The SGWB produced by MHD turbulence in the early universe

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ARP *et al.*, *Geophys. Astrophys. Fluid Dyn.* **114**, 130 (2020), arXiv:1807.05479
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020), arXiv:1903.08585
ARP, C. Caprini, A. Neronov, D. Semikoz, *Phys. Rev. D* **105**, 123502 (2022), arXiv:2201.05630.

`https://github.com/AlbertoRoper/GW_turbulence` [CosmoGW]

Probing the early Universe with GWs

Cosmological (pre-recombination) GW background

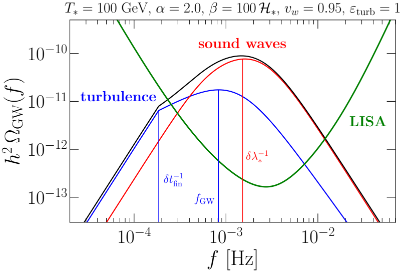
- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.
- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries, ... $\sim 10^7, 10^8$ GeV (untested physics).
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array (PTA) frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV
- From inflation
  - B-modes of CMB anisotropies ($f_c \sim 10^{-18}$ Hz).
  - Can cover all f spectrum, depending on end-of-reheating T, and blue-tilted (beyond slow-roll inflation).

MHD sources in the early universe

- Magnetohydrodynamic (MHD) sources of GWs:
  - Sound waves generated from first-order phase transitions.
  - (M)HD turbulence from first-order phase transitions.
  - Primordial magnetic fields.

- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.

- Other sources of GWs include
  - Bubble collisions.
  - Cosmic strings.
  - Primordial black holes.
  - Inflation.



Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions.
- The magnetic fields are strongly coupled to the primordial plasma and inevitably lead to MHD turbulence.[1]
- Present magnetic fields can be amplified by primordial turbulence via dynamo.[2]

[1] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).
[2] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Primordial magnetic fields

- Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.[3]
- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.[4]
- Additional constraints from CMB, Faraday Rotation, ultra-high energy cosmic rays (UHECR).

[3] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

[4] V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

Generation of primordial magnetic fields

- Bubble collisions and velocity fields induced by first-order phase transitions can generate magnetic fields.
- Parity-violating processes during the EWPT are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields.[5]
- Axion fields can amplify and produce magnetic field helicity.[6]
- Magnetic fields from inflation can be present during phase transitions (non-helical[7] and helical[8]).
- Low-scale (QCD and EWPT) inflationary magnetogenesis.[9]
- Chiral magnetic effect.[10]

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[5] T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991), T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001), J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

[6] M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

[7] M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).

[8] M. Giovannini, *Phys. Rev. D* **58**, 124027 (1998).

[9] R. Sharma, *Phys. Rev. D* **97**, 083503 (2018).

[10] M. Joyce and M. E. Shaposhnikov, *PRL* **79**, 1193 (1997).

How do we compute these signals?

- Direct numerical simulations using the PENCIL CODE[11] to solve:
  1. Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
  2. Gravitational waves equation.

- *In general*, large-scale simulations are necessary to solve the MHD (analytical estimates require simplifying assumptions on, e.g., the UETC).

[11] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`

Contributions to the stress-energy tensor

$$T^{\mu\nu} = \left(p/c^2 + \rho\right) U^\mu U^\nu + p g^{\mu\nu} + \pi^{\mu\nu} + F^{\mu\gamma} F^{\nu}{}_{\gamma} - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma}$$

- From fluid motions:

  $$T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$$

  - Ultrarelativistic EoS: $p = \rho c^2/3$
  - Viscous stresses: $\pi_{ij} = \nu(p/c^2 + \rho)(u_{i,j} + u_{j,i})$

- From magnetic fields:

  $$T_{ij} = -B_i B_j + \delta_{ij} B^2/2$$

- 4–velocity $U^\mu = \gamma(c, u^i)$
- 4–potential $A^\mu = (\phi/c, A^i)$
- 4–current $J^\mu = (c\rho_{\mathrm{e}}, J^i)$
- Faraday tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Conservation laws

$$T^{\mu\nu}{}_{;\nu} = 0$$

We assume subrelativistic motions:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[12]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}\left(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho\right) + \frac{1}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta\boldsymbol{J}^2\right],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3}\mathbf{u}\left(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta J^2\right]$$

$$-\frac{1}{4}\boldsymbol{\nabla}\ln\rho + \frac{3}{4\rho}\boldsymbol{J}\times\boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla}\cdot(\rho\nu\boldsymbol{S}),$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{u}\times\boldsymbol{B} - \eta\boldsymbol{J}), \quad \boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B},$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,{\rm phys}}, u_i$, and conformal time t ($\mathrm{d}t = a\mathrm{d}t_c$).

[12] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).

GW equation for a flat expanding Universe

- Assumptions: isotropic and homogeneous Universe.
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2\delta_{ij}$.
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\rm phys}\right), \quad |h_{ij}^{\rm phys}| \ll |g_{ij}|$$

- GW equation is[13]

$$\left(\partial_t^2 - \frac{a''}{a} - c^2\nabla^2\right) h_{ij} = \frac{16\pi G}{\mathbf{a} c^2} T_{ij}^{\rm TT}$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\rm phys}$.
  - Comoving spatial coordinates $\nabla = a\nabla^{\rm phys}$.
  - Conformal time $\mathrm{d}t = a\,\mathrm{d}t_c$.
  - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\rm phys}$.
  - Radiation-dominated epoch such that $a'' = 0$.

[13] L. P. Grishchuk, *Sov. Phys. JETP* **40**, 409 (1974).

Numerical results for decaying MHD turbulence[14]

Initial conditions

- Initial stochastic magnetic field with fractional helicity $\sigma_{\rm M}$.

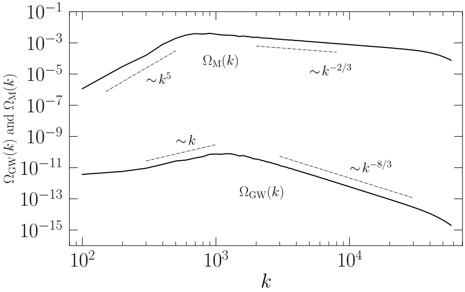
$$kB_i(\boldsymbol{k}) = \left(\delta_{ij} - \hat{k}_i\hat{k}_j - i\sigma_{\rm M}\varepsilon_{ijl}\hat{k}_l\right) g_j \sqrt{2\Omega_{\rm M}(k)/k}$$

- Batchelor spectrum for magnetic (or vortical velocity) fields, i.e., $\Omega_{\rm M} \propto k^5$ for small $k < k_* \sim \mathcal{O}(\xi_{\rm M}^{-1})$.
- Kolmogorov spectrum in the inertial range, i.e., $\Omega_{\rm M} \propto k^{-2/3}$.

[14] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. D* **96**, 123528 (2017).
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).
ARP *et al.*, *JCAP* **04** (2022), 019.
ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Numerical results for decaying MHD turbulence[15]

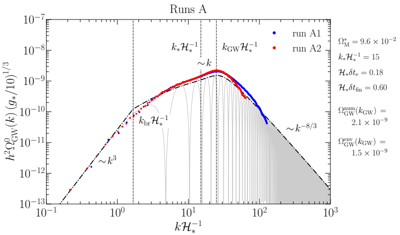
$$1152^3, k_* = 2\pi \times 100, \Omega_{\rm M} \sim 10^{-2}, \sigma_{\rm M} = 1$$

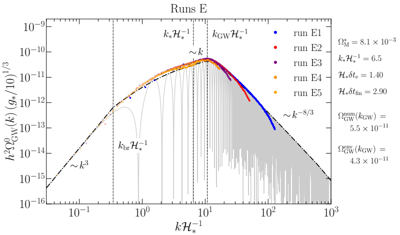


- **Novel k scaling in the subinertial range for the GW spectrum.**
- k^3 expected at large scales $k < k_*$.

[15] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).

Numerical results for nonhelical decaying MHD turbulence[16]





run	$\Omega_{\rm M}^*$	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_{\rm e}$	$\mathcal{H}_*\delta t_{\rm fin}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	$\mathcal{H}_*L$	$\mathcal{H}_*t_{\rm end}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}

[16] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Analytical model for decaying turbulence

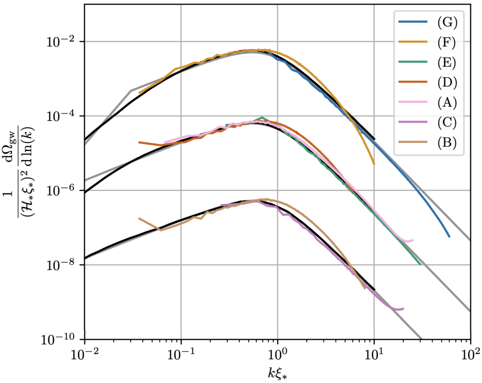
- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations[17] of $\Omega_{\rm GW}(k)$.

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 {\Omega_{\rm M}^*}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)} \, p_\Pi\left(\frac{k}{k_*}\right) \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\,\delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\,\delta t_{\rm fin} \geq 1. \end{cases}$$

- Improved models with turbulence decay [with Caprini] and fractional helicity [with Caprini, Midiri] are in preparation for publication.

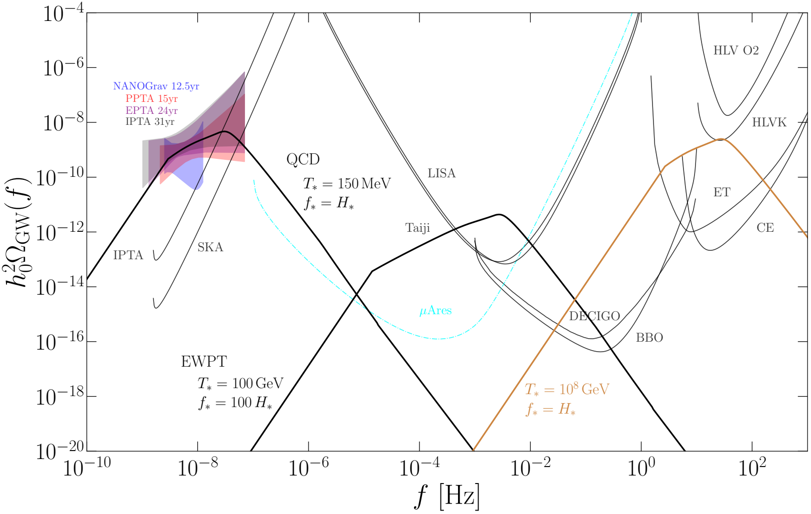
[17] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Numerical results for decaying HD vortical turbulence[18]



[18] P. Auclair *et al.*, *JCAP* **09** (2022), 029.

Gravitational spectrum (turbulence from PTs)[19]



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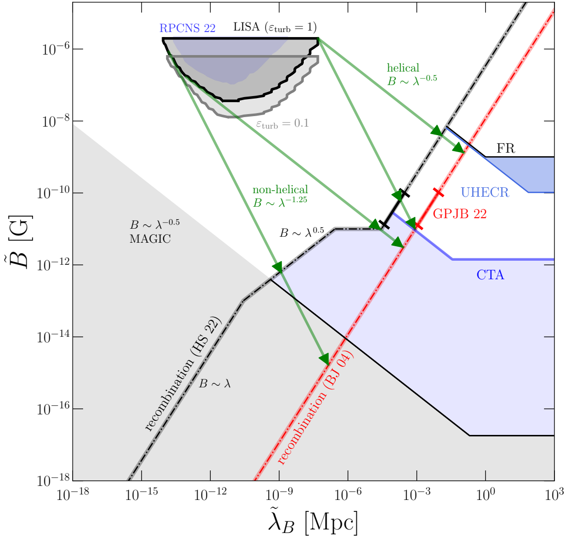
[19] ARP, C. Caprini, A. Neronov, D. Semikoz, *PRD* **105**, 123502 (2022)
A. Neronov, ARP, C. Caprini, D. Semikoz, *PRD* **103**, L041302 (2021).

Primordial magnetic fields constraints with LISA **[unpublished]**



[19] with T. Boyer, C. Caprini, A. Neronov, D. Semikoz, arXiv:2306.xxxx.

Conclusions 1/2

- Sources of MHD turbulence in the early universe can contribute to the stochastic GW background (SGWB).
- To study MHD turbulence we need, in general, to perform high-resolution numerical simulations.
- Since the SGWB is a superposition of different sources, it is extremely important to characterize the different sources, to be able to extract clean information from the early universe physics.
- The interplay between sound waves and the development of turbulence is not well understood. It plays an important role on the relative amplitude of both sources of GWs.
- LISA (and PTA) could potentially probe the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology.

Conclusions 2/2

- A main caveat of our analytical template for MHD turbulence is that it only considers the phase of turbulence decay, not the production phase.
- We have performed simulations of MHD turbulence (vortical and acoustical) from cosmological phase transitions, showing that a period of turbulence forcing can impact the SGWB shape and amplitude.
- Coupling magnetogenesis dynamics with our MHD simulations is one of the main objectives of my SNSF Ambizione project with C. Caprini, A. Midiri and other collaborators, please contact me if you are interested!

The End

Thank You!

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github.com/AlbertoRoper/GW_turbulence

cosmology.unige.ch/users/alberto-roper-pol