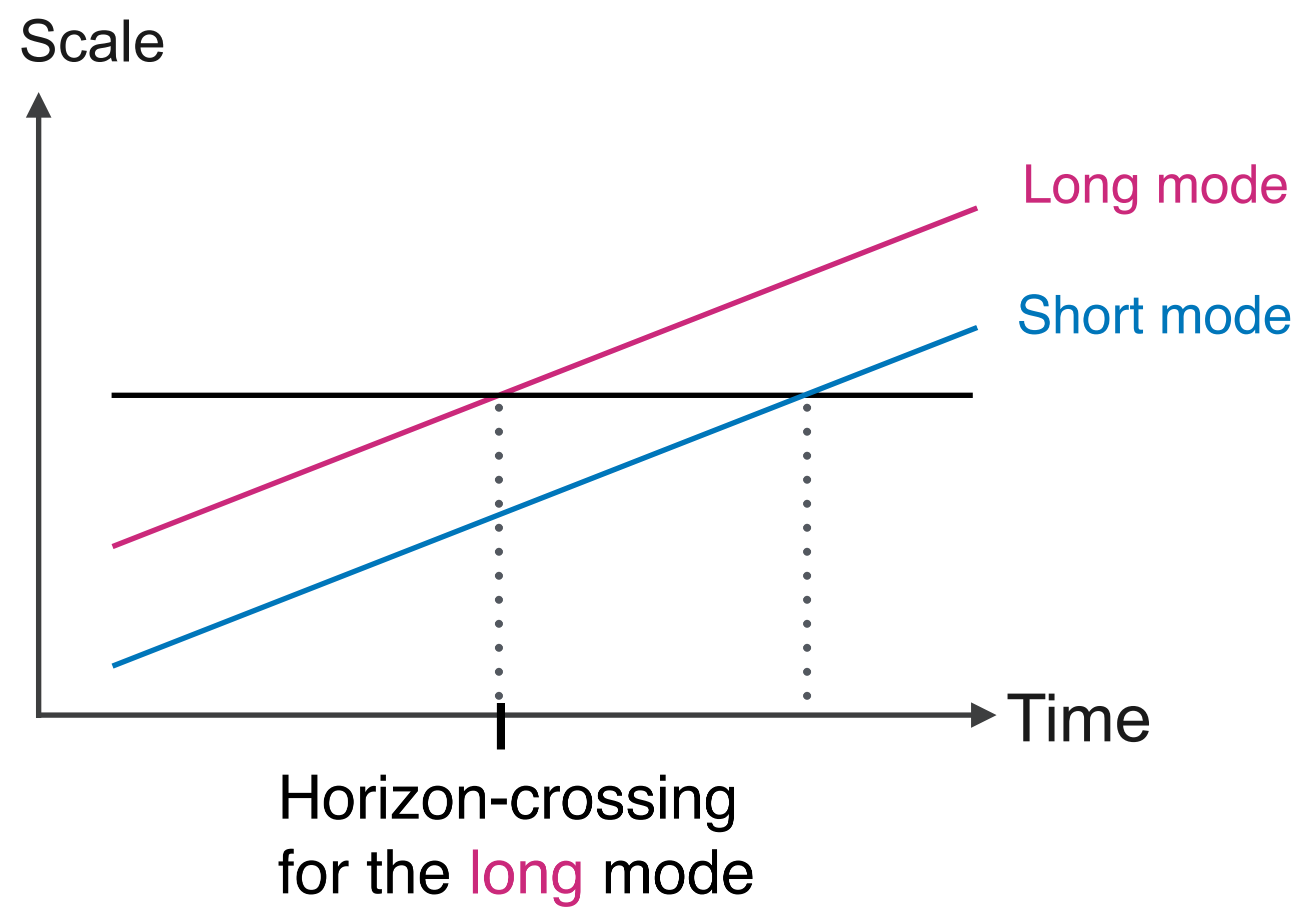
Generation of squeezed non-Gaussianity
(we focus on the scalar-tensor-tensor interaction)

2. Non-Bunch-Davies (NBD) states

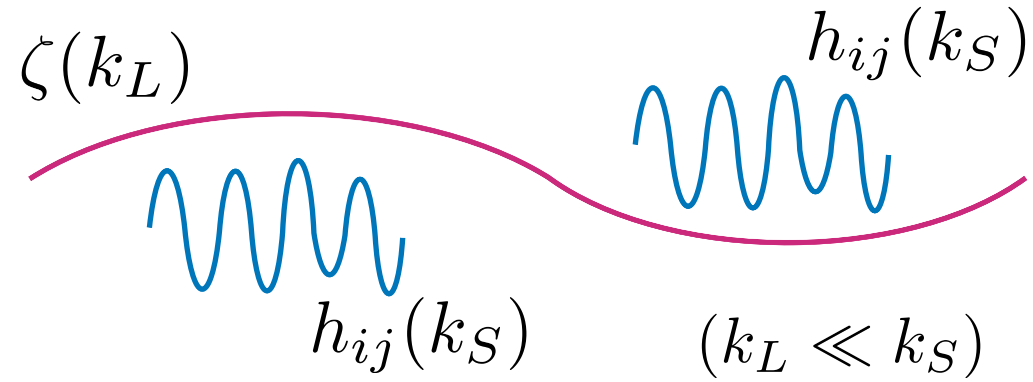
$$\langle h(k_S)h(k_S)\zeta(k_L)\rangle|_{\rm NBD} - \langle h(k_S)h(k_S)\zeta(k_L)\rangle|_{\rm BD} \sim \int_{\eta_0}^{0} {\rm d}\eta(-\eta)^n e^{-i(k_L+k_S-k_S)\eta}$$

consequences of the interaction between the positive and negative frequency modes

$$\sim (\alpha u_{s,{\rm BD}})(\alpha_k^{(s)} u_{h,{\rm BD}}^{(s)})(\beta_k^{(s')} u_{h,{\rm BD}}^{(s')*})$$



Primordial non-Gaussianities



Generation of squeezed non-Gaussianity
(we focus on the scalar-tensor-tensor interaction)

2. Non-Bunch-Davies (NBD) states

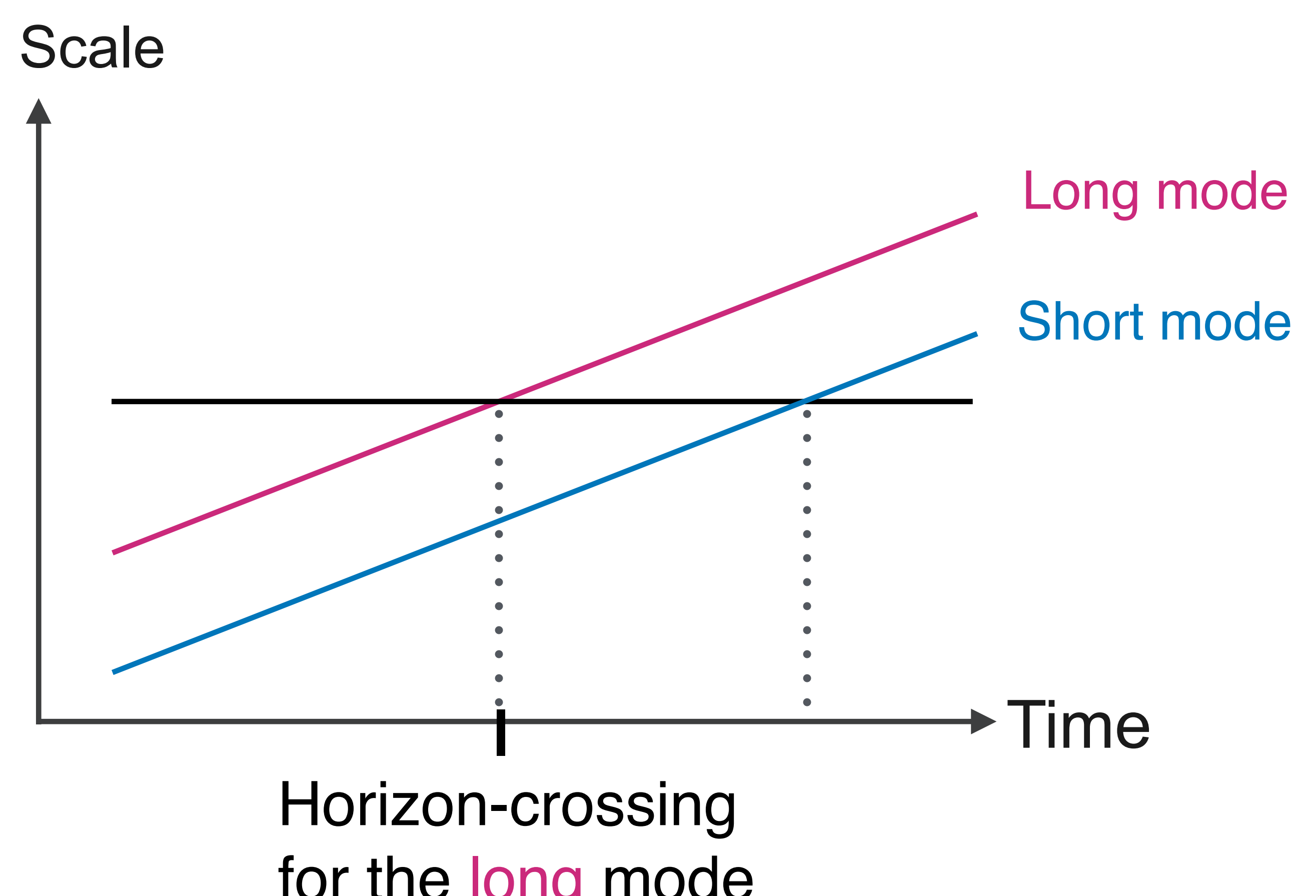
$$\langle h(k_S)h(k_S)\zeta(k_L)\rangle|_{\rm NBD} - \langle h(k_S)h(k_S)\zeta(k_L)\rangle|_{\rm BD} \sim \int_{\eta_0}^{0} d\eta(-\eta)^n e^{-i(k_L+k_S-k_S)\eta}$$

Oscillation stops around $\eta_* \sim -1/k_L$

WKB approximation is broken for the long mode.

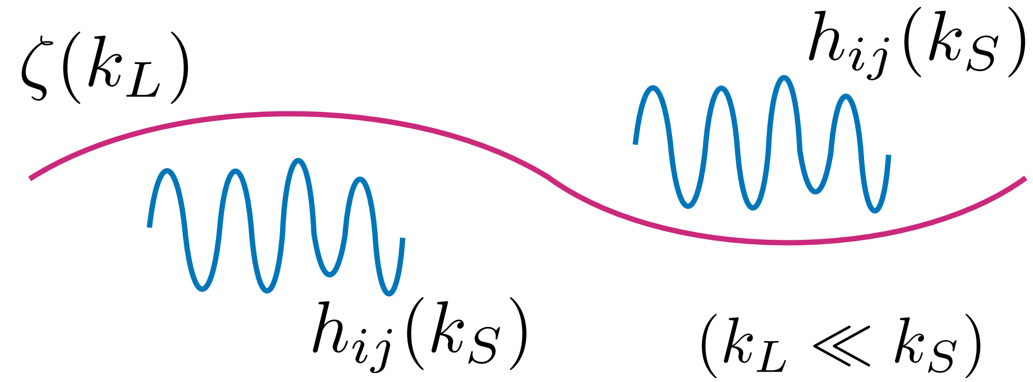
↓

Non-Gaussianity is generated after the long mode crosses the horizon

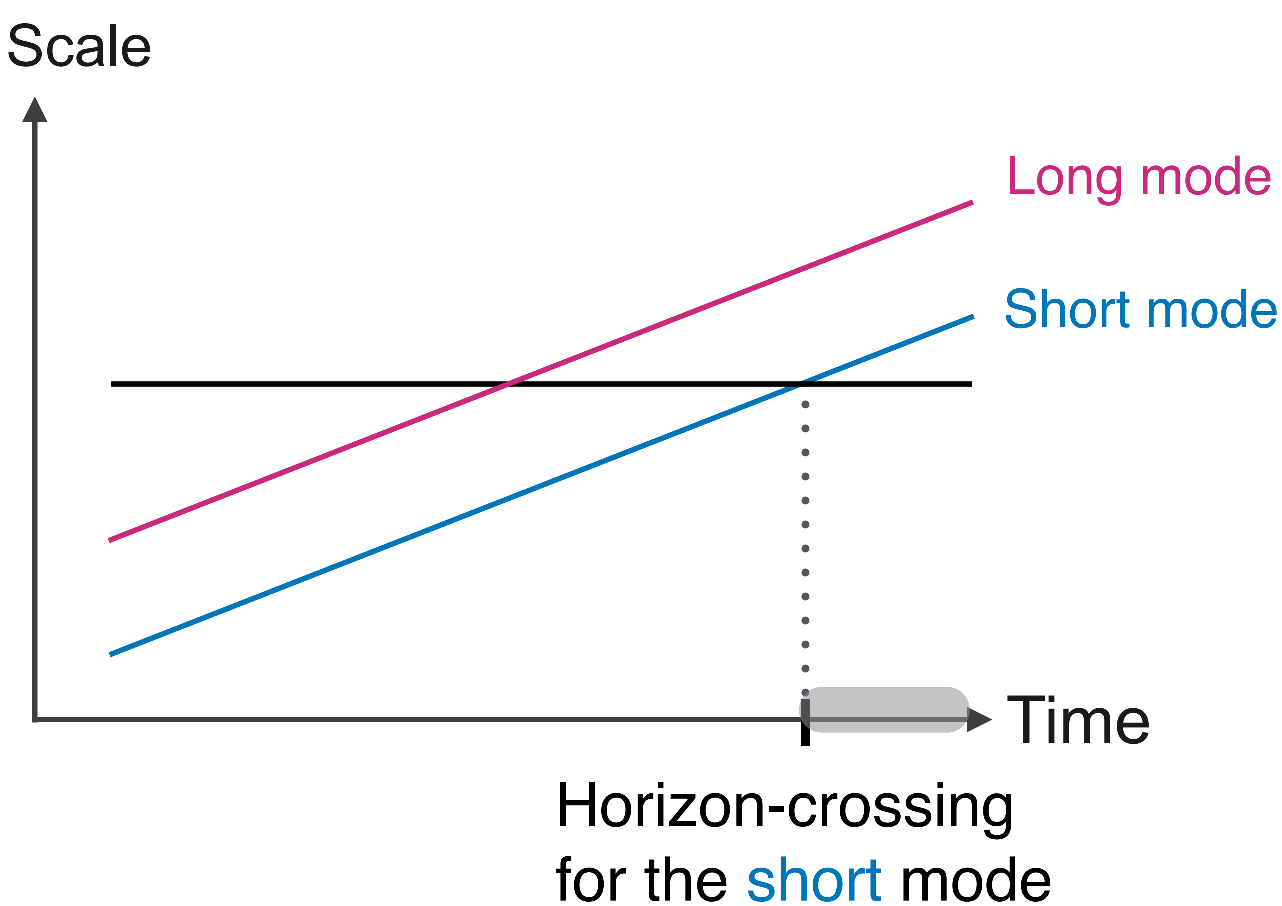


Primordial non-Gaussianities

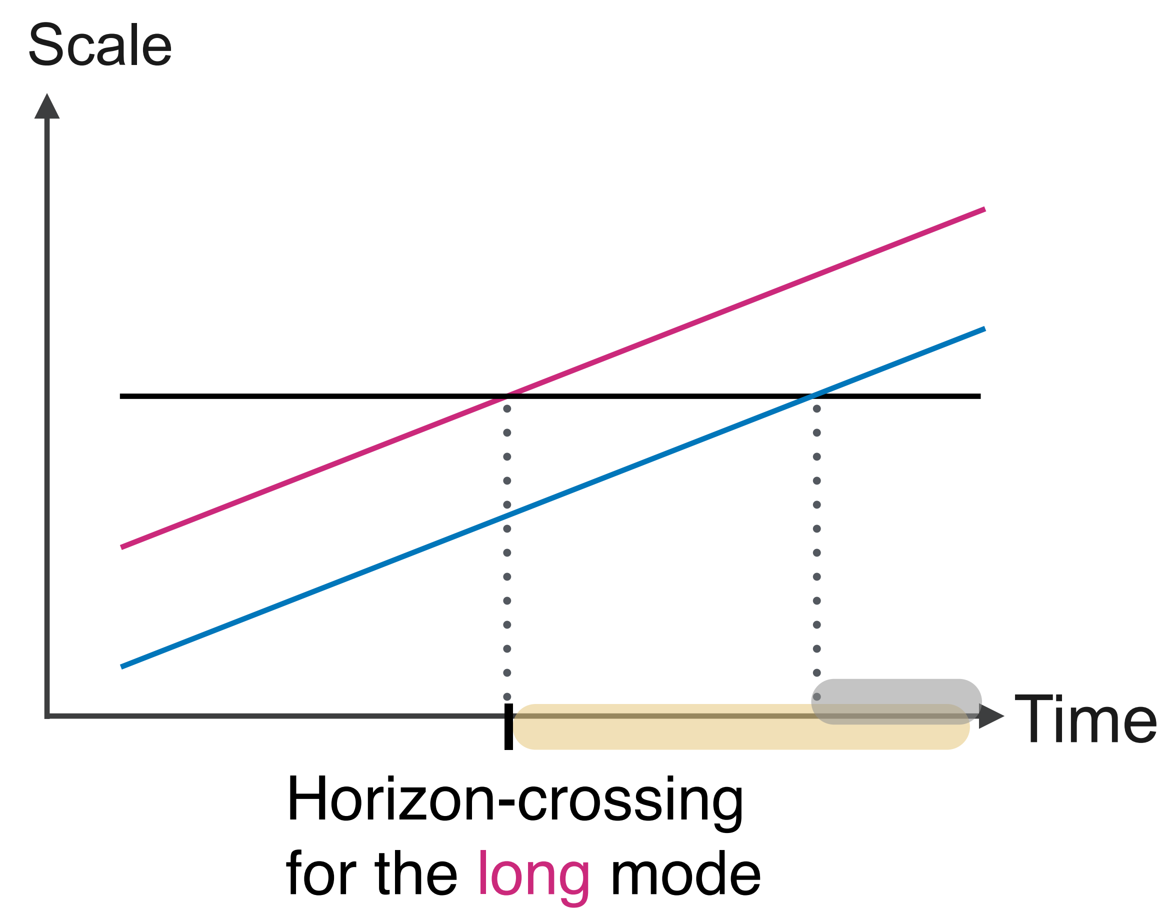
Generation of squeezed non-Gaussianity
(we focus on the scalar-tensor-tensor interaction)



1. Bunch-Davies (BD) state



2. Non-BD state



Additional non-Gaussianities →

- **violation of the consistency relation**
- **enhancement of non-G**

Squeezed non-G in the Horndeski theory

Horndeski theory

Horndeski (1974), Kobayashi et al.(2011)

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right], \quad X := -\frac{1}{2}(\partial\phi)^2$$

→most general 2nd-order field equations

Perturbations

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a^2 e^{2\zeta}(e^h)_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

tensor-tensor-scalar interaction terms

Gao et al. (2012)

$$\mathcal{L}_{shh} = a^3\left[b_1\zeta\dot{h}_{ij}^2 + \frac{b_2}{a^2}\zeta h_{ij,k}h_{ij,k} + b_3\psi_{,k}\dot{h}_{ij}h_{ij,k} + b_4\dot{\zeta}\dot{h}_{ij}^2 + \frac{b_5}{a^2}\partial^2\zeta\dot{h}_{ij}^2\right.$$
$$\left.+b_6\psi_{,ij}\dot{h}_{ik}\dot{h}_{jk} + \frac{b_7}{a^2}\zeta_{,ij}\dot{h}_{ik}\dot{h}_{jk}\right] + E_{shh},$$

Squeezed non-G in the Horndeski theory

Primordial bispectrum from inflation with non-BD (Preliminary)

$$\mathcal{L}_{shh} = a^3 \left[b_1 \zeta \dot{h}_{ij}^2 + \frac{b_2}{a^2} \zeta h_{ij,k} h_{ij,k} + b_3 \psi_{,k} \dot{h}_{ij} h_{ij,k} + b_4 \dot{\zeta} \dot{h}_{ij}^2 + \frac{b_5}{a^2} \partial^2 \zeta \dot{h}_{ij}^2 + b_6 \psi_{,ij} \dot{h}_{ik} \dot{h}_{jk} + \frac{b_7}{a^2} \zeta_{,ij} \dot{h}_{ik} \dot{h}_{jk} \right] + E_{shh},$$

$$\mathcal{B}_{shh}^{s_2 s_3} = \frac{4}{k_1^3 k_2^3 k_3^3} \frac{H^6}{\mathcal{F}_S \mathcal{F}_T^2 c_s c_h^2} b_5 \mathcal{V}_{s_2 s_3}^{(5)} \mathcal{I}^{(5)}, \qquad (3)$$

where

$$\mathcal{V}_{s_2 s_3}^{(5)} := \frac{k_1^2}{16 k_2^2 k_3^2} [k_1^2 - (s_2 k_2 + s_3 k_3)^2]^2, \qquad (4)$$

$$\mathcal{I}^{(5)} := 2 c_h^4 k_2^2 k_3^2 \left[\frac{3 c_s k_1 + K'}{K'^4} - \frac{3 c_s k_1 + \tilde{K}'}{\tilde{K}'^4} \mathrm{Re}[\beta_{k_2}^{(s_2)}] - \frac{3 c_s k_1 + \tilde{K}''}{\tilde{K}''^4} \mathrm{Re}[\beta_{k_3}^{(s_3)}] \right], \qquad (5)$$

with

$$K' := c_s k_1 + c_h (k_2 + k_3), \qquad (6)$$

$$\tilde{K}' := c_s k_1 - c_h (k_2 - k_3), \qquad (7)$$

$$\tilde{K}'' := c_s k_1 - c_h (-k_2 + k_3). \qquad (8)$$

Enhanced at the squeezed limit:

$$k_1 \ll k_2 = k_3$$

$\mathcal{O}\left(\left(\frac{k_{GW}}{k_{CMB}} \right)^n \right)$ **-enhancement**

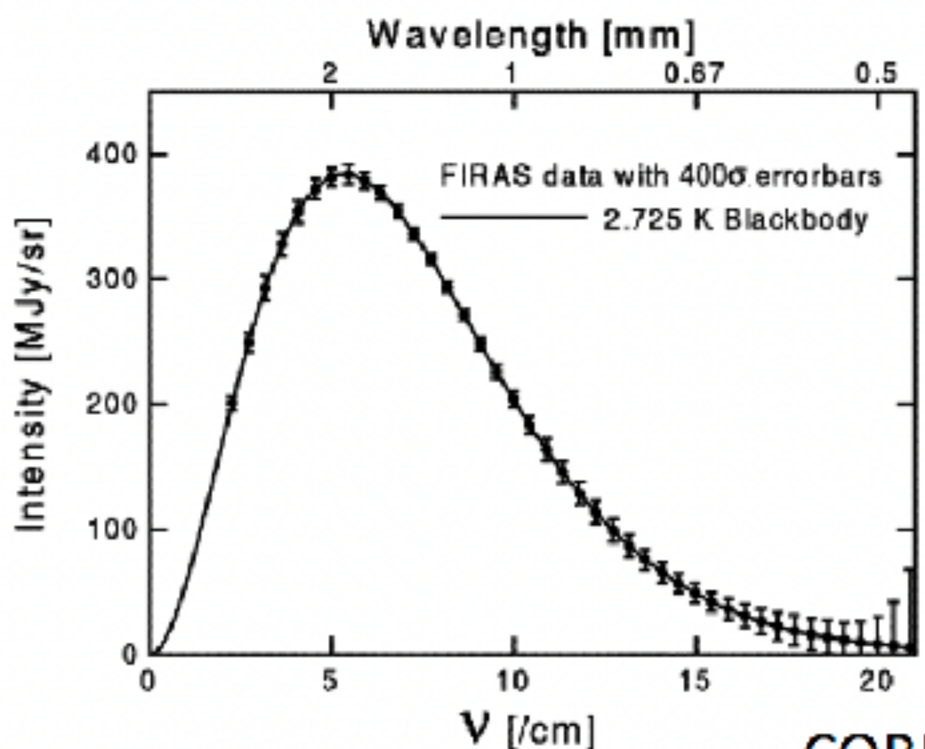
Impacts on SGWB anisotropies

SGWB anisotropies

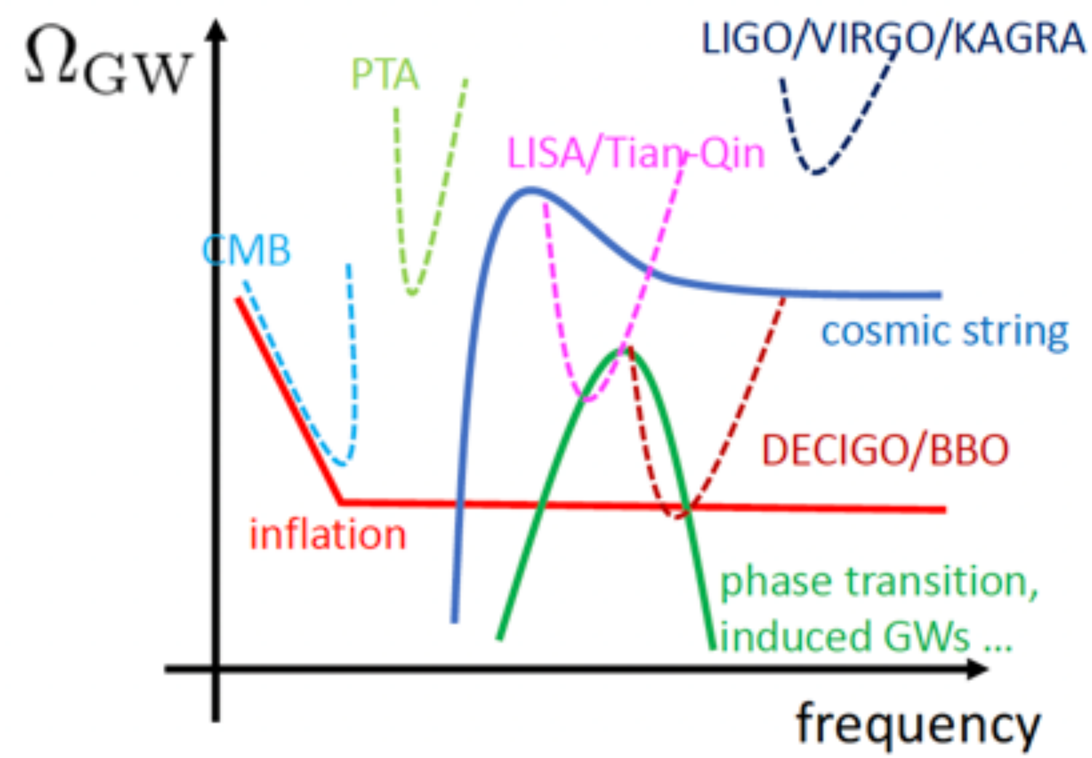
Cosmic Microwave background (CMB)

GW Backgrounds (GWB)

CMB black body spectrum ⟷ **SGWB spectrum**



COBE/FIRAS



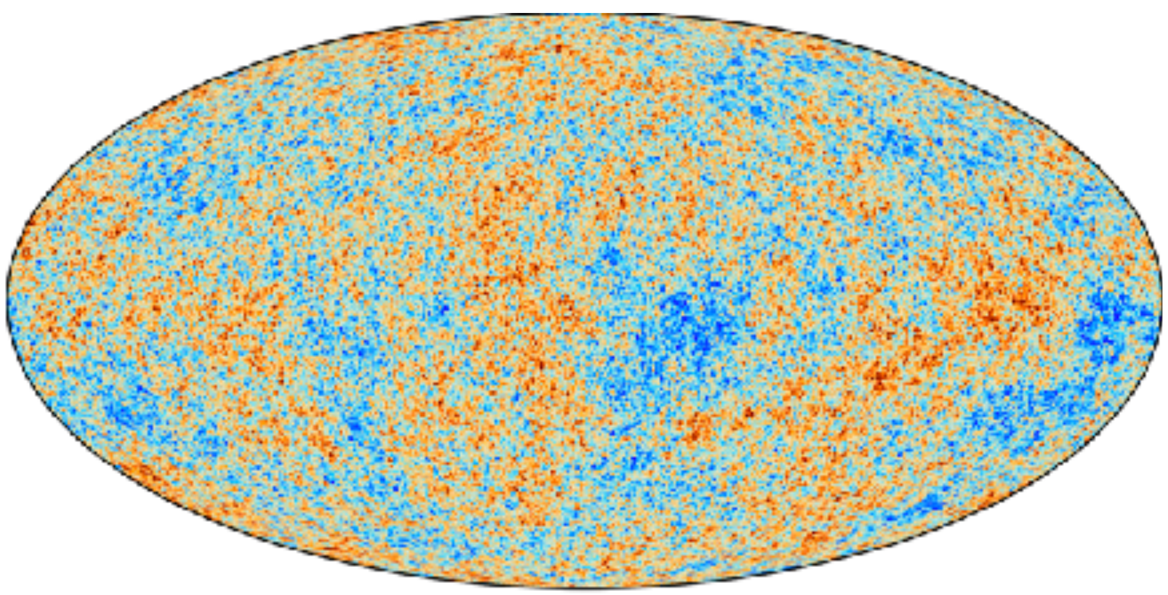
"cosmological" stochastic GWB

SGWB anisotropies

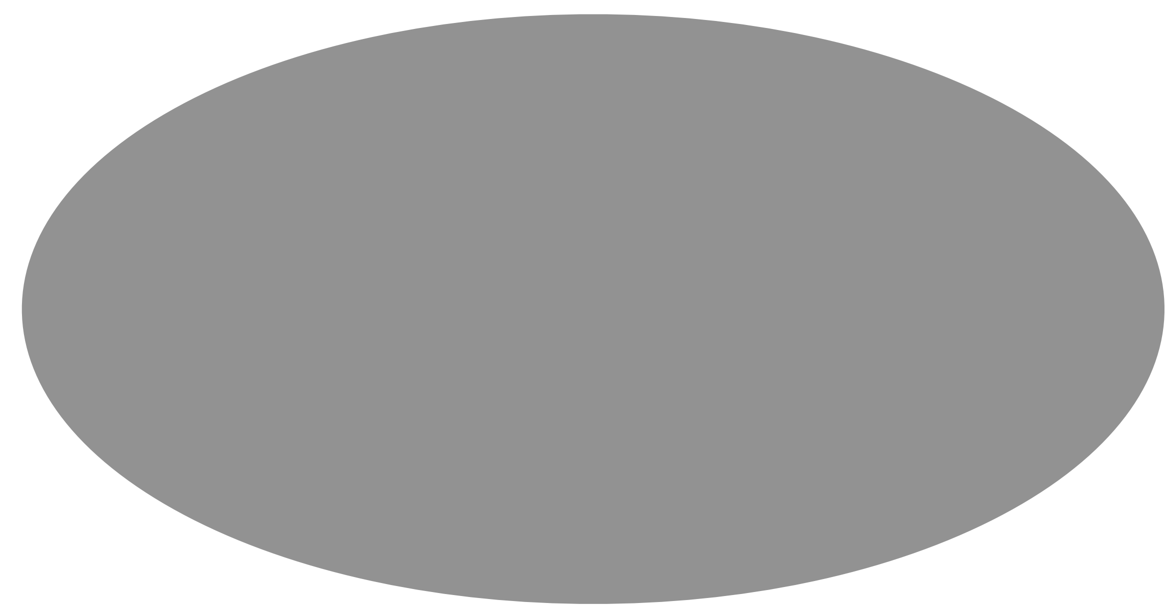
Cosmic Microwave background (CMB)

Stochastic GW Backgrounds (SGWB)

CMB anisotropies ⟷ **SGWB anisotropies**



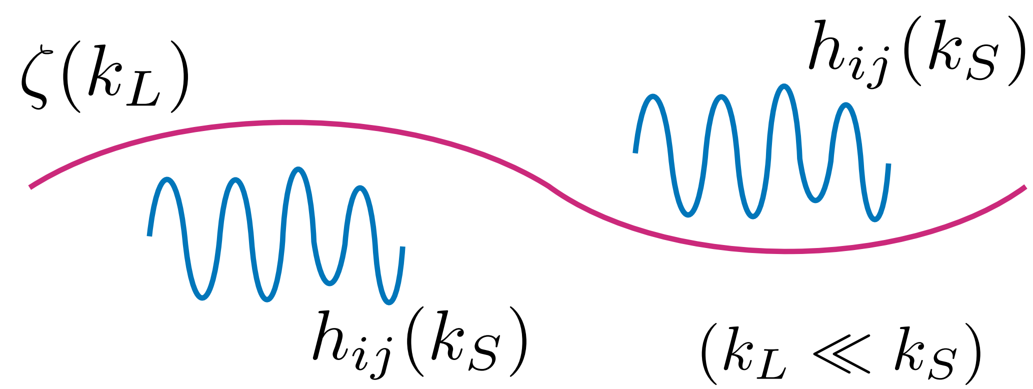
©ESA Planck team



SGWB anisotropies

1. Primordial non-Gaussianity

Bartolo et al, (2019), Malhotra et al, (2020), Orland (2022), …



Power spectrum of short-wavelength modes
can fluctuate due to the long-wavelength mode:

$$\mathcal{P}_h(\vec{k}, \vec{x}) = \mathcal{P}_h(k)\left[1 + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} F_{\rm NL}^{\lambda\lambda{\rm tts}}(\vec{k}, \vec{q})\zeta(\vec{q})\right]$$

where $F_{\rm NL}^{\rm tts}(\vec{k}, \vec{q}) := \dfrac{\mathcal{B}^{\lambda\lambda}_{tts}(\vec{k} - \vec{q}/2, -\vec{k} - \vec{q}/2, \vec{q})|_{q\to 0}}{P_h(k)P_s(q)}$

$$(\vec{x} = d\hat{n})$$

$$d = \eta_0 - \eta_{\rm entry} \simeq \eta_0$$

SGWB anisotropies

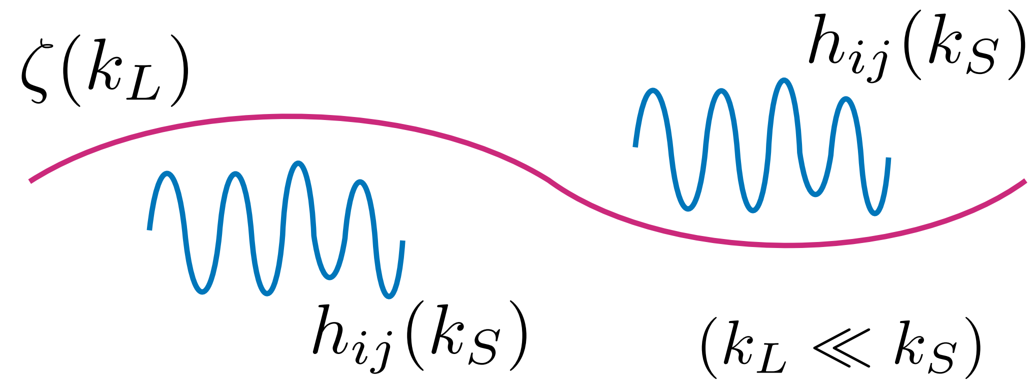
$$\Omega_{\rm GW}(\vec{k}, \hat{n}) = \Omega_{\rm GW}(k)\left[1 + \delta_{\rm GW}(k, \hat{n})\right]$$

SGWB anisotropies

1. Primordial non-Gaussianity

Bartolo et al, (2019), Malhotra et al, (2020), Orland (2022), …

Power spectrum of short-wavelength modes can fluctuate due to the long-wavelength mode:



SGWB anisotropies

$$\Omega_{\rm GW}(\vec{k}, \hat{n}) = \Omega_{\rm GW}(k)\left[1 + \delta_{\rm GW}(k, \hat{n})\right]$$

2. "Late-time" metric perturbations (SW, ISW,..)

Bartolo et al, (2019), Malhotra et al, (2020)

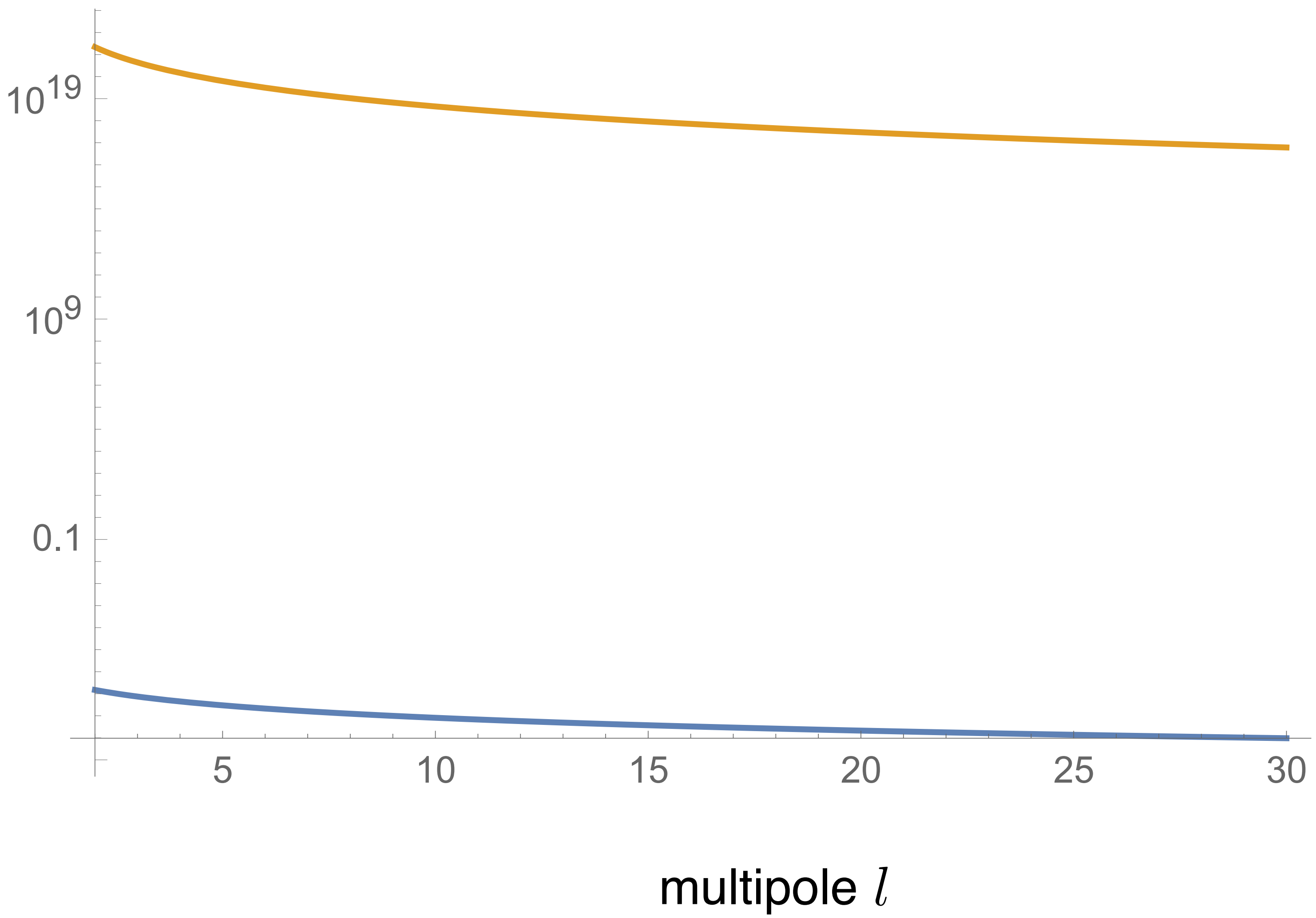
distribution function of graviton changes during its propagation in the presence of the metric perturbations

(c.f. distribution func. of photon changes in the same way)

SGWB anisotropies

Power spectrum $C_l = \frac{1}{2l+1}\sum_m \langle \delta_{lm}^{\mathrm{GW}} \delta_{lm}^{\mathrm{GW}*} \rangle$

Preliminary result



Roughly.. $C_l \sim f_{\mathrm{NL}} \times \mathcal{O}\left(\frac{k_{\mathrm{GW}}}{k_{\mathrm{CMB}}}\right)$

$(f_{\mathrm{NL}} \propto \mathcal{P}_h b_i \beta_{k_S}^{(s)})$

SGWB anisotropies

$f_{\rm NL}$ **must be tiny..?** $(f_{\rm NL} \propto \mathcal{P}_h b_i \beta^{(s)}_{k_S})$ b_i : coefficients in the cubic actions

1. Perturbativity

$$\mathcal{L}^{(2)}_h > \mathcal{L}^{(3)}_{shh} > \ldots$$

2. Backreaction constraint SA et al. (2020)

$$\rho_{\rm excited} \sim \frac{c_h}{a^4} \int^{k_S} |\beta^{(s)}_k|^2 k^3 {\rm d}^3 k < \rho_{\rm inf} \sim M_{\rm pl}^2 H^2$$

$$\longrightarrow \; f_{\rm NL} \ll 1$$

$$\longrightarrow \; C_l^{\rm GW} \ll C_l^{SW}$$

Conclusion

- **Squeezed non-G is enhanced in the presence of the non-Bunch-Davies modes**

$$\langle hh\zeta\rangle \propto \left(\frac{k_S}{k_L}\right)^n$$

- **SGWB anisotropies receive enhancements:**

$C_l \propto \mathcal{O}\left(\frac{k_{\rm GW}}{k_{\rm CMB}}\right)$ (in the Horndeski theory)

- **Perturbativity and Backreaction constraints suppress the amplitudes of the anisotropies**

Future direction

- enhanced non-G is generated when the perturbations are on the subhorizon scales
- higher-derivative cubic operators are more important
- $C_l \propto \mathcal{O}\left(\left(\frac{k_{\rm GW}}{k_{\rm CMB}}\right)^n\right), (n > 1)$ in more extended theories?