Gravitational wave signals from cosmological phase transitions and cosmic strings

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10th LISA Cos-WG workshop, Stavanger, 6 VI 2023

POLSKIE POWROTY
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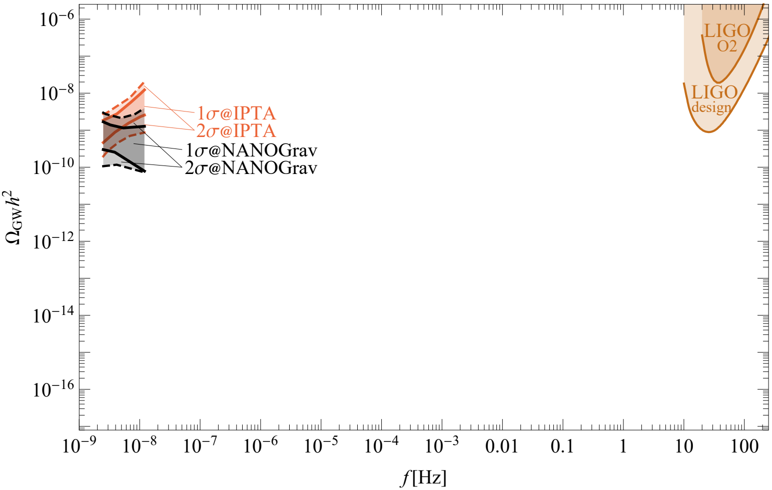

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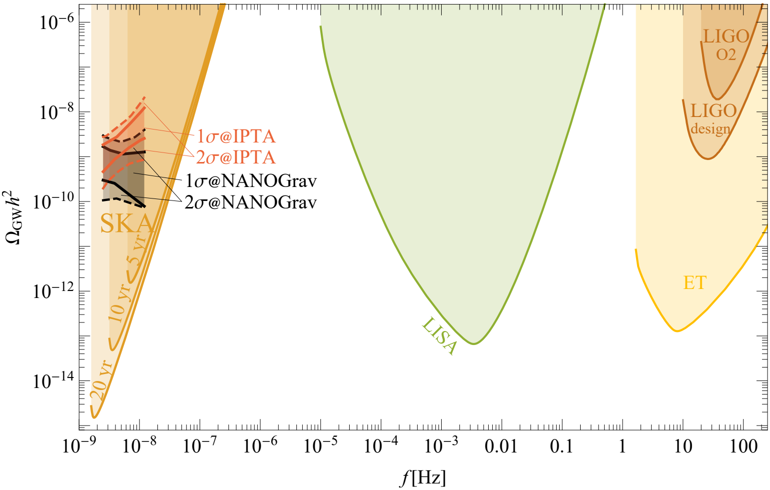
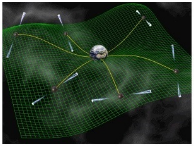
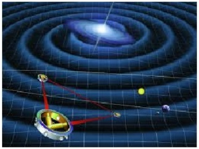
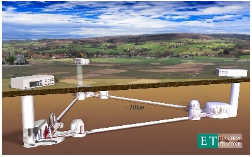




Pulsar Timing
[David Champion/NASA/JPL]



LISA
wiki/Laser_Interferometer_Space_Antenna



Einstein Telescope
www.et-gw.eu

First Order Phase Transition: bubble nucleation

- Temperature corrections to the potential

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4$$

- EOM $\rightarrow$ bubble profile

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial \phi} = 0,$$

$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

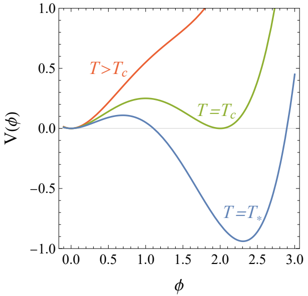
- $\mathcal{O}(3)$ symmetric action

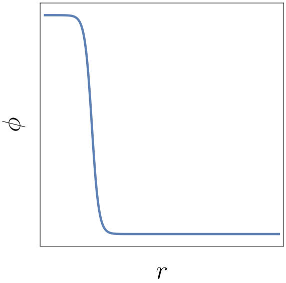
$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- nucleation temperature

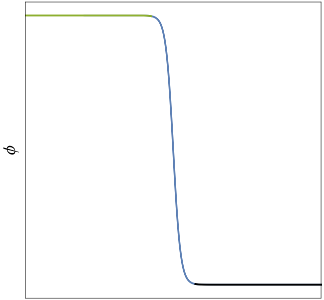
$$\frac{\Gamma}{H^4} \approx \left(\frac{T}{H}\right)^4 \exp\left(-\frac{S_3(T)}{T}\right) \approx 1$$

Linde '81 '83

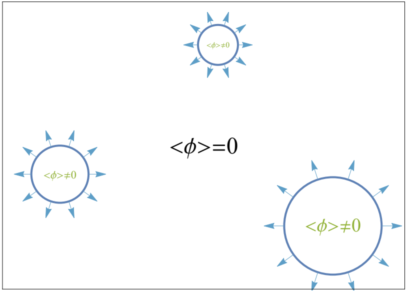




First Order Phase Transition



$\Longrightarrow$



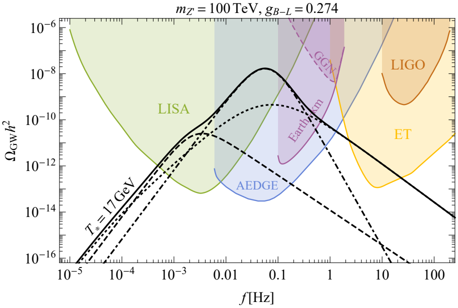
- Strength of the transition

$$\alpha \approx \left. \frac{\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}}{\rho_R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Average size of bubbles upon collision (Characteristic scale)

$$H R_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H} \right)^{-1}$$

Gravitational waves from a PT



- Gravitational wave signals are produced by three main mechanisms:
  - collisions of bubble walls $\Omega_{\rm col} \propto \left(\kappa_{\rm col}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)^2$

    Kamionkowski '93, Konstandin '08 '17, Hindmarsh '18 '20, Lewicki '19 '20 '22,
  - sound waves $\Omega_{\rm sw} \propto \left(\kappa_{\rm sw}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)(H\tau_{sw})$

    Hindmarsh '13 '15 '17 '19 '21 '22, Ellis '18 '19 '20, Jinno '20 '22 Lewicki '22
  - turbulence $\Omega_{\rm turb} \propto\ ?$

    Caprini '06 '09 '20, Brandenburg '10 '12 '17, Roper-Pol '17 '19 '21, Ellis '19 '20

Sound Waves

- Simulation of a scalar coupled to the plasma

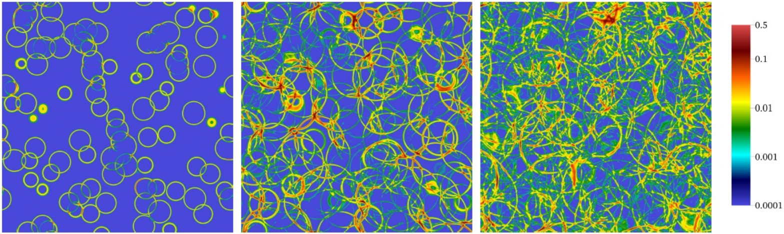


FIG. 4. Slices of fluid kinetic energy density E/T_c^4 at $t = 500\,T_c^{-1}$, $t = 1000\,T_c^{-1}$ and $t = 1500\,T_c^{-1}$ respectively, for the $\eta/T_c = 0.15$, $N_b = 988$ simulation.

- Fit to the GW spectrum

$$\Omega_{\rm gw} \propto \left(\frac{f}{f_p}\right)^3 \left(\frac{7}{4+3\,(f/f_p)^2}\right)^{\frac{7}{2}}$$

Hindmarsh, Huber, Rummukainen, Weir, arXiv: 1504.03291, 1704.05871

Sound Waves

- Higgsless simulation of the plasma

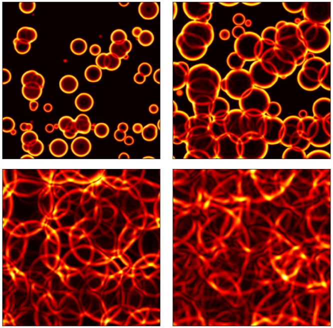
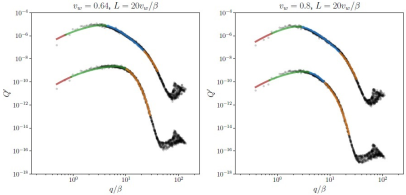


Figure 4: Kinetic energy v^2 in different simulation snapshots: $t = 2.7/\beta$ (top left), $5.4/\beta$ (top right), $10.8/\beta$ (bottom left) and $20.1/\beta$ (bottom right). We use box size $L = 40v_w/\beta$, weak transitions and $v_w = 0.8$.

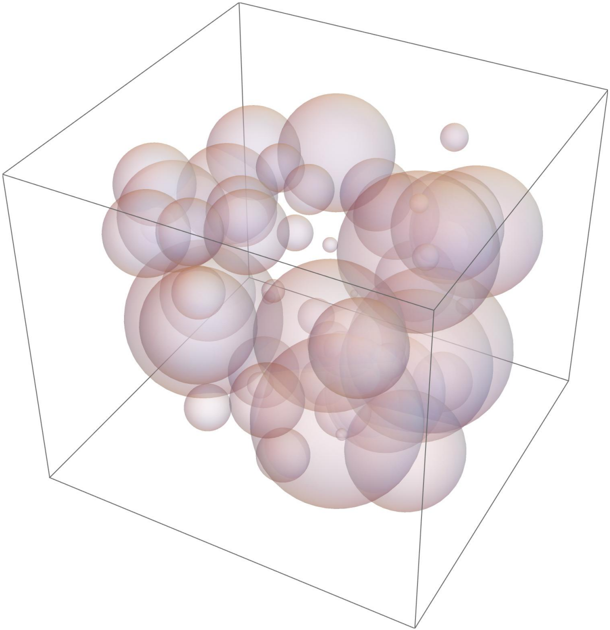


- Fit to the GW spectrum

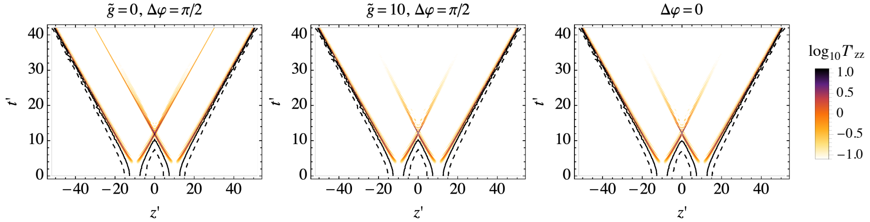
$$\Omega_{\rm gw} \propto \frac{(f/f_1)^3}{1 + (f/f_1)^2[1 + (f/f_2)^4]}, \quad f_2/f_1 \approx 1/\xi_{\rm shell}$$

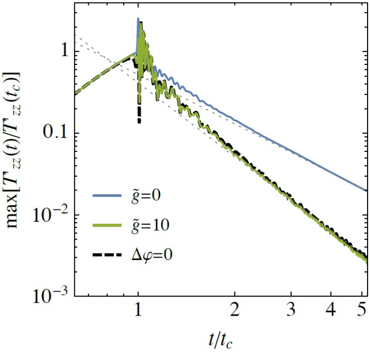
Jinno, Konstandin, Rubira, Stomberg, arXiv: 2209.04369

Strong transitions: computation of the GW spectrum



Abelian Higgs Model: Energy Scaling





- scaled gauge coupling:

$$\tilde{g} = \frac{gv^2}{\sqrt{\Delta V}}$$

- Global Symmetry breaking:

$$T_{zz} \propto R^{-2}$$

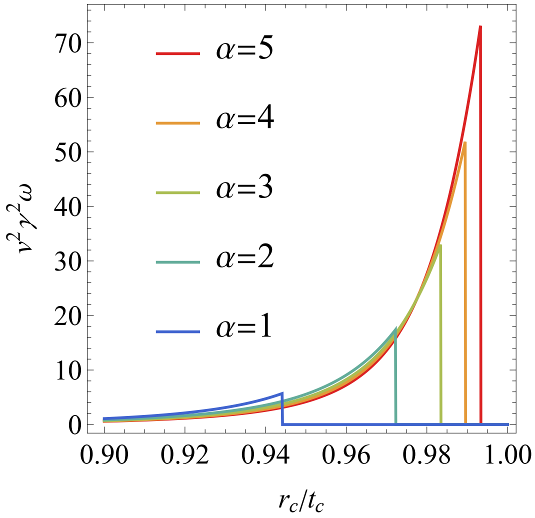
- Gauge Symmetry breaking:

$$T_{zz} \propto R^{-3}$$

ML, Ville Vaskonen arXiv: 2007.04967

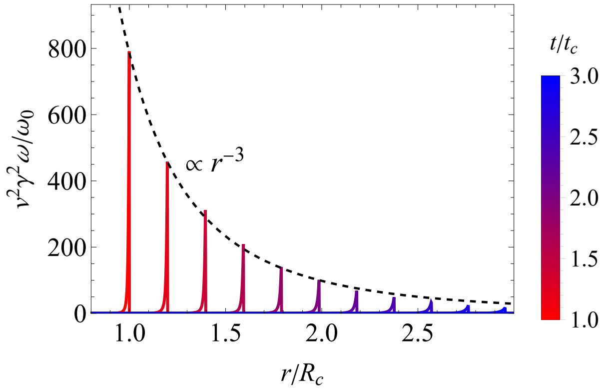
Fluid Shells

- Plasma profiles for $v_w \gtrsim v_J$



Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 20$ and $\gamma_w = 50$



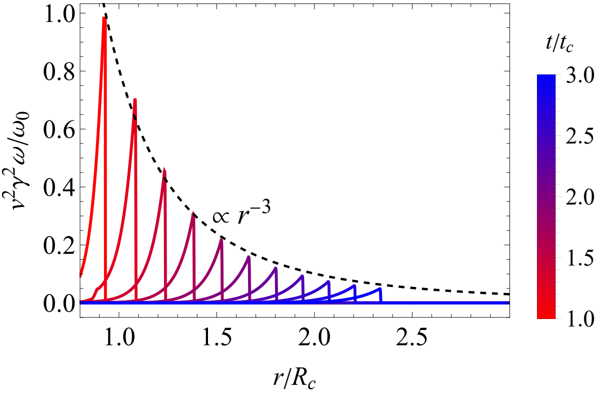
- Fluid shells with $\alpha \gg 1$:

$$T_{zz} \propto R^{-3}$$

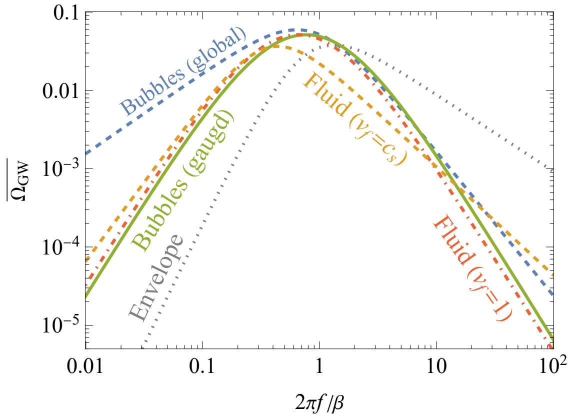
ML, Ville Vaskonen arXiv:2208.11697

Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 0.5$ and $\gamma_w = 3$



ML, Ville Vaskonen arXiv:2208.11697



- Resulting spectrum:

$$\overline{\Omega_{GW}} = \frac{A\,(a+b)^c}{\left[b\left(\frac{f}{f_p}\right)^{-\frac{a}{c}} + a\left(\frac{f}{f_p}\right)^{\frac{b}{c}}\right]^c}$$

	Bubbles		Fluid	
	Global($T \propto R^{-2}$)	Gauged ($T \propto R^{-3}$)	$v_{\rm fluid} = 1$	$v_{\rm fluid} = c_s$
100 A	5.93 ± 0.05	5.13 ± 0.05	5.14 ± 0.04	3.64 ± 0.02
a	1.03 ± 0.04	2.41 ± 0.10	2.36 ± 0.09	2.02 ± 0.08
b	1.84 ± 0.17	2.42 ± 0.11	2.36 ± 0.09	1.38 ± 0.06
c	1.91 ± 0.29	1.45 ± 0.34	3.69 ± 0.48	1.48 ± 0.32
$2\pi f_p/\beta$	1.33 ± 0.19	0.64 ± 0.09	0.66 ± 0.04	0.44 ± 0.04

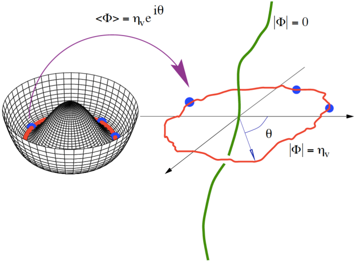
ML, Ville Vaskonen arXiv:2208.11697
ML, Ville Vaskonen, arXiv: 2007.04967

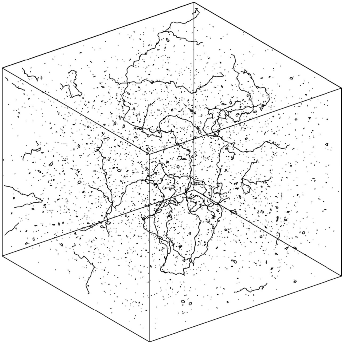
Cosmic Strings

- Charged complex scalar field

$$V = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

- Horizon size at early time (high temperature) $d_H \propto M_p/T^2$





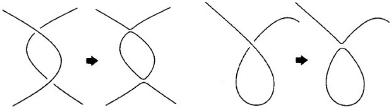
Christophe Ringeval (Adv.Astron. 2010)   Vilenkin and Shellard ’94

Cosmic String network evolution

- Static string network would red-shift as

$$\rho_\infty \propto a^{-2}$$

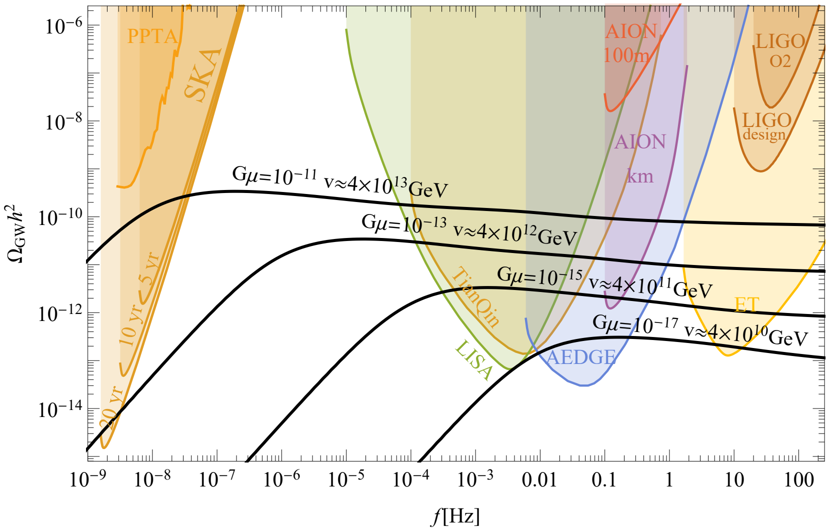
- strings intercommute on collision



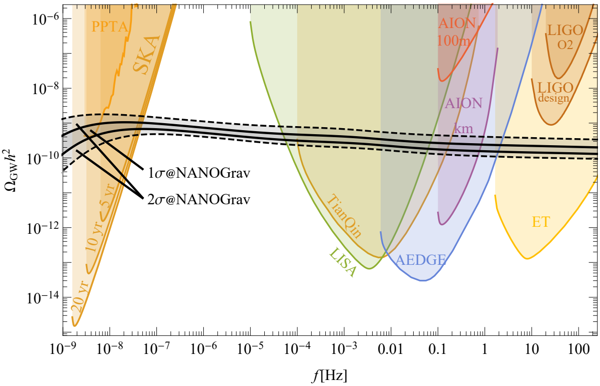
- overall energy density of the network scales with total energy density

$$\frac{\rho_\infty}{\rho_{\rm tot}} \propto G\mu \propto \frac{v^2}{M_p^2}$$

Stochastic GW background from Cosmic Strings



Cosmic String fit to NANOGrav data



- results within the 68% CL

$$G\mu \in (4 \times 10^{-11}, 10^{-10})$$

- results within the 95% CL

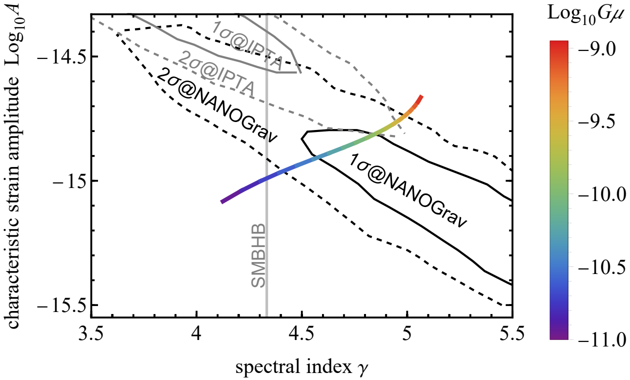
$$G\mu \in (2 \times 10^{-11}, 3 \times 10^{-10})$$

John Ellis, ML arXiv: 2009.06555

power-law fit to PTA data

- power-law fit to the data

$$\Omega(f) = \frac{2\pi^2}{3H_0^2} A^2 f_{yr}^2 \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}$$



- Data

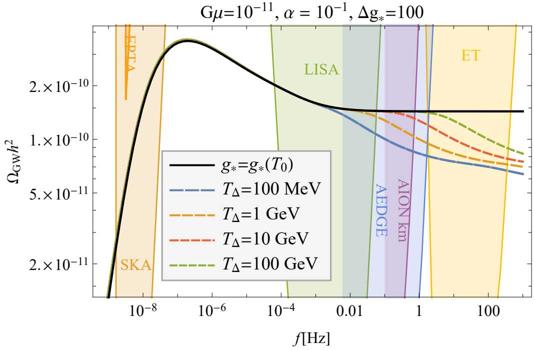
NANOGrav 2009.04496 PPTA 2107.12112 EPTA 2110.13184 IPTA 2201.03980

Cosmic Strings GW signal and expansion history

- We add Δg_* new degrees of freedom at T_Δ

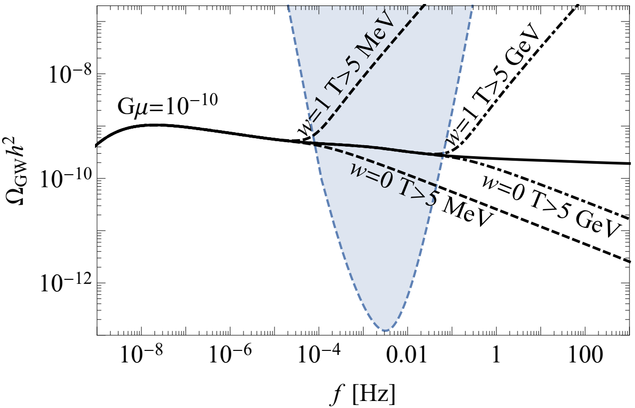
$$g_*(T) = \begin{cases} g_*(T_0) & \text{for} \quad T < T_\Delta \\ g_*(T_0) + \Delta g_* & \text{for} \quad T > T_\Delta \end{cases}$$

- An example with $\Delta g_* = 100$



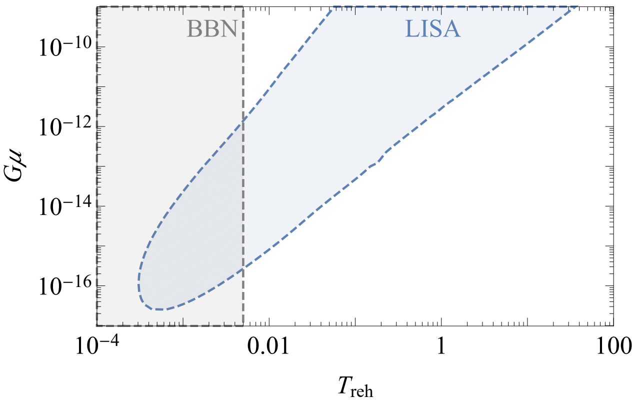
Y. Cui, ML, D. E. Morrissey, J. D. Wells, arXiv: 1711.03104, 1808.08968, 1912.08832

- More dramatic modifications of the expansion for example an early kination ($w = 1$) and early MD era ($w = 0$)



LISA Cosmology Working Group arXiv: 2204.05434

- Reach of LISA in terms of the temperature of the modification



LISA Cosmology Working Group arXiv: 2204.05434

Conclusions

Phase transitions

- Sound waves produce a broken power law spectrum in very strong transitions and a double broken power law in weak transitions.

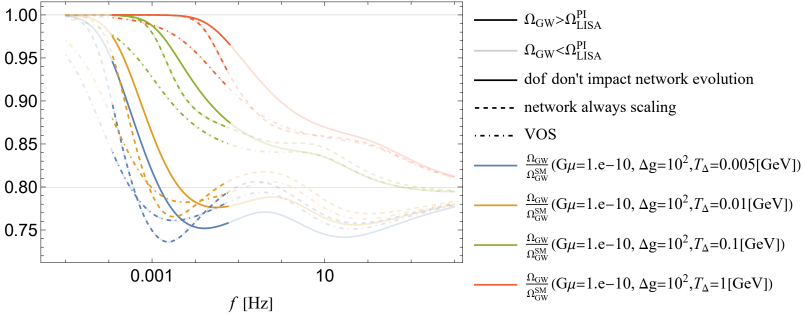
Cosmic strings

- Cosmic strings provide a very good fit to the 12.5yr NANOGrav data.
- LISA will be able to verify this and if confirmed provide a powerful tool for probing the cosmological evolution to time well before the currently available BBN data.

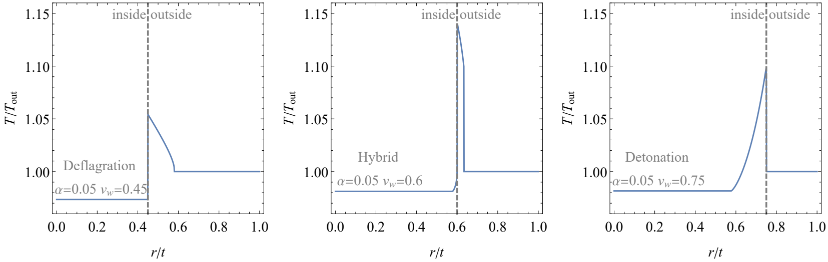
Thank you for your attention!

Backup slides

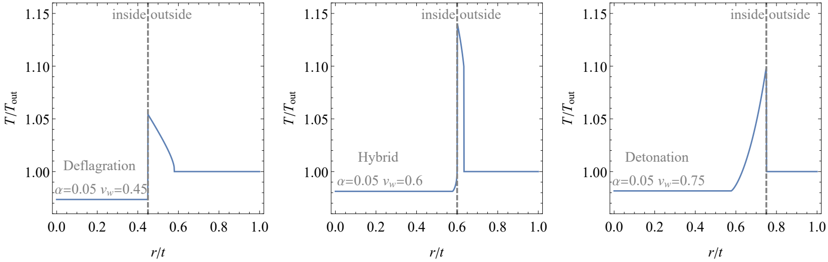
- impact of network modeling on the imprint of cosmological modification in the spectrum



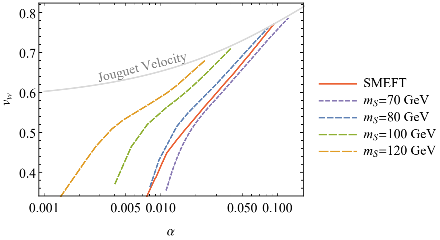
Wall Velocity



Wall Velocity



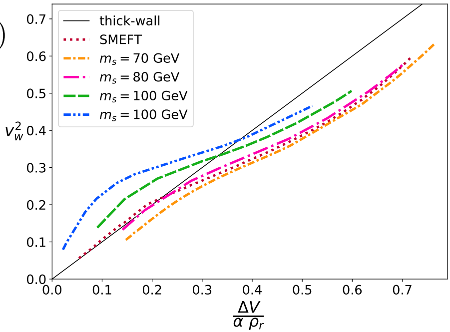
- No solutions found beyond $v_J = \frac{1}{\sqrt{3}} \frac{1+\sqrt{3\alpha^2+2\alpha}}{1+\alpha}$.



ML, Marco Merchand, Mateusz Zych, JHEP **02** (2022) 017, arXiv: 2111.02393

Wall Velocity analytic approximation

$$v_w = \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_R}} & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_R}} < v_J(\alpha) \\ 1 & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_R}} \geq v_J(\alpha) \end{cases}$$



- Here: $\alpha = \frac{1}{\rho_R}\left(\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}\right)$
- Formula does not require solving transport equations
- Only the form of the potential is important

ML, Marco Merchand, Mateusz Zych, JHEP **02** (2022) 017, arXiv: 2111.02393

John Ellis, ML, Marco Merchand, José Miguel No, Mateusz Zych arXiv:2210.16305

Lattice realisation

- Simple high temperature expansion

$$V(\phi, T) = \frac{1}{2}\left(T^2 - T_0^2\right)\phi^2 - \frac{1}{3}\delta T\phi^3 + \frac{1}{4}\lambda\phi^4$$

- The energy-momentum tensor for the field and the fluid:

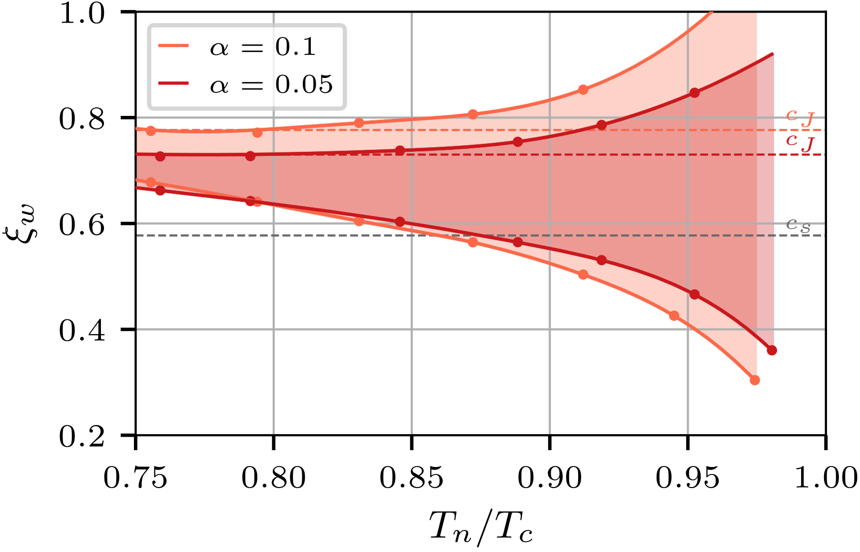
$$T^{\mu\nu}_{\text{field}} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi\right)$$
$$T^{\mu\nu}_{\text{fluid}} = wu^\mu u^\nu + g^{\mu\nu}p$$

- effective coupling of the fluid and scalar:

$$\nabla_\mu T^{\mu\nu}_{\text{field}} = -\nabla_\mu T^{\mu\nu}_{\text{fluid}} = \frac{\partial V(\phi, T)}{\partial\phi}\partial^\nu\phi + \eta u^\mu\partial_\mu\phi\partial^\nu\phi$$

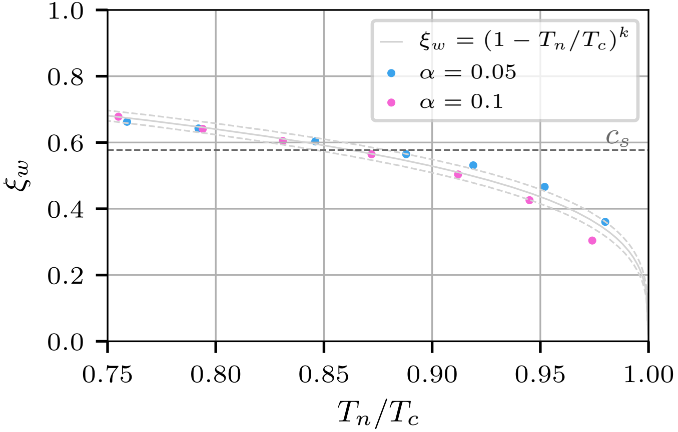
Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

Hydrodynamical obstruction: can all v_w be realised?



Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

Hydrodynamical obstruction: numerical fit

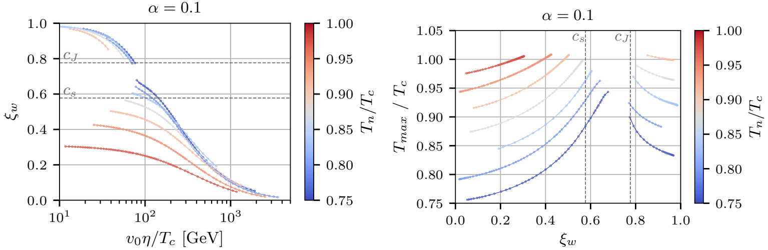


- Simple numerical fit accurate for relatively strong PTs

$$v_w = \left(1 - \frac{T_n}{T_c}\right)^k, \quad \text{with} \quad k = 0.2768 \pm 0.0055$$

Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

Hydrodynamical obstruction: numerical fit



Hydrodynamical obstruction: numerical fit