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Primordial Gravitational Waves in non-Minimally Coupled Chromo- Natural Inflation

Martino Michelotti

University of Groningen, Van Swinderen Institute

Based on: E. Dimastrogiovanni, M. Fasiello, **MM**, L. Pinol, 2023, *arXiv:2303.10718*

Outline

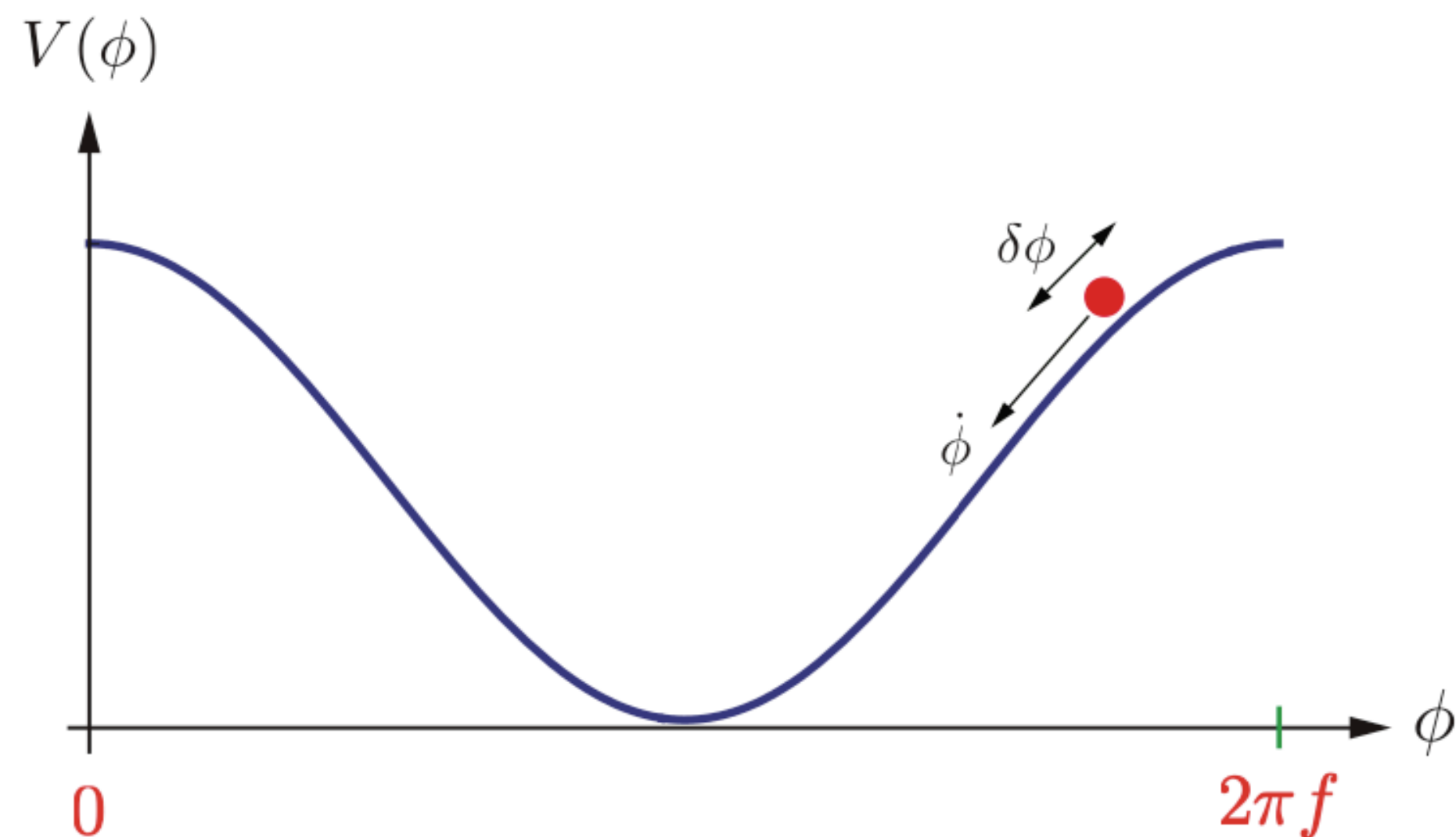
- * Motivations
- * Background and perturbations evolution
- * CMB observables
- * Sourced background of primordial GWs

Motivations

Natural inflation

Freese, Frieman, Olinto, 1990

shift symmetry protects from
large quantum corrections ✓



Motivations

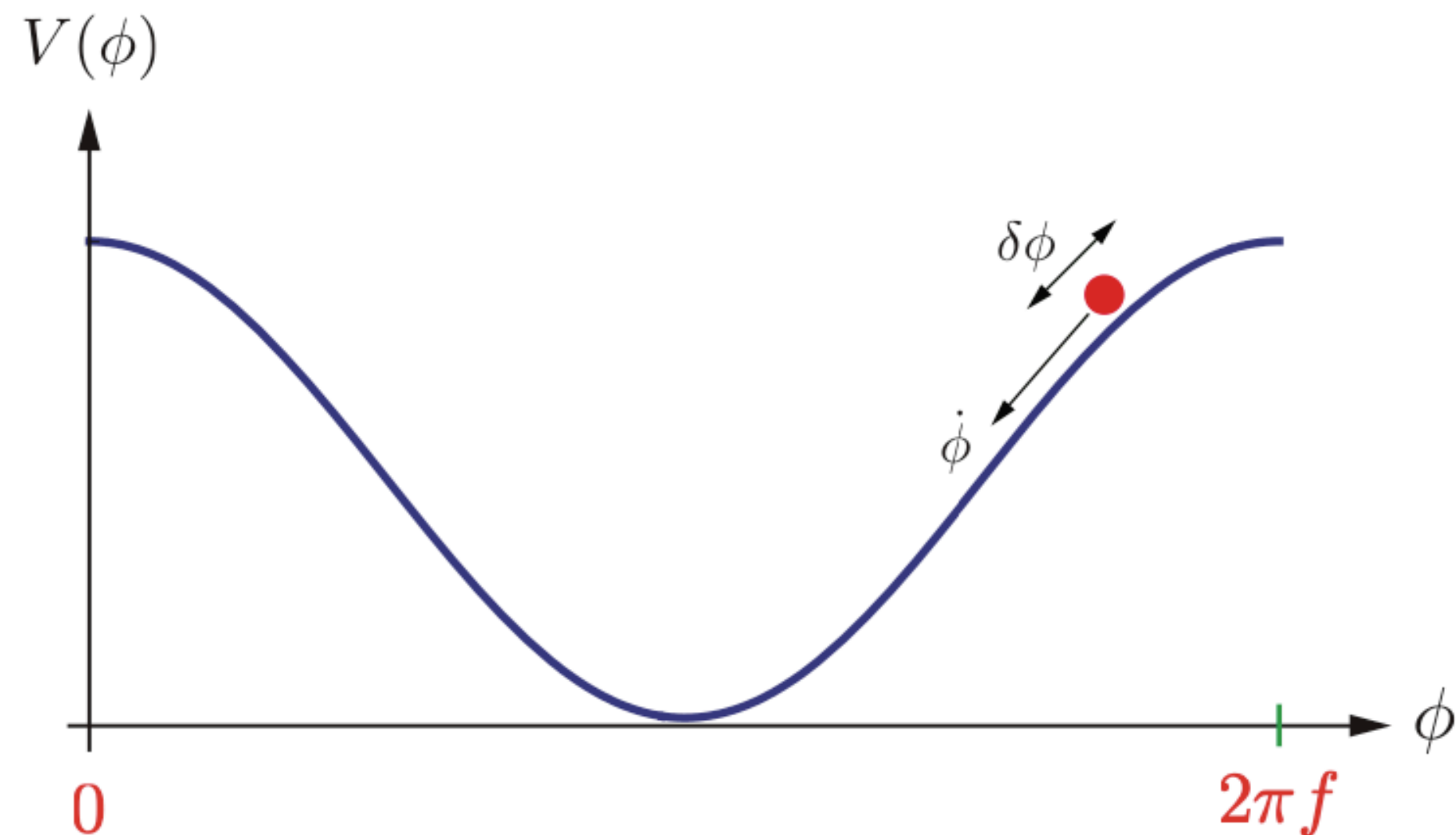
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tension with CMB measurements ✗



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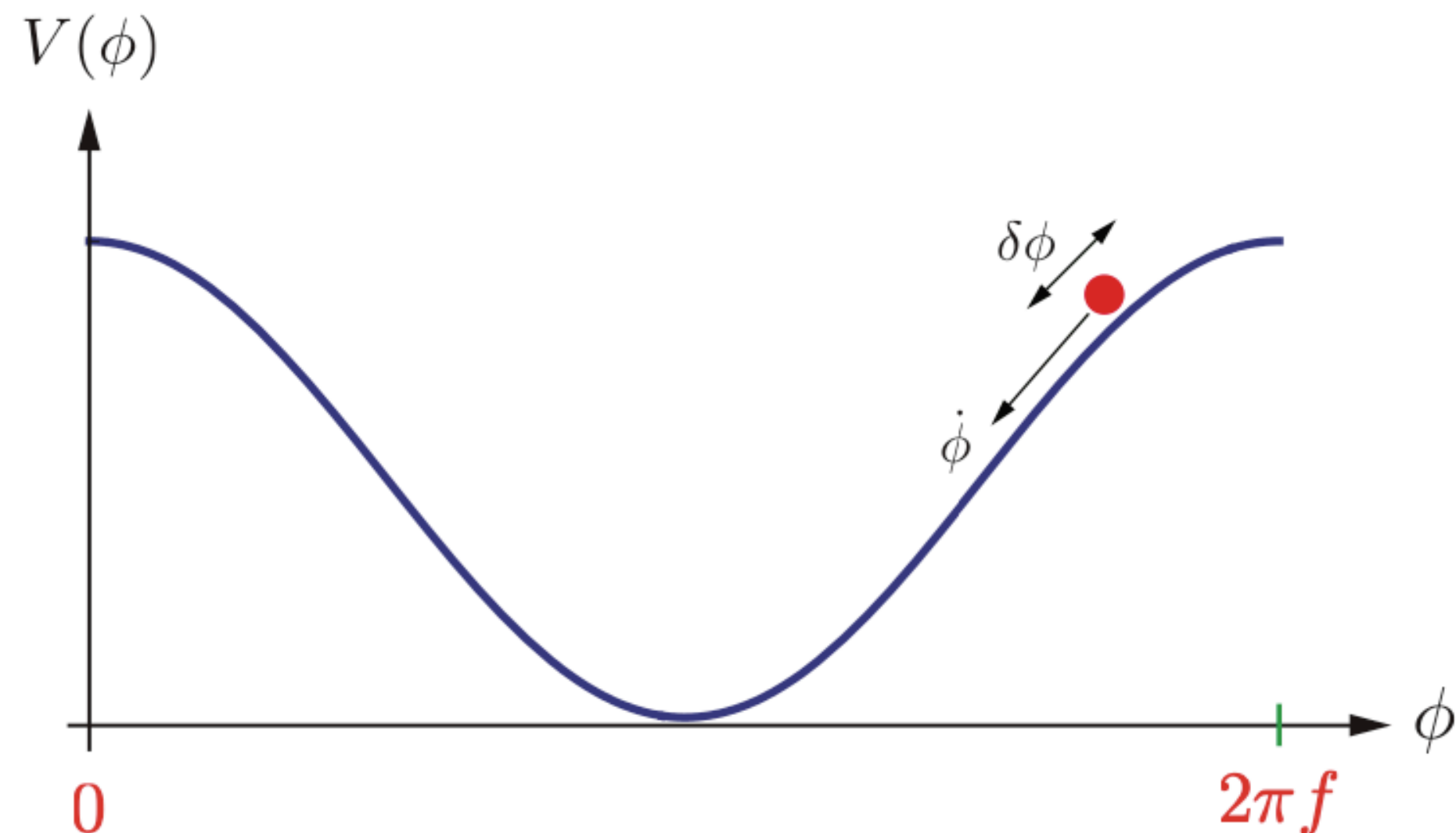
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D. Baumann, 2009



additional friction from gauge fields

Anber, Sorbo, 2010

Chromo-natural inflation

Adshead, Wyman, 2012

blue-tilted and chiral GW signal



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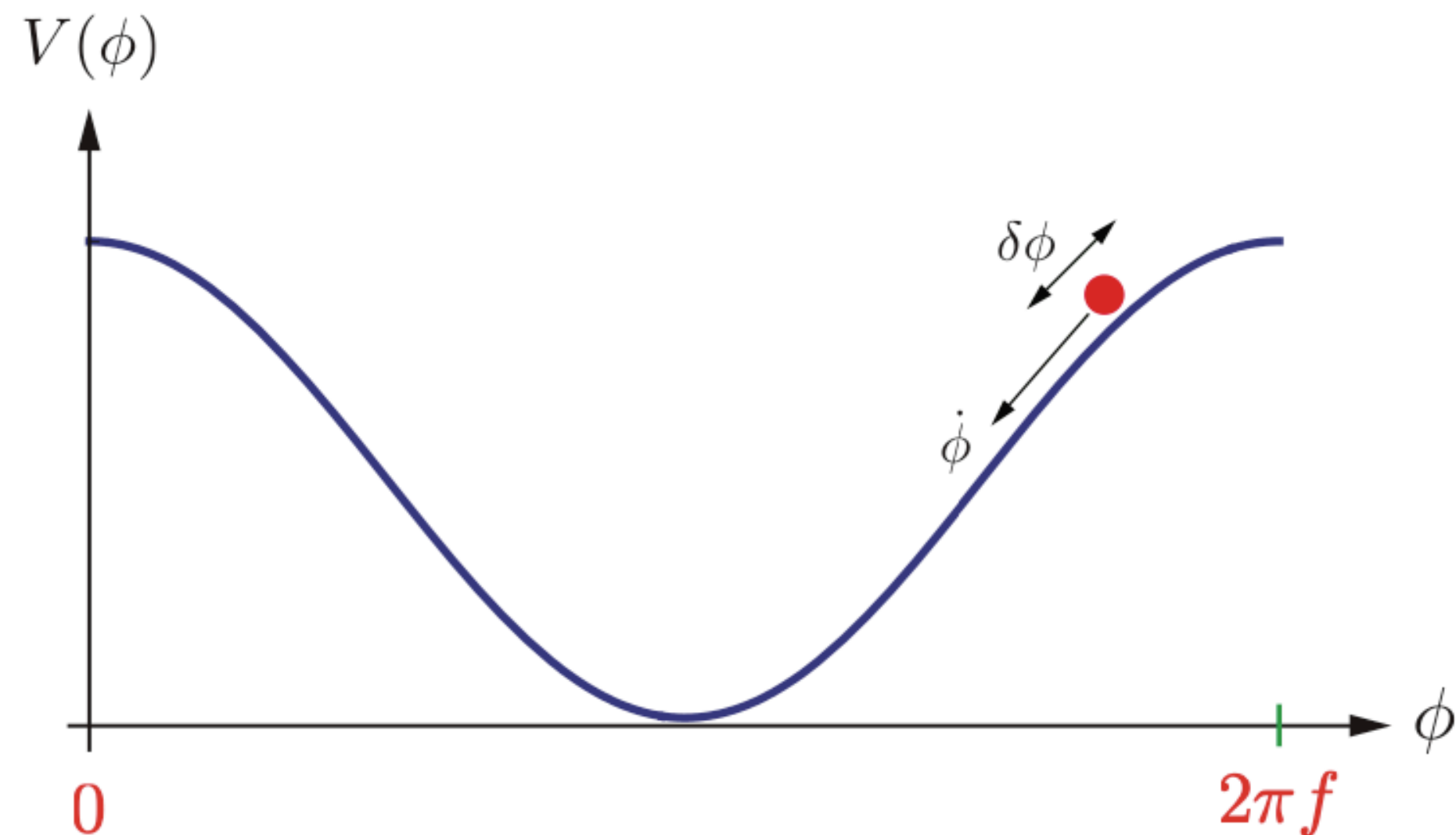
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GW overproduction

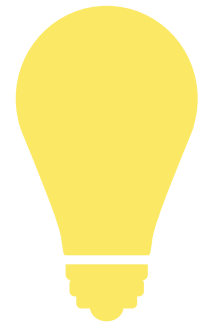


further step is needed



See Matteo's talk...

The model



Idea: non-minimal coupling with the Einstein tensor

first introduced in UV-protected natural inflation

Germani, Kehagias, 2011

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc} A_\mu^b A_\nu^c$$

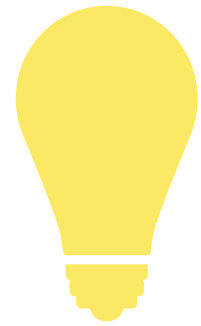
$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \partial_\mu \chi \partial_\nu \chi - V(\chi) - \frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a + \frac{\lambda\chi}{8f\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a$$

$$V(\chi) = \mu^4 \left(1 + \cos \frac{\chi}{f} \right)$$

Chern-Simons coupling

M^{-2} : strength of non-minimal coupling $\longleftrightarrow M \rightarrow \infty$: chromo-natural limit

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\longrightarrow additional friction unrelated to GW production



Background dynamics

Axion background: $\chi(t)$

gauge friction

$$\left(1 + \frac{3H^2}{M^2}\right) \ddot{\chi} + 3H \left(1 + \frac{3H^2}{M^2}\right) \dot{\chi} + V'(\chi) = -\frac{3g\lambda}{f} H Q^3$$

→ additional friction from the non-minimal coupling if $\frac{H}{M} \gg 1$

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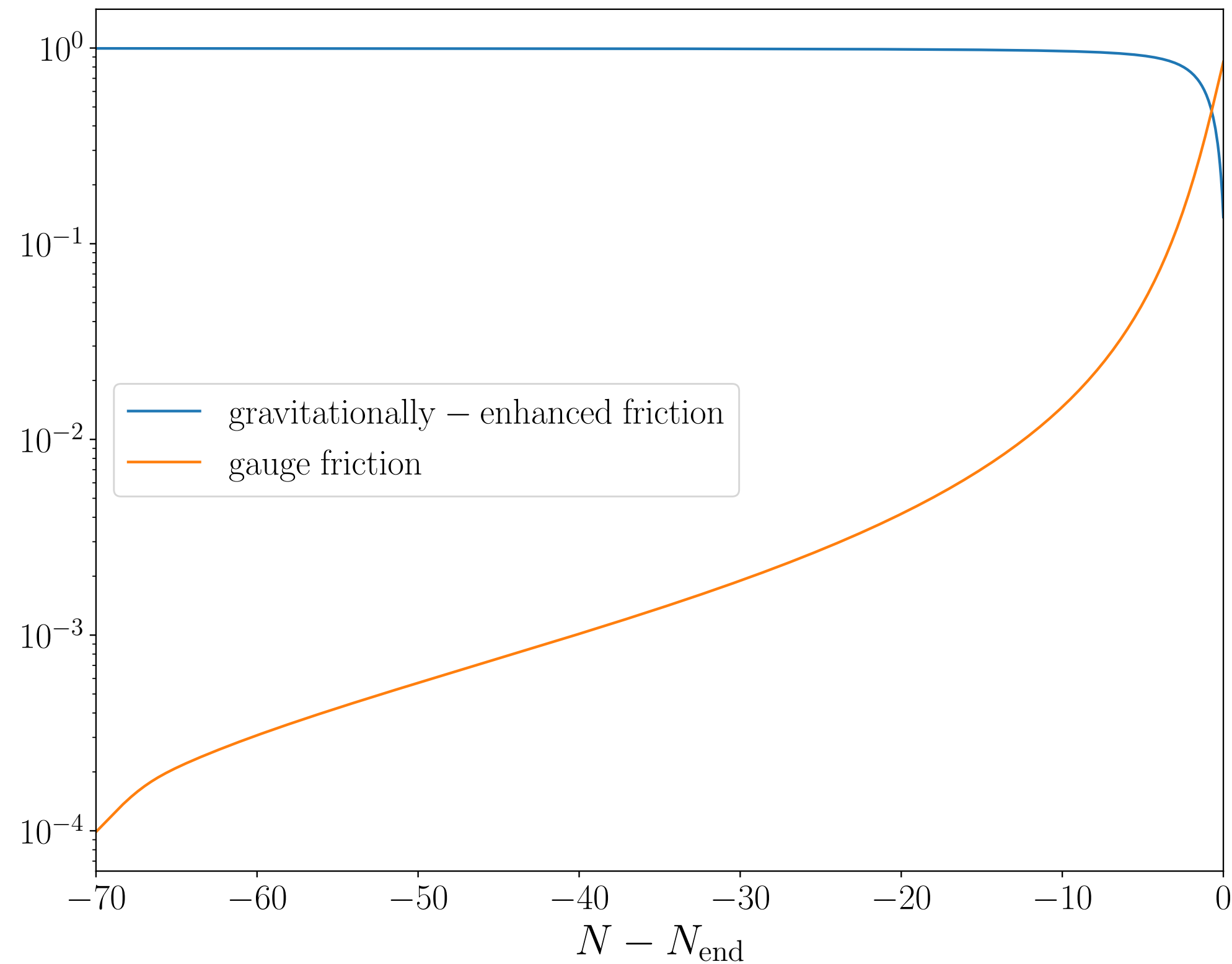
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Gauge field background: $A_0^a = 0, \quad A_i^a = \delta_i^a a(t) Q(t)$

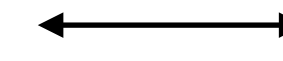
$$\ddot{Q} + 3H\dot{Q} + 2H^2Q + 2g^2Q^3 = \frac{g\lambda}{f} \dot{\chi} Q^2$$

Gravitationally-enhanced friction



In the limit $\frac{H}{M} \gg 1$

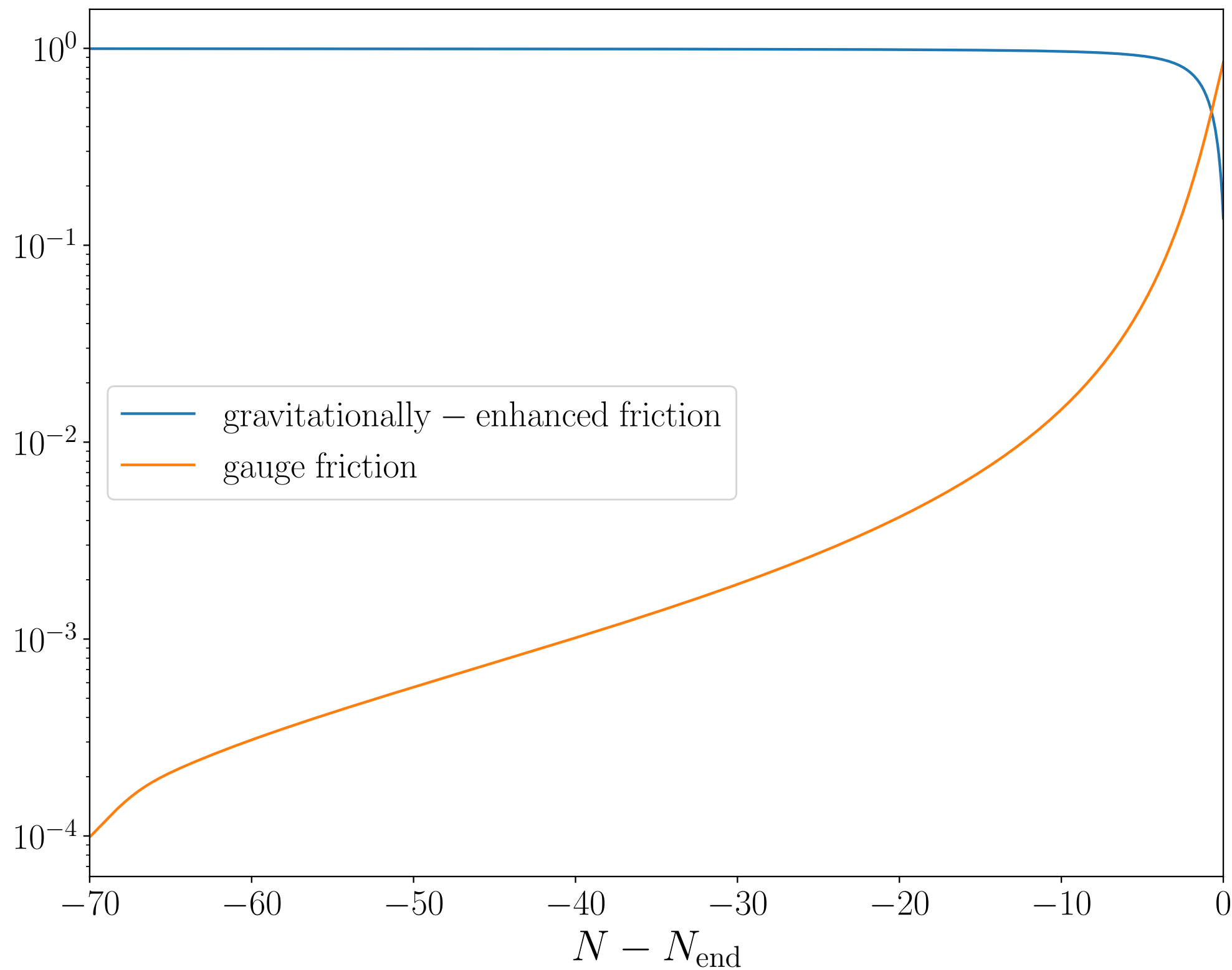
Gravitationally-enhanced friction dominating at large scales



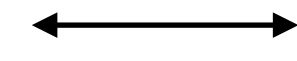
Gauge friction increasing at later times

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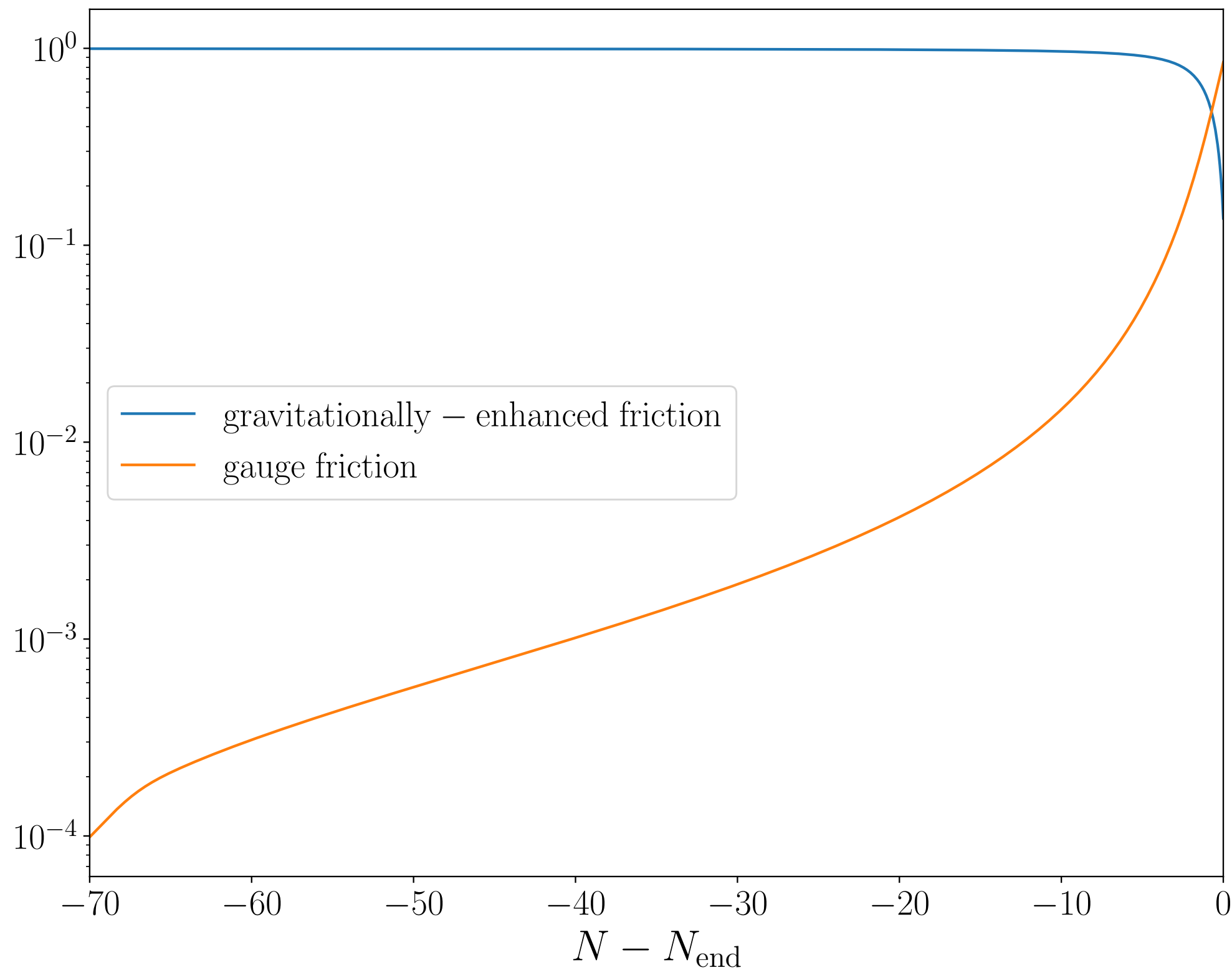
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long-enough
inflation without
GW overproduction

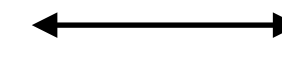


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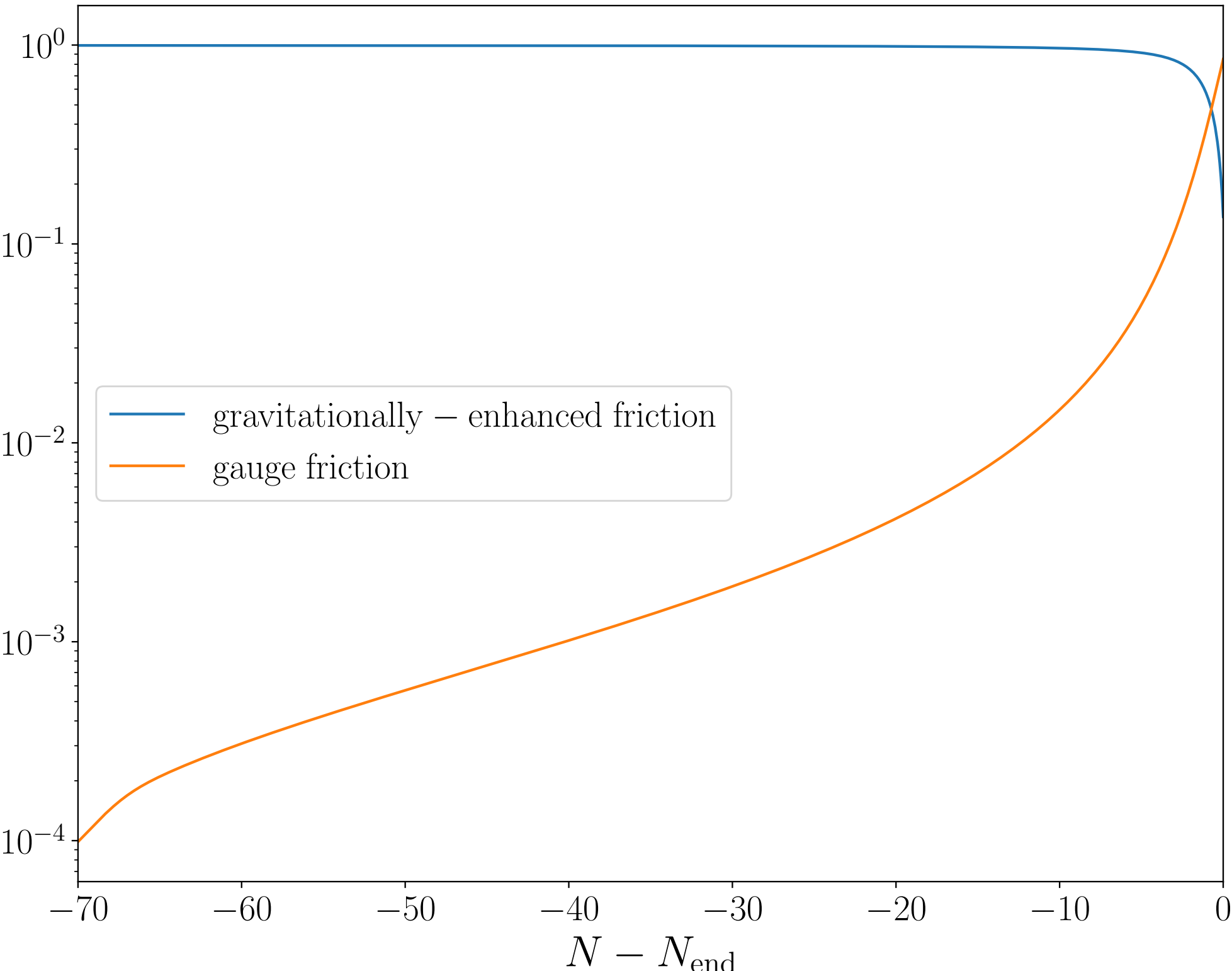
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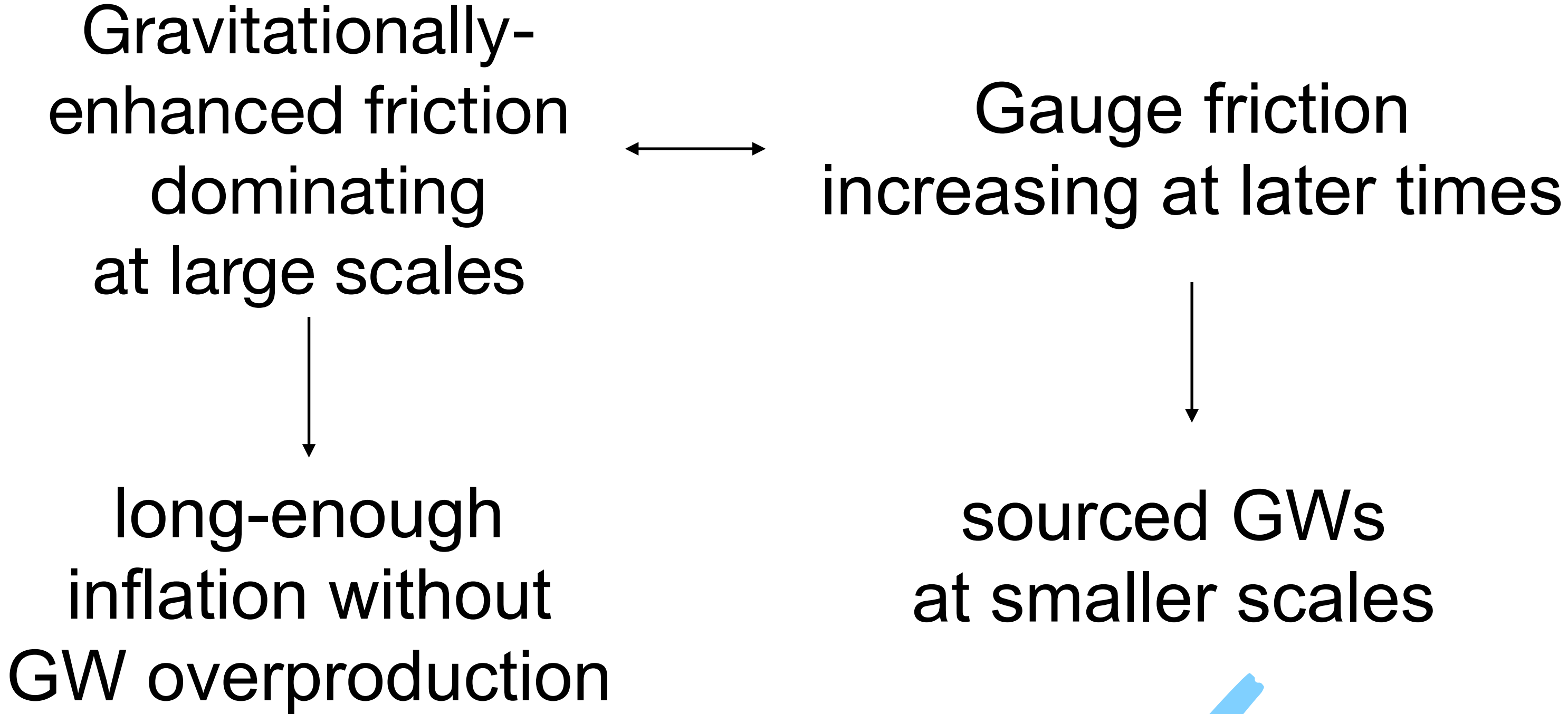
sourced GWs
at smaller scales



Gravitationally-enhanced friction



In the limit $\frac{H}{M} \gg 1$



→ expectation: viable model with detectable GW signal



Chiral GW production

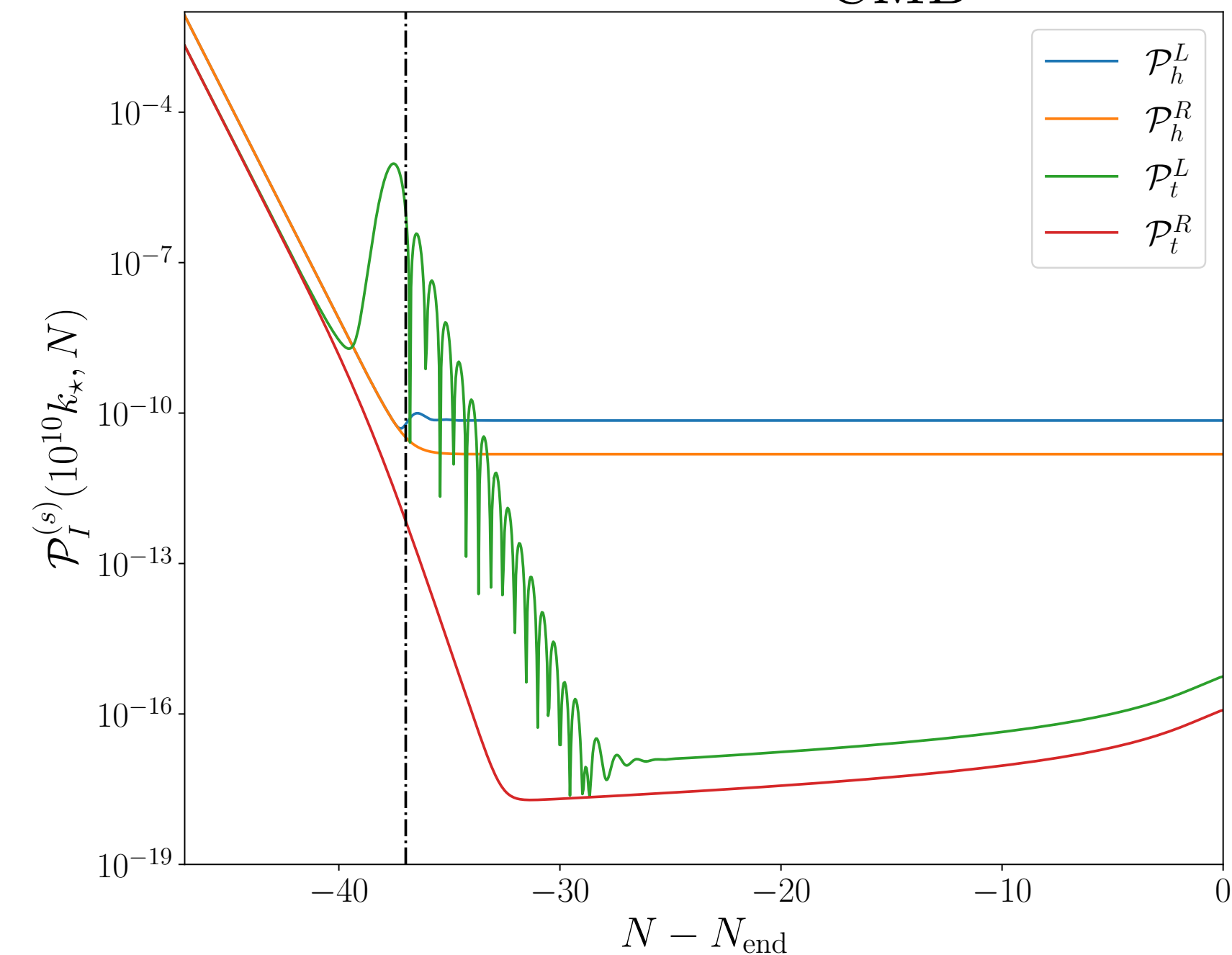
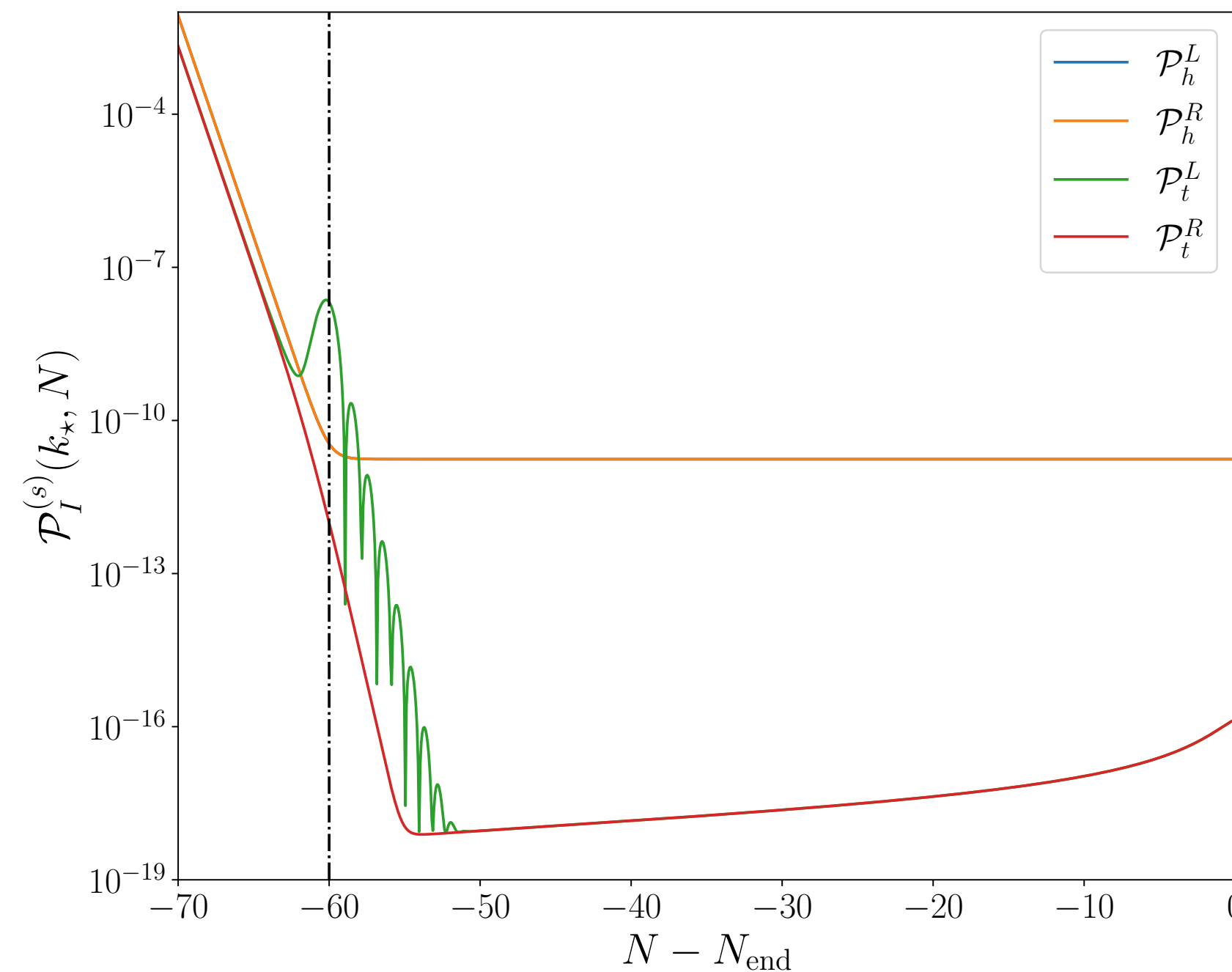
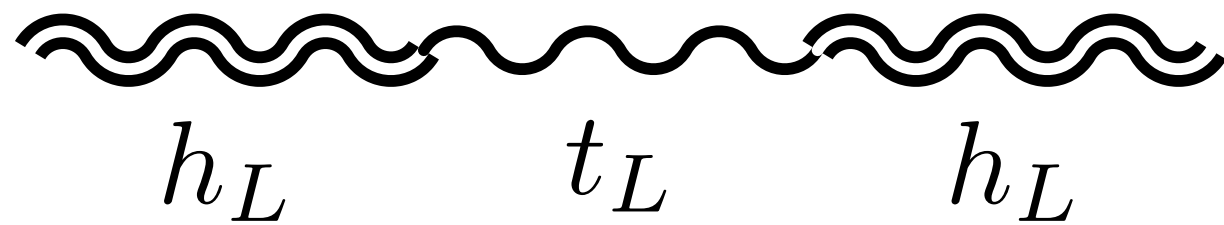
Gauge tensors $\delta A_i^a \supset t_{ia}$:
$$\frac{d^2}{dx^2} t_{L,R} + \left[1 + \frac{2(1 + m_Q^2)}{x^2} \mp \frac{2(2m_Q + m_Q^{-1})}{x} \right] t_{L,R} = 0$$

with $m_Q \equiv \frac{gQ}{H}$ and $\dot{m}_Q > 0 \longrightarrow t_L \propto \exp(m_Q)$ enhanced at small scales

$k = k_{\text{CMB}}$

$k = 10^{10} k_{\text{CMB}}$

Linear coupling
with metric tensors



Constraints from CMB

Scalar spectrum $\langle \zeta(\vec{k})\zeta(\vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \mathcal{P}_\zeta(k)$ with $\zeta \simeq -\frac{H}{\dot{\chi}} \delta\chi$

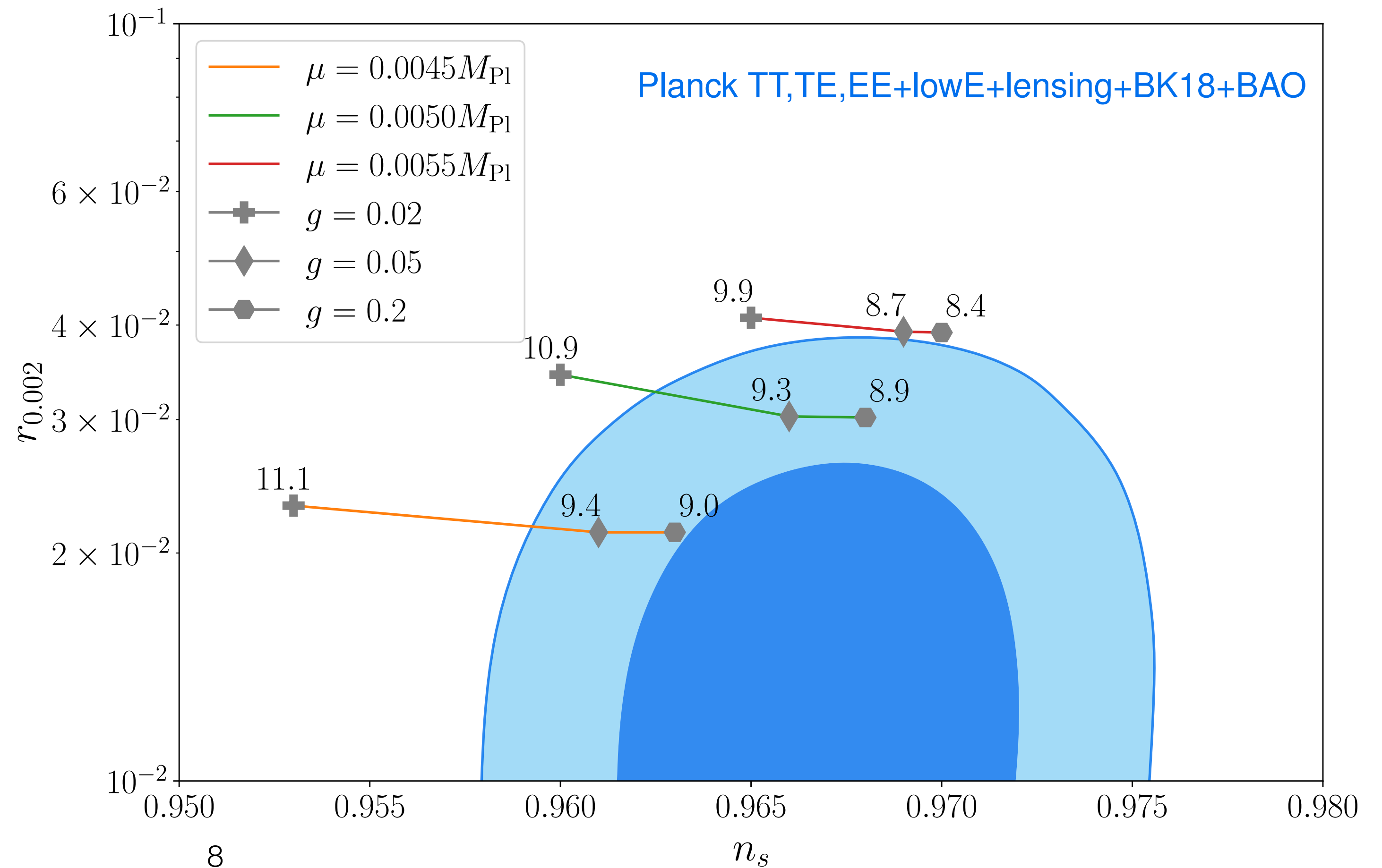
Tensor spectrum $\sum_{i,j} \langle h_{ij}(\vec{k})h_{ij}(\vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \mathcal{P}_h(k)$

Observables

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} \quad n_s - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k}$$

with constraint:

$$\mathcal{P}_\zeta(k_{\text{CMB}}) \simeq 2.1 \times 10^{-9}$$



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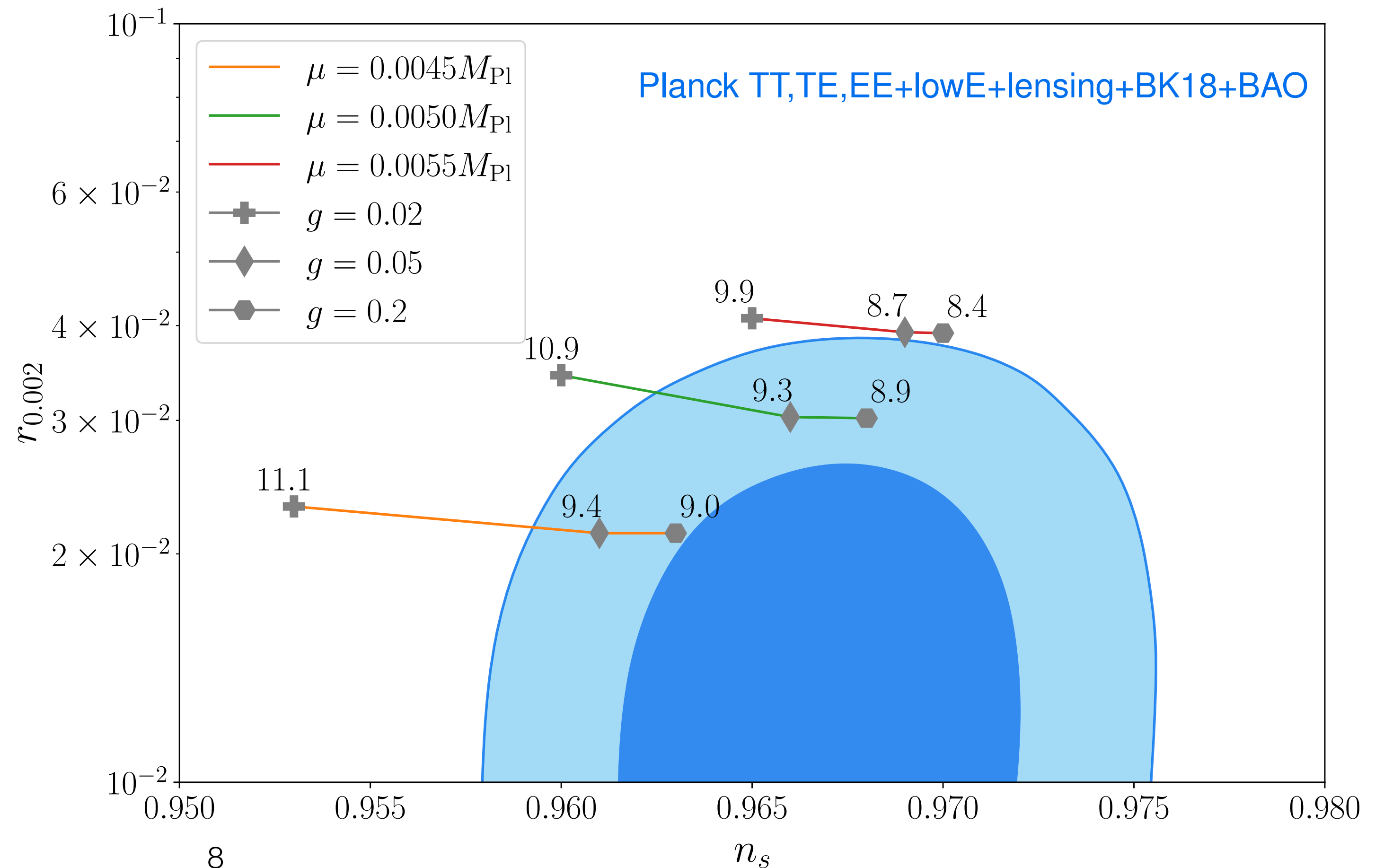
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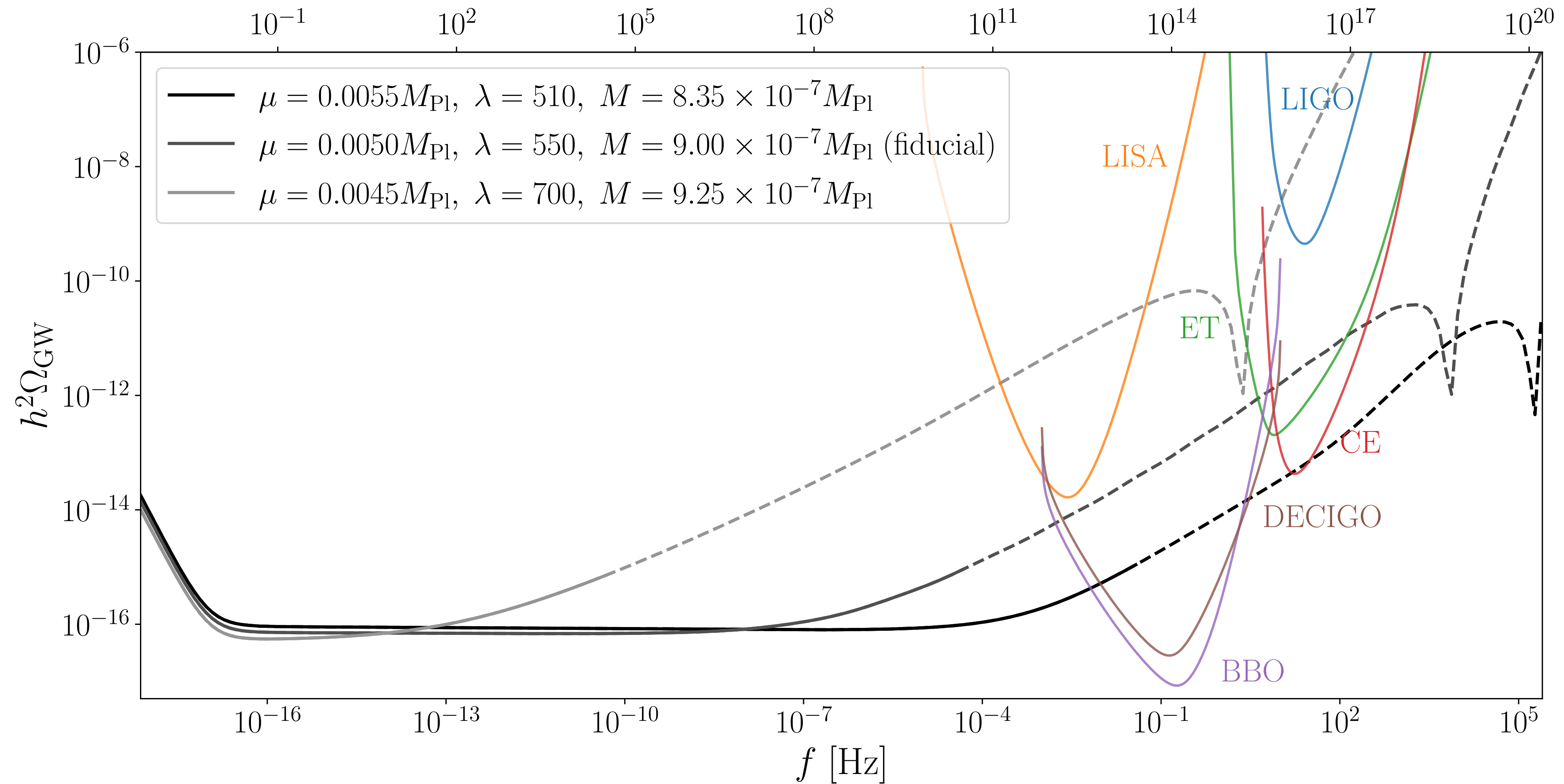
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→ observationally viable



Sourced background of GWs

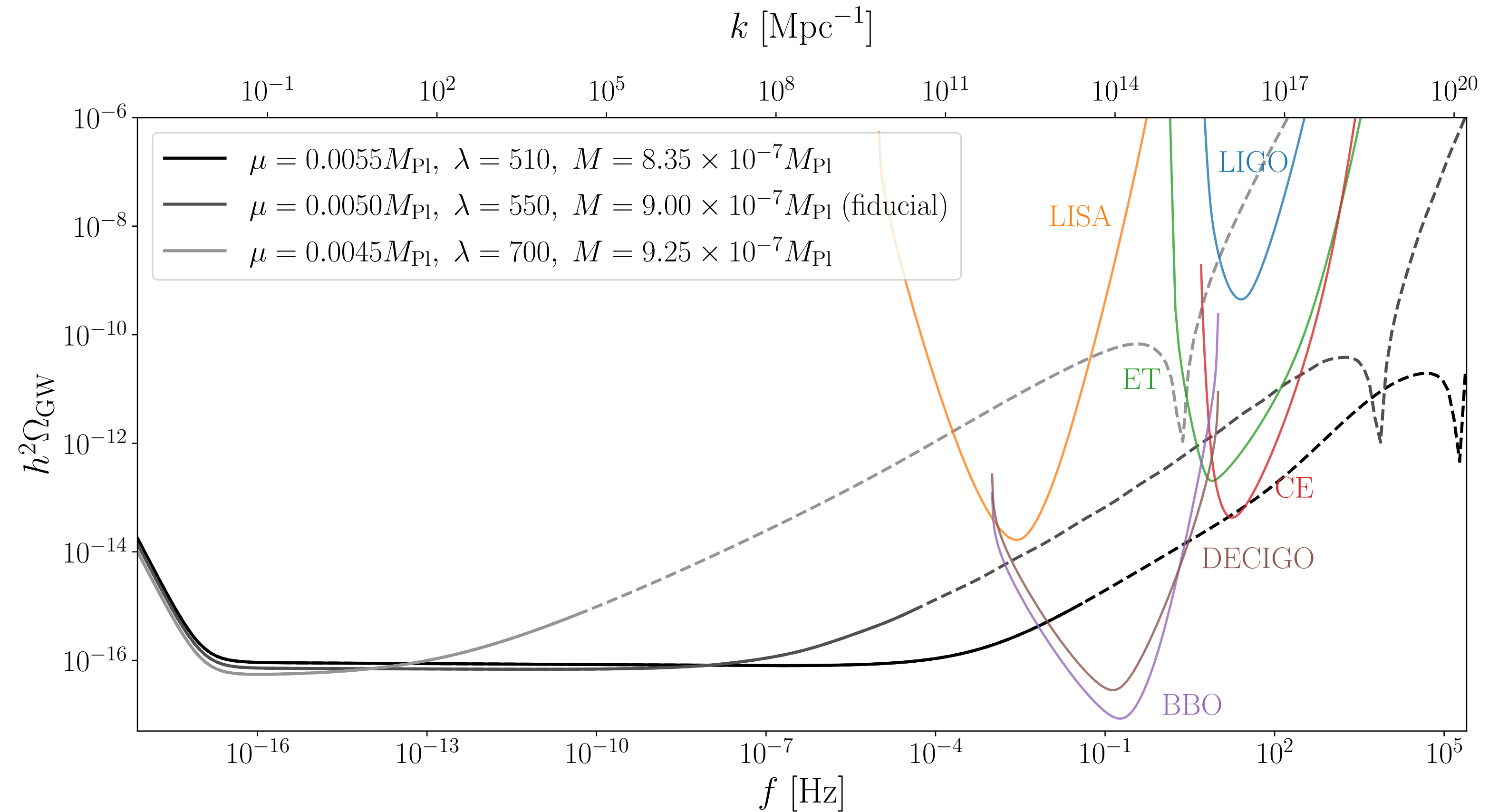
Sourced GWs $\propto \exp(m_Q)$ \longrightarrow blue (and chiral) spectrum
 k [Mpc $^{-1}$]



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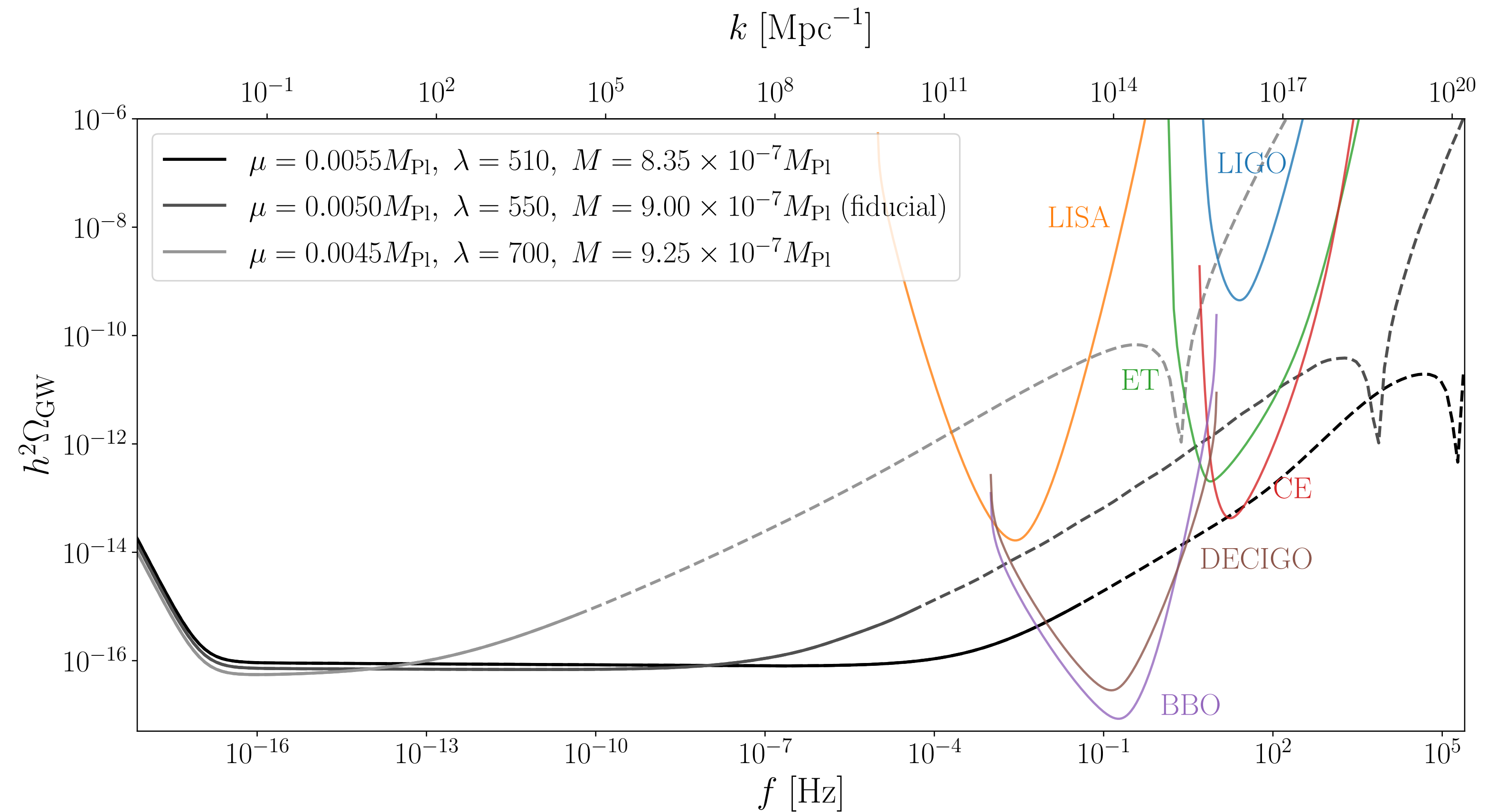
detectable by
future interferometers



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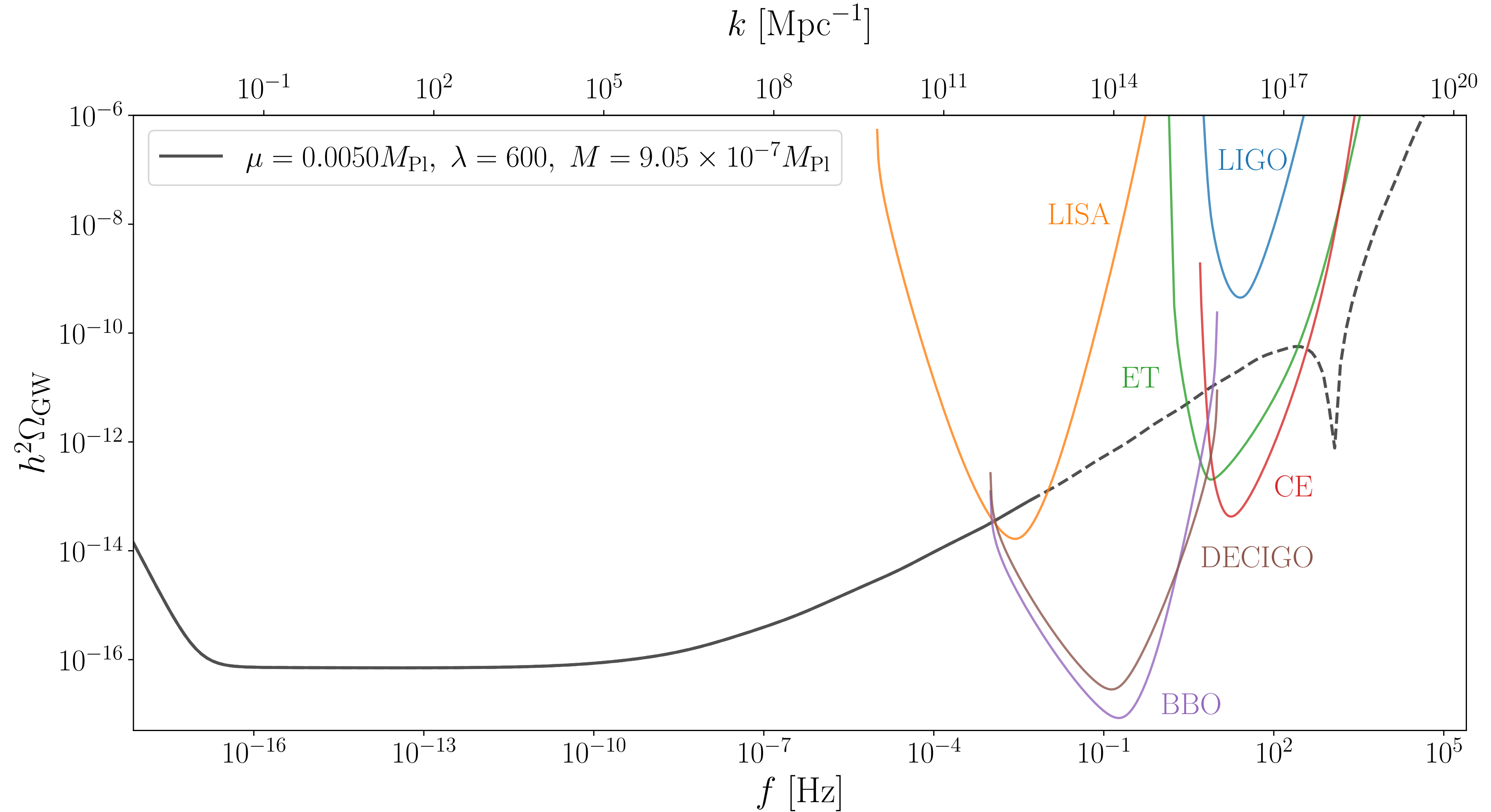
! Very conservative bound on m_Q from backreaction

Dimastrogiovanni et al. 2016

Ishiwata et al. 2022

Peloso, Sorbo, 2022

Sourced background of GWs



(Weaker but still) conservative bound on m_Q

Conclusions

By means of the non-minimal coupling with gravity in CNI:

- * GW overproduction avoided in a long-enough period of inflation
- * Distinct signature: blue and chiral GW spectrum (possibly detectable)

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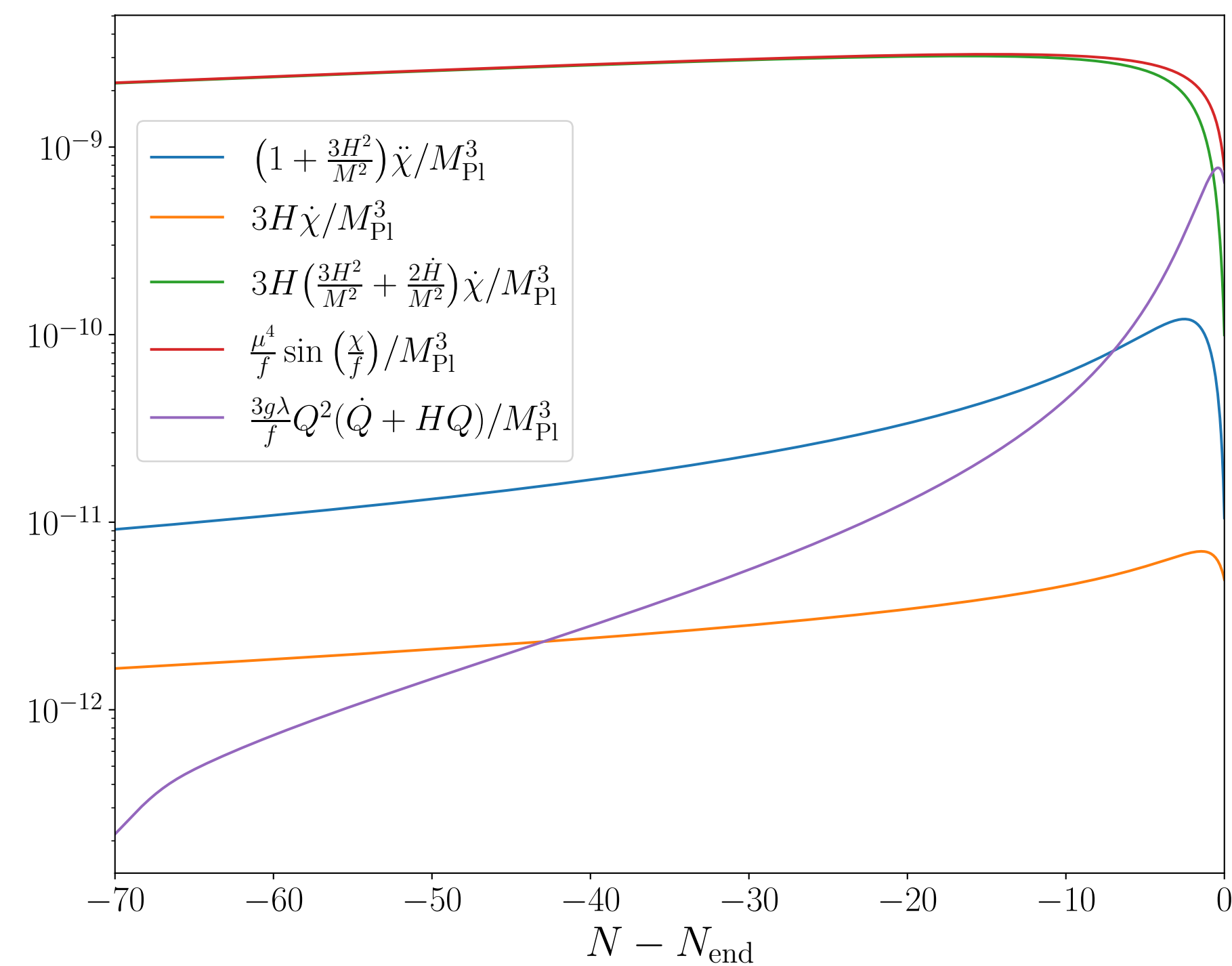
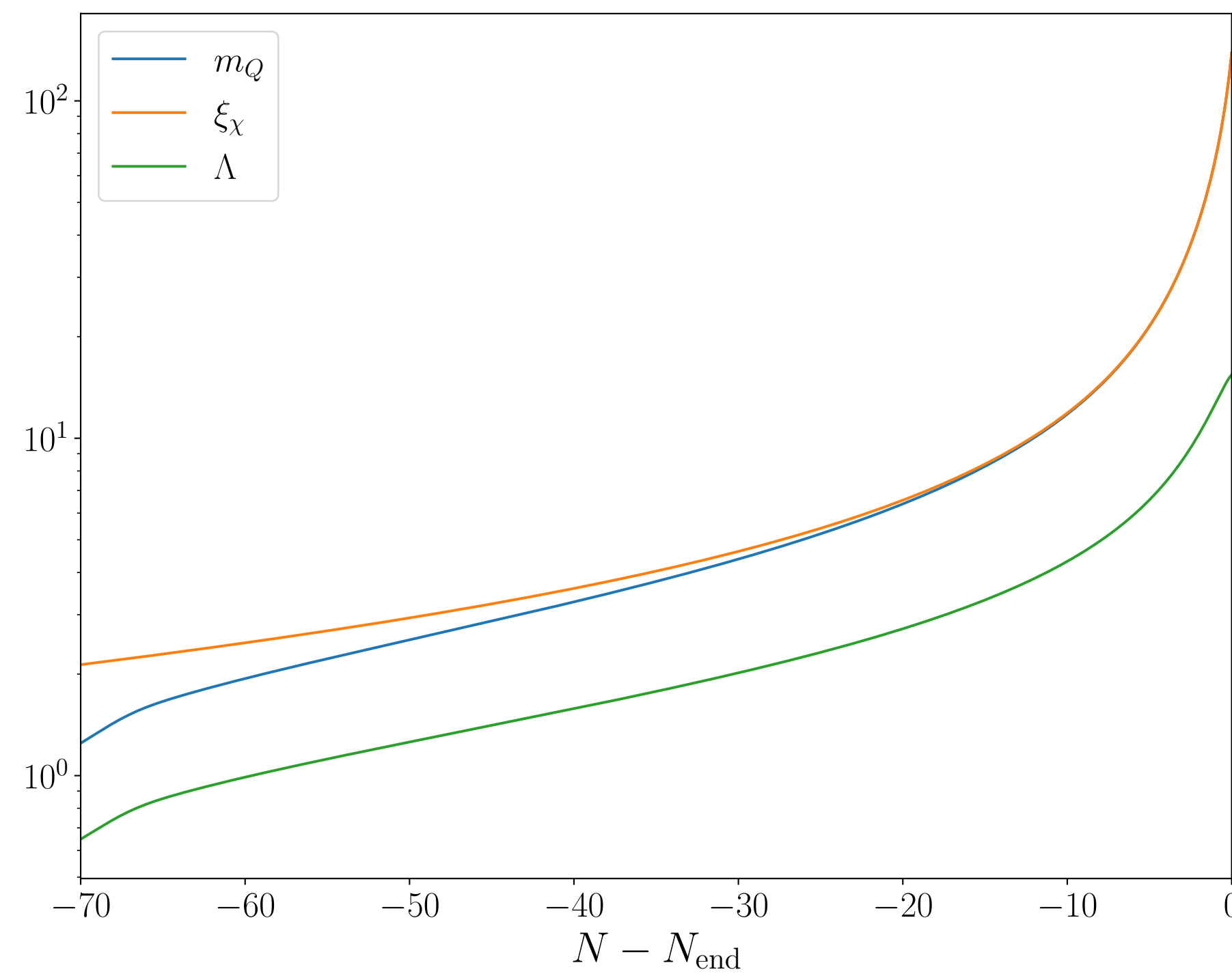
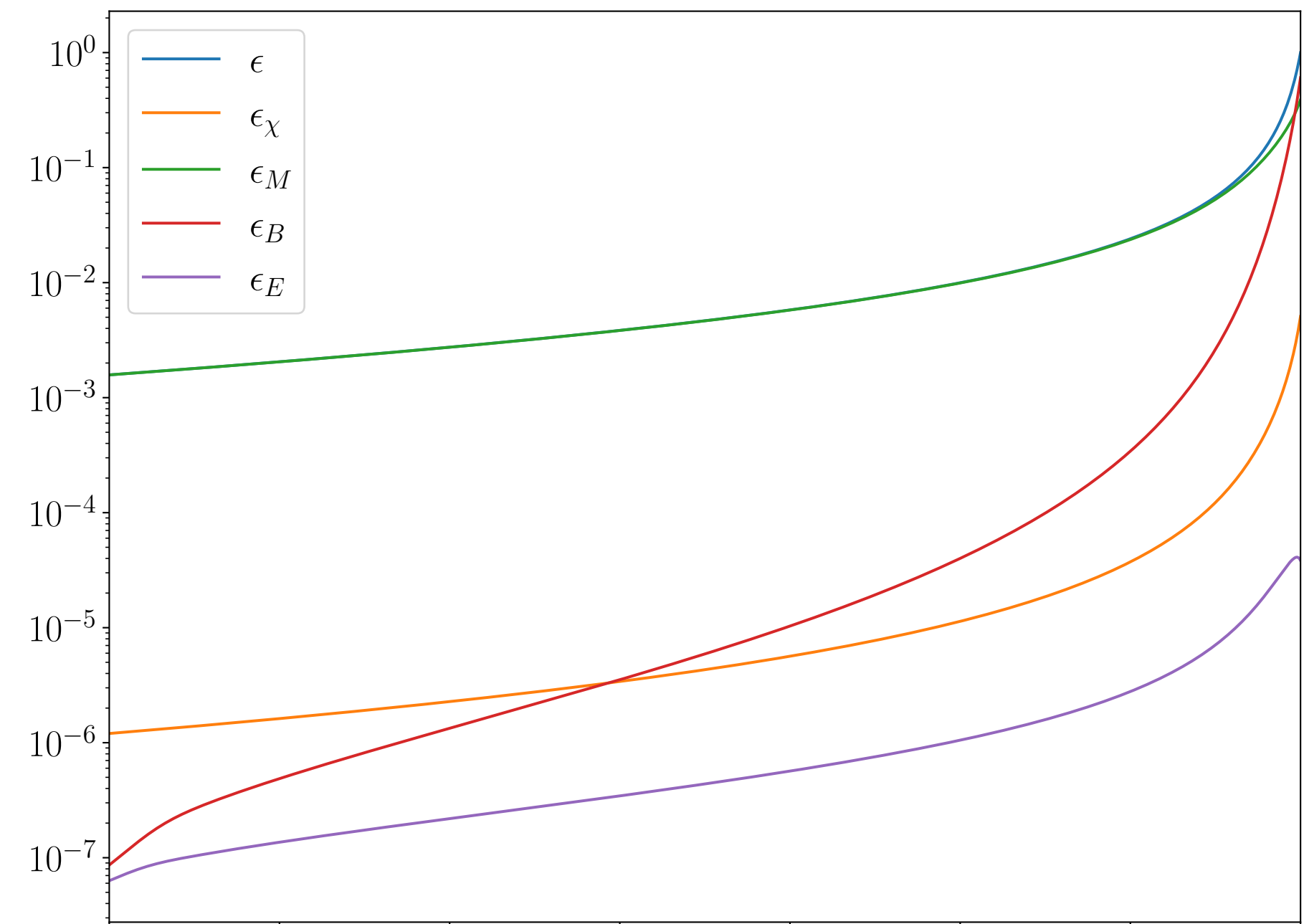
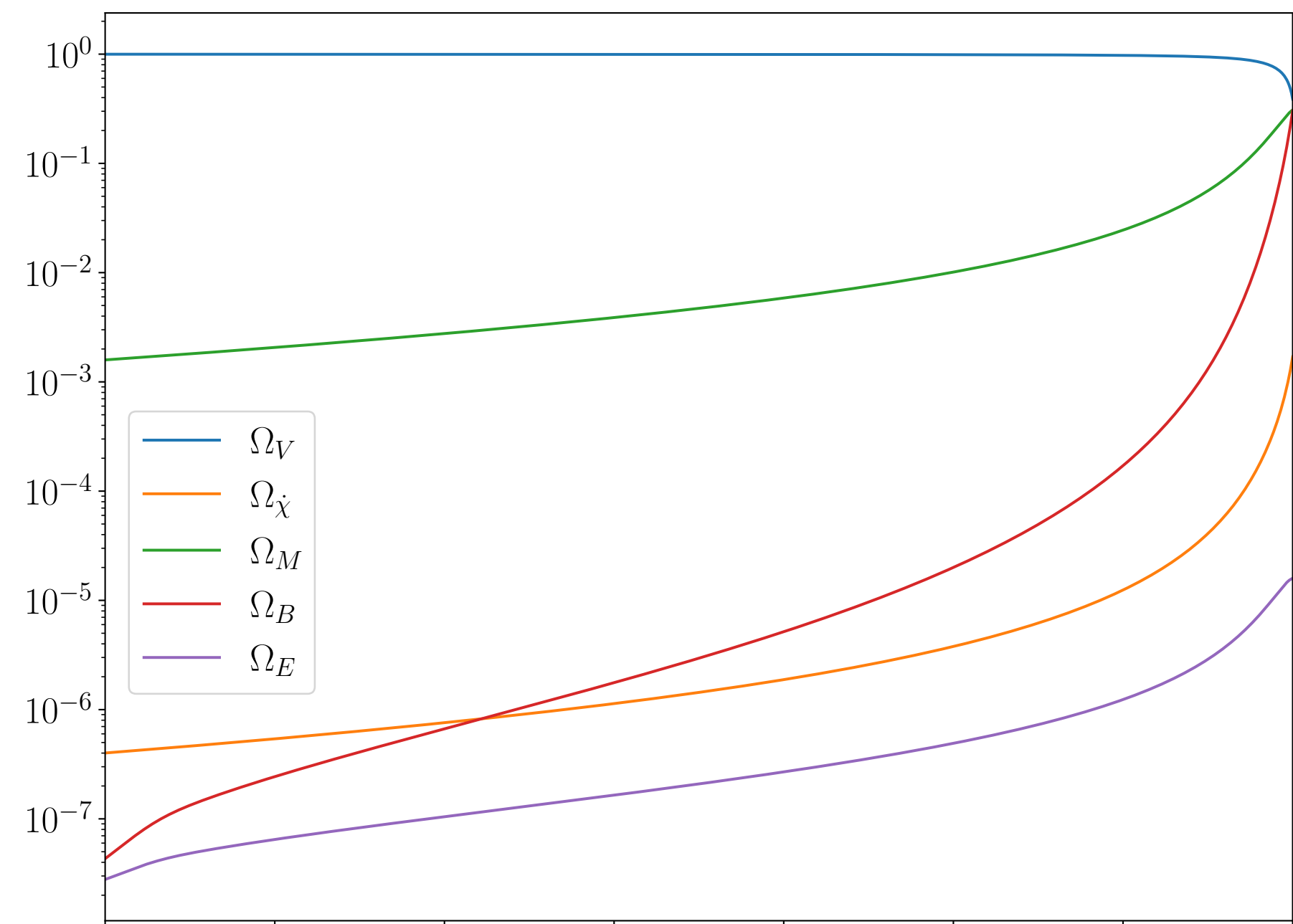
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Thank you for the attention!



Tensor perturbations

$$\partial_x^2 h_{L,R} + \left(1 - \frac{2}{x^2}\right) h_{L,R} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{L,R} + \frac{2\sqrt{\epsilon_B}}{x^2} (m_Q \mp x) t_{L,R}$$

$$\partial_x^2 t_{L,R} + \left[1 + \frac{2}{x^2} (m_Q \xi \mp x(m_Q + \xi))\right] t_{L,R} = -\frac{2\sqrt{\epsilon_E}}{x} \partial_x h_{L,R} + \frac{2}{x^2} [(m_Q \mp x)\sqrt{\epsilon_B} + \sqrt{\epsilon_E}] h_{L,R}$$

Canonical normalization: $h_{L,R} \equiv \frac{aM_{\text{Pl}}}{2} (h_+ \pm ih_\times)$ $t_{L,R} \equiv a (t_+ \pm it_\times)$

Gauge-field background evolution

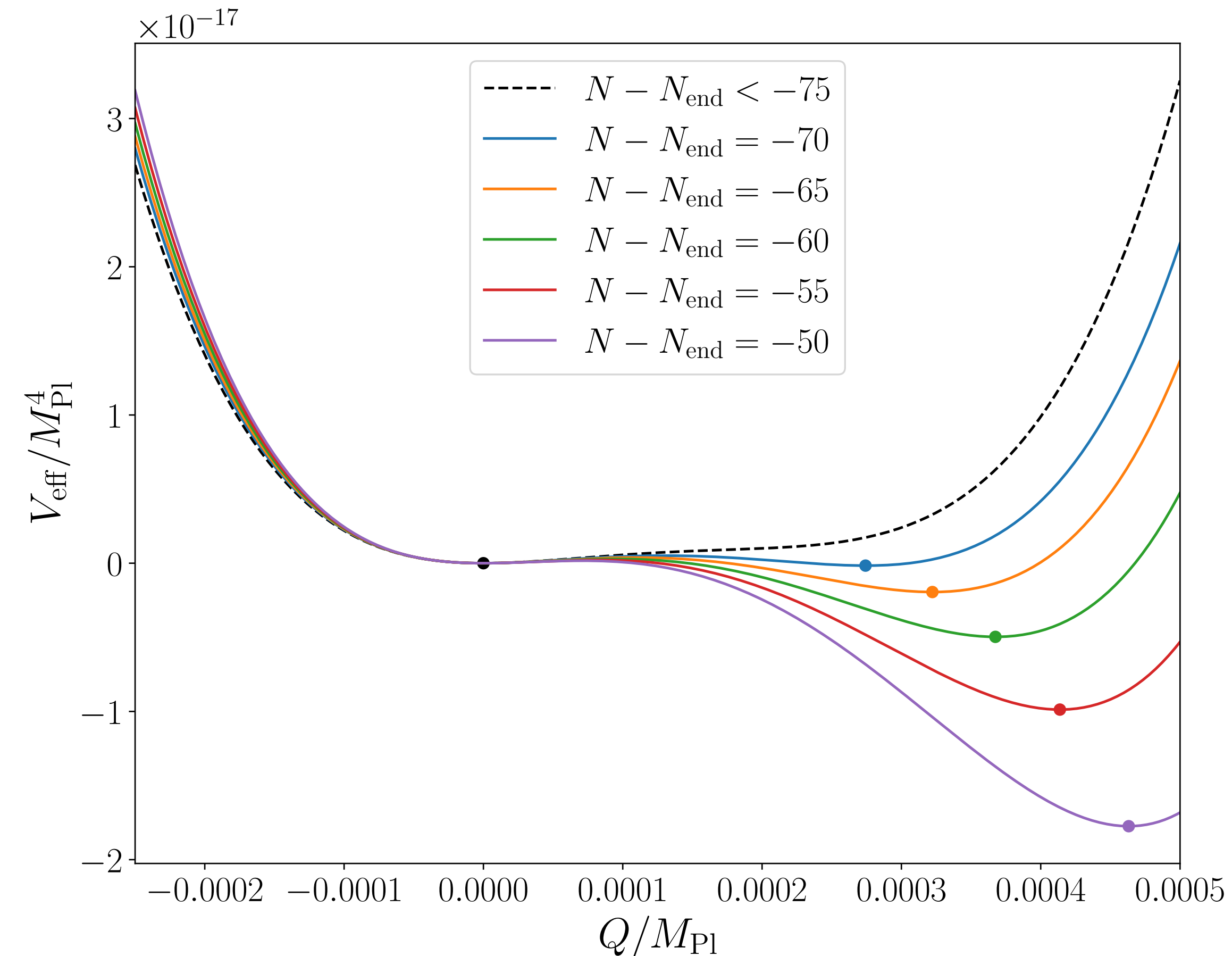
Effective potential:
$$V_{\text{eff}}(Q) = H^2 Q^2 - \frac{g\lambda\mu^4}{27f^2 H} \frac{M^2}{H^2} \sin \frac{\chi}{f} Q^3 + \frac{g^2}{2} Q^4$$

$Q_{\text{min}} \neq 0$ develops and increases with time



Lower bound on M :
$$\frac{H^2}{M^2} < \frac{\lambda\mu^4}{36f^2 H^2} \sin \frac{\chi}{f}$$

→
$$Q_{\text{min}} \simeq \mathcal{O}(1) \frac{\lambda\mu^4 M^2}{18gf^2 H^3} \sin \frac{\chi}{f}$$



Perturbations

Axion scalar

$$\delta\chi$$

Gauge-field scalars

$$\delta A_i^a \supset \delta Q, \delta M$$

Metric tensors

$$\delta g_{ij} \supset h_{ij} \longrightarrow h_{L,R}$$

Gauge-field tensors

$$\delta A_i^a \supset t_{ia} \longrightarrow t_{L,R}$$

Canonical quantization: $\hat{\Delta}_a(\tau, \mathbf{k}) = \mathcal{D}_{a\alpha}(\tau, k)\hat{a}_\alpha(\mathbf{k}) + \mathcal{D}_{a\alpha}^*(\tau, k)\hat{a}_\alpha^\dagger(-\mathbf{k})$

Equations of motion:

$$\mathcal{D}'' + 2K\mathcal{D}' + (\Omega^2 + K')\mathcal{D} = 0$$