WEIZMANN INSTITUTE OF SCIENCE

מכון ויצמן למדע

Faculty of Physics

הפקולטה לפיסיקה

Update on the Phase Transition Parameter Estimation Project

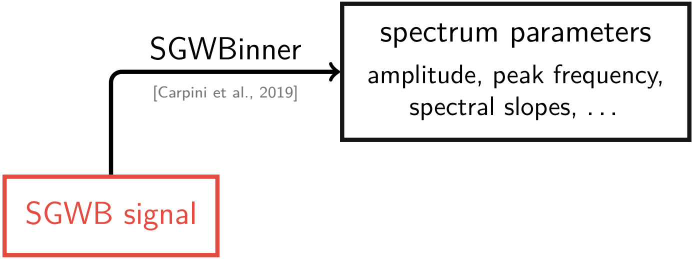
C. Caprini, R. Jinno, M. Lewicki, Eric Madge, M. Merchand, G. Nardini, M. Pieroni, A. Roper Pol, and V. Vaskonen

Parameter reconstruction for cosmological phase transitions

SGWB signal

cf. also Deanna's talk on Wednesday

Parameter reconstruction for cosmological phase transitions



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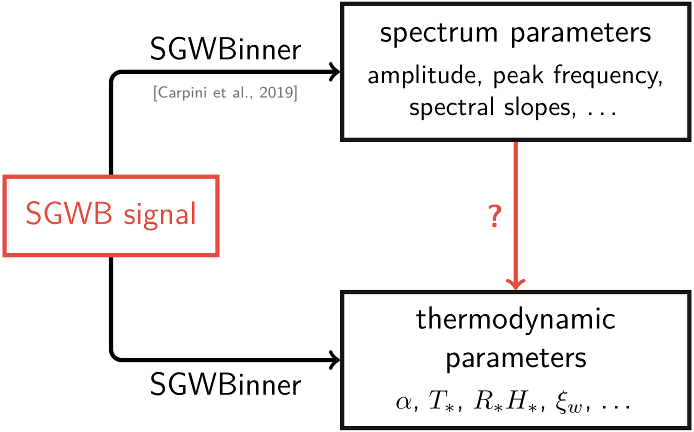

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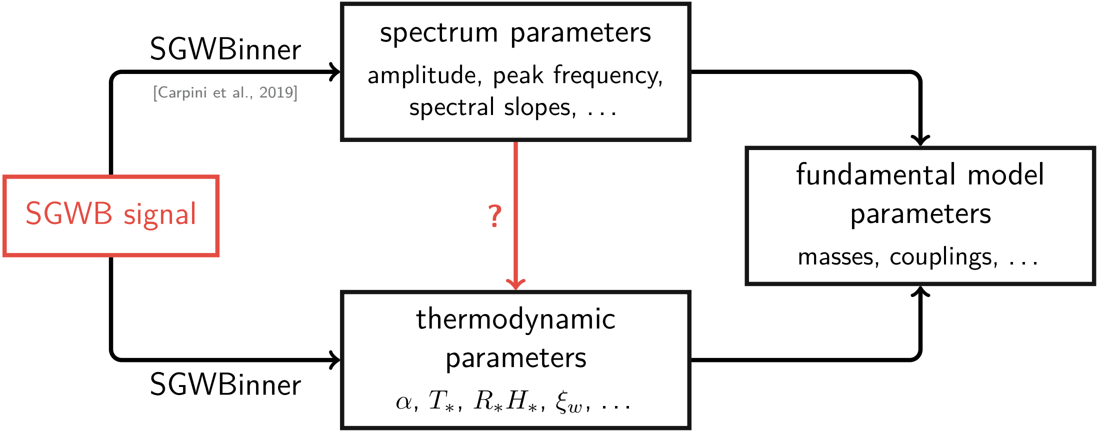

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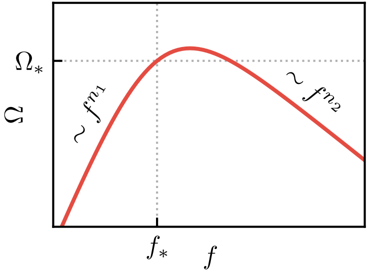
Templates

broken power-law

$$\Omega_* \mathcal{N} \left(\frac{f}{f_*}\right)^{n_1} \left[1 + \left(\frac{f}{f_*}\right)^{a_1}\right]^{\frac{n_2 - n_1}{a_1}}$$



- bubble collisions: [Lewicki & Vaskonen, 2023]
  $(n_1, n_2, a_1) = (2.4, -2.4, 4)$

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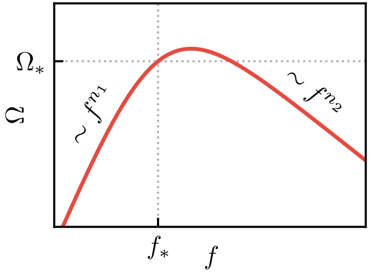

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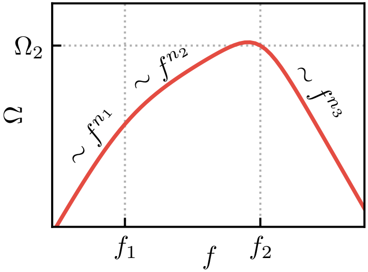
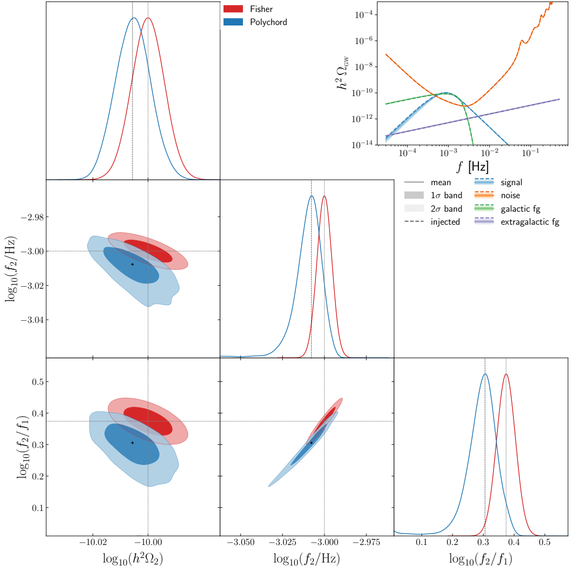
double broken power-law

$$\Omega_2 \mathcal{N}\left(\frac{f}{f_1}\right)^{n_1}\left[1+\left(\frac{f}{f_1}\right)^{a_1}\right]^{\frac{n_2-n_1}{a_1}}\left[1+\left(\frac{f}{f_2}\right)^{a_2}\right]^{\frac{n_3-n_2}{a_2}}$$



- sound waves: [Jinno et al., 2023]
  $(n_1, n_2, n_2, a_1, a_2) = (3, 1, -3, 2, 4)$
- MHD turbulence: [Roper Pol et al., 2022]
  $(n_1, n_2, n_2, a_1, a_2) = (3, 1, -\frac{8}{3}, 4, 2.15)$

Polychord/MCMC vs. Fisher analysis



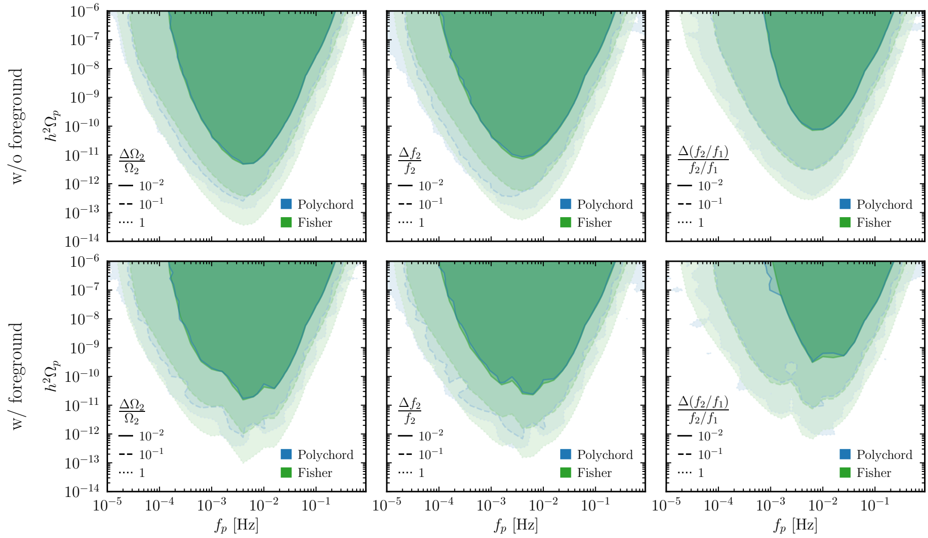
$h^2\Omega_2 = 10^{-10}$, $f_2 = 1\,\mathrm{mHz}$, $\frac{f_2}{f_1} = \frac{3+\sqrt{3}}{2}$

- estimation of parameter reconstruction reach based on generated data + Polychord takes a lot of time
- alternative: Fisher analysis

$$\mathcal{C}_{ij}^{-1} = \mathcal{F}_{ij} = -\frac{\partial^2 \log \mathcal{L}}{\partial\theta_i \partial\theta_j}$$

works well if posteriors are approximately Gaussian

Spectral parameter reconstruction



$$(n_1,\, n_2,\, n_3,\, a_1,\, a_2) = (3,\, 1,\, -3,\, 2,\, 4)$$

Reconstructing thermodynamics parameters

- 3 spectral params.: $\Omega_2,\ f_2,\ f_1$
  4 therm. params.: $\alpha,\ H_*R_*,\ \xi_w,\ T_*$
  $\Longrightarrow$ degenerate

- consider fixed T_*:
  $$\Omega_2 \propto \frac{\xi_{\text{shell}}}{\xi_w} \begin{cases} K^2\, H_*R_* & \text{if } H_*\tau > 1 \\ K^{3/2}(H_*R_*)^2 & \text{if } H_*\tau < 1 \end{cases}$$
  $$f_2 \propto T_*/(H_*R_*)$$
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- repeat for random $\log_{10}(T_*/\text{GeV})$

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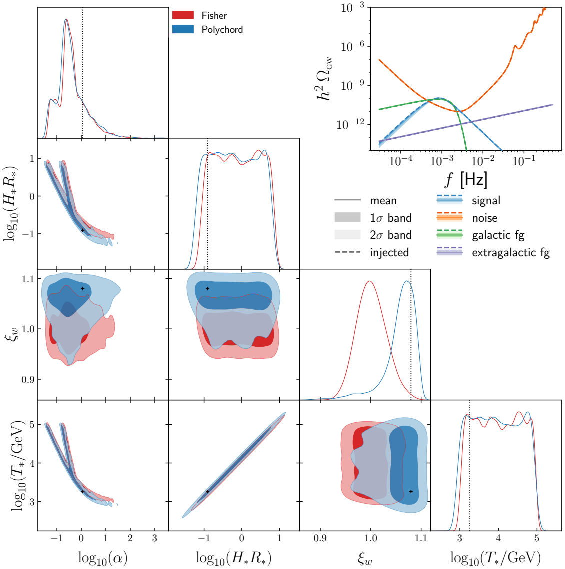

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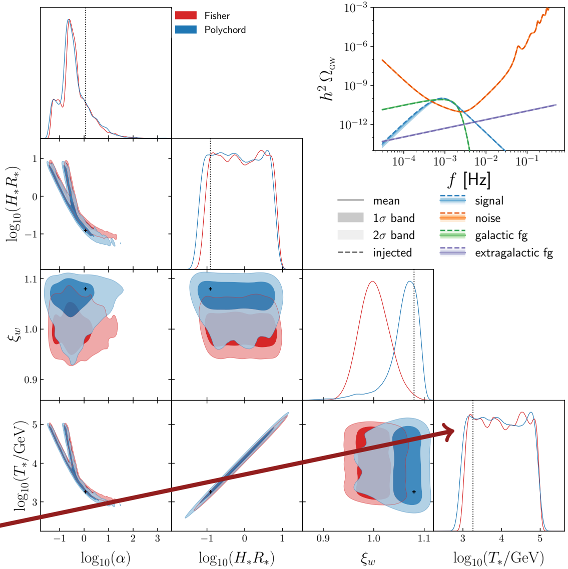

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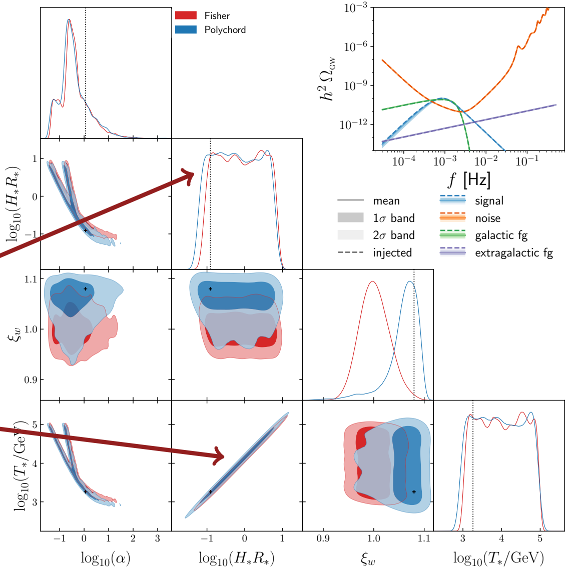

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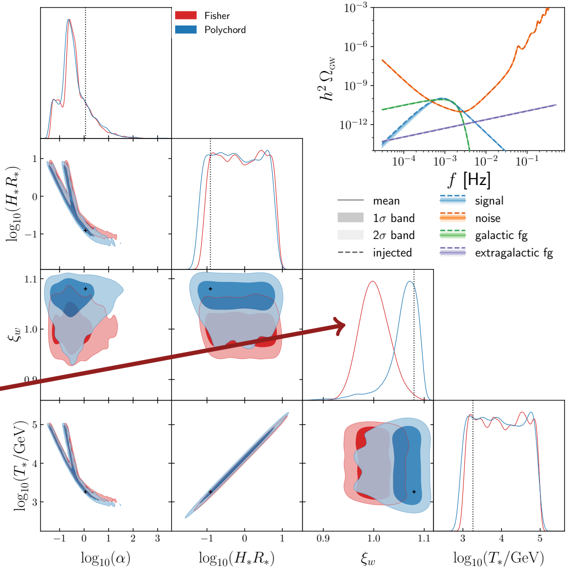

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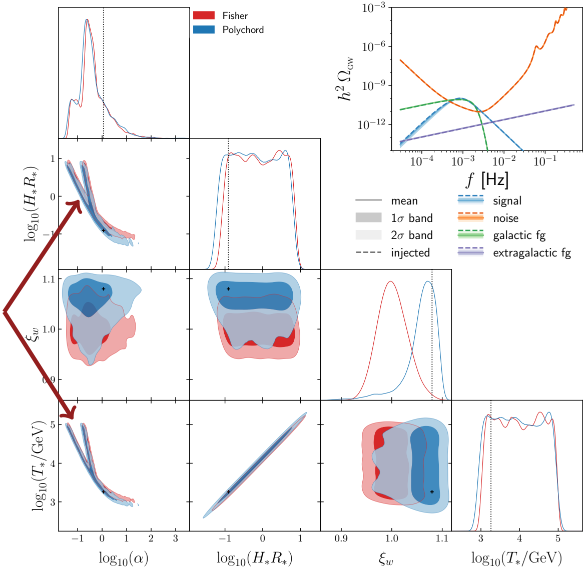

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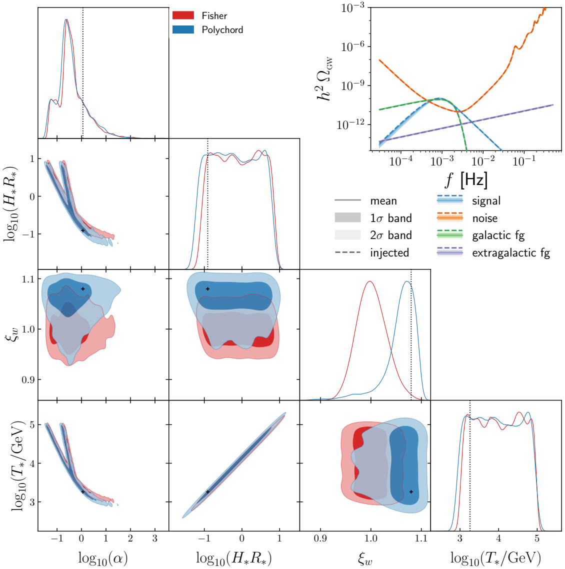

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Soundwaves + turbulence

- sound waves + turbulence
  $\implies$ additional parameter: ϵ
  $\implies$ degeneracy broken?

- analytically:

$$
\begin{aligned}
(f_1^{\text{turb}}/f_2^{\text{turb}})^2 &\propto \epsilon\, K \\
\Omega_2^{\text{turb}} &\propto \epsilon^2 K^2 (H_* R_*)^2
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$$

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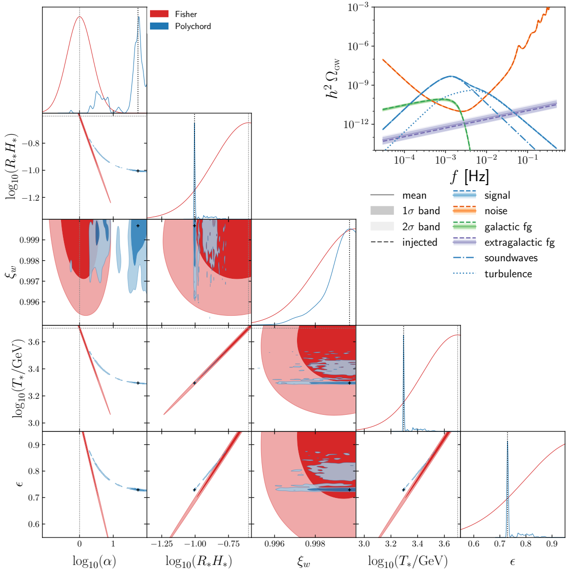
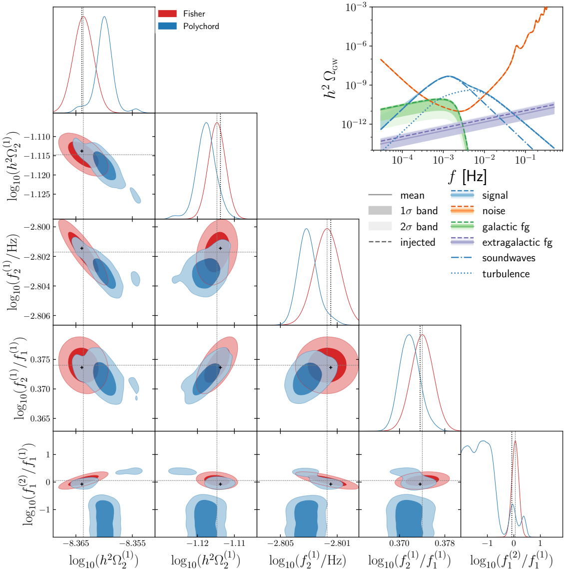
- BUT:
  - degeneracies remain
  - inputs not reconstructed
  - Fisher and Polychord disagree



$\alpha = 1,\ R_* H_* = 0.25,\ \xi_w = 1,\ T_* = 500\ \text{GeV},\ \epsilon = 1$

Two double broken power-laws



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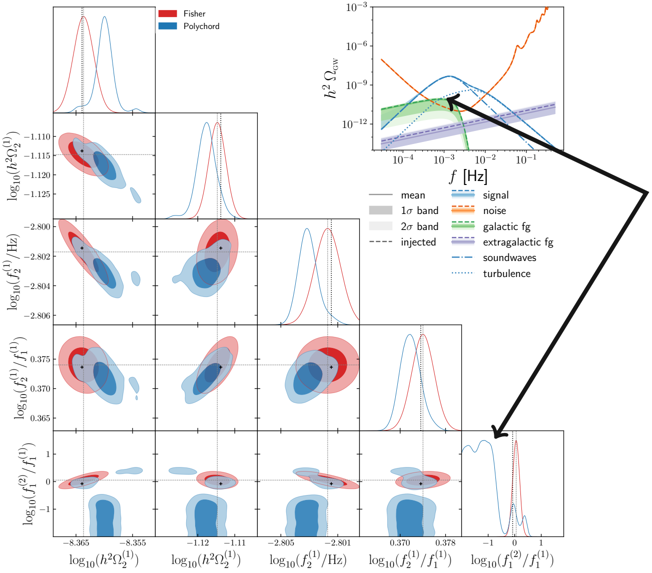
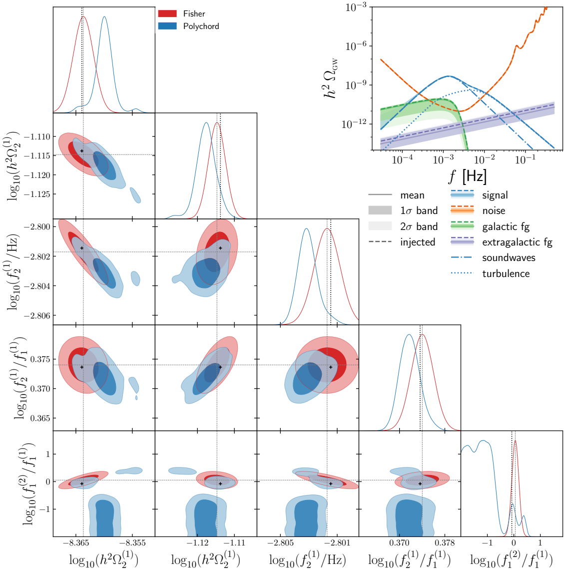
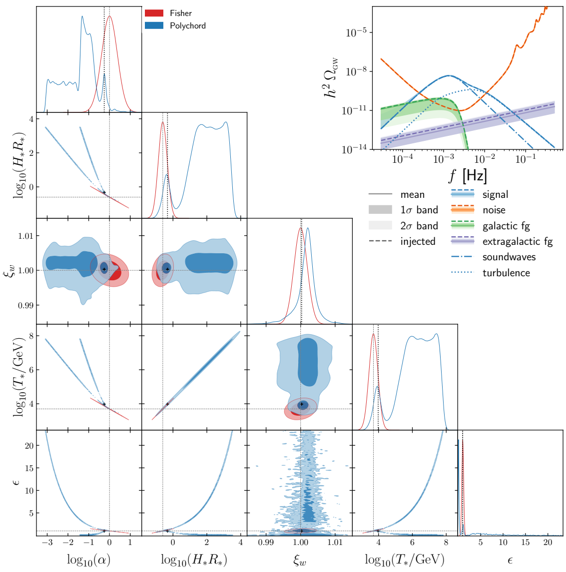


First break of turbulence spectrum hidden under sound waves

$\Longrightarrow$ f_1^{turb} not reconstructed

Two double broken power-laws





Fundamental theories

constraints on spectrum parameters $\xrightarrow{\text{error propagation}}$ constraints model parameters

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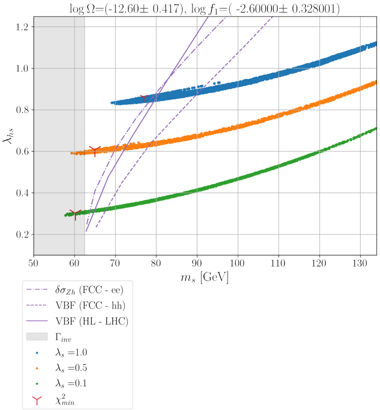
Singlet extension with Z_2



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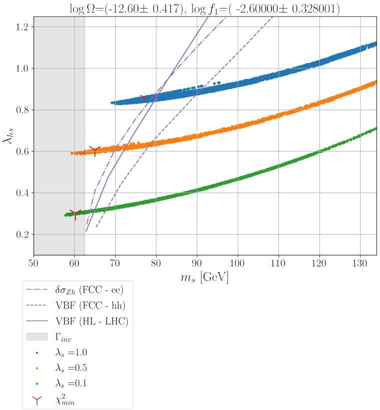
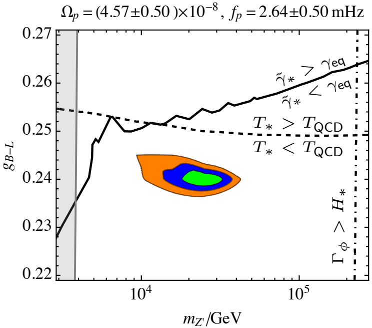


Classically conformal $U(1)_{B-L}$



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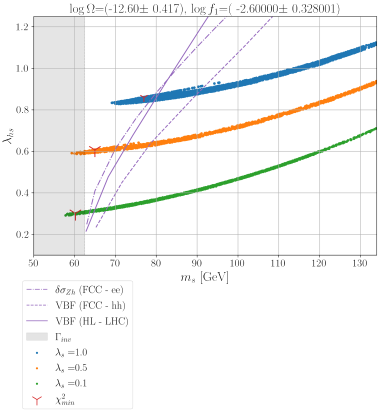

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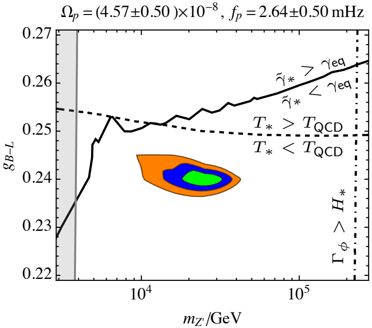
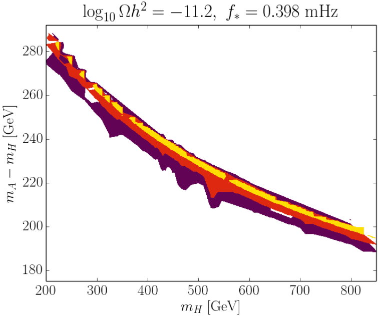


Two-Higgs-doublet model



Conclusions

- for the spectral parameters (amplitude, peak/break frequencies, ...), we can estimate the reach using Fisher analysis
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Thank you for your attention!