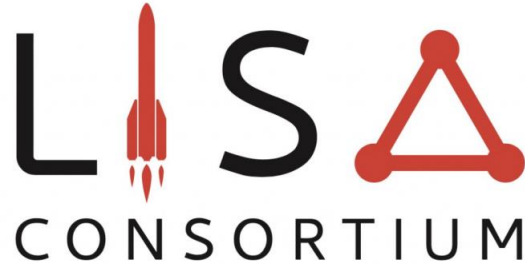


INFLATIONARY STOCHASTIC GRAVITATIONAL WAVE BACKGROUND IN



JACOPO FUMAGALLI (ICCUB)

10th LISA COSWG Workshop

Stavanger 6TH June 2023

Work in Progress with CosWG LISA

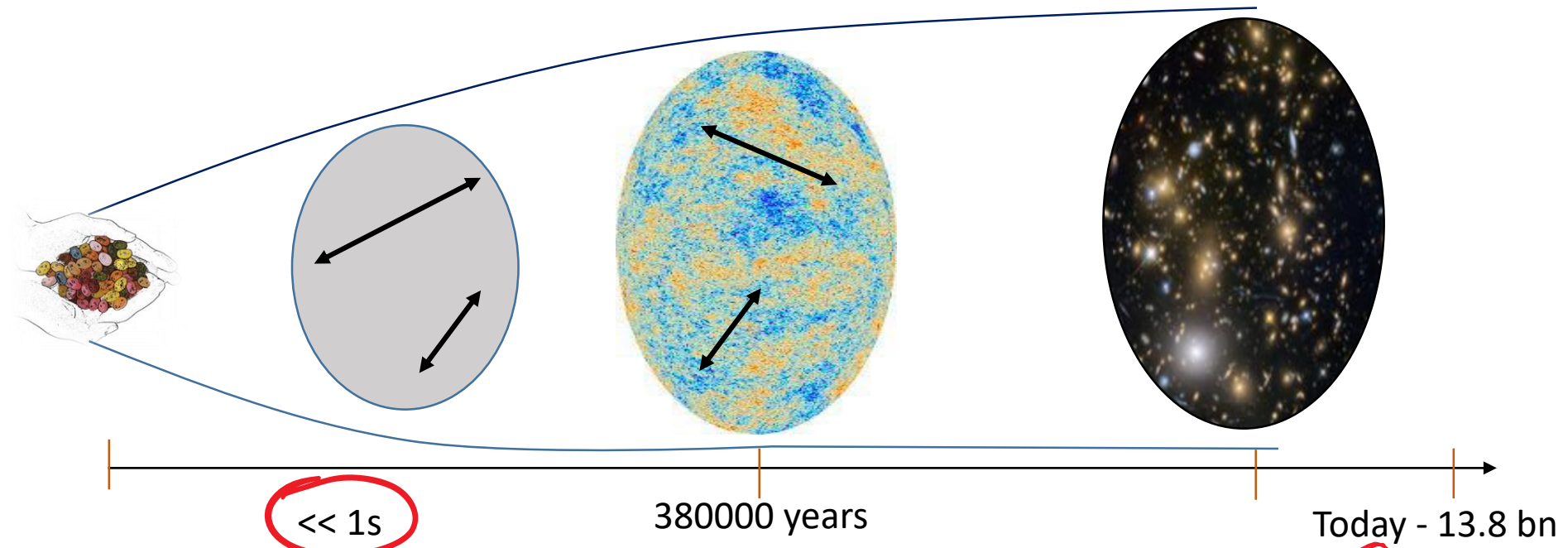


Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA



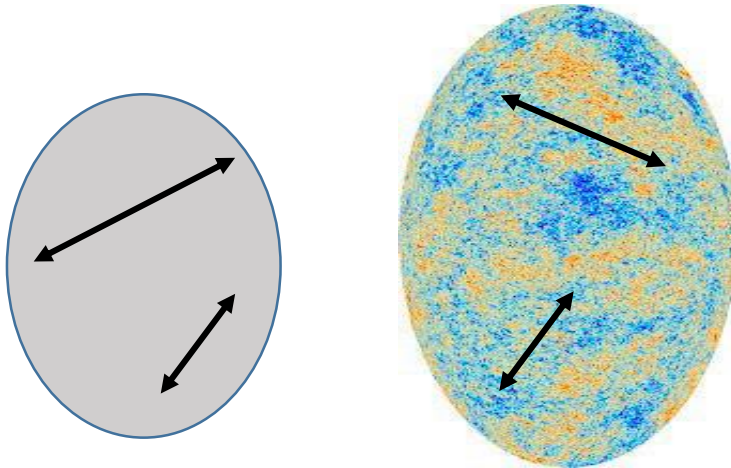
INITIAL CONDITIONS FROM INFLATION

STRUCTURES IN THE UNIVERSE EMERGE FROM
VACUUM QUANTUM FLUCTUATIONS



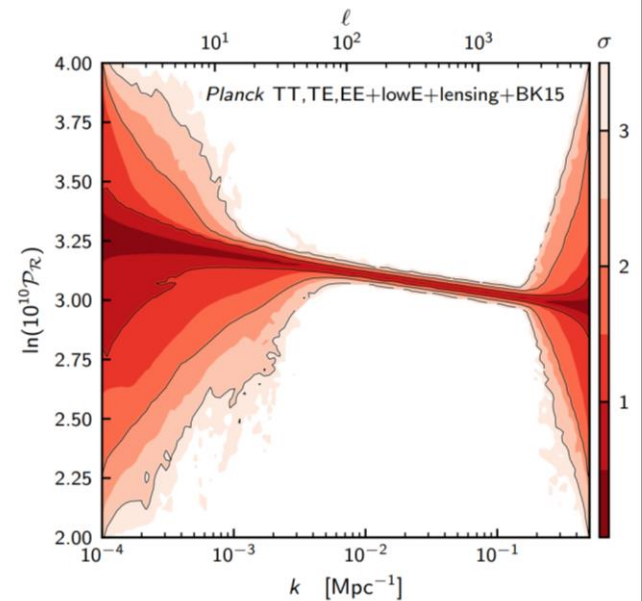
INITIAL CONDITIONS FROM INFLATION

ζ \longleftrightarrow δT



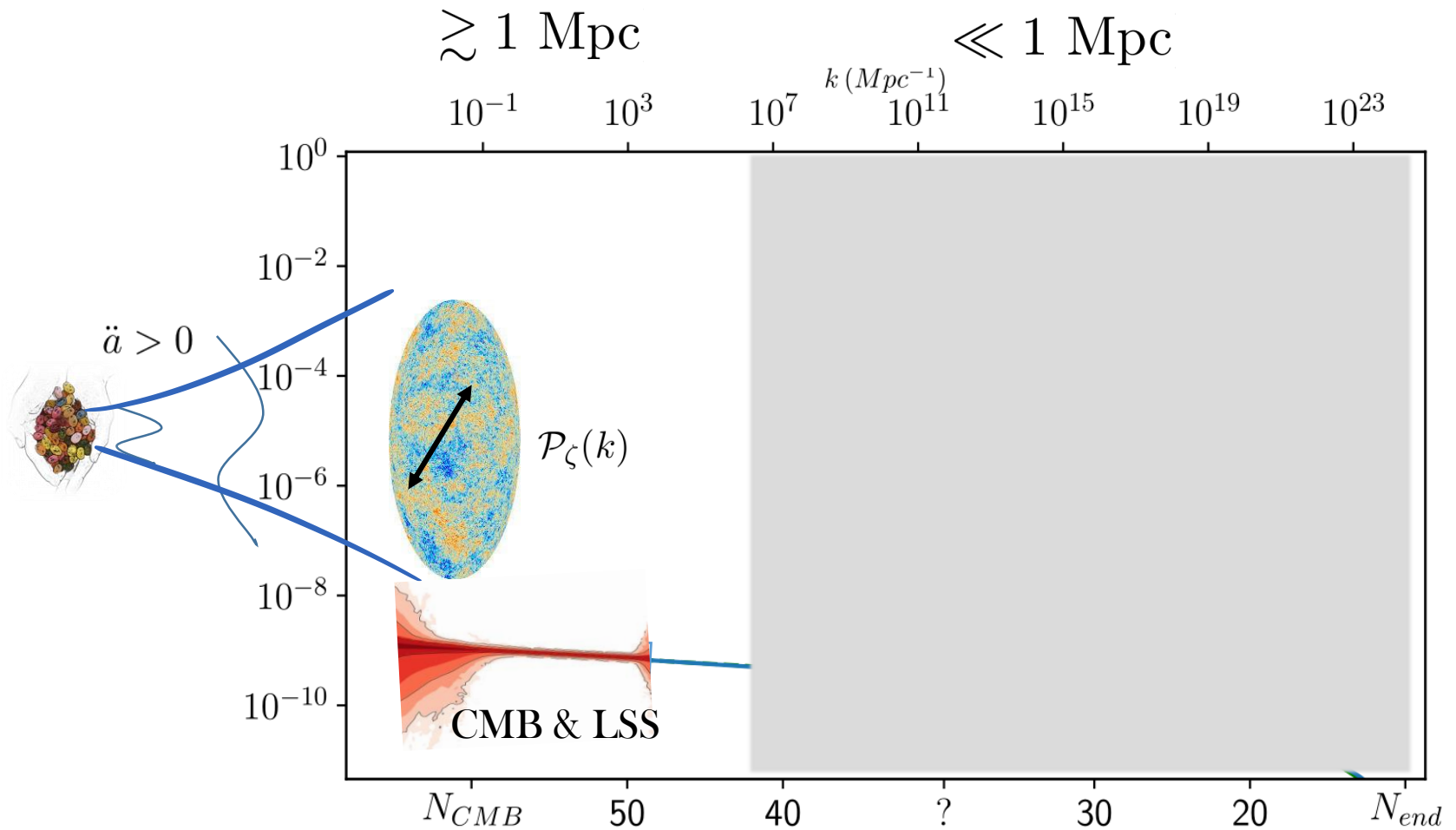
$$\langle \hat{\zeta}(\mathbf{x}, \tau) \hat{\zeta}(\mathbf{x}, \tau) \rangle = \int d \ln k \cdot \mathcal{P}_{\zeta}(k, \tau)$$

Fluctuations:
Almost scale-invariant,
Gaussian, super-Horizon...

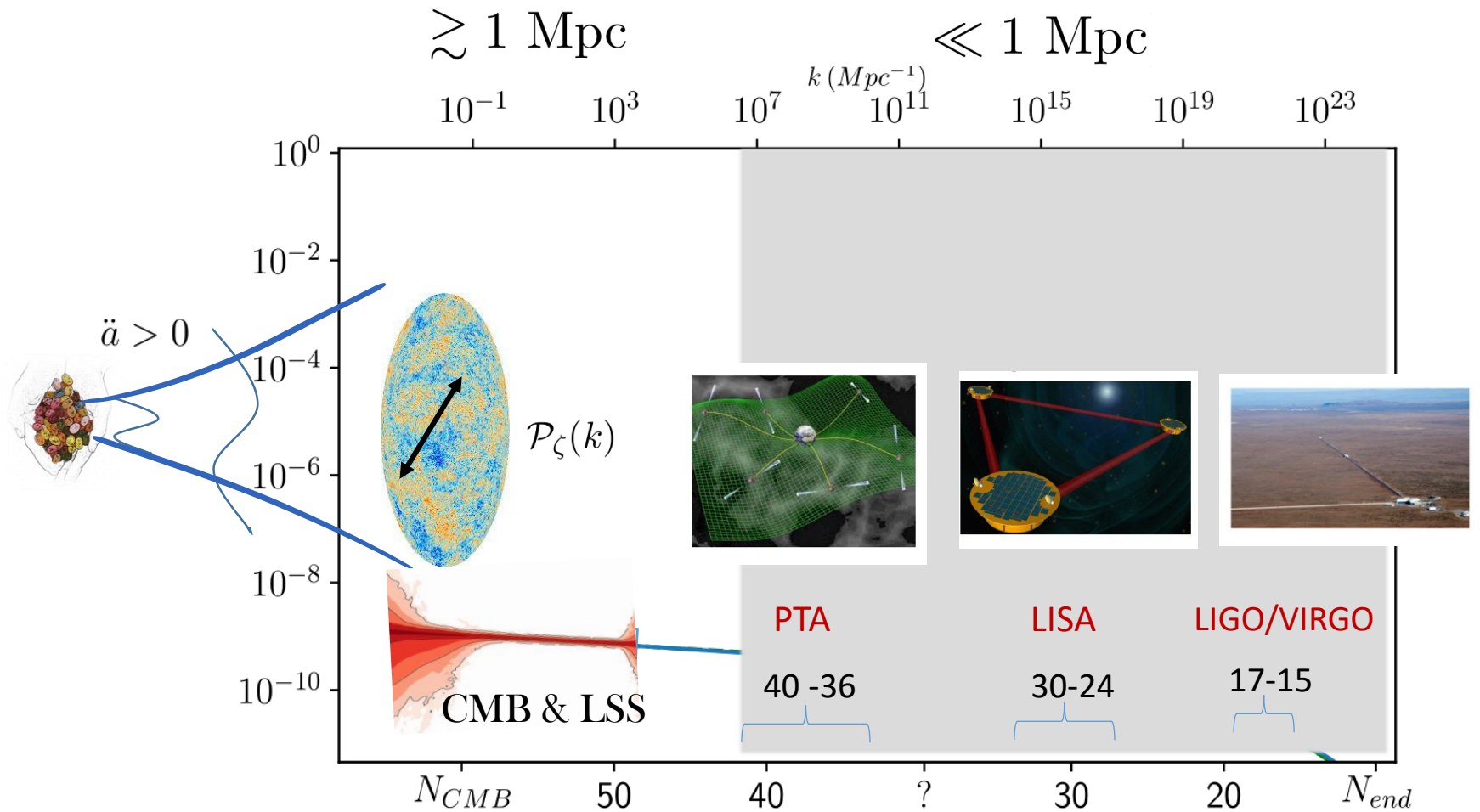


Planck '18

SGWB FROM INFLATION AT SMALL SCALES

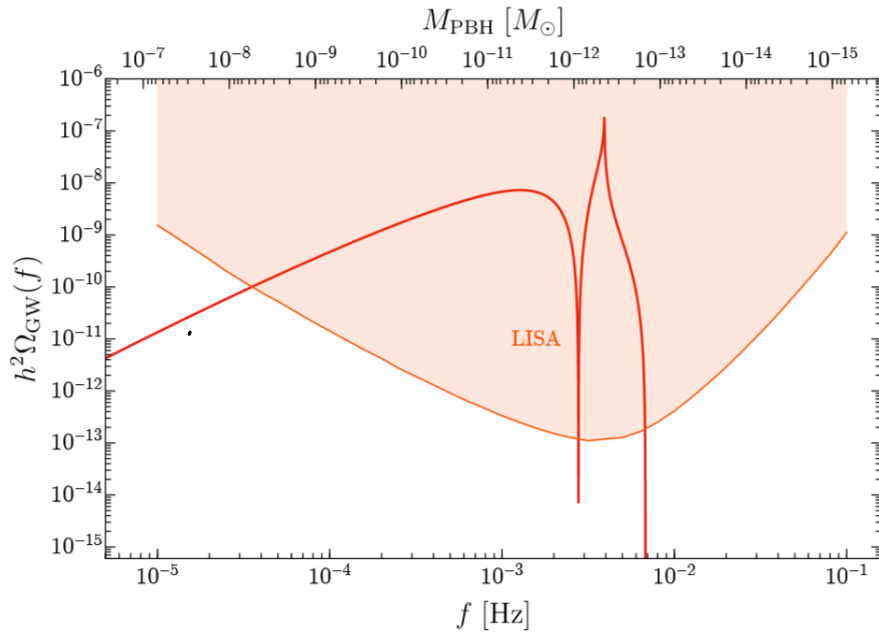


SGWB FROM INFLATION AT SMALL SCALES

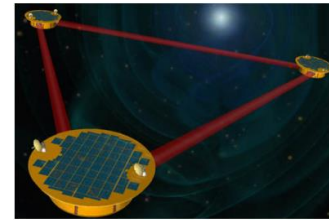


Constrained at large scales

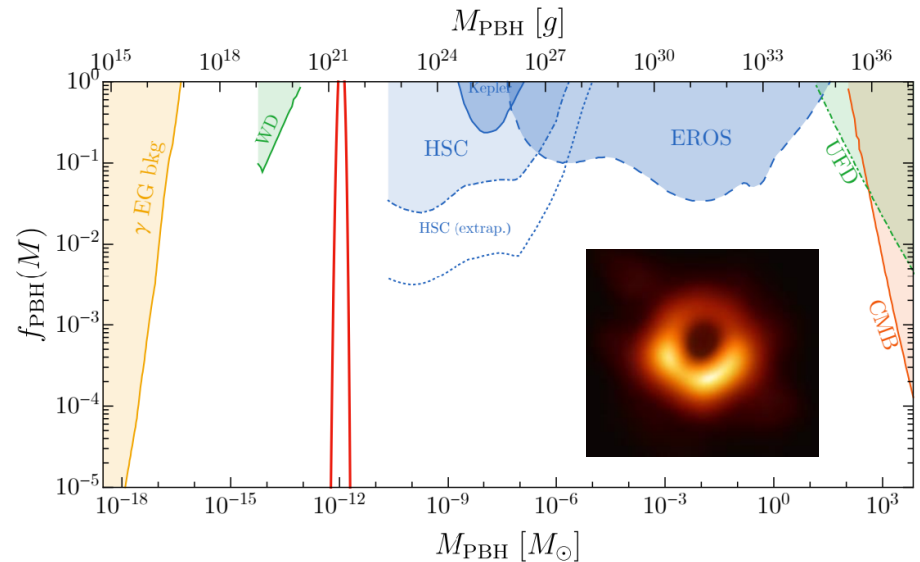
PBH / SGWB & LISA COINCIDENCE



$$\Omega_{\text{GW}}(k) \propto \int \int I(u, v) \mathcal{P}_{\zeta}(ku) \mathcal{P}_{\zeta}(kv)$$



LISA Serendipity
Bartolo, Franciolini, Peloso et al. '18



STOCHASTIC BACKGROUND OF GWS

- Angular size on the sky (today) of a correlated region at the time of production \ll detector resolution

Caprini and Figueroa '18

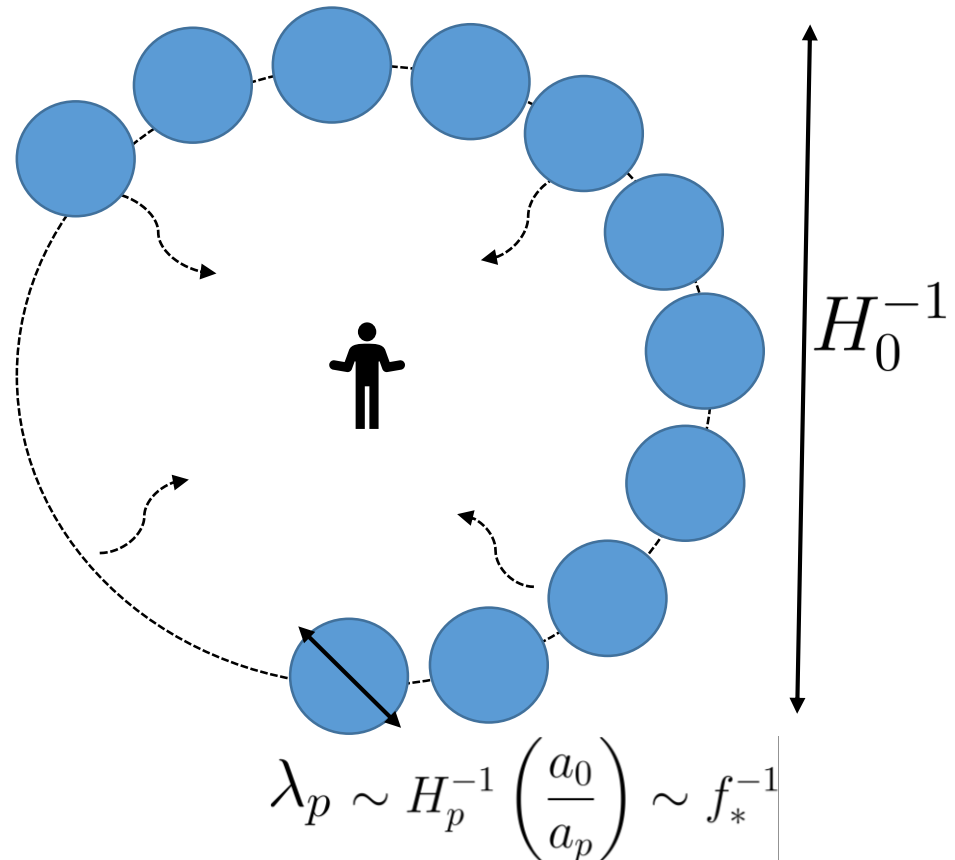
with resolution $\Theta_{\text{Res}} \sim 10 \text{ deg}$

$$\implies z \lesssim 17$$

- Characteristic frequency associated to the time of production/experiment

- $h_{ij} \longleftrightarrow$ STOCHASTIC VARIABLE

$$\Omega_{\text{GW}}(k) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln k}$$

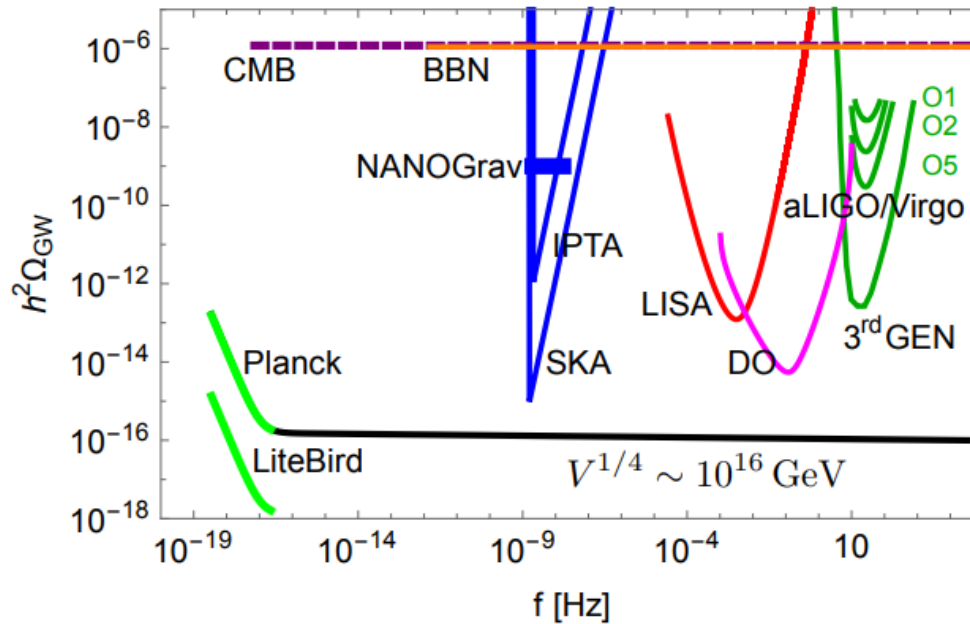


SGWB FROM INFLATION: the Holy Grail of Theoretical cosmology



Standard: enhancement of the vacuum fluctuations due to the inflationary exp. Expansion

$$\square h_{ij} = 0$$



$$\frac{2}{\pi} \frac{H^2}{M_{\text{pl}}^2} n_T \simeq -2\epsilon$$

Too tiny to be seen directly in future GW detector !

STOCHASTIC BACKGROUND OF GWS



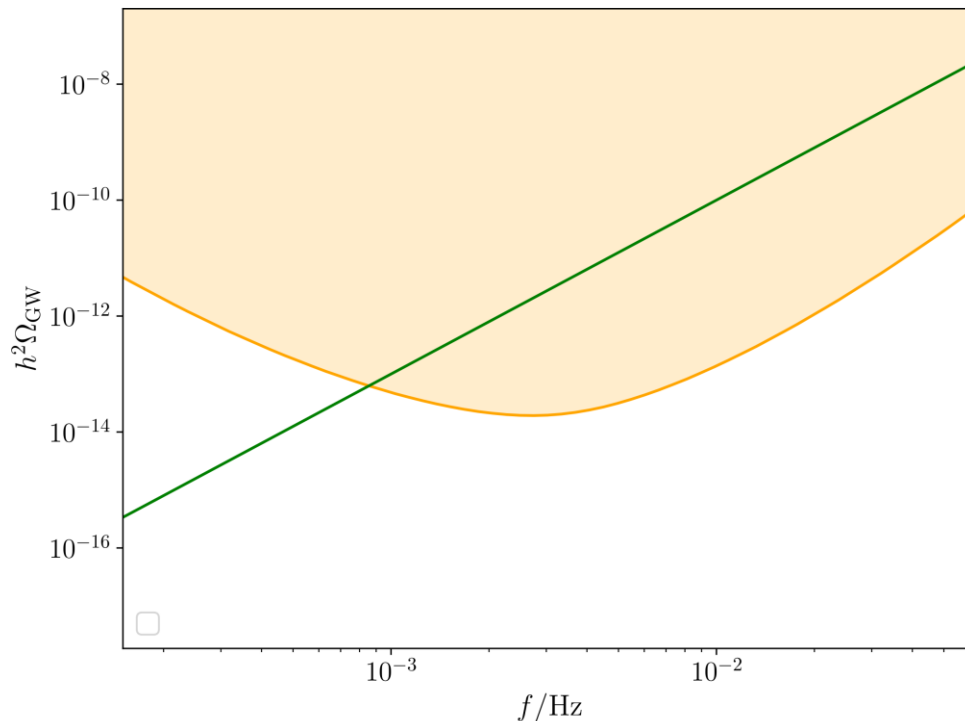
TEMPLATE DATABANK

THE INFLATIONARY MENU



POWER LAW

$$h^2 \Omega_{\text{GW}}^{\text{PL}}(f, \vec{p}) = 10^{\alpha_*} \left(\frac{f}{f_*} \right)^{n_t}$$



- Anything far from production
- Axion inflation

$$\mathcal{L} \supset \frac{\phi F_{\mu\nu} \tilde{F}^{\mu\nu}}{4f_\phi}$$

$$n_t(\dot{\phi}_*/f_\phi, \epsilon_*, \eta_*)$$

N. Barnaby, M. Peloso '10
L. Sorbo '11

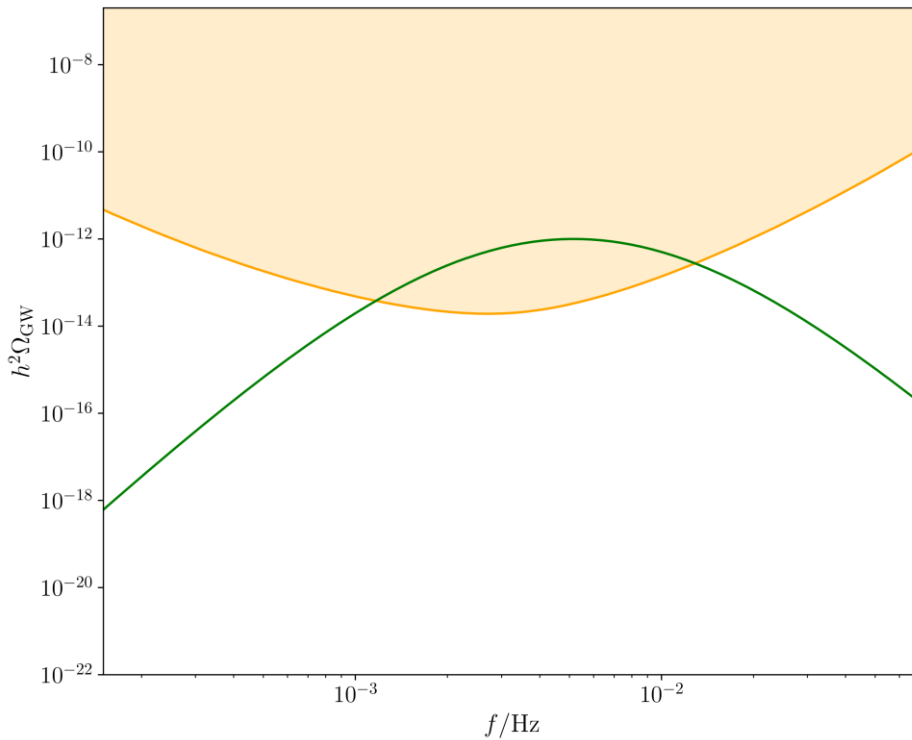
- Massive spin-2 field, time dependent sound speed

$$n_t \simeq \frac{2}{3} \frac{m_h^2}{H_*^2} \quad n_t \simeq |s_2|(2\nu - 1)$$

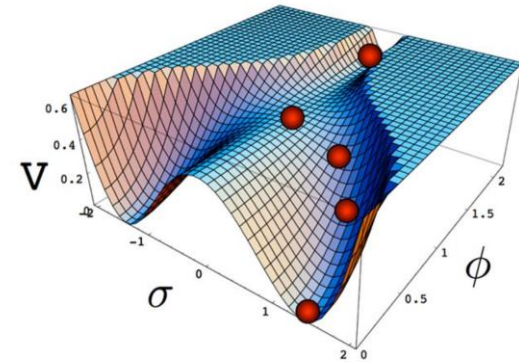
N. Bartolo et al. '16, D. Cannone '15,
L. Iacconi '20, E. Dimastrogiovanni '21

BROKEN POWER LAW

$$h^2\Omega_{\text{GW}}(f, \vec{p}) = h^2\Omega_* \frac{\left(\frac{f}{f_*}\right)^{n_{t,1}}}{\left[\frac{1}{2} \left(1 + \frac{f}{f_*}\right)\right]^{n_{t,1}-n_{t,2}}}$$



- **Hybrid mild waterfall phase**



S. Clesse, J. Garcia-Bellido '15

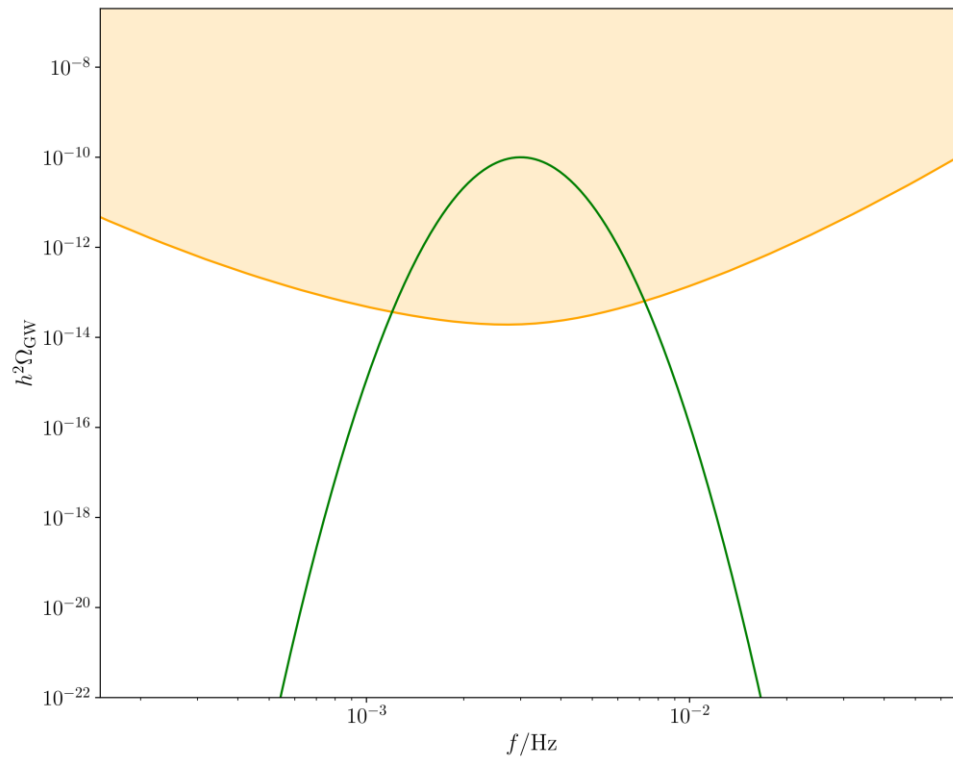
- **Non-standard thermal history**
Matter Domination / Kination...

$$n_{t,1} = n_{T,\text{prim}} + 2(3w_1 - 1)/(3w_1 + 1)$$

$$n_{t,2} = n_{T,\text{prim}} + 2(3w_2 - 1)/(3w_2 + 1)$$

GAUSSIAN BUMP

$$h^2 \Omega_{\text{GW}}^{\text{GBp}}(f, \vec{p}) = 10^{\alpha_*} \exp \left[-\frac{1}{2 \times 10^{2\Delta}} \log_{10}^2 \left(\frac{f}{f_*} \right) \right]$$



- **Producing phenomenon persisting during inflation**

- **Axion spectator**

R. Namba et al. '16

P. Campeti et al. '22

...

...

DOUBLE PEAK

$$\Omega_{\text{GW}}(f) = \beta \Omega_* \left(\frac{f}{\kappa_1 f_*} \right)^\alpha \left(\frac{\sqrt{\frac{2}{3}} - \frac{f}{f_*}}{\sqrt{\frac{2}{3}} - \kappa_1} \right)^{\frac{\alpha}{\kappa_1} (\sqrt{\frac{2}{3}} - \kappa_1)} \theta \left(\sqrt{\frac{2}{3}} - \frac{f}{f_*} \right) + \Omega_* \exp \left[-\frac{1}{2\rho^2} \ln^2 \left(\frac{f}{\kappa_2 f_*} \right) \right] \left\{ 1 + \text{Erf} \left[-\zeta \ln \left(\frac{f}{\kappa_2 f_*} \right) \right] \right\}$$

Scalar-induced GWs

Acquaviva et al. '02; Mollerach, Harari, Matarrese '03; Ananda, Clarkson, Wands '06;

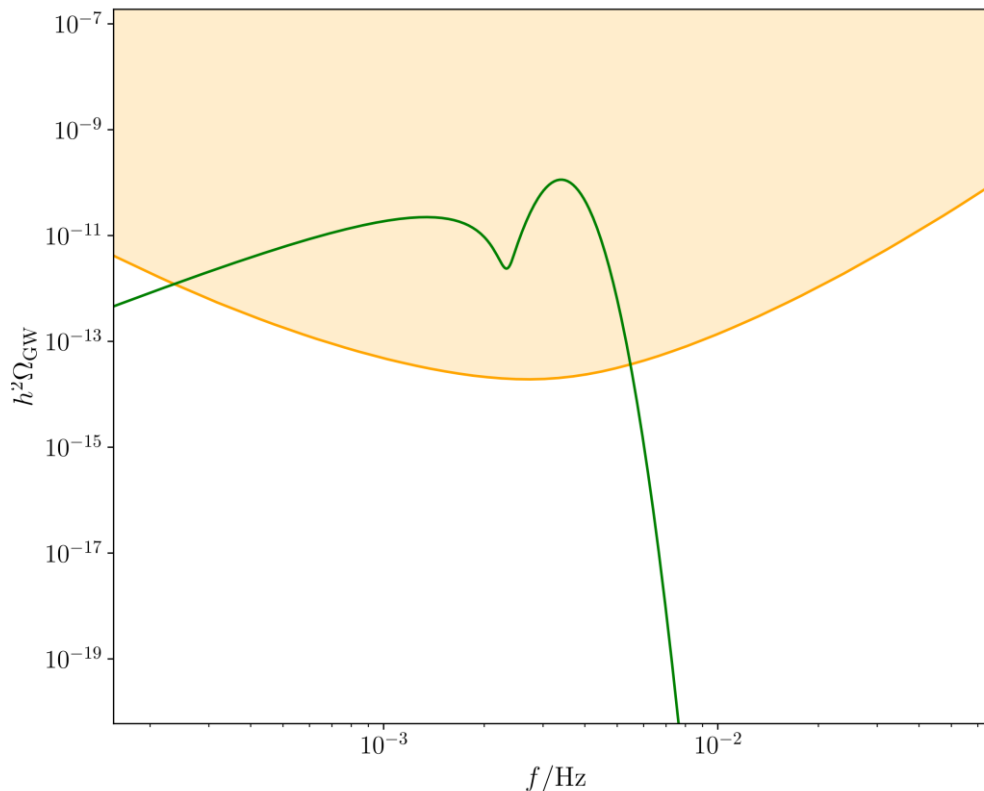
Log-normal Pzeta (resonance etc..)

$$\mathcal{P}_\zeta(k) \simeq \mathcal{P}_\zeta(k)^{\text{ln}} = \mathcal{A}_s \exp \left[-\frac{1}{2\Delta^2} \ln^2 \left(\frac{k}{k_*} \right) \right]$$

e.g. Cai et al. 10

Broken Power law Pzeta (Ultra-slow-roll..)

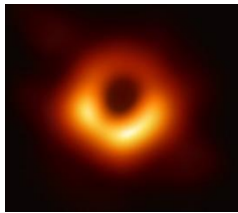
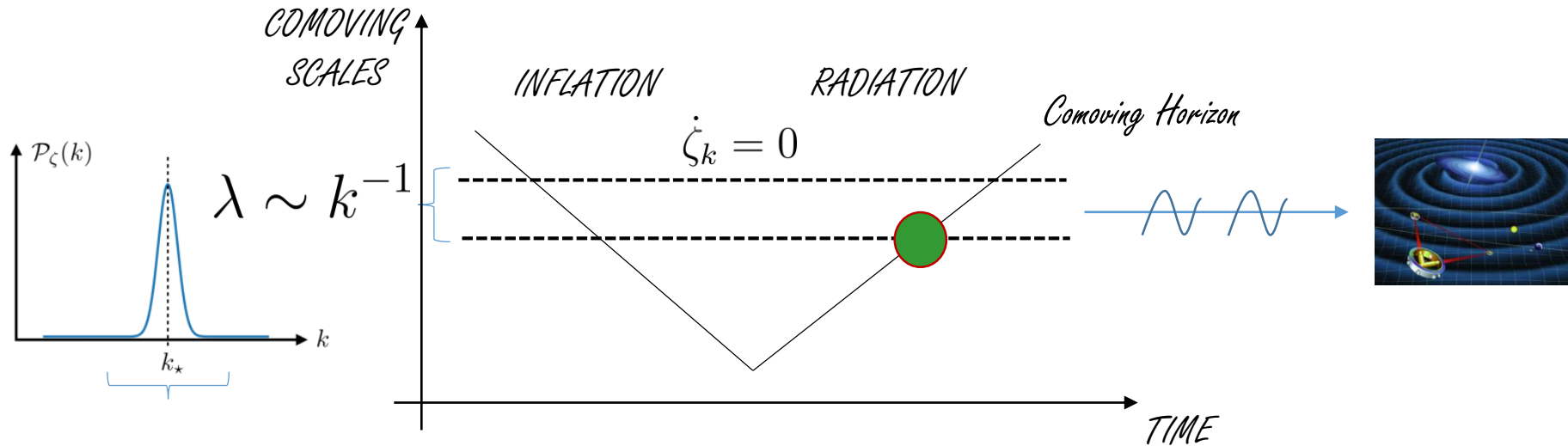
$$\mathcal{P}_\zeta(k) \simeq \mathcal{P}_\zeta(k)^{\text{bpl}} = \frac{\mathcal{A}_s (p_1 + p_2)}{\left[p_2 \left(\frac{k}{k_*} \right)^{-p_1} + p_1 \left(\frac{k}{k_*} \right)^{p_2} \right]}$$



SCALAR INDUCED (post-inflationary) SGWB

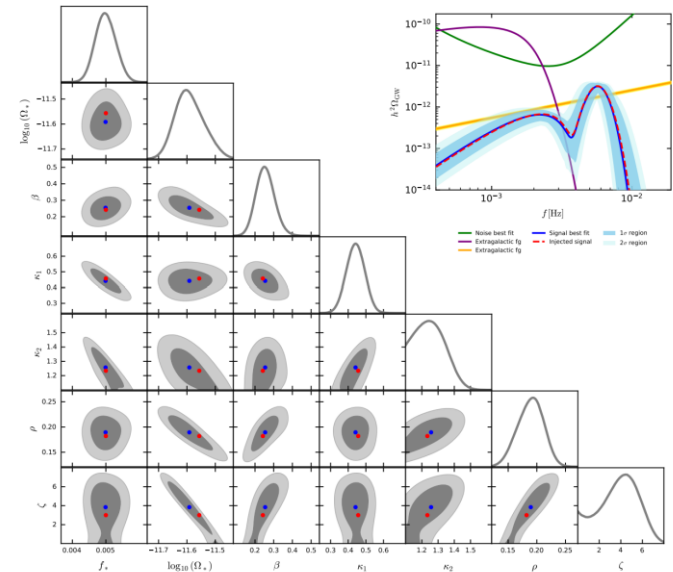
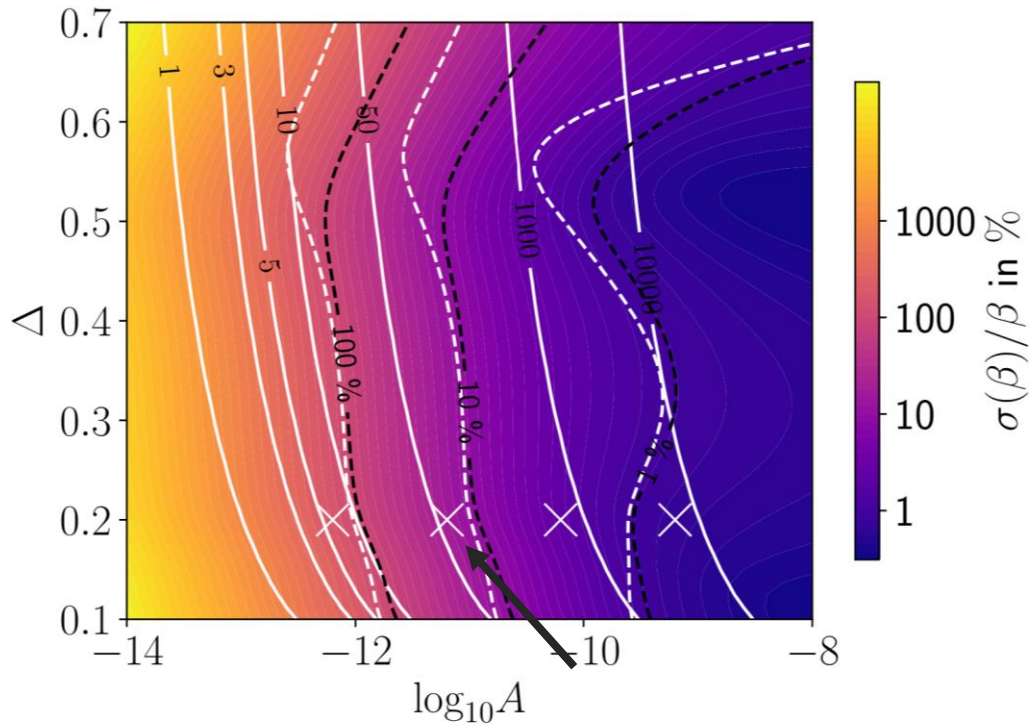
SCALAR PERTURBATIONS ACT AS A TENSOR SOURCE

$$\square h_{ij} = S_{ij}^{TT} \sim (\partial\zeta)^2$$



$$\Omega_{\text{GW}}(k) = \int \int \mathcal{T}(u, v) \mathcal{P}_\zeta(ku) \mathcal{P}_\zeta(kv)$$

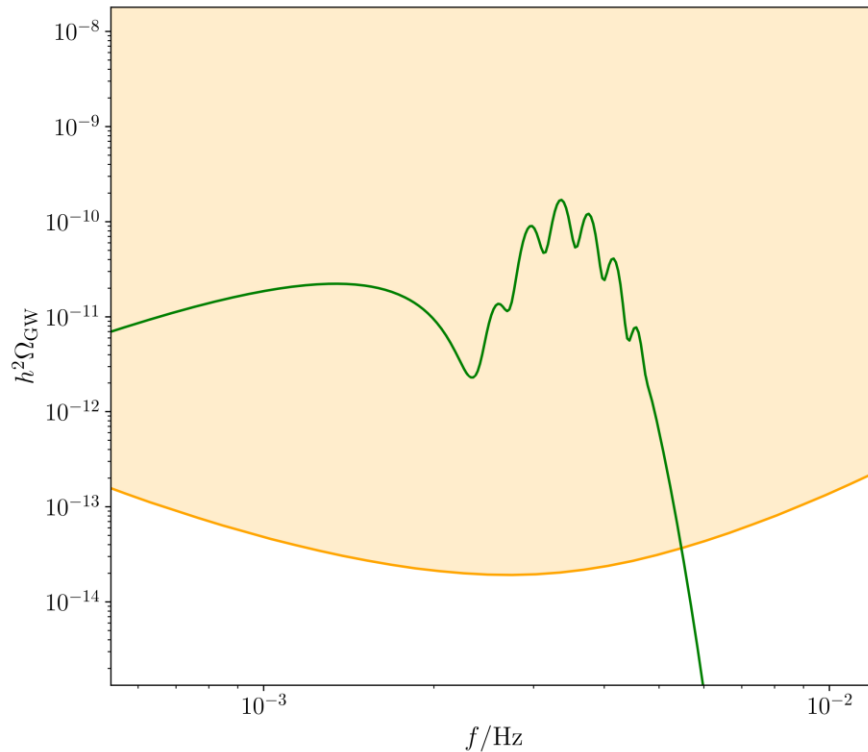
DOUBLE PEAK FISHER FORECAST



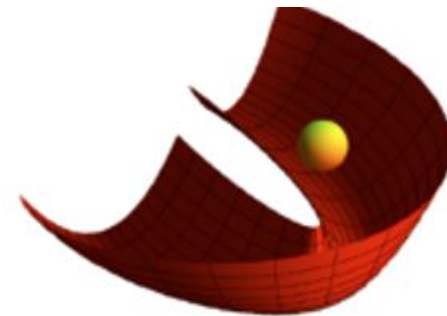
Direct link with ‘Fundamental physics’ is hard to achieve
 Extra layer of complexity wtr to phase transition, see E.Madge talk

LINEAR OSCILLATIONS

$$h^2 \Omega_{\text{GW}}^{\text{LO}}(f, \vec{p}) = \left[1 + \mathcal{A}_{\text{lin}} \cos(\omega_{\text{lin}} f + \theta_{\text{lin}}) \right] h^2 \Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$



Sharp feature - Localized Event
(Step in the potential / 2-stage / turn in field-space etc..)

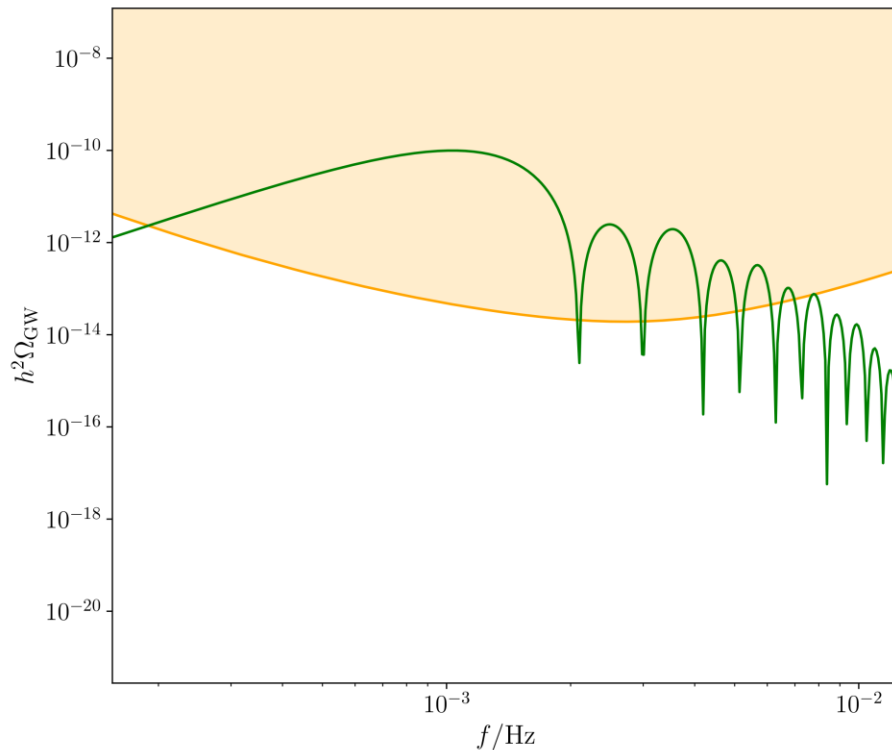


$$\omega_{\text{lin}} \sim 2c_s^{-1} / \bar{f}$$

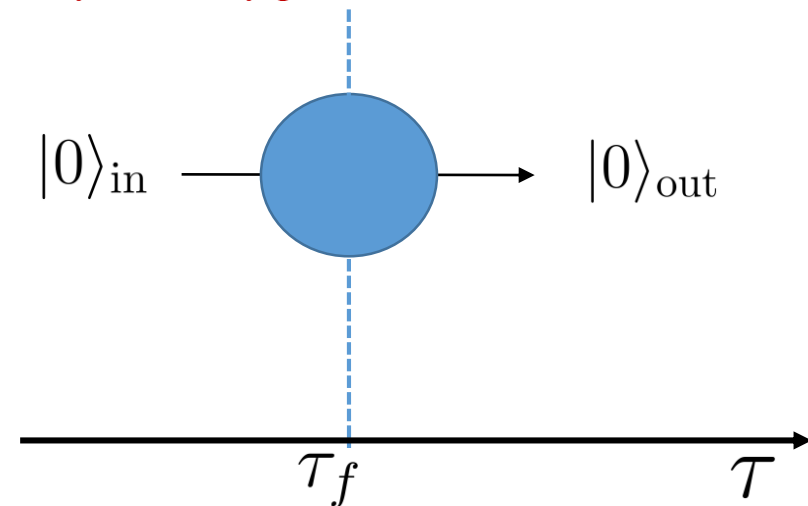
J. Fumagalli, S.Renaux-Petel, L.Witkowski '20,
M. Braglia, X. Chen, D.K. Hazra '20
I. Dalianis et al. '21

EXCITED STATES

$$h^2 \Omega_{\text{GW}}^{\text{DB}}(f, \vec{p}) = \frac{10^{\log_{10} \Omega_*}}{0.0065} \frac{1}{(\omega f)^3} \left(1 - \frac{(\omega f)^2}{16\gamma^2}\right)^2 \cdot \left(\sin(\omega f/2) - 4 \frac{(1 - \cos(\omega f/2))}{\omega f}\right)^2$$



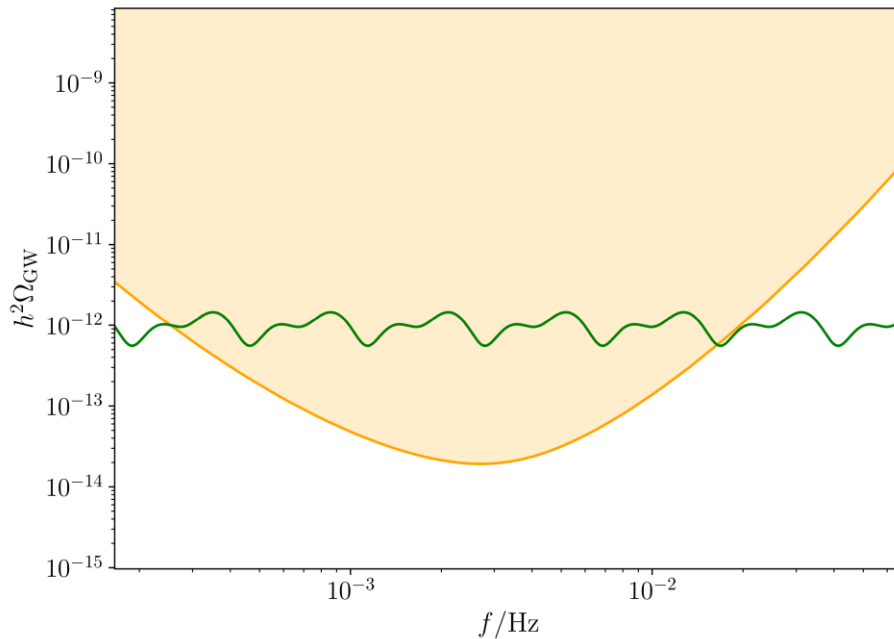
Dynamically generated excited state



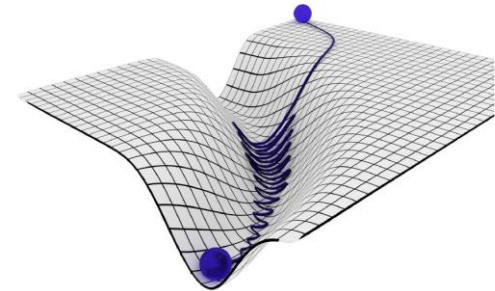
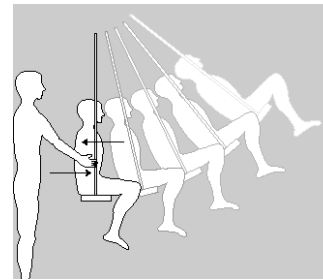
J. Fumagalli, et al. '21,
Z. Peng et al '21
K. Inomata '21

RESONANT LOGARITHMIC OSCILLATIONS

$$h^2\Omega_{\text{GW}}^{\text{RO}}(f, \vec{p}) = \left[1 + \mathcal{A}_1(A_{\log}, \omega_{\log}) \cos(\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,1}) + \mathcal{A}_2(A_{\log}, \omega_{\log}) \cos(2\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,2}) \right] h^2\Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$

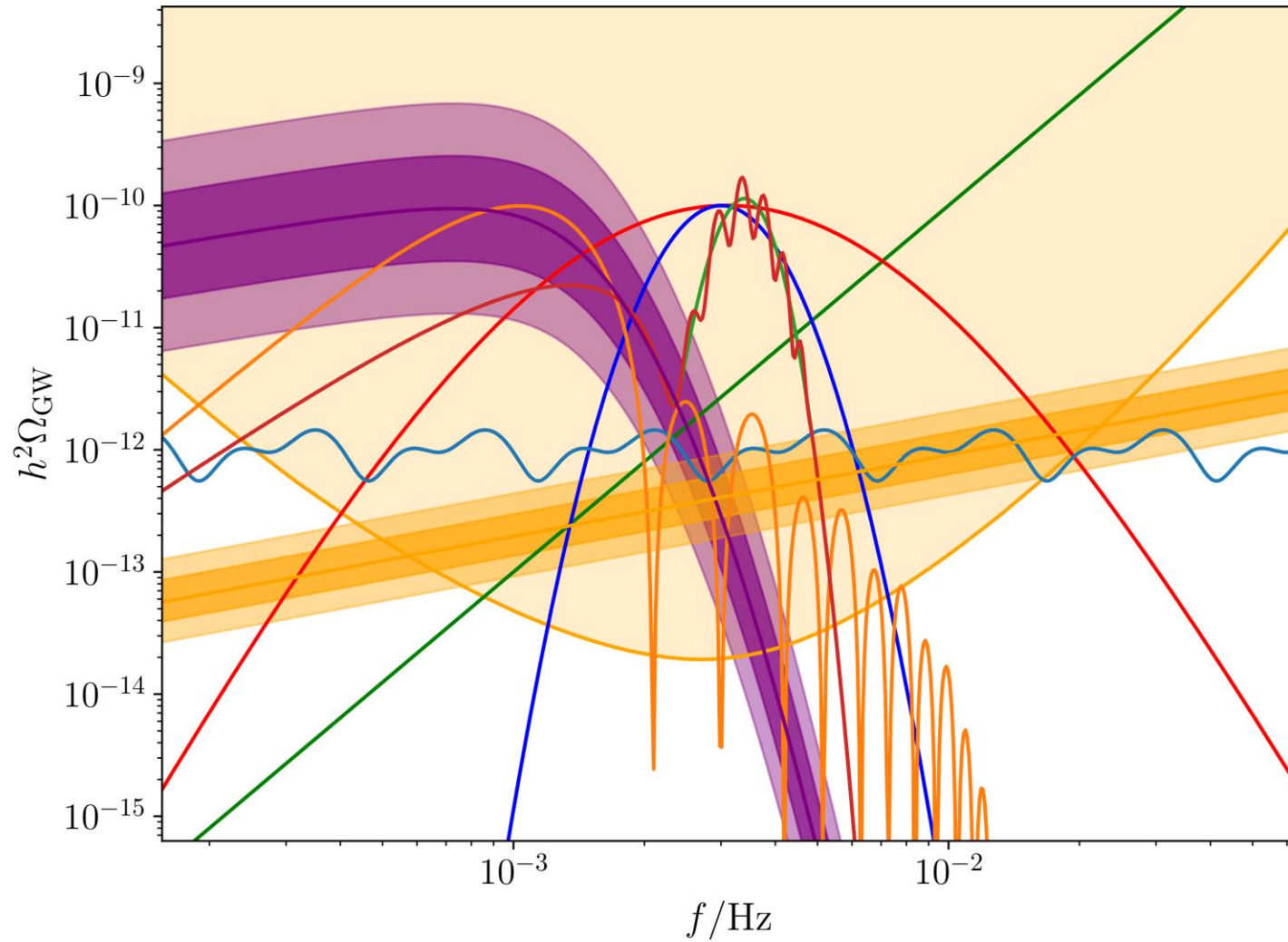


Resonant Features– Oscillations of BkG
(Ex. Monodromy inflation, double turn, in-out horizon..)



J.Fumagalli et al. '21, Battacharya, Zavala '22,
N. Mavromotos, V. Spanos, I. Stamou '22 ..

ALL TOGETHER



CONCLUSIONS

- Stochastic gravitational wave background has the potential to unveil new physics not reachable by any other means
- It provides a new window to test inflation at small scales.
Reasons to go beyond the vanilla inflationary scenarios lead to features in the SGWB
- Building a template databank: The List is not exhaustive (and will never be)
- Often not a direct link between fundamental parameters and spectral shapes
 - Simple \rightarrow Degeneracy
 - Complicated \rightarrow Weak link
- Effort in making the connection with fundamental physics explicit
- Effort in identifying pattern which belongs ‘only’ to inflation